

Determination of Dynamic Progressive Collapse Resistances for Deteriorating Multi-story RC Frames Subjected to Column Removal Scenarios via the Energy-based Method

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ABSTRACT: The alternate path method is widely employed to perform progressive collapse analyses of reinforced concrete (RC) structures subjected to sudden column removal scenarios. To avoid cumbersome direct dynamic analyses, the energy-based method (EBM) is an alternative efficient approach to predict the maximum dynamic responses. However, the EBM cannot directly determine dynamic collapse limit states with respect to multi-story RC frames subjected to column removal scenarios, because the deteriorating behavior or the development of cumulative damage in the structural systems. In this contribution, an EBM-intersection point (IP)-based method is proposed to determine the collapse limit states when EBM is adopted to calculate maximum dynamic responses. Considering both an edge column loss case and a middle column loss case with regard to a six-story six-bay RC frame, both deterministic and stochastic analyses are carried out to validate the proposed approach and to calibrate the associated coefficients. According to the results, the proposed method is verified to be effective. Moreover, a Gumbel distribution $Gu(0.982, 0.009)$ is adopted to fit the PDF of the coefficient related to the displacements at the collapse limit states, while a lognormal distribution $LN(1.031, 0.007)$ is adopted to capture the PDF of the coefficient associated with the resistances.

1. INTRODUCTION

Progressive or disproportionate collapses due to accidental loads are low-probability but high-consequence events [1-5]. In order to mitigate the collapse risk of reinforced concrete (RC) structures, it is of great importance to design structures to be robust. Dynamic effects may have significant influence on progressive collapse resistances of RC building structures subjected to a sudden column removal scenario [1; 3; 4]. Generally, direct nonlinear dynamic analyses can yield the most accurate results but it is very time-consuming in particular in the context of structural reliability or robustness evaluation [1].

Alternatively, the energy-based method (EBM) [1; 4] in which no cumbersome direct dynamic analyses or nonlinear time history analyses (NTHA) are required is found to be a promising approach for the calculation of maximum

dynamic progressive collapse responses. The EBM completely depends on the result of a nonlinear quasi-static pushdown analysis, i.e. the pushdown curve, considering the conservation of energy. Therefore, dynamic effects are not involved in the EBM, nor dynamic failure criteria or collapse limit states (CLS). However, structural collapses of multi-story building RC structures are usually induced by the gradual deterioration in the stiffness and strength of some structural members [6]. In these cases, the occurrence of system collapse is usually after the peak point (static resistance) in the pushdown curve. Namely, it may be not correct if the dynamic resistance is calculated from the static resistance, in which the post-peak behavior in the pushdown curve should be accounted for (or deteriorating behavior) [6; 7]. In this case, the information of CLS is unknown

when the EBM is directly employed, although the EBM gives a dynamic load-displacement curve.

The determination of the CLS of a RC structure both in the earthquake engineering [6] and in the field of progressive collapse is not easy and relevant studies are ongoing. In terms of the CLS in the context of progressive collapse for RC structures subjected to column removal scenarios, different dynamic failure criteria have been adopted. For example, in the NTHAs both for performance limit states studies for collapse analysis of RC framed buildings [8] and for progressive collapse assessment of European RC buildings under multi-column loss scenarios [3], vertical drift of a beam equal to an ultimate drift level corresponding to the beam end chord rotation of 15% (0.26 rad) was taken as one of the failure conditions (others are bar fracture, loss of system equilibrium, and loss of numerical convergence). Yu et al. [7] used the EBM to calculate the maximum dynamic responses with respect to multi-story RC structures subjected to middle column loss scenarios, in which the deformation corresponding to the 20% reduced post-peak capacity was approximately selected as the ultimate deformation of the damaged structure. In the research [1] regarding the simplified method for quantifying the progressive collapse fragility of multi-story RC frames using the EBM, where the failure of the structures under column removal scenarios was determined when the displacement of the joint of the removed column exceeded the displacement corresponding to the beam end chord rotation of 0.20 rad. It can be found that different collapse criteria were taken in these studies and further studies are still necessary.

In light of the above background, in order to determine the CLS of deteriorating multi-story RC frames, the focus of this work is put on the determination of dynamic progressive collapse resistances and displacements by using the EBM.

2. EBM-IP-BASED METHOD

When the RC frame in Figure 1 is subjected to the sudden column removal scenario (i.e. the alternate path method), three major parts of energy can be taken into account. They are the released potential

energy (PE) due to the unbalanced loads, the accumulated strain energy (SE) stored in the deformed structure, and the kinetic energy (KE) [4]. Moreover, the three parts of energy should satisfy the law of energy conservation, i.e. $PE = SE + KE$.

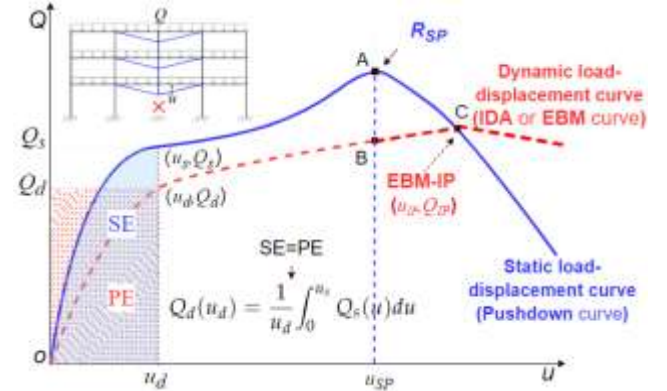


Figure 1: The energy-based method.

The EBM is based on the aforementioned energy balance concept, where the PE equals to the SE when the KE is zero. In this regard, an assumption that the response is governed by a dominant deformation mode and a moment with zero kinetic energy exists if collapse can be averted [4]. The SE can be calculated by a pushdown analysis, where the area under the pushdown curve is the SE. For example, the hatched area under the pushdown curve up to the displacement u_s in Figure 1 is the SE. Considering the SE equals to the released PE (the hatched rectangular area at u_d in the dynamic load-displacement curve in Figure 1), the correspond load Q_d in the dynamic load-displacement (or EBM) curve can then be calculated:

$$Q_d(u_d) = \frac{1}{u_d} \int_0^{u_s} Q_s(u) du \quad (1)$$

where Q_d is the load in the dynamic load-displacement curve, Q_s is the load in the static load-displacement curve, u_d is the peak dynamic deflection, and u_s is the static deflection corresponding to the load Q_s .

Regarding multi-story RC frame structures subjected to a sudden column removal scenario, the failure of the structural system is determined

by the collapse or loss of stability of the structural system. Namely, a collapse criterion should be used. If the EBM is applied, the CLS or the collapse criterion is not known, as no direct dynamic analyses are performed and the EBM curve is completely based on the pushdown curve. In the pushdown curve, the static resistance R_{SP} however maybe does not correspond to the CLS. It is because that the existence of a decreasing stage after the peak point R_{SP} in the pushdown curve due to the development of accumulated local damage (or deteriorating behavior) in the structural system, in which no collapse occurs until the CLS is reached. Namely, it may be the curve OAC rather than the curve OAB in Figure 1.

Although both the EBM and the pushdown analysis cannot directly determine the CLS, it is easy to understand that the CLS corresponds to one point in the EBM curve, as well as in the pushdown curve. In view of this, an EBM-IP-based method is proposed in order to tackle the issue in relation to the determination of CLS when the EBM is used to calculate the dynamic ultimate responses of multi-story RC frames subjected to a sudden column loss scenario. In this approach, the intersection point (EBM-IP) between the EBM curve and the pushdown curve is taken (Figure 1) and a correction is further made by comparing to the accurate results of CLS by Incremental Dynamic Analyses (IDA-CLS). Specifically, the approach determines the dynamic resistance R_d and the ultimate displacement u_d as follows:

$$R_d = \alpha_R Q_{IP} \quad (2)$$

$$u_d = \alpha_u u_{IP} \quad (3)$$

$$\alpha_R = \frac{R_{IDA}}{Q_{IP}} \quad (4)$$

$$\alpha_u = \frac{u_{IDA}}{u_{IP}} \quad (5)$$

where Q_{IP} is the value of load at the intersection point EBM-IP; u_{IP} is the value of the displacement at the EBM-IP; R_{IDA} is the accurate dynamic resistance obtained by the IDA, i.e. that at IDA-

CLS; u_{IDA} is the accurate ultimate displacement at the IDA-CLS; α_R and α_u are coefficients used to determine the dynamic resistance R_d and ultimate displacement u_d through making a correction to the values at the EBM-IP, respectively.

3. NUMERICAL MODELING STRATEGIES

3.1. Numerical modeling techniques

The finite element (FE) software package OpenSees [9] is used to create numerical models and to execute progressive collapse analyses in this work. For beam and column elements, the force-based fiber beam-column element considering 5 integration points is employed [9]. The cross-section of the fiber element is discretized into different fibers, in which different stress-strain relationships can be assigned to different fibers. For concrete fibers, the ConcreteD uniaxial plastic-damage model in OpenSees is applied [9; 10]. In order to consider the confinement effect caused by stirrups on the compressive behavior to the core concrete, the widely used Mander model [11] is adopted. The Steel02 material model [9] is adopted for reinforcing steel. As large deformations are expected, the co-rotational transformation is adopted to consider the geometrical nonlinearity in the progressive collapse simulations [8]. More details can be found in relevant references [2; 12].

3.2. Validation

In order to validate the numerical modeling techniques, the 1/3-scale three-story four-bay RC frame tested by Yi et al. [13] is adopted. The configuration of the frame is shown in Figure 2a. In the experimental test, the middle column in the ground floor was replaced by a jack and was gradually removed. Meanwhile, a concentrated load P was imposed at the top of the column in the third floor. The difference between the imposed load P and the axial load N in the jack, i.e. P-N, against the vertical displacement at the top of the removed column was recorded, which is shown in Figure 2b.

A FE model using the previously mentioned modeling techniques is created. The material

parameters and the loading procedure according to the actual test are adopted in the numerical analysis. The simulated result P-N against the vertical displacement is presented in Figure 2b (the curve 'FEM'). Good agreement is found between the experimental and the simulated curves, which means that the numerical modeling approach is effective.

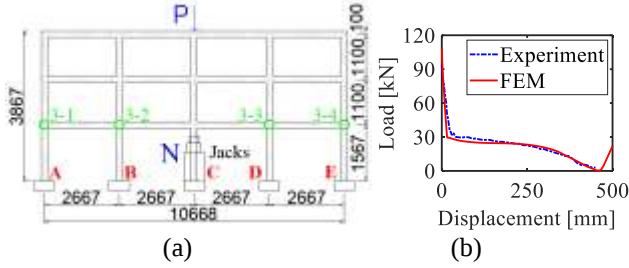


Figure 2: RC frame: (a) layout (dimensions in mm); (b) load-vertical displacement relationship.

4. NUMERICAL EXAMPLE: AN RC FRAME

4.1. Description

A six-story six-bay RC frame 6S6B [5] is adopted to illustrate the proposed EBM-IP-based method and to calibrate the associated coefficients. Layout of the frame and details of beam and column cross-sections are shown in Figure 3. Columns in the ground floor are labeled as A, B, C, D, E, F, and G from the left to the right hand side. In this work, two column loss cases in the ground floor are considered, which are the loss of the edge column A and the loss of the middle column D. Mean values of material mechanical properties for both concrete and reinforcing rebar are summarized in Table 1.

4.2. Deterministic response analyses

For each of the two column removal cases for the multi-story RC frame 6S6B (Figure 3), both nonlinear static pushdown analysis and IDA (i.e. a series of NTHAs) are carried out, considering the mean values of the material mechanical properties. In the static pushdown analysis, a displacement-controlled loading approach is adopted, where the controlled point is at the top of the removed column. The vertical displacement at the controlled point against the imposed

uniformly distributed load on all beams is recorded, which is the pushdown curve (Figure 1). Regarding every NTHA of the IDA, uniformly distributed loads are first imposed on all the beams. Next, the column is suddenly removed, where the column removal duration is less than 1/10 of the first natural period of the damaged structure. The Rayleigh damping with a damping ratio 5% is assigned to the structures [3]. The explicit Koly-Ricles-alpha algorithm in OpenSees [9] is used to carry out the NTHA, where the time step is set as 0.001s. The time history response of the vertical displacement at the top of the removed column is recorded.

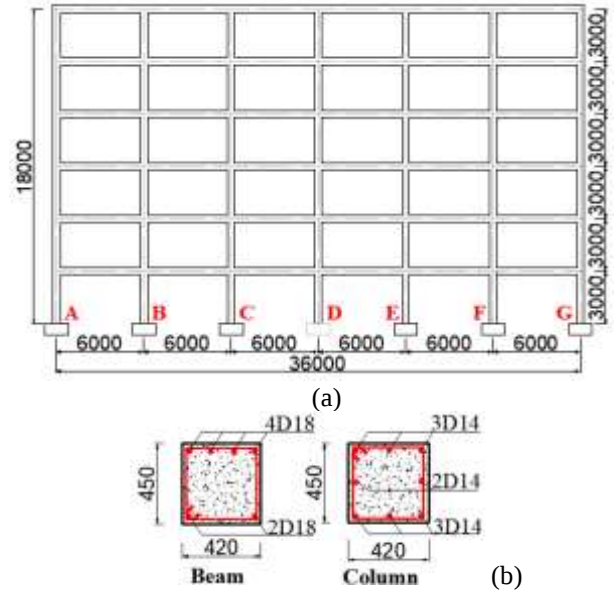


Figure 3: RC frame 6S6B: (a) layout; (b) cross-sections of beams and columns. (dimensions in mm)

Table 1: Probabilistic models for random variables.

Random variable	Unit	Dis.	Mean	CoV
Concrete cover c	mm	Beta	30	0.17
Concrete compressive strength f_c	MPa	LN*	38	0.1
Concrete peak strain ϵ_{cu}	%	LN	0.23	0.15
Concrete tensile strength f_{ct}	-	LN	2.9	0.1
Rebar elastic modulus E_s	GPa	N**	200	0.08
Rebar yielding strength f_y	MPa	LN	555	0.03
Rebar ultimate strength f_t	MPa	LN	605	0.03
Rebar fracture strain ϵ_u	%	LN	7.5	0.15

Note: *Lognormal; ** Normal.

Time histories of displacement responses for the two column removal cases are shown in Figure 4. It can be observed that the dynamic displacements become larger with increasing loads. The oscillations of the displacement responses decay fast due to the damping effects. The peak values of the time history displacement responses against the imposed loads is the dynamic load-displacement or IDA curves in Figure 5.

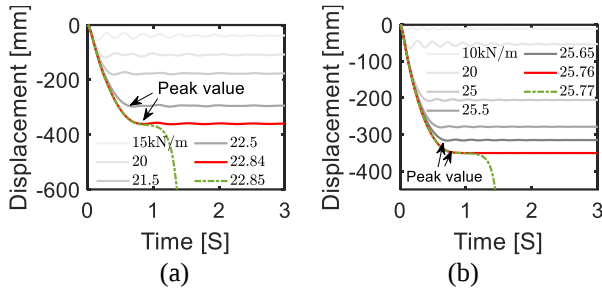


Figure 4: Time-history responses of displacement: (a) loss of column A; and (b) loss of column D.

Results for the two column loss cases are presented in Figure 5, including pushdown curves yielded from the pushdown analyses, IDA curves obtained from the IDAs, and EBM curves calculated from the pushdown curves through using Eq. (1). Figure 5a show the results for the case of column A loss, while Figure 5b show the results for the case of column D removal scenario. Generally, the dynamic load-displacement (i.e. IDA and EBM) curves are significantly lower than the pushdown curves, which means the dynamic effects have great influence on the dynamic responses. Comparing to the IDA curves, only a slight difference is found for the EBM curves regarding both the edge and the middle column removal scenarios. It indicates that the EBM curves approximate the IDA curves well and the EBM has a good performance. The pushdown curves first increase against displacements but decrease after the peak points R_{SP} , which is due to the development of cumulative damage (rebar rupture or concrete crushing at different beam ends) in the structural system. It is easy to see that the dynamic ultimate displacements at IDA-CLS in the IDA curves are

significantly larger than the displacements at the peak points R_{SP} in the pushdown curves for the two cases, which means that it will underestimate the CLS if the peak points R_{SP} in the pushdown curves are employed to calculate the CLS by using the EBM. As expected, the intersection points EBM-IP between the EBM curves and the pushdown curves are close to the points of collapse limit state IDA-CLS in the IDA curves.

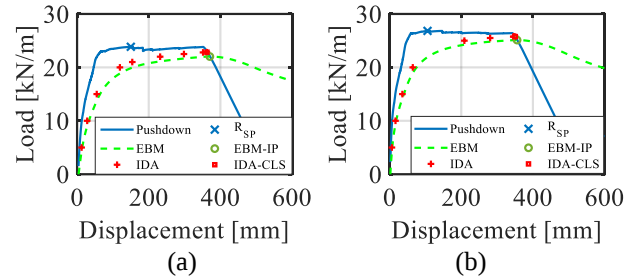


Figure 5: Results for RC frame 6S6B: (a) loss of column A; and (b) loss of column D.

According to the results (Figure 5), the EBM shows good performance when it is used to calculate the approximate dynamic load-displacement curve. Regarding the CLS in the EBM curve, it can be determined by the proposed EBM-IP-based method, considering Eqs. (2) and (3). Ratios between the values at IDA-CLS and the values at EBM-IP are calculated by Eqs. (4) and (5), i.e. the coefficients α_u and α_R . Results for the two column removal cases are summarized in Table 2. The values with respect to the loss of column A are 0.978 and 1.037 for α_u and α_R , respectively. The values for the loss of column D are 0.986 and 1.026 for α_u and α_R , respectively. The values are close to unity, which means the EBM-IP is almost identical to the IDA-CLS. Namely, a correction on the EBM-IP can be used to determine the CLS for the RC frames, which indicates the proposed EBM-IP-based method is effective.

4.3. Stochastic response analyses

4.3.1. Probabilistic modeling

Many uncertainties involved in the structural analysis may have significant influence on the global system behavior, which is not taken into

account in the previous deterministic analyses. In order to calibrate the coefficients in a probabilistic view of point, the multi-story RC frame 6S6B is further used to perform stochastic analyses. For this purpose, eight parameters are selected as random input variables to quantify the uncertainty propagation when applying the proposed EBM-IP-based method. Based on previous investigations [5; 12], details of the probabilistic models for the eight random variables are presented in Table 1. The other input parameters are considered deterministic. The Latin Hypercube Sampling (LHS) technique is adopted in combination with the developed numerical model to carry out stochastic analyses. According to the probabilistic models, 60 Latin Hypercube samples are generated. The two column removal cases of the RC frame share the same samples. Eventually, each set of sampled realizations is used as an input for the corresponding numerical model to calculate the responses.

Table 2: Coefficients of α_R and α_u .

Case			α_u	α_R
Deterministic	Loss of column A		0.978	1.037
	Loss of column D		0.986	1.026
Stochastic	Loss of column A	Mean	0.981	1.035
		St.D.	0.008	0.005
	Loss of column D	Mean	0.983	1.025
		St.D.	0.009	0.004
	All	Mean	0.982	1.031
		St.D.	0.009	0.007

4.3.2. Stochastic analyses

For each of the two column removal cases with respect to the multi-story RC frame 6S6B, all 60 Latin Hypercube samples are used to conducting both pushdown analyses and IDAs. For the former, only one pushdown analysis is executed for each sample. For the latter, a series of NTHAs is carried out for each sample. Namely, the same procedure as that in the previous deterministic analysis is adopted.

Regarding the RC frame subjected to the loss of the edge column A, Figure 6a shows stochastic results yielded from the pushdown analyses, the IDAs, and the EBM. Generally, large influence on

the responses due to the uncertainties in the input parameters is observed, which highlights the importance of taking uncertainties into account. Peak points R_{SP} in the pushdown curves show great differences to EBM-IPs (Figure 6a). Again, the EBM-IPs are situated near the points of IDA-CLS, which is consistent to that observed in the previous deterministic analysis. Figures 6b & c further present comparisons of displacements and resistances between the EBM-IP and the IDA-CLS for all 60 results, respectively. Meanwhile, their mean values are shown as well. The displacement from the IDA-CLS is generally slight smaller (Figure 6b), while the resistance from the IDA-CLS is a little larger (Figure 6c). Overall, discrepancies between the EBM-IP and the IDA-CLS regarding both the displacements and the resistances are not significant. Details of three typical results, i.e. realizations No. 1, No. 33, and No. 52, are presented in Figures 6d-f, respectively. Despite different curves are obtained due to different failure modes or progressive collapse behavior, the EBM-IP is always close to the IDA-CLS.

Figure 7a shows stochastic results yielded from the pushdown analyses, the IDAs, and the EBM when the RC frame subjected to the loss of the middle column D. Likewise, large influence on the responses due to the uncertainties is observed. A great difference between peak points R_{SP} in the pushdown curves and EBM-IPs can be seen (Figure 7a). On the contrary, the EBM-IPs are situated near the points of IDA-CLS, which is similar to that observed before. Figures 7b & c further present comparisons of displacements and resistances between the EBM-IP and the IDA-CLS for all 60 results respectively, as well as their mean values. The displacement and the corresponding mean value from the IDA-CLS are slight smaller (Figure 7b). The resistance and the corresponding mean value from the IDA-CLS are slight larger (Figure 7c). Generally, the difference between the EBM-IP and the IDA-CLS is not significant, which can also be confirmed from details of three typical results, i.e. realizations No. 4, No. 30, and No. 58, as shown in Figures 7d-f,

respectively. Although the three results are observed to be quite different due to different failure modes in the structural system, the EBM-IP is close to the IDA-CLS.

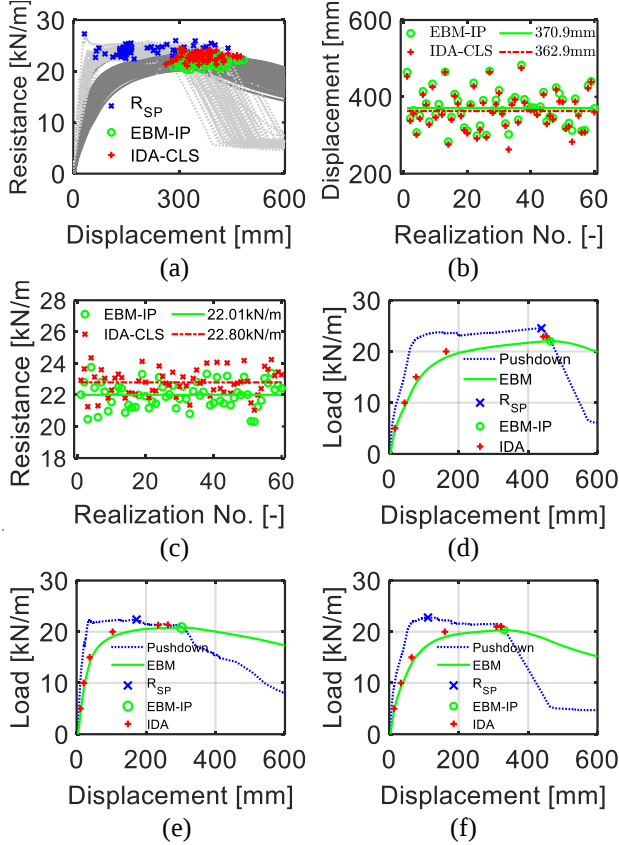


Figure 6: Results for column A loss: (a) stochastic analysis results; (b) comparison of displacement between EBM-IP and IDA-DLS; (c) comparison of resistance between EBM-IP and IDA-DLS; (d) No. 1; (e) No. 33; and (f) No. 52.

Ratios between the values at IDA-CLS and the values at EBM-IP, i.e. the coefficients α_u and α_R , are calculate. Results for the two column removal cases are summarized in Table 2. The mean values (standard deviations or St.D.) with respect to the loss of column A are 0.981 (0.008) and 1.035 (0.005) for α_u and α_R , respectively. The mean values (St.D.) for the loss of column D are 0.983 (0.009) and 1.025 (0.004) for α_u and α_R , respectively. The mean values are close to unity and the St.D. are small, which means the EBM-IP is almost identical to the IDA-CLS. These results

are consistent to those from the deterministic analysis.

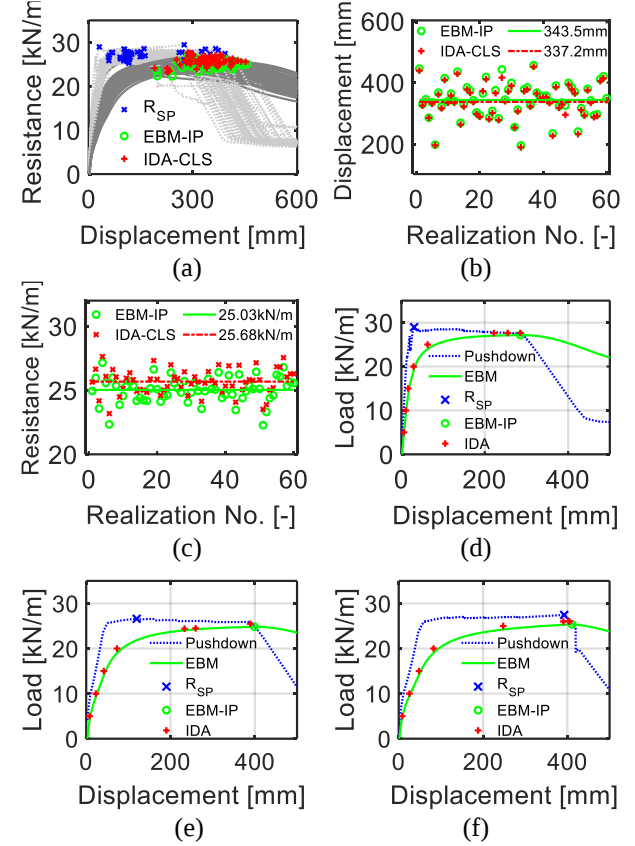


Figure 7: Results for column D loss: (a) stochastic analysis results; (b) comparison of displacement between EBM-IP and IDA-DLS; (c) comparison of resistance between EBM-IP and IDA-DLS; (d) No. 4; (e) No. 30; and (f) No. 58.

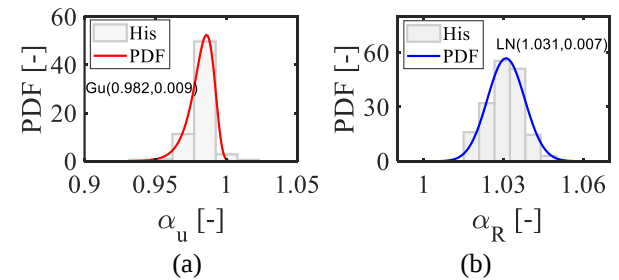


Figure 8: Histograms and PDFs for the coefficients considering all data: (a) α_u ; and (b) α_R .

Considering the difference of the results between the two cases is not significant, all data are collected as a new data set ‘all’. The statistical information for the coefficients α_u and α_R regarding the new data set is calculated and

summarized in Table 2. Figure 8 shows histograms and PDFs for the two coefficients α_u and α_R . Regarding the coefficient α_u , a Gumbel distribution Gu(0.982,0.009) is used to fit the PDF (Figure 8a). Meanwhile, a lognormal distribution LN(1.031,0.007) is adopted to represent the PDF of the coefficient α_R (Figure 8b).

5. CONCLUSIONS

In this work, an EBM-IP-based method is proposed for the determination of collapse limit states of multi-story RC frames subjected to sudden column removal scenarios when the EBM is used. Considering both loss of an edge column and loss of a middle column regarding a six-story six-bay RC frame, both deterministic and stochastic analyses are carried out to verify the proposed method and to calibrate the associated coefficients. According to the results, it is found that the collapse limit state is situated close to the intersection point of the pushdown curve and the EBM curve, which means the proposed EBM-IP-based method is effective. Moreover, correction coefficients to the intersection points are calculated according to the results, where the Gumbel distribution Gu(0.982,0.009) can be adopted to fit the PDF of the coefficient α_u (displacements) and the lognormal distribution LN(1.031,0.007) can be adopted to represent the PDF of the coefficient α_R (resistances).

It must be emphasized that the results obtained herein are based on a specific numerical example. Further research, such as for structural systems with different layouts, shall be conducted.

6. ACKNOWLEDGEMENT

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