Adaptable robust traffic signal control for urban road networked systems with uncertain capacity

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ABSTRACT: In order to effectively mitigate congestion and propagate uncertainty in road networked systems, a robust signal control with adaption to traffic dynamics is presented. A Stochastic Traffic Model (STM) is introduced to appropriately address spatial evolution of traffic congestion inside road links. An Identification of Robust Family (IRF) for Stochastic Performance Index (SPI) is proposed. A stochastic program can be proposed and efficiently solved by a variant of Benders' decomposition. While time-varying SPI value functions can be improved progressively by proposed adaption algorithm, the feasibility of constraints against high-consequence realization of capacity uncertainty is recursively enforced for unknown probability distribution of occurrence. Numerical experiments are performed at a real-data city road network. As compared to recently proposed the state-of-the-art traffic signal control, obtained results showed that the proposed robust signal control can exhibit sufficient gain of achieving road network effectiveness while attenuating time-varying congestion in the presence of uncertain capacity at links downstream.

For urban road networked systems, the Model Predictive Control (MPC) [1]-[4] real-time signal control [5]-[9] has received growing attention. The objective of MPC traffic signal control system is to balance the number of vehicles between links and to minimize total travel time incurred by road users [10]-[12]. Most existing MPC network-wide traffic control relies on long term deterministic converged states of links rather than allows for parametric changes in the presence of data uncertainty. Among which [13] proposed a Robust MPC signal control (RMPC) considering parametric uncertainty and disturbance in road networks. In order to reduce parametric uncertainty, a scenario-based MPC [14]-[16] and adaptive MPC [17]-[18] aiming to augment the model set and refine the feasible region have received increasing attention. In this regard, [17] proposed a control algorithm for constrained linear systems with multiple inputs and outputs. In the presence of measurement noise and output disturbances, the proposed adaptive controller consists of an iterative set of all candidate models at each time step, and a MPC controller that enforces both input and output constraints for entire models inside the set. As expected, such adaptive MPC design may incur excessive computational requirements in light of arbitrary uncertain parameters and disturbances. While MPC traffic signal control using simple store-and-forward traffic models [5]- [9], [13] to represent time-dependent traffic states on links, the spatial change of traffic phenomena inside links can be more appropriately handled by MFD (Macroscopic Fundamental Diagrams) traffic control strategies [19]-[20].

A sudden capacity drop can occur at downstream links due to upstream queue spillback. Considering stochastic effect of random vehicle arrival and departure, a number of studies for stochastic capacity at links downstream have also raised ample attention [21]-[23]. For example, [24] proposed a quasi-dynamic control setting where partial traffic state information is used to adjust the green and red light times grounded on a given queue content threshold being reached. However, in the presence of parametric uncertainty, none of aforementioned work has allowed for insight into optimizing the effect of random capacity on time-varying traffic signal control system. In order to effectively improve mobility for road networked systems, a novel robust traffic signal control system adapting to uncertain capacity is proposed and can be illustrated in Fig. 1.

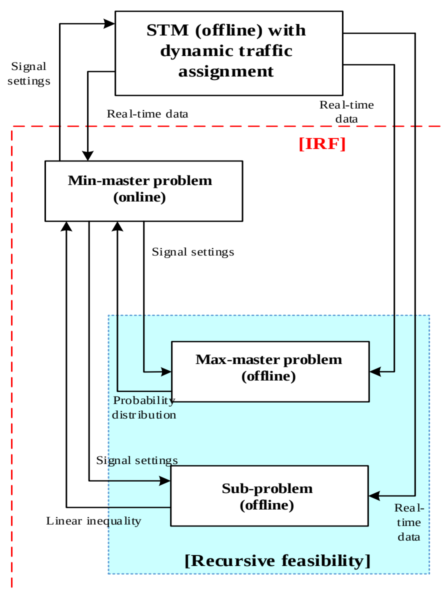

Fig.1 Overall schema for adaptable robust traffic signal control

As shown in Fig. 1, a Stochastic Traffic Model (STM) is proposed to evaluate time-varying traffic dynamics. For random capacity at links downstream, the time-varying outflow can be readily determined. Also, the time-varying signal delay incurred by road users can be obtained. In the presence of uncertain capacity, a Stochastic Performance Index (SPI) is proposed, and can be represented by a weighted sum of delay rates and number of stops. Since SPI value function is unknown, an Identification of Robust Family (IRF) is proposed. A stochastic program for IRF can be given. In order to find a most non-conservative linear approximation of SPI value function, a Benders' decomposition is employed. As shown in Fig. 1, the stochastic program can be decomposed into three parts: a minimum master (min-master) problem, a maximum master (max-master) problem, and a sub-problem. For a min-master problem, it generates on-line optimal signal settings based on real-time data transmission from STM. For a max-master problem, it generates a worst-case probability distribution for random link capacity under current signal control and progressively augments the SPI value function set offline. Based on real-time data transmission from STM and signal control, a sub-problem aiming to find a most non-conservative approximation of SPI value function can be presented if current signal settings are not optimal. Contributions of this paper are summarized as follows.

(1). A Stochastic Traffic Model (STM) is presented to evaluate traffic dynamics for random link capacity.

(2). A Stochastic Performance Index (SPI) is proposed to represent system performance for random capacity.

(3). In order to optimize traffic signal control for uncertain capacity, an Identification of Robust Family (IRF) of SPI value function is proposed.

(4). A stochastic program is proposed to minimize total SPI values and solved by a Benders' decomposition variant.

(5). Numerical experiments are performed at a real-data city road network. Obtained results showed that the proposed signal control can exhibit substantial gain of achieving effectiveness for road networked systems with uncertain capacity.

The rest of this paper is organized as follows. Section II presents a problem formulation that includes a Stochastic Traffic Model (STM) for random capacity, a dynamic traffic assignment with equilibrium flow, and a stochastic program. In Section III, a proposed algorithm termed as IRF is presented. In Section IV, numerical experiments are performed at a real-data city road network for uncertain capacity. Comparisons are numerical made with the stae-of-the-art traffic control under varying traffic conditions. Conclusions and discussions of this paper are briefly made in Section V together with future interest of relevant issues. Notation used throughout this paper can be summarized in Table I.

TABLE I. Notation

Sets	
N	set of nodes with index *i*, $\forall i \in \mathrm{N}$.
A	set of directed links with index pair (i, j) , $\forall i, j \in \mathrm{N}$.
T	set of consecutive time steps with index *t*.
Ξ	set of scenarios for link random saturation flow with index *s*.
$\mathrm{H}(i)$	set of head nodes of outgoing links from a node *i*.
$\mathrm{T}(i)$	set of tail nodes of incoming links to a node *i*.
Parameters	
Γ^t	uncertainty budget for saturation flow.
$z_{(i,j)}^{s,t}$	scaled deviation for link saturation flow in scenario *s* at time step *t*.
$D_{(i,j)}^{s,t}$	stochastic link delay rate in scenario *s* at time step *t*.
$S_{(i,j)}^{s,t}$	stochastic link number of stops in scenario *s* at time step *t*.
$d_{(i,j)}^{s,t}$	stochastic link delay in scenario *s* at time step *t*.
$F_{(i,j)}^{t}$	cumulative link inflow at time step *t*.
$G_{(i,j)}^{s,t}$	cumulative stochastic link outflow in scenario *s* at time step *t*.
$c_{(i,j),0}^{t}$	link cruise travel time at time step *t*.
$c_{(i,j)}^{t}$	link travel time at time step *t*.
$\pi_{(i,j)}^{t}$	minimum link travel time at time step *t*.
$\lambda_{\min}^{t}$	minimum green at time step *t* .
$s_{(i,j)}^{t}$	nominal link saturation flow rate at time step *t*.
$\hat{s}_{(i,j)}^{t}$	maximum link saturation flow rate at time step *t*.
$\Sigma_i^t(a,b)$	collections of 0 and 1 for groups *a* and *b* at junctions *i* at time step *t*.
υ_{abi}^{t}	clearance time between groups *a* and *b* at junctions *i* at time step *t*.
$\kappa_{(i,j)}$	link jam density.
$\bar{C}_{(i,j)}^{t}$	capacity at links upstream at time step *t*.
$L_{(i,j)}$	link length.
$v_{(i,j)}$, $w_{(i,j)}$	link free flow and shock wave speeds.
W_D , W_S	weight for delay rate and number of stops.
M_D , M_S	monetary factor associated to delay rate and stops.
Decision variables	
$\Psi^t = (\zeta^t, \theta^t, \phi^t)$	vector of time-varying signal settings.
ζ^t	reciprocal of time-varying common cycle time.
ζ_1^t , ζ_2^t	minimum and maximum reciprocal of common cycle time.
θ_{ai}^{t}	starts of green for group *a* at junctions *i*, as proportions of cycle time.
ϕ_{ai}^{t}	green duration of group *a* at junctions *i*, as proportions of cycle time.
$\lambda_{(i,j)}^{t}$	effective link green splits.
$\mu_{(i,j)}^{s,t}$	stochastic link capacity in scenario *s*.
$f_{(i,j)}^{t}$	link inflow rate.
$g_{(i,j)}^{s,t}$	stochastic link outflow in scenario *s*.
$p_{(i,j)}^{s,t}$	probability of stochastic capacity in scenario *s*.

1. PROBLEM FORMULATION

1.1. *Time-Varying Stochastic Traffic Model (STM)*

For congested road networked systems with capacity uncertainty, a time-varying stochastic traffic queue at links downstream can be developed. Due to variation in saturation flow rate, a random link flow $\xi_{(i,j)}^{s,t}$ belongs to a given interval $[s_{(i,j)}^t, s_{(i,j)}^t + \hat{s}_{(i,j)}^t]$ of nominal saturation flow rate $s_{(i,j)}^t$ with scaled deviation $z_{(i,j)}^{s,t}$

$$0 \le z_{(i,j)}^{s,t} \le 1 \tag{1}$$

by a pre-determined budget of uncertainty Γ^t ,

$$\sum_{(i,j)\in \mathrm{A}} \sum_{s\in\Xi} z_{(i,j)}^{s,t} \le \Gamma^t \tag{2}$$

such that in scenarios *s*, it implies

$$\xi_{(i,j)}^{s,t} = s_{(i,j)}^t + z_{(i,j)}^{s,t} \cdot \hat{s}_{(i,j)}^t \tag{3}$$

By definition, the time-varying stochastic capacity at links downstream can be expressed as

$$\mu_{(i,j)}^{s,t} = \lambda_{(i,j)}^t \cdot \xi_{(i,j)}^{s,t} \tag{4}$$

The underlying probability distributions for random capacity can be defined as

$$\sum_{s\in\Xi} p_{(i,j)}^{s,t} = 1 \text{, } \forall (i,j) \in \mathrm{A} \tag{5}$$

$$\sum_{(i,j)\in\mathrm{A}} p_{(i,j)}^{s,t} = 1 \text{, } \forall s \in \Xi \tag{6}$$

and

$$0 \le p_{(i,j)}^{s,t} \le 1 \text{, } \forall (i,j) \in \mathrm{A} \text{, } \forall s \in \Xi \tag{7}$$

In terms of kinematic waves [25]-[26], a Stochastic Traffic Model (STM) for random link capacity can be presented.

$$D_{(i,j)}^{s,t+1}(\Psi^{t+1}, f_{(i,j)}^{t+1}) = D_{(i,j)}^{s,t}(\Psi^t, f_{(i,j)}^t) + F_{(i,j)}^{t-\frac{L_{(i,j)}}{v_{(i,j)}}} - G_{(i,j)}^{s,t} \tag{8}$$

In (8), the time-varying outflow can be determined by a minimum principle:

$$G_{(i,j)}^{s,t} = Min\left\{ F_{(i,j)}^{t-\frac{L_{(i,j)}}{v_{(i,j)}}}, \quad \mu_{(i,j)}^{s,t} \cdot \triangle t, \right. \\ \left. G_{(j,k)}^{t-\frac{L_{(j,k)}}{w_{(j,k)}}} + L_{(j,k)} \cdot \kappa_{(j,k)} - F_{(j,k)}^t, \forall (j,k) \in \mathrm{A} \right\} \tag{9}$$

For congested links, the stochastic residual queue at links downstream can be defined as

$$RQ_{(i,j)}^{s,t} = F_{(i,j)}^{t-\frac{L_{(i,j)}}{v_{(i,j)}}} - G_{(i,j)}^{s,t} \tag{10}$$

The maximum outflow rate for congested links can be determined accordingly.

$$R\hat{Q}_{(i,j)}^{s,t}\begin{cases} >0 & \Rightarrow \quad \hat{g}_{(i,j)}^{s,t}=\mu_{(i,j)}^{s,t} \\ =0 & \Rightarrow \quad \hat{g}_{(i,j)}^{s,t}\le \mu_{(i,j)}^{s,t} \end{cases} \tag{11}$$

Similarly, spare capacity for congested links can be defined as

$$SC_{(j,k)}^{t}=\kappa_{(j,k)}\cdot L_{(j,k)}+G_{(j,k)}^{t-\frac{L_{(j,k)}}{w_{(j,k)}}}-F_{(j,k)}^{t}\ge 0 \tag{12}$$

The maximum inflow rate at links upstream can be determined as

$$S\hat{C}_{(j,k)}^{t}(\Psi^t)\begin{cases} >0 & \Rightarrow \quad \bar{C}_{(j,k)}^{t}=\hat{f}_{(j,k)}^{t} \\ =0 & \Rightarrow \quad \bar{C}_{(j,k)}^{t}\ge \hat{f}_{(j,k)}^{t} \end{cases} \tag{13}$$

In terms of (11) and (13), the time-varying link outflow rate can be decided.

$$\begin{aligned} & Max \quad g_{(i,j)}^{s,t} \\ & \text{subject to} \quad \begin{cases} g_{(i,j)}^{s,t}\le \hat{g}_{(i,j)}^{s,t} \\ g_{(i,j)}^{s,t}\le \hat{f}_{(j,k)}^{t}, \forall (j,k)\in \mathrm{A} \end{cases} \end{aligned} \tag{14}$$

Therefore, the time-varying signal delay at links downstream can be determined as

$$d_{(i,j)}^{s,t}(\Psi^t, f_{(i,j)}^{t})=\frac{D_{(i,j)}^{s,t}(\Psi^t, f_{(i,j)}^{t})}{g_{(i,j)}^{s,t}} \tag{15}$$

The time-varying link travel time can be expressed as follows.

$$c_{(i,j)}^{t}(\Psi^t, f_{(i,j)}^{t})=c_{(i,j),0}^{t}+\sum_{s\in\Xi} p_{(i,j)}^{s,t}\cdot d_{(i,j)}^{s,t}(\Psi^t, f_{(i,j)}^{t}) \tag{16}$$

1.2. A Time-Varying Traffic Assignment

According to Wardrop's first principle of route choice in time-varying road networks, the travel times incurred by traffic on all routes used between every origin-destination (OD) pair at each time are equal and less than those that would be on any unused route at that time. A costly route between every OD pair carries no flow. In terms of (16), the time-varying traffic assignment can be determined by the following condition:

$$f_{(i,j)}^{t}\begin{cases} >0 & \Rightarrow c_{(i,j)}^{t}(\Psi^t, f_{(i,j)}^{t})=\pi_{(i,j)}^{t} \\ =0 & \Rightarrow c_{(i,j)}^{t}(\Psi^t, f_{(i,j)}^{t})\ge \pi_{(i,j)}^{t} \end{cases} \tag{17}$$

1.3. A Stochastic Program

In order to mitigate the effect of uncertainty on link travel times, a number of studies like [13], [27] using robust optimization has received growing attention in its own right. Although robust control in [17]-[18], [13], [27] can hedge against the worst-case instances of uncertainty, the solutions found appear excessively conservative for real-life implementation. To the best knowledge of author, none of them allows for insight into the impact on traffic dynamics for uncertain capacity. In the presence of random capacity at links downstream due to congestion, a stochastic program can be readily given as follows. We firstly denote Π^t a feasible set for time-varying signal setting constraints. For time-varying cycle time constraint, it can be expressed as

$$\zeta_1^t \le \zeta^t \le \zeta_2^t \tag{18}$$

For time-varying phase a green time at signal-controlled junctions j, it can be expressed as

$$g_{\min}^{t}\cdot\zeta^t \le \phi_{aj}^{t}\le 1, \ \forall j\in \mathrm{N} \tag{19}$$

Let $\mathrm{E}_p[\cdot]$ denote a mathematical expectation operator with respect to some probability distributions p. In terms of (4), the maximum stochastic capacity for link flows leading to signal-controlled junctions j can be expressed as

$$f_{(i,j)}^{t}(\Psi^t)\le \mathrm{E}_{p_{(i,j)}^{t}}\left[\mu_{(i,j)}^{s,t}\right]=\lambda_{(i,j)}^{t}\cdot\sum_{s\in\Xi} p_{(i,j)}^{s,t}\cdot\xi_{(i,j)}^{s,t} \tag{20}$$

For time-varying clearance time for incompatible signal groups a and b at junctions j, it can be expressed as

$$\theta_{aj}^{t}+\phi_{aj}^{t}+\upsilon_{abj}^{t}\cdot\zeta^t\le\theta_{aj}^{t}+\Sigma_j^t(a,b),\forall a,b \tag{21}$$

In order to evaluate traffic dynamics, a time-varying Stochastic Performance Index (SPI) for road networked systems can be taken as a weighted sum of a following linear combination of rate of delay and number of stops per unit time. We firstly denote Z a SPI value function short for $Z(\Psi^t)$ as

$$\begin{aligned} Z(\Psi^t)=&\sum_{t\in \mathrm{T}}\sum_{(i,j)\in \mathrm{A}}\mathrm{E}_{p_{(i,j)}^{t}}\left[D_{(i,j)}^{s,t}(\Psi^t, f_{(i,j)}^{t}(\Psi^t))\right]\cdot W_D\cdot M_D+ \\ &\sum_{t\in \mathrm{T}}\sum_{(i,j)\in \mathrm{A}}\mathrm{E}_{p_{(i,j)}^{t}}\left[S_{(i,j)}^{s,t}(\Psi^t, f_{(i,j)}^{t}(\Psi^t))\right]\cdot W_S\cdot M_S \end{aligned} \tag{22}$$

A stochastic program can be readily presented as to

$$\underset{\Psi^t, f^t}{Min} \quad Z(\Psi^t) = \sum_{t\in T}\sum_{(i,j)\in A}\left(E_{p^t_{(i,j)}}\left[D^{s,t}_{(i,j)}\right]\cdot W_D \cdot M_D + E_{p^t_{(i,j)}}\left[S^{s,t}_{(i,j)}\right]\cdot W_S \cdot M_S\right)$$

$$subject \quad to \begin{cases} \zeta_1^t \le \zeta^t \le \zeta_2^t \\ g^t_{\min}\cdot \zeta^t \le \phi^t_{aj} \le 1, \forall j \in N \\ f^t_{(i,j)}(\Psi^t) \le E_{p^t_{(i,j)}}\left[\lambda^t_{(i,j)}\cdot \xi^{s,t}_{(i,j)}\right], \forall (i,j)\in A \\ \theta^t_{aj} + \phi^t_{aj} + \upsilon^t_{abj}\cdot\zeta^t \le \theta^t_{aj} + \Sigma^t_j(a,b), \forall a,b,\forall j\in N \end{cases} \tag{23}$$

In terms of (22), the first order sensitivity for SPI value function with respect to signal settings can be derived as

$$\nabla Z(\Psi^t) = \sum_{t\in T}\sum_{(i,j)\in A} E_{p^t_{(i,j)}}\left[\nabla_{\Psi^t} D^{s,t}_{(i,j)} + \nabla_{f^t_{(i,j)}} D^{s,t}_{(i,j)}\cdot \nabla_{\Psi^t} f^t_{(i,j)}\right]\cdot W_D\cdot M_D + \sum_{t\in T}\sum_{(i,j)\in A} E_{p^t_{(i,j)}}\left[\nabla_{\Psi^t} S^{s,t}_{(i,j)} + \nabla_{f^t_{(i,j)}} S^{s,t}_{(i,j)}\cdot \nabla_{\Psi^t} f^t_{(i,j)}\right]\cdot W_S\cdot M_S \tag{24}$$

2. ADAPTABLE STRATEGY FOR ROBUST TRAFFIC SIGNAL CONTROL

For random capacity with unknown probability distribution of occurrence, the SPI value function set in (22) is unknown. In order to effectively enrich SPI value function set, an Identification of Robust Family (IRF) strategy can be proposed as follows. Let Π^t_p denote a feasible probability set with respect to conditions in (5)-(7). The stochastic program in (23) can be re-formulated as a following min-max model:

$$\underset{\Psi^t\in\Pi^t}{Min}\underset{p^t\in\Pi^t_p}{Max} \quad Z(\Psi^t, p^t) = \sum_{t\in T}\sum_{(i,j)\in A}\left(E_{p^t_{(i,j)}}\left[D^{s,t}_{(i,j)}\cdot W_D\cdot M_D + S^{s,t}_{(i,j)}\cdot W_S\cdot M_S\right]\right) \tag{25}$$

Let Σ denote a level set of signal settings with unknown probability distribution as

$$\Sigma = \left\{\Psi^t \in \Pi^t, p^t\in\Pi^t_p : Z_{LB} \le Z(\Psi^t, p^t) \le Z_{UB}\right\} \tag{26}$$

2.1. *Identification of Robust Family (IRF) adaptable strategy*

In terms of (25), an identification of robust family for SPI value in the worst case can be proposed.

$$Z^k_{UB} = \underset{p^{t,k}_{(i,j)}}{Max} \quad \sum_{t\in T}\sum_{(i,j)\in A}\left(E_{p^{t,k}_{(i,j)}}\left[D^{s,t,k}_{(i,j)}\cdot W_D\cdot M_D + S^{s,t,k}_{(i,j)}\cdot W_S\cdot M_S\right]\right) \tag{27}$$

Based on (24), the worst-case SPI value function in (27) for iterations $k = 0,1,2,...$ can be linearly approximated by

$$Z^k_{UB}(\Psi^t) = Z^k_{UB} + \nabla Z^k_{UB}\cdot(\Psi^t - \Psi^{t,k}) \tag{28}$$

For the robust family of feasible region in (25), it can be readily determined by a following min-max model.

$$\underset{\Psi^t\in\Pi^t}{Min}\underset{p^t\in\Pi^t_p}{Max}\underset{0\le i\le k}{Max} \quad \left\{Z^i + \nabla Z^i\cdot(\Psi^t - \Psi^{t,i})\right\} \tag{29}$$

2.2. *Benders' decomposition*

For the min-max model (29), it can be decomposed into three parts by Benders' decomposition: a max-master, min-master, and a sub-problem as illustrated in Fig. 1. For objective value function with signal settings $\Psi^{t,k}$, a max-master problem can be constructed first.

$$Z^k_{UB} = \underset{p^{s,t,k}_{(i,j)}}{Max} \quad \sum_{t\in T}\sum_{(i,j)\in A} E_{p^{t,k}_{(i,j)}}\left[D^{s,t,k}_{(i,j)}\cdot W_D\cdot M_D + S^{s,t,k}_{(i,j)}\cdot W_S\cdot M_S\right]$$

$$subject \quad to \quad \sum_{s\in\Xi} p^{s,t,k}_{(i,j)} = 1, \forall (i,j)\in A$$

$$and \quad \sum_{(i,j)\in A} p^{s,t,k}_{(i,j)} = 1, \forall s\in\Xi \tag{30}$$

Next, a min-master problem can be optimized with respected to signal settings over past iterates.

$$Z^k_{LB} = \underset{\Psi^t\in\Pi^t}{Min} \quad Z(\Psi^t)$$
$$subject \quad to \quad Z^i_{UB} + \nabla Z^i_{UB}\cdot(\Psi^t - \Psi^{t,i}) \le Z(\Psi^t), 0\le i\le k-1 \tag{31}$$

Finally, a sub-problem to identify a worst-case SPI value function at iteration k can be accordingly constructed.

$$\underset{0\le l\le k}{Max} \quad \left\{Z^l_{UB} + \nabla Z^l_{UB}\cdot(\Psi^t - \Psi^{t,l})\right\} \tag{32}$$

In terms of (30) and (31), we can compare two bounds on optimal SPI values until the following condition holds.

$$Z^k_{UB} \le Z^k_{LB} \tag{33}$$

Otherwise, a newly identified linear inequality from (32) can be added into the family of feasible region for the min-max model (29). Then return to the master problems (30)-(31), and let $k \leftarrow k+1$.

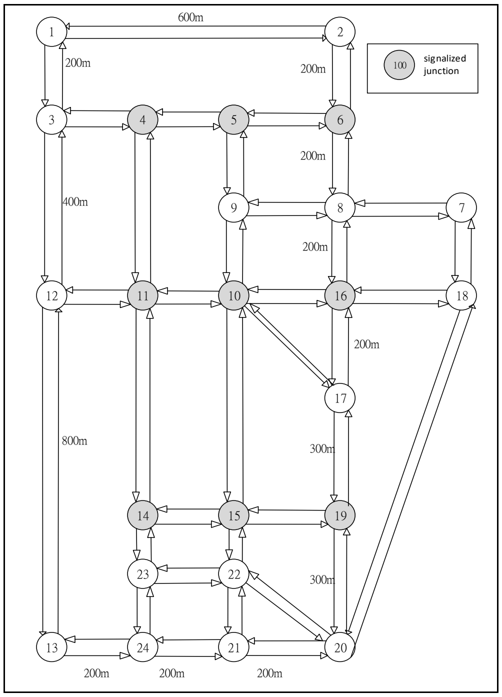

Fig. 2. Sioux Falls city network with signalized junctions

3. NUMERICAL COMPUTATIONS AND RESULTS

To understand computational effectiveness for proposed signal control, numerical simulations are performed for varying traffic conditions. For non-congested traffic state, nominal flow is set 500 veh per lane per direction. For congested traffic state, nominal flow is set 800 veh per lane per direction. Lane stochastic capacity can vary between 500 and 800 veh/lane/direction. To investigate computational efficiency of IRF/QNEW, computations are performed at Sioux Falls city network [28] with 9 signal-controlled junctions as shown in Fig. 2. Initial signal settings are given in Table II. In line with STM implementation, link free flow and shockwave speeds are set 25m/sec, and 5 m/sec. Link jam density is set 0.2 veh/m. Comparisons are also made with the state-of-the-art signal control. In the following, the nominal common cycle times are set 60 seconds (s), and the maximum cycle time is set at 180 s. The minimum green time for each group is 7 s using typical values found in practice, and the clearance times are 5 s between incompatible signal groups. The stopping criterion used in solution scheme is set when the relative gap for (33) is less than 0.5%. Implementations for carrying out computations are made on DELL 7810, Intel Xeon 2.5 GHz processor with 32 GB RAM under Windows 10 using C++ compiler.

3.1. Computational Results

To understand computational effectiveness of proposed IRF/QNEW, numerical experiments are performed with two initial signal settings for non-congested and congested traffic states. Results are summarized in Table II, and detailed in Tables III and IV. For non-congested traffic state, IRF can improve SPI with rate of 52.7% by decreasing initial value of $1123 to $531. For congested traffic state, IRF makes a promising improvement rate of nearly 50% by decreasing initial value of $1319 to $664 with success. In accordance with Benders' decomposition, the max-master problem (30) and the min-master problem (31) can be successfully solved by IRF with QNEW while the gap between upper and lower bounds decreases iteratively both for non-congested and congested traffic states.

TABLE II. SUMMARY RESULTS WITH TWO INITIAL SIGNAL SETTINGS

Signal settings	Non-congested traffic state		Congested traffic state	
	Initials	Results	Initials	Results
$1/\zeta^t$	70	75	100	135
$\varphi^t_{1,4}/\zeta^t$	30	32	45	60
$\varphi^t_{2,4}/\zeta^t$	30	33	45	65
$\varphi^t_{1,5}/\zeta^t$	30	30	45	65
$\varphi^t_{2,5}/\zeta^t$	30	35	45	60
$\varphi^t_{1,6}/\zeta^t$	30	35	45	62
$\varphi^t_{2,6}/\zeta^t$	30	30	45	63
$\varphi^t_{1,10}/\zeta^t$	30	32	45	63
$\varphi^t_{2,10}/\zeta^t$	30	33	45	62
$\varphi^t_{1,11}/\zeta^t$	30	33	45	61
$\varphi^t_{2,11}/\zeta^t$	30	32	45	64
$\varphi^t_{1,14}/\zeta^t$	30	30	45	55
$\varphi^t_{2,14}/\zeta^t$	30	35	45	70
$\varphi^t_{1,15}/\zeta^t$	30	35	45	68
$\varphi^t_{2,15}/\zeta^t$	30	30	45	57
$\varphi^t_{1,16}/\zeta^t$	30	32	45	55
$\varphi^t_{2,16}/\zeta^t$	30	33	45	70
$\varphi^t_{1,19}/\zeta^t$	30	32	45	56
$\varphi^t_{2,19}/\zeta^t$	30	33	45	69
SPI	1123	531	1319	664
CPU (in s)		251		254

where $1/\zeta^t$ denotes the common cycle time and φ^t_{aj}/ζ^t denotes the green duration in sec for signal group *a* at junction *j*.

TABLE III. COMPARISON RESULTS FOR SPI AND IMPROVEMENT RATE OVER MPC

Level	AROSS	MPC	RMPC	IRF/QNEW
I	635(0.94%)	641(-)	640(-0.16%)	**633(1.25%)**
II	701(-1.3%)	692(-)	688(-0.58%)	**677(2.17%)**

*Level I and II represents light and severe traffic congestion

TABLE IV. COMPARISON RESULTS FOR SPI AND IMPROVEMENT RATE OVER MPC UNDER LIGHT CONGESTION WITH VARYING BUDGET OF UNCERTAINTY

Γ^c	AROSS	MPC	RMPC	IRF/QNEW
0	635(0.94%)	641(-)	640(0.16%)	**633(1.25%)**
1.5	648(1.67%)	659(-)	671(-1.82%)	**638(3.19%)**
2	661(1.78%)	673(-)	678(-0.74%)	**645(4.16%)**
2.5	665(2.35%)	681(-)	682(-0.15%)	**651(4.41%)**

TABLE V. COMPARISON RESULTS FOR SPI AND IMPROVEMENT RATE OVER MPC UNDER SEVERE CONGESTION WITH VARYING BUDGET OF UNCERTAINTY

Γ^c	AROSS	MPC	RMPC	IRF/QNEW
0	701(-1.3%)	692(-)	688(0.58%)	**677(2.17%)**
1.5	711(0.56%)	715(-)	713(0.28%)	**681(4.76%)**
2	720(0.28%)	722(-)	720(0.28%)	**689(4.57%)**
2.5	729(0.82%)	735(-)	728(0.95%)	**695(5.44%)**

TABLE VI. CPU TIME (IN SEC) FOR LIGHT/SEVERE CONGESTION WITH VARYING UNCERTAINTY BUDGET

Γ^c	AROSS	MPC	RMPC	IRF/QNEW
0	435/655	613/812	523/791	**252/261**
1.5	439/662	616/815	531/795	**250/262**
2	442/667	619/822	535/809	**255/265**
2.5	445/670	621/826	537/811	**253/267**

3.2. Numerical Comparisons with MPC Signal Controls for Varying Traffic Conditions and Uncertainty Budget

To investigate computational efficiency of IRF/QNEW, numerical comparisons are made with MPC [5], RMPC [13], and AROSS [27]. Results are summarized in Tables III-VI. As shown in Table III, IRF/QNEW can achieve comparably better improving rate of 2.17% than those done by MPC for severe traffic congestion. To understand real-time implementation advantage of IRF/QNEW, numerical experiments are performed with varying budget of uncertainty for link capacity. As shown in Tables IV and V, IRF/QNEW accounting for travelers' real-time decisions on route choice can achieve substantial improvement rates of more than 4%. RMPC without full consideration of travelers' response achieved far less SPI improvement rate as compared to those done by AROSS and IRF/QNEW. CPU times for varying traffic conditions are given in Table VI. As seen in Table VI, MPC and RMPC using traffic prediction data incur significant amount of CPU times. IRF/QNEW using stochastic traffic model can exhibit a strong competitive advantage on computation for real-time implementation in practice.

4. CONCLUSION AND DISCUSSIONS

In this paper, a novel adaptable robust traffic signal control was presented for urban road networked systems with uncertain capacity. An Identification of Robust Family (IRF) was practically proposed to contain the maximum outer approximation of Stochastic Performance Index (SPI). Numerical experiments were performed using a real-data city road network. As compared to most existing MPC signal control in the literature, the proposed IRF/QNEW has practically demonstrated significant gain of achieving better improvement in all cases. By contrast with the state-of-the-art signal control, the proposed IRF/QNEW also achieved comparably better performance in SPI improvement for varying random link capacity with far less computational overheads, which leads to promising advantage for practical real-time implementation.

For urban road networks with persistent congestion, most existing centralized traffic controls become a challenge both empirically and theoretically due to curse of dimensionality. A distributed system design can shed some light toward real-time network-wide implementation [8], [9], [12]. For most existing hierarchical traffic control [10]-[11], a IRF/QNEW distributed controller can be further explored in conjunction with border control for area-wide congested traffic road networked systems. For real time implementation with connected vehicles on large-scale road networked systems [29]-[30], a data-driven signal control can be readily investigated based on this paper in a foreseeable future. We will discuss these issues of interest in practice in subsequent papers.

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