

1 **A SENSITIVITY ANALYSIS OF A DYNAMIC RESTRICTED EQUILIBRIUM MODEL**
2 **TO EVALUATE THE TRAFFIC NETWORK RESILIENCE**

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ABSTRACT

Extreme weather events have a devastating impact on traffic networks. Therefore, mathematical tools that are able to measure systematically the impacts of extreme weather events, i.e. to evaluate its resilience, should be developed. With the aim of improving the resilience of a traffic network when affected by a hazard, a profound knowledge of the model to evaluate resilience is necessary. Consequently, the model parameters should be analyzed, since these parameters represent the characteristics of the network and this analysis will permit to identify those characteristics that should be improved to reach a more resilient system. This paper develops a sensitivity analysis that reduces the number of studied point needed due to its statistical approach using a global technique (Latin Hypercube), without losing efficiency, as a local technique (One-At-a-Time) is applied too. This analysis confirms that the model to evaluate the resilience represents the real behavior of the traffic network. The results show that the intensity of the hazards is the most sensitive parameter. When hazard intensity is low, the impedance of the system becomes the most sensitive parameter.

Keywords: Hazards, Extreme Weather, Vulnerability, Stress Level, Latin Hypercube, On-At-a-Time, Local Sensitivity, Global Sensitivity.

1 INTRODUCTION

2 A variety of extreme weather events, including river floods, rain induced landslides, droughts,
3 winter storms, wildfire, and hurricanes, have threatened and damaged many different regions
4 worldwide. These events have a devastating impact on critical infrastructure systems resulting in
5 high social, economical and environmental costs. For this reason, it is imperative to develop a
6 mathematical tool that is able to measure systematically the impacts of extreme weather events
7 on transport networks.

8 The concept that evaluates the behavior of a traffic network when a perturbation takes place is
9 known as resilience. The most common definition of this holistic feature was given by Foster
10 (1993) as “the capacity to absorb shocks gracefully”.

11 With the goal of improving the resilience of a network when affected by a hazard, the
12 weaknesses of the system should be identified. Therefore, the knowledge of the most influential
13 variables involved in the model to evaluate the resilience, allows the enhancement of this
14 characteristic. With this aim, a sensitivity analysis is carried out.

15 A sensitivity analysis identifies the influence of each parameter on the outputs of the model,
16 permitting a profound knowledge of its behavior. In addition, the definition of the inputs will be
17 more efficient after studying how these parameters modify and influence the model.

18 Different methodologies to analyze the sensitivity have been developed previously. These
19 methods can be differentiated between local methods and global methods, the former focuses on
20 estimating the local impact of a parameter on the model outputs.

21 Whereas global techniques are based on sampling methods which scan, in a random or
22 systematic way, the complete range of the parameters involved in the model. Selection of the
23 sampling strategy is crucial to the sensitivity analysis.

24 The paper is organized as follows; Section 2 describes the model and the parameters involved.
25 Section 3 presents the analysis of sensitivity and its application. Finally, in Section 4 some
26 conclusions and future research lines are drawn.

29 THE MODEL AND THE PARAMETERS INVOLVED

30 When a disaster occurs, the traffic network is affected mainly through two different ways,
31 namely, (a) user travel costs (generally time) increase and (b) users become aware of these
32 greater costs and try to reduce them by changing their route choices, generating a certain stress
33 level in the network. When the alteration stops and the initial state is recovered, the travel costs
34 are recuperated and users eventually return to their initial route choices. On the other hand, if the
35 alteration stops but the initial state is not recovered, users will find other route choices that
36 minimize their costs, though these costs will be greater than before. The explained performance
37 is measured by the concept of resilience. Resilience can be defined as the capacity of a
38 transportation network (a) to absorb disruptive events, maintaining its level of service, and (b) to
39 return to a level of service equal to or greater than the pre-disruption level of service within a
40 reasonable time frame, Freckleton et al.(2012). In this paper only the resilience in the
41 perturbation stage is analyzed.

42 The assessment of the traffic network resilience requires a dynamic approach. With this aim,
43 Nogal et al. (2015) propose a “Dynamic Equilibrium-Restricted Assignment Model” (DERAM),
44 which allows the simulation of the network behavior when a disruptive event occurs. This
45 approach permits the inclusion of the stress level of the system together with the extra cost
46 generated by the hazard. This model proposes that the network behavior is restricted by a system
47 impedance, α .

The perturbation resilience is defined between (0, 100), 100% being the optimum value. Moreover, a cost threshold is included to assume the system break-down. This value restricts the perturbation resilience and is the limit-state associated with the failure of the travel cost network due to the extreme overcost generated by a strong perturbation. Although the system could theoretically recover, it would imply an unacceptable effort by the system. Furthermore, a travel cost function including climatological events has to be considered. More precisely, Nogal et al. (2014) propose the following expression.

$$\tau_a(t) = \tau_{0a} \left\{ 1 + m_a \exp \left[\frac{\left(\frac{S_a(t)}{\beta_a} + p_a h(t) \right) \exp(-\gamma_a)}{1 - h(t)} \right] \right\},$$

(1)

where τ_a and τ_{0a} are denoted as the actual travel time and the free travel time, respectively; m_a , β_a and γ_a are parameters related to the traffic characteristics; $S_a(t)$ is the saturation degree computed as the ratio between the actual flow and the capacity; $h(t)$ is the hazard intensity whose range is (0,1), and p_a is the specific sensitivity of each link to a given hazard. For instance, in the case of pluvial flooding, p_a depends on the catchment area, slope of the road, type of pavement, existence of element of protection, etc. Subscript a implies association with link a .

SENSITIVITY ANALYSIS

Methodology

The applied methodology is an integration of a local into a global sensitivity methodology. Based on the complexity of the resilience model defined previously, this paper presents a combination of One-At-a-Time (OAT) for the local sensitivity and Latin Hypercube (LH) sampling for a global approach (Griensven et al (2006)). According to the OAT technique, the analysis is performed by varying each parameter while the rest remains constant. This local method is as simple as efficient, however this process can become quite intensive with larger models. Then, instead of applying it in a large number of points to cover the entire range of the parameters, a global methodology has been chosen to obtain a sample of points that represents the different variables. The selected global sampling procedure (LH) allows the reduction of the sample size. Due to the importance of the pairing procedure, the method Translational Propagation algorithm proposed by Viana et al (2010) has been implemented. The main advantage of this methodology is that it requires virtually no computational time. When the sample is obtained, the local sensitivity analysis can be accomplished as follows. Considering that the total space is covered and the sample is a reliable and robust representation of the global, the resilience is evaluated for each point of the sample. Then, every variable in each sample point is modified in a percentage to calculate the corresponding resilience in that close point. It is important to modify only one variable each time, in this way, the behavior of the modified variable is identified. Measuring the variation, according the OAT methodology, the sensitivity in each point is captured.

The percentage applied for modifying the variable is also a critical point. Since, on the one hand, small values can show the instabilities of the model, being this behavior not according with the real tendency of the model. On the other hand, if this value is too large, the derivative loses its meaning.

The formulation to assess the sensitivity is based on the concept of derivative, that is

$$\xi = \frac{R_d^+(x, Y) - R(Z)}{d}, \quad x, Y \in Z, \quad (2)$$

where Z is the set of variables involved in the model, x , is the modified variable and Y , the subset of variables which remain constant. R is the resilience calculated for the initial parameter set Z and R_d^+ is the resilience calculated when one parameter has been increased by a percentage, d .

Sensitivity, denoted by ξ , is a dimensionless parameter.

Application of the Methodology

This methodology is applied in a simple traffic network to analyze the sensitivity of the set of variables defined in section 2, that is $Z = \{\alpha, p_a, h(t), \beta_a, \gamma_a\}$.

This traffic network consists of 5 nodes, 16 links and 9 routes, (see Figure1). The sensitivity analysis is carried out by modifying the model parameters as shown in Table 1. It is noted that only characteristics of links 13 and 16 change in each evaluation with the aim of distinguish the effect. In this case, the sample size is 50 and the percentages of variation, d are 1, 5, 10%.

Figures 1-6 show the results associated with the percentage of 10%, as the results for the other percentages follow a similar tendency.

TABLE 1 Values and statistical distributions of the parameters.

Links	Parameters	Min	Max	Distribution	Distribution Parameters
-	α	0	1	BETA	$\alpha = 2, \beta = 5$
-	$h(t)$	0	1	BETA	$\alpha = 4, \beta = 4$
1-12,14-15	p_a	-	-	0.1	constant
13,16	p_a	0	1	BETA	$\alpha = 1.2, \beta = 2.5$
1-12,14-15	β_a	-	-	0.83	constant
13,16	β_a	1	4.5	GAMMA	$k = 2.9, \theta = 0.5$
1-12,14-15	γ_a	-	-	4	constant
13,16	γ_a	0	6	GAMMA	$k = 7, \theta = 0.37$

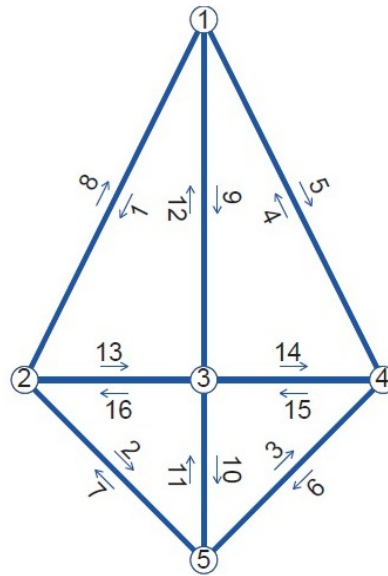


FIGURE 1 Network.

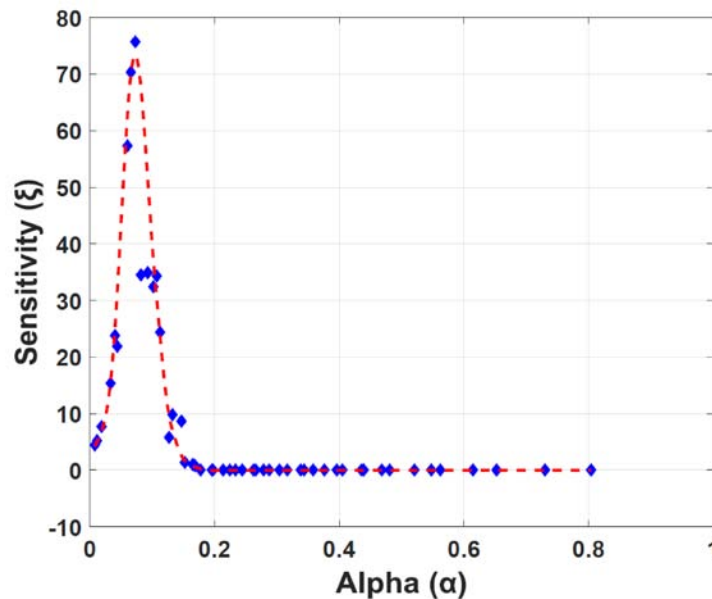


FIGURE 2 Sensitivity of α .

The sensitivity of the parameters is analysed through the development of two cases, since the influence of $h(t)$ in the model is crucial. The first case shows the sensitivity of the variables when the possible values of $h(t)$ are smaller than 0.3, and in the second case, the values of $h(t)$ are higher. When $h(t)$ takes higher values the model tends to reach the break point and the resilience index becomes 0, therefore, the sensitivity of the rest of the parameters is negligible. These cases are shown in the figures with red-circle points.

In lower values of $h(t)$, the most sensitive parameter is α , (see the sensitivity range in Figure 2). The influence of $h(t)$ is larger than the influence of p_a (see the sensitivity range in Figure 3-4). In addition, the sign of the sensitivity in these two cases implies, as expected, that an increase in the parameters produces a decrease in the resilience.

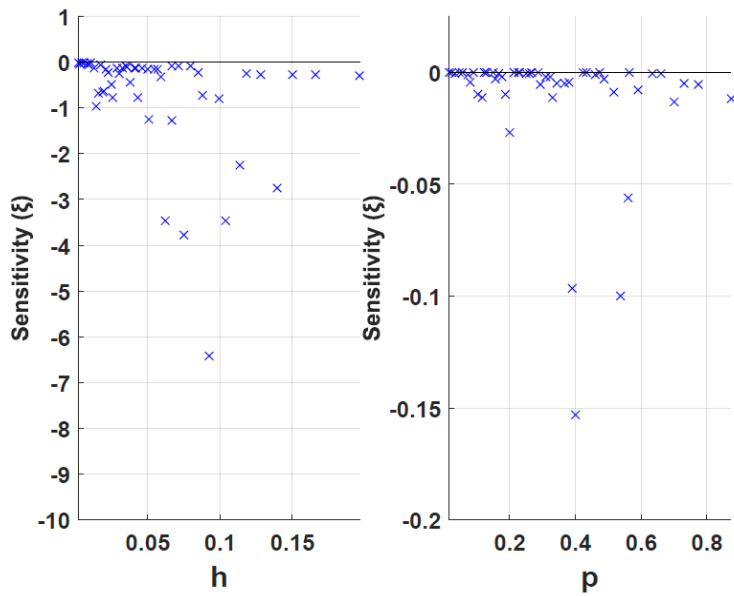


FIGURE 3 Sensitivity of h and p for small values of h.

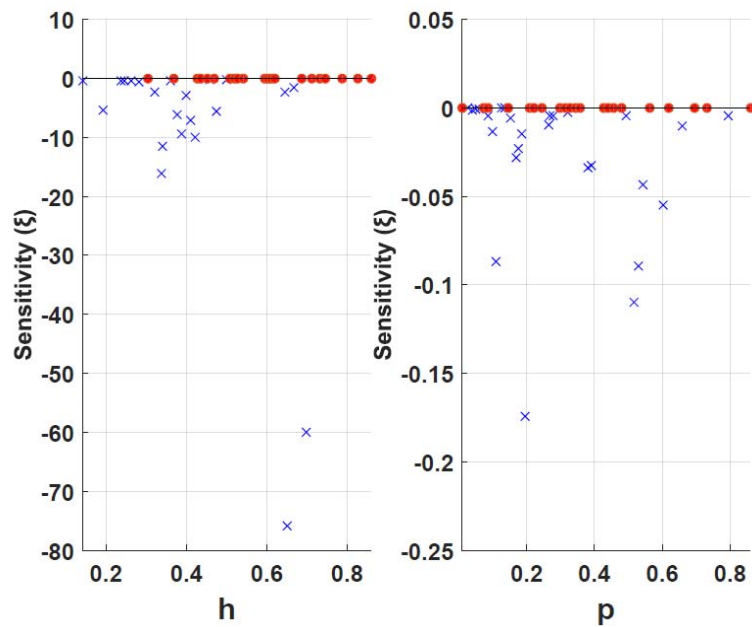


FIGURE 4 Sensitivity of h and p for large values of h.

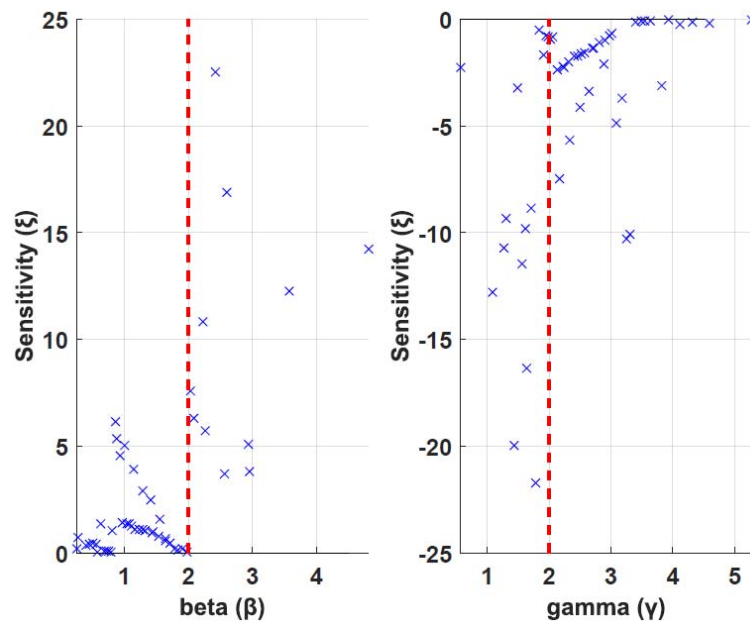


FIGURE 5 Sensitivity of β and γ for small values of h .

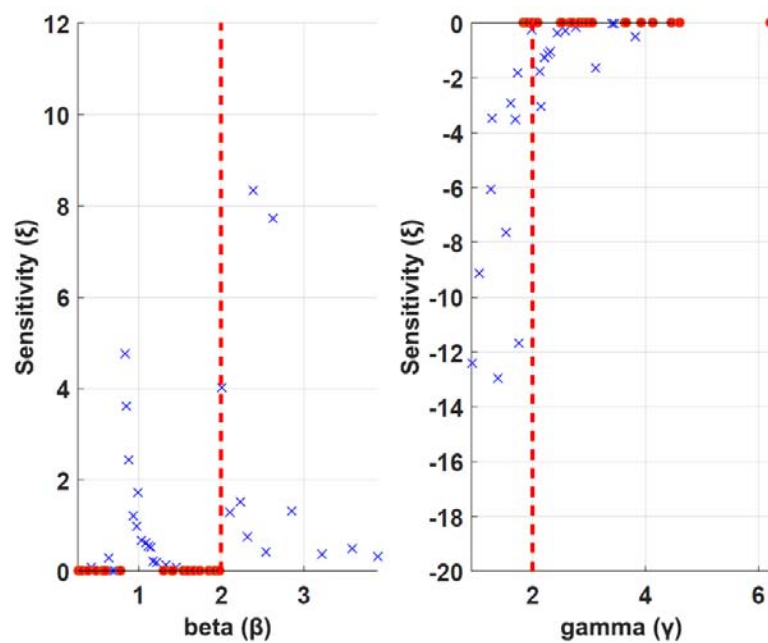


FIGURE 6 Sensitivity of β and γ for large values of h .

With regard to β_a and γ_a (see Figure 5 and 6), their corresponding ranges are similar, being γ_a slightly more sensitive. It is possible to appreciate that in the case of β_a the sensitivity is positive, due to this parameter is modifying the capacity and producing a virtual increment of the link capacity. On the other hand, γ_a takes negative values, since it governs the slope of the cost function in (1) when the saturation degree increases.

CONCLUSIONS

The response of a traffic system is highly non-linear and its analysis becomes difficult, especially when extreme weather is the source of the traffic network disruption. Consequently, the model used to analyze the resilience of a traffic network suffering a weather hazard is a complex one. The selection of an adequate methodology to analyze the sensitivity of the parameters should include a statistical approach to reduce the computational times and the number of chosen points to cover the entire range of the parameters. Additionally, the following conclusions can be drawn from this paper:

- This sensitivity analysis allows the reader to understand how the parameters, which represent the characteristics of a network, influence the resilience index of this system. Therefore, this study allows knowing which properties of a network should be implemented to improve this feature.
- The intensity of the hazard, as expected, is the most sensitive parameter. Values higher than 0.5 increases the probability of reaching the break-down point.
- When the intensity of the hazard takes low values, the impedance of the network reaches its largest sensitivity, mainly when this parameter takes values around 0.1. This impedance is due to the actual capacity of adaptation to the changes, the lack of knowledge of the new situation and the lack of knowledge of the behavior of other users.
- The sensitivity of the parameters related to the traffic characteristics, γ_a and β_a becomes more relevant when the impedance is not very sensitive, that is when users have a considerable capacity of adaptation to changes.
- A mixed methodology to analyze the sensitivity is proposed, which include local (One-At-a-Time) and global techniques (Latin Hypercube). This kind of methodology is justified when a large number of variables are involved, because local methods are very efficient but they do not cover the entire space; whereas, global methods provide a robust and reliable approach but the computational cost could be too high in complex models.
- The pairing procedure known as the Translational Propagation algorithm, has been implemented, which requires minimal computational times.

Future research will provide an extension of this methodology, including aspects such as the topology of the network, the capacity and the demand.

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