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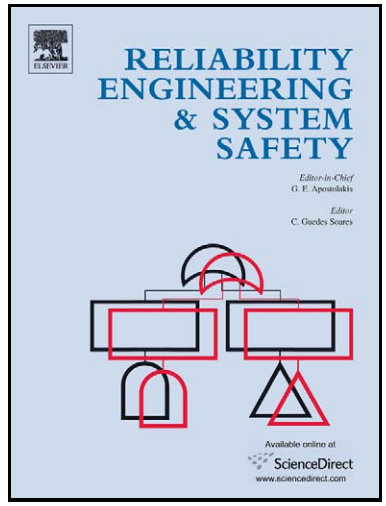

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Development of a two stage inspection process for the assessment of deteriorating infrastructure

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Abstract

Inspection based maintenance strategies can provide an efficient tool for the management of ageing infrastructure subjected to deterioration. Many of these methods rely on quantitative data from inspections, rather than qualitative and subjective data. The focus of this paper is on the development of an inspection based decision scheme, incorporating analysis on the effect of the cost and quality of NDT tools to assess the condition of infrastructure elements/networks during their lifetime. For the first time the two aspects of an inspection are considered, i.e. detection and sizing. Since each stage of an inspection is carried out for a distinct purpose, different parameters are used to represent each procedure and both have been incorporated into a maintenance management model. The separation of these procedures allows the interaction between the two inspection techniques to be studied. The inspection for detection process acts as a screening exercise to determine which defects require further inspection for sizing. A decision tool is developed, which allows the owner/manager of the infrastructural element/network to choose the most cost efficient maintenance management plan based upon his/her specific requirements.

Keywords: Infrastructure, markov deterioration, inspection, maintenance management, repair

1. Introduction

Due to the extent of deteriorating infrastructure in the U.S. (about 5,000 bridges become classed as deficient each year), the estimated cost of rehabilitation and repair has been estimated at $1.3 trillion [1]. "The federally mandated biennial inspection interval is not the most cost-effective maintenance strategy for bridges [2], and bridge repairs are not always performed with life-cycle cost effectiveness in mind" [1]. As a result, over the last decade a lot of research has been conducted into optimisation of the existing infrastructural resource to develop methods of maintenance management which consider the dual constraint of optimal maintenance budget while maximising efficiency for the required remaining service life [3, 4, 5, 6, 7, 8, 9, 10, 11 12, 13, 14, 15, 16]. The main objective is to find the optimal maintenance management plan, thereby optimizing the life-cycle cost of the structure. Many of these methods rely on quantitative data from inspections, rather than qualitative and subjective data. Consequently, monitoring and inspections are key aspects in this process [17] as the information from these tests can be used to update deterioration models and to derive the optimal economic maintenance strategy for the remaining lifetime of the structure.

The main focus of this paper is on the development of an inspection based decision scheme, incorporating analysis of the effect of the cost and quality of NDT tools to assess the condition of infrastructure elements/networks over their lifetime. Two aspects of an inspection, i.e. detection and sizing are considered here. The aim is not to compare existing strategies but to suggest a new systematic approach which facilitates quantification of cost and predicts the required maintenance budgets as a function of time. There have been many studies which focus only on the detection stage of an inspection, using various sets of parameters such as Probability of Detection and Probability of False Alarm [18, 19], Probability of Detection and Probability of False Indications [20] or Probability of Detection and False Call Probability [21, 22] to assess the quality of a particular inspection method. In this study a distinction has been made between an inspection carried out to detect a defect, and an inspection carried out to size a defect. Since each stage of an inspection is carried out for a distinct purpose, different parameters are used to represent each procedure and both have been incorporated into a maintenance management model. By separating these two procedures, an optimal maintenance management plan can be developed by choosing the most suitable inspection technique for each stage of the inspection, whether it is for detection or sizing, rather than using the same inspection technique for both procedures.

As part of the new process the first part of an inspection is concerned with the detection of existing defects. The Probability of Detection (PoD) and the Probability of False Alarm (PFA) are used in this study, for a particular NDT tool used in the assessment, to indicate the quality of the inspection method for detection. The second part of an inspection deals with the assessment of the size of the defect knowing that it has already been detected. For this part of the analysis, two new parameters are introduced, Probability of Good Assessment (PGA) and Probability of Wrong Assessment (PWA). In this context it has been necessary to introduce a distinction between good and wrong sizing assessments that lead to repair (PGA_R, PWA_R), and those which lead to no repair (PGA_{NR}, PWA_{NR}).

Using the methodology developed in Rouhan and Schoefs [19], an events based decision theory is subsequently introduced to look at the effects of an individual good/bad inspection performance. Based on the inspection results, for detection or sizing, a decision is made whether a further inspection should be carried out, or to repair. For evaluating the cost of the system, and to find the optimum costs, it is useful to investigate whether the decision to carry out a sizing assessment or a repair is correct/incorrect. On this basis, a decision scheme is introduced which considers four inspection events for each of the two stages of an inspection. The probability of these events are evaluated using Bayes Theorem and are subsequently introduced as parameters into cost functions which are used to investigate the effect of cost overrun due to inaccurate inspection results.

In addition to this, for a particular set of input parameters the optimum time between inspections, which results in the lowest annual cost, is determined. By varying the quality of the inspection techniques, the sensitivity of the optimal inspection time to changes in the modelled parameters is assessed, allowing the optimum combination of techniques to be determined for the constraint of optimisation of performance with respect to available budget.

The purpose of this study is to develop a method which simulates the deterioration, inspection, repair and failure of structures over time using Markov matrices, with the ability to consider many different forms of defect growth and deterioration kinetics (i.e. gradual and abrupt growth, linear and non-linear), allowing for different materials, environments (i.e. passive and aggressive) and limit states (i.e. Ultimate, Fatigue and Serviceability) to be studied. For example, this method can be used to simulate the deterioration and repair of a structure in a marine environment. By varying the growth parameters, the effects of: (i) different environments (mild or aggressive) and concrete mixes on the growth rate of the cracks and rate of spalling in reinforced concrete [15], (ii) corrosion-fatigue crack growth in steels [23] or (iii) decay of timber structures [24] can be modelled. In all cases the optimum inspections techniques for each stage of deterioration for each environment can be determined.

2. Probabilistic Modelling of Inspection Results

When carrying out an inspection or any non destructive measurement, one has to consider measurement error associated with the signal output from the measurement instrument. Measurement error is caused by imperfect instruments, quality of the protocol (which is dependent on the inspector) and sample disturbance when a quantity is observed. It generally involves two components: (i) a systematic error associated with the bias in the measurements, and (ii) a random error associated with the precision in the measurements [25]. The precision of the measurement depends on the equipment, on the expertise of the *measurer* and on the conditions (e.g. meteorology) during measurement. This concept is also valid in the case of visual inspection, since the expert's brain and eyes are no more than a sensor. By knowing the true value and performing a statistical study, a distribution of noise is available. The information obtained through inspection (e.g. cover depth, crack size etc.) is thus only an estimate of the true reality ("signal of reference"), the difference between the two being due to bias and noise. Therefore, due to the inherent uncertainty associated with inspections, many of the variables involved are modelled stochastically, and the simulation of inspection results should be performed in a probabilistic sense. In the following, the systematic bias will be neglected and only the noise will be considered.

For a given defect size, and the inspection method being used, there is a certain probability of detection [4, 26, 27]. On this basis, probabilistic methods are described below which are used to model inspection results for detection and sizing assessment, taking this uncertainty into account. Note that the quantification of the on-site performance of inspection is difficult. Generally specific inter-calibration campaigns are needed as were initiated in the offshore field during the ICON project [28]. Other recent works provide data for the probability of detection of the corrosion initiation in concrete [29, 30], the probability of detection and false alarm for uniform [31, 32] or localized [33] corrosion of steel structures. Expert judgment can also be introduced in this regard [34].

2.1. Stage 1 - Detection

It is assumed that every ΔT years, an inspection is carried out. The first part of an inspection is concerned with the detection of existing defects. For an individual defect, it is assumed that detection of a defect by the first inspection leads to a further inspection to assess the size of the defect, and that no detection leads to no further action. In this study, the Probability of Detection (PoD) and the Probability of False Alarm (PFA) are the parameters chosen to indicate the quality of an inspection method for detection and are used to assess if a defect will be detected or not when an inspection is carried out, Figure 1. The PoD is the probability that a defect is detected by the inspection, given that a defect is present, Equation 1, and the PFA is the probability that a defect is detected by the inspection, given that no defect greater than the detection threshold is present, Equation 2.

$$\text{PoD} = P\left(\hat{d}_1 \geq d_{min} \mid d \geq d_{min}\right) \quad (1)$$

$$\text{PFA} = P\left(\hat{d}_1 \geq d_{min} \mid d < d_{min}\right) \quad (2)$$

The results of an inspection (and the ability of a method to detect a defect) depend on many different factors, such as the NDT method, the detection threshold (d_{min}), the environment and several conditions of the structure [35], the skill/experience of the operator, the characteristics of the defect (e.g. the deterioration mechanism such as chloride-induced corrosion, fatigue etc.) and primarily on the size of the defect. For a given test, the PoD depends on the defect size (for example the average defect size), the detection threshold and noise. The PFA, however, is independent of the size of the defect and, depends only on the detection threshold and noise.

Therefore, the PoD is the probability that the signal “signal+noise” is greater than the detection threshold, d_{min}, and the PFA is the probability that the signal “noise” is greater than the detection threshold [19]. For the general form of this problem, two probability distributions are needed to model the results of an inspection. The “noise” distribution represents the error due to environmental conditions, human interference and the nature of what is being measured. The distribution of signal represents the physical uncertainty of the inspection, and the distribution of the population of defects at the time of inspection, Figure 1.

2.2. Stage 2 - Sizing Assessment

The second part of an inspection deals with the assessment of the size of a defect. This assessment is only carried out if the previous inspection has indicated that a defect exists. For this analysis, two new probabilities are defined, the Probability of Good Assessment (PGA) and the Probability of Wrong Assessment (PWA). A repair of the defect is carried out if the inspection indicates that the size of the defect is greater than the critical defect size d_c. The value of d_c will be fixed by the owner/manager, depending on the safety level he/she wants to ensure. It can for instance be related to the annual probability of failure. There is also a distinction made between good and wrong assessments that lead to repair (subscript $_R$), and those which lead to no repair (subscript $_{NR}$), Equations 3-6.

$$PGA_R = P\left(\hat{d}_2 \geq d_c \mid d \geq d_c \ \&\ \hat{d}_1 \geq d_{min}\right) \tag{3}$$

$$PGA_{NR} = P\left(\hat{d}_2 < d_c \mid d < d_c \ \&\ \hat{d}_1 \geq d_{min}\right) \tag{4}$$

$$PWA_R = P\left(\hat{d}_2 \geq d_c \mid d < d_c \ \&\ \hat{d}_1 \geq d_{min}\right) \tag{5}$$

$$PWA_{NR} = P\left(\hat{d}_2 < d_c \mid d \geq d_c \ \&\ \hat{d}_1 \geq d_{min}\right) \tag{6}$$

Again, for this inspection, the accuracy of the results can depend on many different factors, and the noise can be due to effects of environmental conditions, human interference and the nature of what is being measured. In this case however, for a given inspection, both the PGA and the PWA depend on the defect size, the detection threshold and noise. Therefore, the inspection can be modelled using just one distribution, as shown in Figure 2, where $\overline{d}_i$ is the mean defect size within a group i. The PGA_R is the probability that the "signal+noise" is greater than the critical defect size (leading to repair), given that the actual defect is greater than the critical defect size d_c, Figure 2(a), and the PGA_{NR} is the probability that the "signal+noise" is less than the critical defect size (leading to no repair), given that the defect is less than the critical defect size, Figure 2(b). Similarly, the PWA_R is the probability that the "signal+noise" is greater than the critical defect size (leading to repair), given that the actual defect is less than the critical defect size, Figure 2(b), and the PWA_{NR} is the probability that the "signal+noise" is less than the critical defect size (leading to no repair), given that the defect is greater than the critical defect size, Figure 2(a). An example of the interaction between PGA_R, PWA_R, PGA_{NR}, PWA_{NR} and the critical defect size is illustrated in Figure 3.

3. Events Based Decision Theory

As described in Rouhan and Schoefs [19], an events based decision theory can be used to look at the effects of a good/bad inspection performance. Since there can be various sources of error when performing an inspection, it is useful to investigate the probability that each of the decisions taken (e.g. to carry out a further assessment for sizing or to repair) are correct/incorrect. In this study, a similar method is implemented for detection and sizing, considering four inspection events for each of the two stages of an inspection. The probability of these events are evaluated using Bayes Theorem and are subsequently introduced as parameters into cost functions which are used to investigate the effect of cost overrun due to inaccurate inspection results.

Firstly, in the case of an inspection to detect a defect, a decision on whether to carry out a further assessment is made based on the inspection result $\hat{d}_1$. It is assumed that detection of a defect by the first inspection leads to a further inspection to assess the size of the defect, and that no detection leads to no further action. This decision on whether or not to carry out a further assessment can never be taken with certainty, and the level of uncertainty depends on the quality of the inspection and the level of the other sources of noise associated with the inspection. To assess this risk, four events are defined for the detection stage of an inspection, labelled E_{1D}, E_{2D}, E_{3D} and E_{4D} respectively. The question is, knowing that something is detected or not detected, what is the probability that there is a defect present or no defect present.

- E_{1D} is associated with a good decision, where the inspection indicates that there is no defect, when no defect greater than the detection threshold (d_{min}) actually exists, in which case no further sizing assessment is carried out, Equation 7.

$$P(E_{1D}) = P\left(d < d_{min} \middle| \hat{d}_1 < d_{min}\right) \tag{7}$$

- E_{2D} is associated with a bad decision, where the inspection indicates that there is a defect, when no defect greater than the d_{min} actually exists, in which case an unnecessary sizing assessment is carried out, with an associated inspection cost, Equation 8.

$$P(E_{2D}) = P\left(d < d_{min} \middle| \hat{d}_1 \geq d_{min}\right) \tag{8}$$

- E_{3D} is also associated with a bad decision, where the inspection indicates that there is no defect, when a defect greater than the d_{min} actually exists, in which case no sizing assessment is carried out, but there is an associated failure risk cost, Equation 9.

$$P(E_{3D}) = P\left(d \geq d_{min} \middle| \hat{d}_1 < d_{min}\right) \tag{9}$$

- E_{4D} is associated with a good decision, where the inspection indicates that there is a defect, when a defect greater than the d_{min} actually exists, in which case a necessary sizing assessment is carried out, resulting in an associated optimal spend, Equation 10.

$$P(E_{4D}) = P\left(d \geq d_{min} \middle| \hat{d}_1 \geq d_{min}\right) \tag{10}$$

The calculation of these probabilities is based on the PoD, PFA and on the parameter γ, which is defined as the probability that the actual defect size is greater than the detection threshold, Equation 11. This process can be understood with reference to Figure 4.

$$\gamma = P(d \geq d_{min}) \tag{11}$$

The probabilities of these events are then evaluated using Bayes Theorem, Equation 12-15.

$$P(E_{1D}) = \frac{(1 - PFA(d))(1 - \gamma)}{(1 - PoD(d))\gamma + (1 - PFA(d))(1 - \gamma)} \tag{12}$$

$$P(E_{2D}) = \frac{PFA(d)(1 - \gamma)}{PoD(d)\gamma + PFA(d)(1 - \gamma)} \tag{13}$$

$$P(E_{3D}) = \frac{(1 - PoD(d))\gamma}{(1 - PoD(d))\gamma + (1 - PFA(d))(1 - \gamma)} \tag{14}$$

$$P(E_{4D}) = \frac{PoD(d)\gamma}{PoD(d)\gamma + PFA(d)(1 - \gamma)} \tag{15}$$

3.1. Events at Stage 2 – Sizing Assessment

For consistency in sizing assessment, the same methodology is employed. It is assumed that a repair is carried out if the size of the defect from the second inspection ($\hat{d}_2$) is larger than the critical defect size, d_c, and that no repair is carried out if the defect size is smaller than d_c. Again, this decision on whether or not to carry out a repair can never be taken with certainty. Therefore, four events are also defined for the sizing assessment stage of an inspection, E_{1A}, E_{2A}, E_{3A} and E_{4A} (using equations similar to Equations 7-15). Again, the question is, knowing that a defect has been sized greater than or less than the critical defect size and will be repaired or not repaired, what is the probability that it should have been repaired or not repaired.

For the second stage of an inspection, the sizing assessment, the calculation of these probabilities is based on the PGA, PWA and the parameter λ, which is defined as the probability that the size of the actual defect is greater than the specified critical defect size and as such requires repair, Equation 16.

$$\lambda = P(d \geq d_c) \tag{16}$$

4. Development of Maintenance Management Model

Depending on the limit state being considered, the group of defects being inspected may be all on the same structure or at the same point on different structures. It is assumed, for this methodology, that the population of defects being inspected are all assessed under the same limit state. For example, when considering the serviceability limit state, there may be many points on one structure that require regular inspection and maintenance. In this case, for one structure alone there may be quite a large population of defects to be considered at each inspection interval (e.g. when crack width is critical limit state). However, when considering the ultimate limit state, only the critical structural elements of the structure are of importance, meaning that only a few points on a structure need to be inspected and maintained on a regular basis (e.g. if moment capacity at mid span is the limit state being considered for a group of bridges, the mid span of each bridge will be inspected). Therefore, it is assumed that the law describing the probability of failure and the consequence of failure is the same for all defects within the population being considered in this model, where failure is defined as exceedance of a critical limit state. Similar to Scherer and Glagola [36], it is assumed that all structures within a certain class have the same deterioration characteristics, and therefore, the same maintenance actions are also carried out on these structures.

When managing a structure or a group of structures it is important to be aware of and to have an accurate estimate of the growth of the population of defects present in the structure over time. Assuming that the state of the structure in each time period only depends on the state of the structure and the action applied to it in the preceding period [37], a Markov process may be employed to simulate the growth/evolving deterioration and repair of a population of defects over time [36, 38]. A Markov decision process can be a useful tool for controlling and finding the optimal strategy when managing a large scale system [39, 40, 41].

For the purpose of this assessment the total range of defect sizes is broken into defect groups, and a record is kept each year of the number of defects within each group. Based on the growth rate, and the kinetics of the growth, the probability of moving from one defect group to a larger defect group is assessed. In Mori and Ellingwood [42] it was concluded that the optimum inspection/repair intervals were almost uniform in all examples considered. Therefore, uniform/fixed inspection intervals (i.e. every ΔT years) are assumed in this paper for practicality. Also, similar to other studies [4, 43, 44, 45, 46], it was decided to assume repairs are perfect (i.e. component returns to the 'as new' condition following a repair) to reduce the number of possible outcomes. Having ΔT as a parameter of the optimisation has the advantage that (i) in the case where a federally mandated inspection frequency is specified, e.g. ΔT = 2 years then the optimum combination of NDT techniques can be chosen, for inspection phase 1 (i.e. detection) and phase 2 (i.e. sizing), which minimise the probability of failure (i.e. limit state exceedance) with respect to assessment budget or (ii) facilitates determination of the optimal inspection interval ΔT for a particular deterioration mechanism based upon the available inspection budget (and associated budget for the two phases of this inspection), or upon available NDT techniques.

Two Markov matrices are required (size NxN), one to simulate the growth, repair and failure of the defects at an inspection year, and another to simulate the growth and failure of the defects between inspections [17]. It is assumed that defects return to the smallest group after repair/failure. Therefore, at an inspection year, the first column in the matrix is controlled by the probability of repair and the probability of failure given that no repair is carried out, whereas, between inspections, this column is controlled by the probability of failure alone.

4.1. Simulation of the Growth of a Defect

The objective here is to develop the upper triangular part (growth part) of the Markov transition matrix of size NxN using the specified growth parameters. Therefore, given an initial population of defects in each group, the growth of the defects, and the movement of the defects into larger defect groups can be modelled over time.

There are two parameters in the model which define the growth of a defect over time and from which all transition probabilities for the growth matrix, P_{ij_GROWTH}, are calculated. The first is α, which describes the growth rate of a defect, which therefore controls how quickly a defect moves from one defect group to the next. A higher growth rate means that there is a lower probability that the defect will remain in the same defect group after each time step and a lower growth rate increases the likelihood that it will stay in the same group after the same period of time. For example, crack growth in a reinforced concrete structure in an aggressive marine environment is more likely to develop at a faster rate than a structure inland where the exposure to chlorides is minimal, and will therefore be more likely to move to a larger defect group within a certain time interval. The other parameter is g, which determines how gradual or sudden the growth of an individual defect is. This parameter controls whether defects develop gradually (e.g. crack growth in reinforced concrete due to steel corrosion or carbonation) and just move from one defect group to the next, or whether the growth of a defect is more abrupt (e.g. crack growth due to overloading), making it possible to move from one group to a defect group several sizes larger, Figure 5. This allows many different forms of deterioration mechanism, which are associated with different environments and materials, to be simulated using this approach. Depending on the limit state being considered, the owner/manager of a structure will be concerned with different forms of deterioration (e.g. in relation to SLS, the owner/manage may be concerned with crack growth due to reinforced concrete corrosion in concrete structures), and based on field data or experimental results in the laboratory, these parameters can be estimated to predict the deterioration of a structure over time. Therefore, both parameters are used to calculate the entries in the Markov matrix which simulates growth only, P_{ij_GROWTH}. Figures 6-7 illustrate the effect of the two growth parameters, α and g, on the growth part of the Markov matrix. The parameters can be seen to be functions both of the deterioration mechanism under consideration and its aggressivity and also of the structure mechanical characteristics (i.e. in a reinforced concrete bridge the rate of crack opening will also be a function of the modulus of rupture of the concrete, while the time to initiation of cracking and of reinforcement corrosion will be a function of the permeability of the concrete, where blended cements will have considerable lower permeability than standard OPC concretes).

4.2. Simulation of Failure between Inspections

If a defect continues to grow, without any repairs being carried out, failure will eventually occur. Therefore, failure must be simulated between inspections. For each defect group, the probability of failure is calculated to assess the probability that a defect will fail and subsequently be repaired, and will therefore return to the smallest defect group (i.e. P_{i1}). The annual probability of failure, p_f, is calculated using the Weibull cumulative distribution function [47, 48], based on the mean size of the defects in each group, $\overline{d}_i$, Equation 17,

$$p_f(\bar{d}_i) = 1 - \left[\exp - \left(\frac{\bar{d}_i - d_1}{d_{ref\_pf}} \right)^m \right] \quad (17)$$

When considering the time between inspections, there is no chance of repair being carried out before failure. Therefore, using Equation 17, the probability of failure for each group (with mean defect size $\bar{d}_i$) is calculated, and then used to determine the values for the first column in the Markov matrix, which simulates the behaviour of a population of defects between inspections.

4.3. Simulation of Repair and Failure at Inspection Year

Over the lifetime of infrastructural elements/networks, inspections and repairs are carried out and, in some cases, failure can occur. However, at an inspection year, it is assumed that failure will only occur if a repair is not carried out. In this case, P_{i1} is calculated using a combination of the probability of repair, and the probability of failure given that repair has not been carried out. Using the parameters associated with the inspection techniques, and the failure of the components, the probability of repair and the probability of failure are calculated for each group. These values, along with the mean size and standard deviation of the defects in each group, are used in the calculation of the P_{i1} column for the Markov matrix simulating the behaviour of a population of defects at an inspection year.

To calculate the probability of repair of defects in each group it is necessary to assess the PoD/PFA and PGA/PWA for each group. The PoD and PFA are estimated for each defect group, given the mean and standard deviation of the defects in the group and the quality of the inspection method assumed to be being used, Q_1. It is assumed that the defect size and the noise are normally distributed and non-correlated, and that for detection the quality (and hence cost) of the inspection method is related to the distribution of the noise, σ_{ND}, Equation 18. It is recognised by the authors that further work must be carried out to correlate these parameters with actual inspection techniques.

$$\frac{\sigma_{ND}}{d_{ref}} = \frac{1}{Q_1} \quad (18)$$

The mean value of the noise depends on environmental conditions and human interference, and is assumed to be the same for each defect group. Therefore, the PoD_i and PFA_i are estimated for each group, i, using Equations 19-20. For the purpose of this study a cumulative normal distribution is used to model the PoD and PFA, although it is recognised that other distributions may also be used.

$$PoD_i = \Phi\left(\frac{\bar{d}_i - d_{min}}{\sqrt{\sigma_d^2 + \sigma_{ND}^2}}\right) \quad (19)$$

$$PFA_i = \Phi\left(\frac{n_{mean} - d_{min}}{\sigma_{ND}}\right) \quad (20)$$

Similarly, for each defect group the values of PGA and PWA are estimated, given the mean and standard deviation of the defects in the group and the quality inspection method being used for sizing. It is assumed for assessment also that the quality of the inspection method is related to the distribution of the noise, σ_{NA}, Equation 21,

$$\frac{\sigma_{NA}}{d_{ref}} = \frac{1}{Q_2} \quad (21)$$

While the form of distribution can be different, for the purpose of this example, a normal distribution is assumed to describe the error on sizing of the inspection technique. In this case the noise has the properties $N(0,\sigma_{NA})$. This is similar to the signal response method, as described by Chung et al. [21], where the recorded signal response is related to the actual crack size using an error term which is modelled using a normal distribution, with some standard deviation and a mean value of 0. Note that other models are available but are specific to a given study case [49]. Therefore, the PGA_i and PWA_i are estimated for each group, i, using Equations 22-23.

$$PGA_{R\_i} = \Phi\left(\frac{\bar{d}_i - d_c}{\sqrt{\sigma_d^2 + \sigma_{NA}^2}}\right) \qquad \left(\text{for } \bar{d}_i \geq d_c\right) \quad (22)$$

$$PWA_{R\_i} = \Phi\left(\frac{\overline{d}_i - d_c}{\sqrt{\sigma_d^2 + \sigma_{NA}^2}}\right) \qquad \left(\text{for } \overline{d}_i < d_c\right) \tag{23}$$

When the first inspection is carried out to detect a defect, there can be two decision outcomes. One is to carry out a further assessment, and the other is to do nothing. Similarly, when a second inspection is carried out to assess the size of a defect, there can also be two decision outcomes. One is to repair, in which case the defect returns to the initial defect group, and the other is to carry out no repair. The process is described in Figure 8. If at the detection stage, no further assessment is carried out, or if at the assessment stage, no repair is carried out, there is still a remaining probability of failure (which will be larger in the second case as the defect is larger). Similar to the event of repair, if failure occurs the defect returns to the initial defect group. These probabilities are calculated analytically, and are combined to assess P_{i1} for each defect group, which is illustrated in Figure 9 (N=5).

At this stage the Markov growth matrix for deterioration has been developed and the values for the probability of repair/failure at an inspection year and the values for the probability of failure between inspections have been calculated for each group. These values need to be added to the first column of the growth matrix (whether it be for an inspection year, or a year between inspections) to develop the two complete Markov matrices which can then be used in the maintenance management procedure. The methodology described assumes that in any year of simulation, the inspection, repair and failure occur at the end of the year in question. Therefore, when modifying the matrix it is assumed that defects may grow throughout the year before inspections, repair or failure occur.

4.4. Stabilisation of Process over Time

In this study it is assumed that inspections are carried out at regular intervals, every ΔT years. For this reason, two Markov matrices have been developed, one to simulate the growth and failure of a defect between inspections, and another to simulate growth, repair and failure of a defect at an inspection year. Once both matrices have been formulated, they are used to simulate the growth and repair of a population of defects over time. Each defect group is assumed to have an initial population of defects. Using this methodology, the number of defects in each group is calculated on a yearly basis using the relevant Markov matrix, and the number of defects in each group from the previous year (k-1), Equation 24.

$$\#d_{j@Y_k} = \sum_{i=1}^{N} \left(\#d_{i@Y_{k-1}} * P_{ij} \right) \quad (24)$$

To find the optimum time between inspections or to carry out a costing analysis using this model, the number of defects in each group is assumed to reach a steady state. Figure 10 illustrates the stabilisation of the coefficient of variation of defect size for all of the groups for an inspection period of 1 year, considering growth, repair and failure for different growth parameters. However, since two different matrices are used in this study simultaneously (for an inspection interval greater than 1 year), the number of defects in each group will not converge to one value over time, but will converge to a set of ΔT values, one value for each year in the ΔT cycle.

When the Markov matrices were developed it was assumed that within each time interval of one year, the defects can grow from the groups they were in at the beginning of the year to other groups before the inspections, repairs or failures for that year occur. To calculate the number of inspections, repairs and failures based on the calculated probabilities in Sections 4.2 and Section 4.3 it is necessary to have the stabilised number of defects in each group directly before the events of inspections, or failure. Therefore, once the stabilised number of defects (at yearly intervals) is evaluated, it is necessary to compute the number of defects in each group (for each year) directly before inspection or failure. This is evaluated using the number of defects at the beginning of each year, and using the relevant Markov matrix to calculate the number of defects in each group after growth alone for that year (i.e. before inspection, repair or failure). This set of values is then used to calculate the expected annual costs of the structure.

4.5. Cost Functions

In the proposed methodology the expected cost of inspections ($E(C_{I\_TOTAL})$), repair ($E(C_{R\_TOTAL})$) and failure ($E(C_{F\_TOTAL})$) are considered, which are summed to find the expected annual total cost of the structure ($E(C_{TOTAL})$).($E(C_{TOTAL})$). These are the direct costs associated with maintenance management of a structure or network of structures. The cost analysis developed as part of this study is used to compare the relative implications of different management decisions (e.g. different inspection quality, inspection intervals etc.) and it is recognised that these cost models are subjective and should only be used to provide an indication of the relative benefits of different management strategies. Further work needs to be carried out to further develop these cost functions, taking indirect costs (such as user delay costs and penalty costs for reduced serviceability) into account.

Once the cost of inspection, repair and failure for an individual defect in a group is calculated, the expected total cost is calculated by multiplying by the number of defects in the group and the probability that inspection, repair or failure occurs. In relation to inspection and repair, the expected costs are calculated using the stabilised number of defects in each group at an inspection year. The expected failure cost at an inspection year must also be calculated each year using the stabilised number of defects in each group at an inspection year, the P(Failure | No Assessment) and the P(Failure | No Repair). The expected failure cost between inspections, however, must be calculated using the stabilised number of defects in each group, for each year between inspections. The various expected costs for each group are then divided into ratios (depending on the four inspection events for detection and assessment) to assess which costs are due to good/bad decisions, Figure 8.

Once the number of defects in each group, and the expected number of inspections to be carried out at each inspection year are determined, the expected annual inspection cost of the structure can be calculated. An initial inspection or first inspection of each defect is carried out every ΔT years. If the first inspection results in detection (with a probability $P(\hat{d}_{1\_i} \geq d_{min})$), then a second inspection takes place to size the detected defect. The cost of an individual inspection for detection and sizing is evaluated using Equations 25-26, respectively.

$$CI1 = C_o k_I \, Q_1 / Q_{ref} \tag{25}$$

$$CI2 = C_o k_I \, Q_2 / Q_{ref} \tag{26}$$

The expected annual total inspection cost of the structure ($E(C_{I\_TOTAL})$) is then calculated by summing the expected annual cost of inspection for detection and inspection for sizing assessment for each group i.

With regards to repair, the sizing assessment takes place if the first inspection results in detection. If the second inspection indicates that the defect is larger than the critical defect size, then it is assumed that a repair is carried out. If the inspection indicates that the defect size is smaller than the critical defect size, then no further action is taken. Again, the expected annual costs can be calculated, knowing the number of defects in each group i, and the cost of an individual repair for a defect in each group, Equation 27.

$$CR_i = C_0 k_R \left(\frac{\overline{d}_i}{d_{ref}} \right) \tag{27}$$

When a repair is carried out, this can be due to a good assessment (i.e. the real defect size is greater than d_c, event E_{4A}) or due to a wrong assessment (i.e. the real size of the defect is actually less than d_c, event E_{2A}). Using the probability of each event and the expected annual cost of repair, the expected cost of repair due to a good assessment and the expected cost due to a bad assessment (or an expected cost overrun) can be evaluated.

For any defect greater than the limit defect size, d_1, there is some probability of failure, depending on the size of the defect. As described previously, the probability of failure for each defect group is calculated based on the mean size of the defects in the group i, Equation 17. In this study, the cost of an individual failure at an inspection year is equal to the cost of an individual failure between inspections. This cost is calculated using Equation 28.

$$CF = C_o k_F \tag{28}$$

Knowing the number of failures at the detection stage, the number of failures at the sizing assessment stage (at an inspection year) and the total number of failures between inspections, the expected total cost of failure is calculated.

The expected total cost of the structure is calculated by summing these costs. These cost are calculated over one ΔT cycle, therefore, the expected annual costs, $(E(C_{I\_TOTAL}))$, $(E(C_{R\_TOTAL}))$, $(E(C_{F\_TOTAL}))$ and $(E(C_{TOTAL}))$ are found by dividing by ΔT.

5. Results

Using this new methodology, a maintenance management model has been developed, an example of the application of which is presented here. For purpose of this study, it was chosen to divide the defect growth into 10 groups. This value was chosen to correspond with the Federal Highways Agency inspection recommendations in the US, where each element in a structure is assigned one of 10 of the National Bridge Inventory Condition Ratings [50]. The possible range of defect sizes for each group is modelled statistically with an assumed mean and standard deviation, as outlined in Table 1. The defect size is assumed to vary from 0-1.0, with a 0.1 defect range for each group (e.g. crack width of a reinforced concrete structure varying from 0-1.0mm). The standard deviation of the groups represents the scatter of the range of actual sizes of the defects in the group, and is not related to the error in sizing of a defect. It was assumed that the standard deviation of the range of defect sizes in a group is independent of the mean defect size of the group, and a constant value was assumed for each group. In addition, it was assumed initially that there were 100 defects in the smallest defect group, and no defects in all other groups (i.e. taken to represent a new structure), Table 1, although the methodology can consider a structure at any stage of its life.

Initially, for each value of ΔT, the two Markov matrices were used to calculate the stabilised number of defects in each group (directly before inspection or failure), for each year in the ΔT cycle, for a given set of input parameters. The methodology outlined was then used to determine the optimum time between inspections, and subsequently to analyse the effect of the interaction of the quality of the inspection techniques for detection and sizing on the optimum time between inspections and the expected annual total costs of the structure. The effect of the quality of inspections on cost overrun, such as unnecessary repairs, was also investigated using the events based decision theory described. This provides a powerful decision tool for infrastructure owners/managers in optimising maintenance budget spend as based upon available funds, structural form, deterioration mechanism, environment and limit state considered, it empowers them, rather than performing inspections on e.g. a bi-annual basis, using a range of NDT tools to identify the best combination of techniques to be employed in assessing condition and associated probability of failure (i.e. limit state exceedance).

5.1. Optimal Time between Inspections

Table 2 shows the set of parameters assumed in the model for the purpose of this exercise. Using these parameters, the optimum time between inspections was determined on the basis of the minimum expected annual total costs of the structure, $E(C_{TOTAL})$, which were assessed according to the cost functions outlined in Section 4.6. It is noted here that these are theoretical parameters selected to demonstrate the operability of the methodology. Research is ongoing to identify realistic parameters to be employed based upon the NDT tool, deterioration mechanism, limit state, cost etc [28, 29, 30, 31, 32, 33, 34]. Figure 11 shows the results of the analysis, illustrating that a period of 4-years represents the optimum inspection interval for the case considered.

As illustrated in Figure 11 the inspection interval has a significant effect on the expected annual total inspection cost, $E(C_{I\_TOTAL})$, and the expected annual total failure cost, $E(C_{F\_TOTAL})$. The expected total inspection cost ranges from 60% of the total cost for a 1 year inspection interval, to just 10% of the total cost for a 10 year inspection interval. As expected, an inverse trend emerges for the total failure cost, with the expected total failure cost ranging from just 1.4% of the total cost at a 1 year inspection interval, to 48% of the total cost for a 10 year inspection interval.

The expected total cost of repair, $E(C_{R\_TOTAL})$, has a significant effect on the expected annual total costs, contributing to 58% of the total cost at the optimal inspection interval ($\Delta T=4$). However, Figure 11 demonstrates that the expected total cost of repair is relatively insensitive to the inspection interval. This is due to the incorporation of the sizing assessment into the analysis, as the second stage of an inspection. Using this methodology it is possible to determine the extent of each repair at the time of an inspection, and to estimate the cost of repair based on the size of the defect, according to Equation 27. For example, if inspections are carried out annually, then it is assumed that large defects are unlikely to develop, and only minor repairs are carried out every year. Whereas if inspections are only carried out every 10 years, it is assumed that quite extensive repairs will be necessary due to larger defects, but these repairs are less frequent. Therefore, there is just a 15% difference between $E(C_{R\_TOTAL})$ for $\Delta T=1$ and $E(C_{R\_TOTAL})$ for $\Delta T=10$.

5.2. Inspection Quality

Using the methodology developed, it is possible to look at the interaction of the inspection methods for detection and sizing, and see how this affects the optimum inspection interval and the expected annual total costs. This provides the owner/manager with a useful decision tool when selecting a combination of inspection techniques to be used as part of a maintenance management plan.

Figures 12-13 illustrate how a different combination of inspection techniques can affect the optimal maintenance management plan, and the expected annual costs of the structure. In relation to the first inspection, a higher quality technique, Q_1, reduces the noise associated with the inspection procedure, and therefore, more accurately determines which defects should be further assessed, which consequently reduces the number of failures due to undetected defects. Figure 12 illustrates a direct relationship between the inspection quality for detection and the optimal inspection interval. A similar trend emerges when the quality of the second technique, Q_2, in increased. A better technique reduces the number of failures, as a higher proportion of defects are sized correctly and repaired when necessary.

The owner/manager has a number of options when using this new decision tool. In the case where a convenient inspection interval has been decided upon, Figure 12 can be used to find a combination of technique qualities with this inspection interval as optimal. Although this can result in a multiple of combinations of techniques, Figure 13 can then be used to determine which of these combinations results in the lowest expected annual costs. For example, if an inspection interval of 4 years is convenient, from Figure 12, there are 6 different combinations of techniques which would be suitable. These 6 combinations are listed in Table 3, with the expected annual total costs for each combination, which are illustrated in Figure 13. In this case, the second option (Q_1=10, Q_2=10) results in the lowest relative cost, and is clearly the most cost efficient combination of techniques for the chosen inspection interval of 4 years.

Alternatively, if the inspection for detection is chosen initially, Figures 12-13 can be used to choose a suitable inspection technique for the sizing assessment. For each inspection quality available for the second assessment it is possible to determine the optimal inspection interval, Figure 12, and the expected annual total costs of the structure, Figure 13. Depending on the structure, it may be more convenient to carry out inspections less often (depending on intangible costs/benefits that have not been incorporated into this model), even though the relative expected annual total cost is higher. Table 4 details a list of options available for a specific inspection quality for detection, Q_1=10. This method interestingly points out that there are two available options (for the quality of the second inspections) that result in an optimal inspection interval of 5 years, yet one option has a relatively lower expected annual total cost than the other, clearly showing it to be the most cost efficient choice. By looking at this interaction of inspection techniques for detection and sizing assessment, an owner/manager can clearly pick the optimum combination of techniques to suit a particular set of requirements.

Furthermore, using the events based decision theory outlined in Section 3, the effect of the quality of inspections on cost overrun, such as unnecessary repair, can also be investigated. Figure 14 illustrates that the total cost of repair is relatively insensitive to the quality of the second inspection, although it is clear that the inspection quality affects the relative breakdown of these costs into necessary and unnecessary repairs. The number of repairs carried out depends on the number of defects that are sized and are found to be greater than the critical defect size, therefore, the cost overrun of unnecessary repairs reduces as more accurate inspections for sizing are carried out. By using a better quality technique, the defects that could lead to failure of a component are repaired, rather than defects that are incorrectly sized, and are not in need of repair. Reducing the number of failures within the structure has the effect of increasing the optimal inspection interval. As discussed previously, although a higher inspection quality results in an increase in the expected annual cost of a structure, Figure 13, the optimal inspection interval is likely to also increase, Figure 12, which can be more convenient for an owner/manager of a structure.

6. Conclusions

This paper presents infrastructure owners/managers with a decision tool based upon the subdivision of the assessment process into two phases, (i) detection and (ii) sizing, which can be used in optimal management of infrastructural elements/networks to minimise the probability of failure (i.e. limit state exceedance) within budgetary constraints.

The paper demonstrates that the choice of inspection techniques for detection and sizing has a significant influence on the optimum time between inspections, and hence the minimum annual total cost of the structure. When carrying out an inspection there are two points of interest, the presence of a defect, and the size of a defect present. Since each stage of the inspection has a different purpose, it is necessary to separate these procedures to accurately model an inspection process which is to be incorporated into a maintenance management plan.

The separation of the inspection process into two stages enables the investigation to study the effect of both stages of the inspection on the expected annual costs of the structure, and the maintenance plan for the structure. The separation of these procedures and the interaction of the two inspection techniques have not previously been considered.

By modelling the two stages of an inspection as separate procedures, using different parameters, the effect of different combinations of techniques can be investigated. The detection process is similar to a screening exercise to determine which defects require further assessment. By producing decision tools similar to Figures 12-13, it is possible to look at the relative benefits of using different quality techniques. Depending on the requirements of the owner/manager, upon the structure considered, its environment and deterioration mechanism, it may be more convenient to use a low quality screening technique for detection and a higher quality inspection technique for sizing to assess which defects should be repaired. The developed methodology allows for the first time, the effect of such decisions to be evaluated quantitatively both with regard to performance (i.e. probability of limit state exceedance) and budgetary cost (i.e. cost of assessment campaign and associated good/bad decisions). The combination of techniques used during an inspection is demonstrated to effect the optimal time between inspections. If an inspection requires partial or total closure of a structure, which can lead to user delays, the owner/manager may prefer to incur a higher annual total cost in return for a longer optimal inspection interval. The results of each combination of techniques can be assessed quantitatively with reference to Figures 12-13, allowing the most cost efficient approach for a given set of requirements to be determined.

Figures 12-13 demonstrates the benefits of the proposed approach. By modelling each stage of an inspection separately, with different parameters, the interaction between these two inspection procedures and the effect of the quality of the individual inspection methods on the optimal maintenance management plan and the annual costs of the structure can be identified.

This methodology can be extended to spatial stochastic fields where inspections can be spatial dependent and the sampling can be different for the kin of inspection. One way to solve this problem, is to base the description of the defect and the error due to NDT on the polynomial chaos expansion [31].

Further work needs to be carried out on the calibration of parameters which were introduced as part of this model, particularly in relating the parameters of the Weibull distribution (which are used to model the probability of failure) to the actual mode of failure being considered (e.g. sudden or progressive failure) and investigating the most suitable number of defect groups (and associated parameters) for the deterioration rate/mechanism being considered. Work is also ongoing to further develop this method to enable the initiation stage of deterioration to be considered as well as the propagation stage (which is particularly relevant for the deterioration of reinforced concrete due to chloride ingress). Once this has been developed, it will also be possible to determine optimum repair materials, considering the properties of the repair materials for both the initiation phase and propagation phase of deterioration.

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Appendix A - Notation

CF = cost of failure for an individual defect

CI1 = cost of an individual inspection for detection

CI2 = cost of an individual inspection for sizing

C_o= initial cost of construction

CR_i = cost of an individual repair

d = actual size of the defect

d_c = critical defect size (a defect size greater than d_c leads to a repair)

di = defect group i

$\overline{d}$ = mean defect size of a group

d_{min} = detection threshold

d_{ref} = reference defect size

d_{ref_pf} = reference defect size for the probability of failure, Weibull law parameter

d_l = limit defect size, Weibull law parameter

$\hat{d}_1$ = size of the detected defect (from inspection 1)

$\hat{d}_2$ = size of the defect from inspection (from inspection 2)

E( ) = annual expectancy of any cost variable

E_{1A} = event 1 for sizing assessment – good sizing, no repair

E_{2A} = event 2 for sizing assessment – wrong sizing, repair

E_{3A} = event 3 for sizing assessment – wrong sizing, no repair

E_{4A} = event 4 for sizing assessment – good sizing, repair

E_{1D} = event 1 for detection – no defect, no detection

E_{2D} = event 2 for detection – no defect, detection

E_{3D} = event 3 for detection – defect, no detection

E_{4D} = event 4 for detection – defect, detection

g = deterioration kinetics parameter

k_F = failure impact coefficient

k_I = inspection cost coefficient

k_R = repair cost coefficient

m = Weibull exponent (to calculate p_f) which determines the spread of the curve

N = total number of groups

NDT= non-destructive technique

PDF = probability density function

PFA = probability of false alarm

PGA_{NR} = probability of a good assessment resulting in no repair

PGA_R = probability of a good assessment resulting in repair

PoD = probability of detection

PWA_{NR} = probability of a wrong assessment resulting in no repair

PWA_R = probability of a wrong assessment resulting in repair

p_f = annual probability of failure

Q_{ref} = reference inspection quality

Q_1 = quality of the inspection method for defect detection

Q_2 = quality of the inspection method for sizing assessment

ΔT = inspection interval in years

α = growth rate of a defect

σ_d = standard deviation of the defect size in a group

σ_{NA} = standard deviation of noise distribution (for assessment)

σ_{ND} = standard deviation of noise distribution (for detection)

$\# d_{j@Y_k}$ = number of defects in group j at year Y_k

References

[1] Enright, M.P. and Frangopol, D. M. (1999). "Maintenance planning for deteriorating concrete bridges." *Journal of Structural Engineering,* 125(12), 1407-1414

[2] Soltani, M. (1995). ''Statistical bridge inspection.'' *Applications of statistics and probability*, Lemaire, Favre, and Mebarki, eds., Balkema, Rotterdam, The Netherlands, 387–391.

[3] Estes, A. C. and Frangopol, D. M. (1999). "Repair optimization of highway bridges using system reliability approach." *Journal of Structural Engineering,* 125(7), 766-775.

[4] Faber, M. H and Sorensen, J. D. (2002). "Indicators for inspection and maintenance planning of concrete structures." *Journal of Structural Safety*, 24(4), 377-396.

[5] Kong, J. S. and Frangopol, D. M. (2004). "Life cycle reliability-based maintenance cost optimisation of deteriorating structures with emphasis on bridges" *Journal of Structural Engineering,* 129(6), 818-828.

[6] Kong, J. S. and Frangopol, D. M. (2004). "Cost-reliability interaction in life-cycle cost optimization of deteriorating structures." *Journal of Structural Engineering,* 130(11), 1704-1712.

[7] Kong, J. S. and Frangopol, D. M. (2005). "Probabilistic optimization of ageing structures considering maintenance and failure costs." *Journal of Structural Engineering,* 131(4), 600-616.

[8] Lauridsen, J., Bjerrum, J., Sloth, M. and Jensen, F. M. (2006). "Principles for a guideline for probability-based management of deteriorated bridges." *IABMAS 2006 Conference, Bridge Maintenance, Safety and Management* (CD-Rom), Porto.

[9] O'Connor A. and Eichinger E., (2007), 'Site-Specific Traffic Load Modelling for Bridge Assessment', *ICE Journal of Bridge Engineering*, 160(4), 185 - 194.

[10] O'Connor A. and Enevoldsen I., (2009), 'Probability based assessment of bridges according to the new Danish guideline', *Structure and Infrastructure Engineering*, 5(2), 157 - 168.

[11] O'Connor A. and O'Brien E., (2005), 'Mathematical Traffic Load Modelling and Factors Influencing the Accuracy of Predicted Extremes', *Canadian Journal of Civil Engineering*, **32**, pp. 270 - 278.

[12] Radojicic, A., Bailey, S. F. and Brühwiler, E. (2001). "Probabilistic models of cost for the management of existing structures." *Life-Cycle Cost Analysis and Design of Civil Infrastructure Systems*, SEI-ASCE, 251-270.

[13] Stewart, M. G. (2001). "Reliability based assessment of ageing bridges using risk ranking and life cycle cost decision analyses" *Reliability Engineering and System Safety,* 74(3), 263 – 273.

[14] Stewart, M. G., Estes, A. C. and Frangopol, D. M., (2004). "Bridge deck replacement for minimum expected cost under multiple reliability constraints." *Journal of Structural Engineering,* 130(9), 1414-1419.

[15] Stewart, M. G. (2005). "Life-cycle cost analysis considering spatial and temporal variability of corrosion-induced damage and repair of concrete surfaces." *International Conference on Structural Safety and Reliability, ICOSSAR*, Rome, Italy.

[16] Stewart, M. G. and Mullard, J. A. (2006). "Reliability based assessment of the influence of concrete durability on the timing of repair for RC bridges." *IABMAS 2006 Conference, Bridge Maintenance, Safety and Management* (CD-Rom), Porto.

[17] Corotis, R.B., Ellis J.H. and Jiang, M. (2005). "Modelling of risk-based inspection, maintenance and life-cycle cost with partially observable Markov decision processes." *Structure and Infrastructure Engineering*, 1(1), 75-84.

[18] Schoefs, F. and Clement, A. (2004). "Multiple inspection modeling for decision making and management of jacket offshore platforms: effect of false alarms." *International Forum on Engineering Decision Making, IFED,* Stoos, Switzerland.

[19] Rouhan, A. and Schoefs, F. (2003). "Probabilistic modeling of inspection results for offshore structures". *Journal of Structural Safety*, 25(4), 379-399.

[20] Straub, D, and Faber, M.H. (2003). "Modeling dependency in inspection performance." *Applications of Statistics and Probability in Civil Engineering*, Rotterdam, 1123-1130.

[21] Chung, H. Y., Manuel, L. and Frank, K. H. (2006). "Optimal inspection scheduling of steel bridges using nondestructive testing techniques." *Journal of Bridge Engineering,* 11(3), 305-319.

[22] Zhang, R. and Mahadevan, S. (2001). "Fatigue reliability using nondestructive inspection." *Journal of Structural Engineering,* 127(8), 957-965

[23] James, L.A. (1997). "Surface-crack aspect ratio development during corrosion-fatigue crack growth in low alloy steels." *Nuclear Engineering and Design*, 172(1-2), 61 – 71.

[24] Pellerin, R.F, Lavinder, J.A., Ross, R.J. and Falk, R.H. (1996). ”Non destructive evaluation of timber bridges.” *Proceedings of SPIE – The International Society for Optical Engineering: Non destructive evaluation of materials and composites*, 2994, 275 – 284.

[25] DNV 2007, Statistical representation of soil data, Recommended practice DNV-RP-C207, Det Norske Veritas, April 2007.

[26] Madsen, H., Skjong, R., Tallin, A. and Kirkemo, F. (1987). “Probabilistic fatigue crack growth analysis of offshore structures, with reliability updating through inspection.” *Marine Structural Reliability Symposium,* Arlington, Virginia, 45-55.

[27] Onoufriou, T. and Frangopol D. M. (2002). “Reliability-based inspection optimisation of complex structures: a brief retrospective.” *Journal of Computers and Structures,* 80(12), 1133-1144.

[28] Barnouin, B., Lemoine, L, Dover, W.D., Rudlin, J., Fabbri, S., Rebourcet, G., Topp, D., Kare, R., Sangouard, D. (1993). “Underwater inspection reliability trials for offshore structures”. In ASME NY, editor. *Proc. of the 12th OMAE conference*. Vol. 2 pp. 883-890.

[29] Bonnet, S., Schoefs, F., Ricardo, J., Salta, M.M. (2008). “Statistical study and probabilistic modelling of error when building chloride profiles”, *Proceeding of 1st International Conference on Applications Heritage and Constructions in Coastal and Marine Environment*, (MEDACHS’08), paper #111, 8 pages, 28-30 January 2008, Lisbon (LNEC), Portugal (2008).

[30] Bonnet, S., Schoefs, F., Ricardo, J., Salta, M. (2009). “Statistical study and probabilistic modelling of error when building chloride profiles for reliability assessment”, to appear in *Material and Structures*, 2009

[31] Schoefs, F. (2008). “Risk analysis of structures in presence of stochastic fields of deterioration: coupling of inspection and structural reliability”, to appear in *Australian Journal of Structural Engineering*, Special “Engineering Decision Making issue” Issue, guest editors Mark Stewart and Stuart Reid.

[32] Schoefs, F., Clément, A., Nouy, A. (2009) ‘‘Assessment of spatially dependent ROC curves for inspection of random fields of defects”, to appear in *Structural Safety*,Ed. 2009

[33] Pakrashi, V., Schoefs, F., Memet, J.B., O’Connor, A. (2008) “An Image Analysis Based Damage Classification Methodology”, *Structure and Infrastructure Engineering. doi: 10.1080 /1573 2470 7017 18072*.

[34] Boéro, J., Schoefs, F., Capra, B. (2008). "Expert Judgement for Combining NDT Tools in RBI context: Application to Marine Structures", P*roc. of 4th International ASRANet Colloquium*, 25–27 June 2008, Athens, Greece, paper 70, 9 pages, proc. on CD-Rom.

[35] Breysse, D., Schoefs, F., Salta, M. and Bonnet, S. (2007) "Assessment updating of corrosion accounting uncertainties in modelling and NDT measurements." *ICASP10 2007 conference*, Tokyo.

[36] Scherer, W.T. and Glagola, D.M. (1994). "Markovian models for bridge maintenance management." *Journal of Transportation Engineering*, 120(1), 37-51.

[37] Ang, A. H-S. and Tang, W. N. (1975) *Probability Concepts in Engineering Planning and Design Volume I, Basic Principles*. John Wiley and Sons, New York.

[38] Roelfstra, G., Hajdin, R., Adey, B. and Brühwiler, E. (2004). "Condition Evolution in Bridge Management Systems and Corrosion-Induced Deterioration". *Journal of Bridge Engineering*, 9(3), 268-277.

[39] Orcesi, A., and Crémona, C. (2006). "Optimisation de la gestion des ponts en béton armé par chaînes de Markov (In French)", "Optimized management of reinforced concrete bridges using Markov chains". *Bulletin des Laboratoires des Ponts et Chausées*, 265, 19-33.

[40] Micevski, T., Kuczera, G. and Coombes, P. (2002). "Markov model for storm water pipe deterioration." *Journal of Infrastructure Systems*, 8(2), 48-56.

[41] Poinard, D., Le Gauffre, P. and Haidar (2003). "Markov model and climate factors for the rehabilitation planning of water networks." *Proc. 17th European Junior Scientist Workshop Rehabilitation Management of Urban Infrastructure Networks*, Neunzehnhain, Germany, September 2003.

[42] Mori, Y. and Ellingwood, B.R. (1994b). "Maintaining reliability of concrete structures. II: Optimum inspection/repair." *Journal of Structural Engineering*, 120(3), 846-862.

[43] Straub, D. and Faber, M.H. (2005). "Risk based inspection planning for structural systems." *Structural Safety*, 27(4), 335-355.

[44] Straub, D. and Faber, M.H. (2004). "System effects in generic risk-based inspection planning." *Journal of Offshore Mechanics and Arctic Engineering*, 126(3), 265-271.

[45] Adey, B., Bernard, O. and Gérard, B. (2003). "Risk-based replacement strategies for deteriorating reinforced concrete pipes." *Proceedings of 2nd International RILEM Workshop on Life Prediction and Aging Management of Concrete Structures*, Paris, France, 373-382.

[46] Kong, J.S. and Frangopol, D.M. (2003). "Life-cycle reliability-based maintenance cost optimisation of deteriorating structures with emphasis on bridges." *Journal of Structural Engineering*, 129(6), 818-828.

[47] Weibull, W. (1951). "A statistical distribution function of wide applicability." *Journal of Applied Mechanics*, 18(3), 293-297.

[48] Ang, A. H-S. and Tang, W. N. (1984) Probability Concepts in Engineering Planning and Design Volume II, Decision Risk and Reliability. John Wiley and Sons, New York.

[49] Bonnet S., Schoefs F., Ricardo J., Salta M. (2009), "Effect of Error Measurement of Chloride Profiles on Reliability Assessment", H. Furuta, D.M. Frangopol & M. Shinozuka (eds), ICOSSAR'09, Osaka, Japan, September 13-19,2009.

[50] Estes, A.C. and Frangopol, D.M. (2001). "Bridge lifetime system reliability under multiple limit states." *Journal of Bridge Engineering*, 6(6), 523-528.

Table Captions

Table 1. Defect group data used in the model

Table 2. Parameter values used in Markov Maintenance model

Table 3. Inspection technique combination 1

Table 4. Inspection technique combination 2

Table 1. Defect group data used in the model

Defect Group	Range		$\bar{d}$	σ_d	Initial Population
	From	To			
d1	0	0.1	0.05	0.02	100
d2	0.1	0.2	0.15	0.02	0
d3	0.2	0.3	0.25	0.02	0
d4	0.3	0.4	0.35	0.02	0
d5	0.4	0.5	0.45	0.02	0
d6	0.5	0.6	0.55	0.02	0
d7	0.6	0.7	0.65	0.02	0
d8	0.7	0.8	0.75	0.02	0
d9	0.8	0.9	0.85	0.02	0
d10	0.9	1	0.95	0.02	0

Table 2. Parameter values used in Markov Maintenance model

Model Properties	Value
Growth rate, α	0.4
Deterioration kinetics parameter, g	3
Reference defect size, d_{ref}	1
Probability of failure exponent, m	3
Limit defect size, d_l	0.4
Reference defect size, d_{ref_pf}	2
Detection threshold, d_{min}	0.35
Quality of inspection for detection, Q_1	10
Mean of noise distribution, n_{mean}	0.3
Critical defect size, d_c	0.62
Quality of inspection for sizing assessment, Q_2	20
Initial construction cost, C_o	1000
Inspection coefficient, k_I	0.01
Reference quality, Q_{ref}	50
Repair coefficient, k_R	0.05
Failure impact coefficient, k_F	1

Table 3. Inspection technique combination 1

$\Delta T=4$		
Q_1	Q_2	$E(C_{TOTAL})$
5	25	446.4
10	10	424.4
15	10	448.4
20	5	473.9
20	10	472.5
25	5	498.4

Table 4. Inspection technique combination 2

Q_1=10		
Q_2	ΔT	$E(C_{TOTAL})$
5	3	417.0
10	4	424.4
25	5	467.7
40	5	508.3
50	6	534.9

Figure Captions

Figure 1. Probabilistic modelling of inspection results for detection

Figure 2. Example of the effect of noise on sizing inspection results

Figure 3. Example of the method used to model the sizing assessment of a defect

Figure 4. Inspection outcomes for detection

Figure 5. Defect growth kinetics

Figure 6. An example of a Markov matrix to simulate defect growth (smooth growth)

Figure 7. An example of a Markov matrix to simulate defect growth (abrupt growth)

Figure 8. Inspection outcomes for a defect group

Figure 9. Schematic of repair process for Markov chain

Figure 10. The stabilisation of the CoV for a population of defects

Figure 11. The effect of the time between inspections on expected annual costs

Figure 12. The effect of inspection quality on optimal inspection interval

Figure 13. The effect of inspection quality on expected annual costs

Figure 14. The effect of the quality of inspections for sizing assessment on the expected annual repair cost

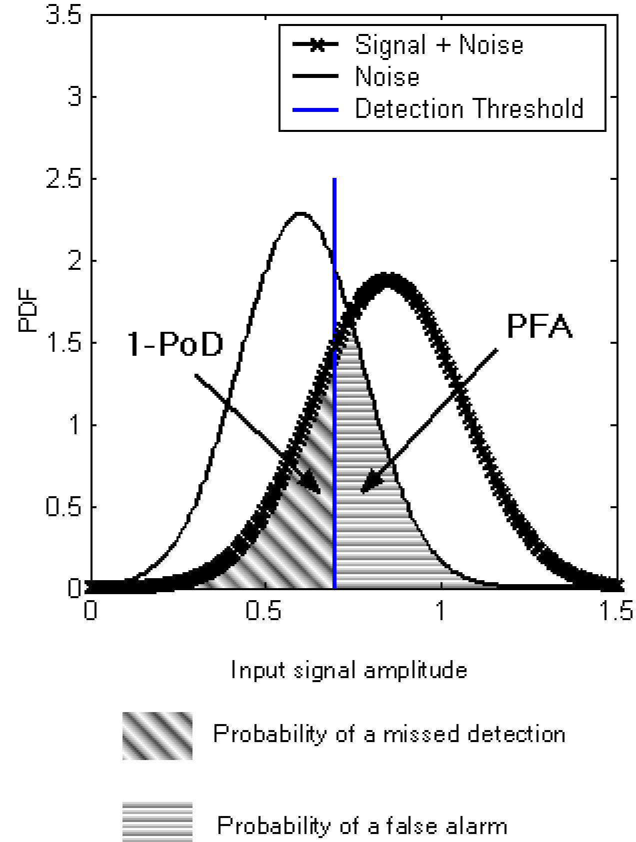

Signal + Noise
Noise
Detection Threshold
PDF
1-PoD
PFA
Input signal amplitude
Probability of a missed detection
Probability of a false alarm

Figure 1

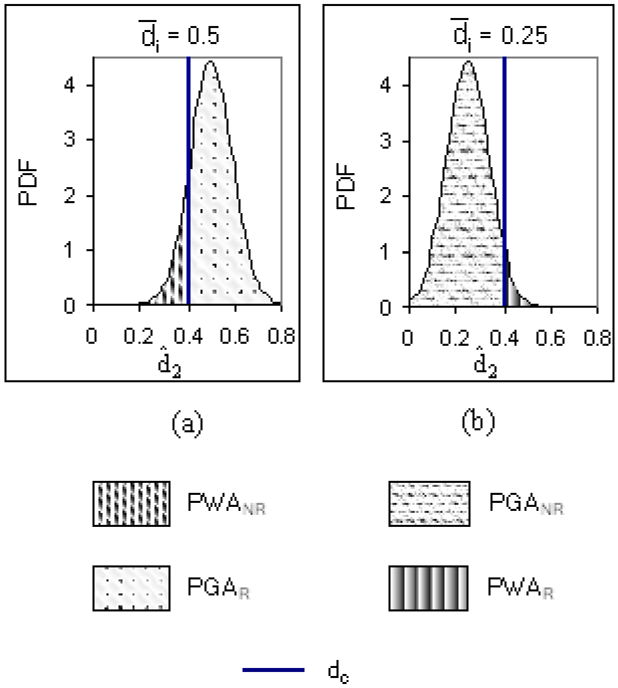

$\overline{d}_i$ = 0.5
$\overline{d}_i$ = 0.25
PDF
$\hat{d}_2$
(a)
(b)
PWA_{NR}
PGA_{NR}
PGA_R
PWA_R
d_c

Figure 2

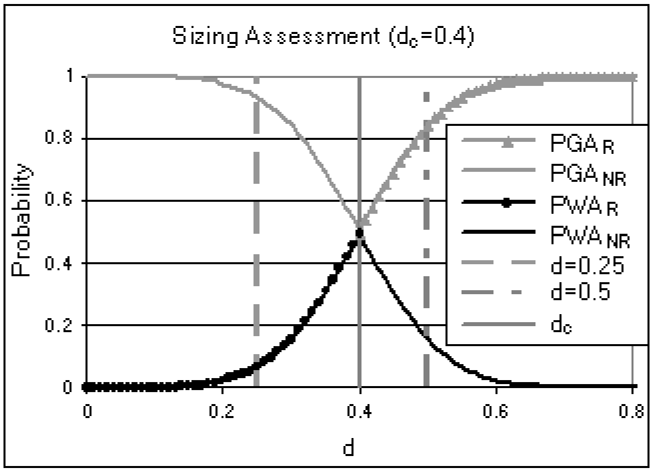

Sizing Assessment (dc=0.4)
Probability
d
0
0.2
0.4
0.6
0.8
1
PGA R
PGA NR
PWA R
PWA NR
d=0.25
d=0.5
dc

Figure 3

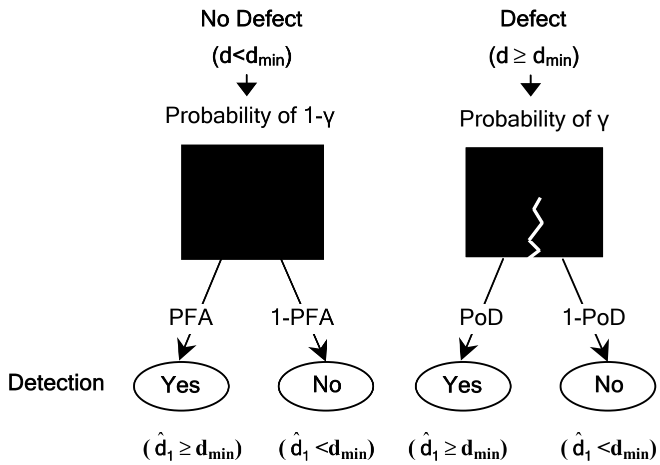
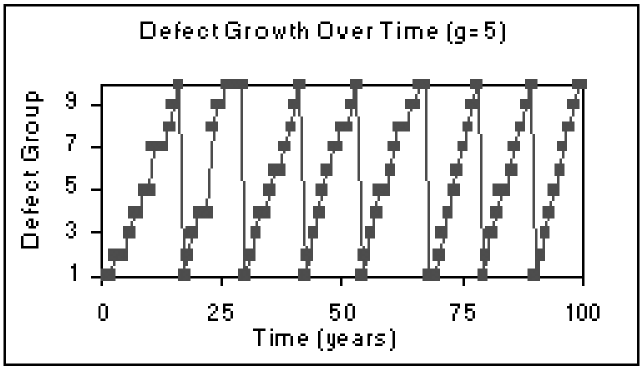
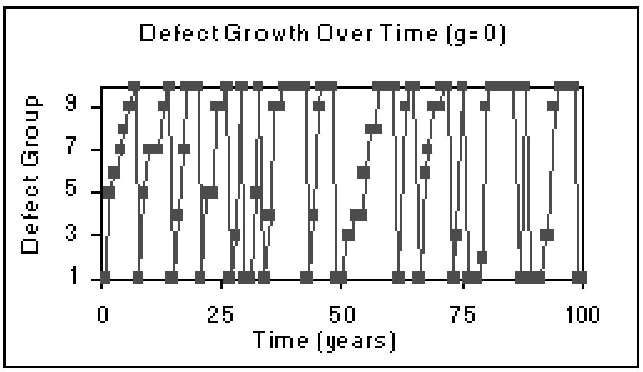

No Defect
(d<d$_{min}$)
Probability of 1-γ
Defect
(d ≥ d$_{min}$)
Probability of γ
PFA
1-PFA
PoD
1-PoD
Detection
Yes
No
Yes
No
(d̂1 ≥ dmin)
(d̂1 <dmin)
(d̂1 ≥ dmin)
(d̂1 <dmin)

Figure 4



(a) Gradual



(b) Abrupt

Figure 5

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10
d1	**0.2**	0.8	0	0	0	0	0	0	0	0
d2		**0.2**	0.8	0	0	0	0	0	0	0
d3			**0.2**	0.8	0	0	0	0	0	0
d4				**0.2**	0.8	0	0	0	0	0
d5					**0.2**	0.8	0	0	0	0
d6						**0.2**	0.8	0	0	0
d7							**0.2**	0.8	0	0
d8								**0.2**	0.8	0
d9									**0.2**	0.8
d10										**1**

$\alpha = 0.8$

g = 10 (smooth growth)

Figure 6

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10
d1	**0.2**	0.28	0.14	0.09	0.07	0.06	0.05	0.04	0.04	0.03
d2		**0.2**	0.29	0.15	0.1	0.07	0.06	0.05	0.04	0.04
d3			**0.2**	0.31	0.15	0.1	0.08	0.06	0.05	0.04
d4				**0.2**	0.33	0.16	0.11	0.08	0.07	0.05
d5					**0.2**	0.35	0.18	0.12	0.09	0.07
d6						**0.2**	0.38	0.19	0.13	0.1
d7							**0.2**	0.44	0.22	0.15
d8								**0.2**	0.53	0.27
d9									**0.2**	0.8
d10										**1**

α = 0.8

g = 1 (abrupt growth)

Figure 7

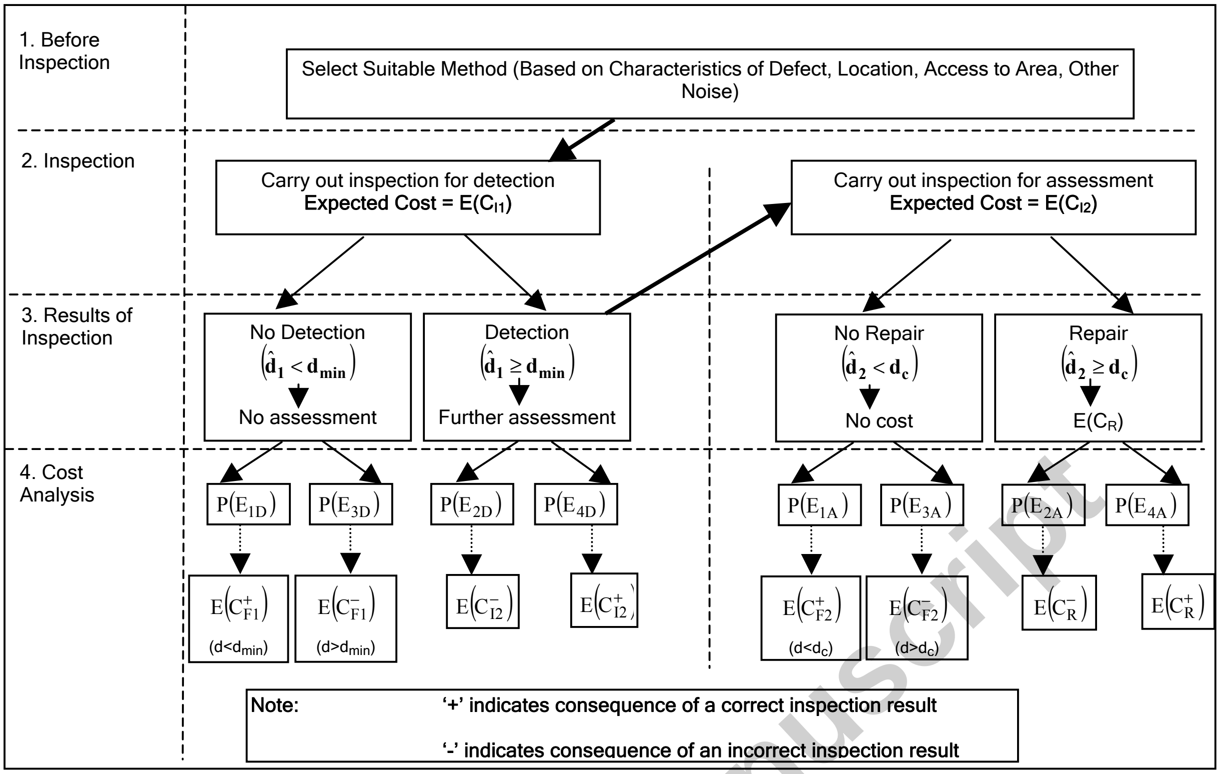

1. Before Inspection
Select Suitable Method (Based on Characteristics of Defect, Location, Access to Area, Other Noise)
2. Inspection
Carry out inspection for detection
Expected Cost = E(CI1)
Carry out inspection for assessment
Expected Cost = E(CI2)
3. Results of Inspection
No Detection
No assessment
Detection
Further assessment
No Repair
No cost
Repair
E(CR)
4. Cost Analysis
(d<dmin)
(d>dmin)
(d<dc)
(d>dc)
Note:
'+' indicates consequence of a correct inspection result
'-' indicates consequence of an incorrect inspection result

Figure 8

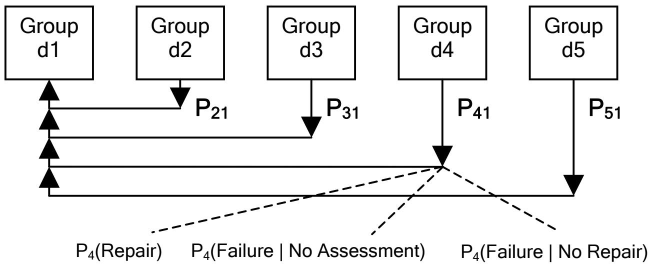

Group d1
Group d2
Group d3
Group d4
Group d5
P21
P31
P41
P51
P4(Repair)
P4(Failure | No Assessment)
P4(Failure | No Repair)

Figure 9

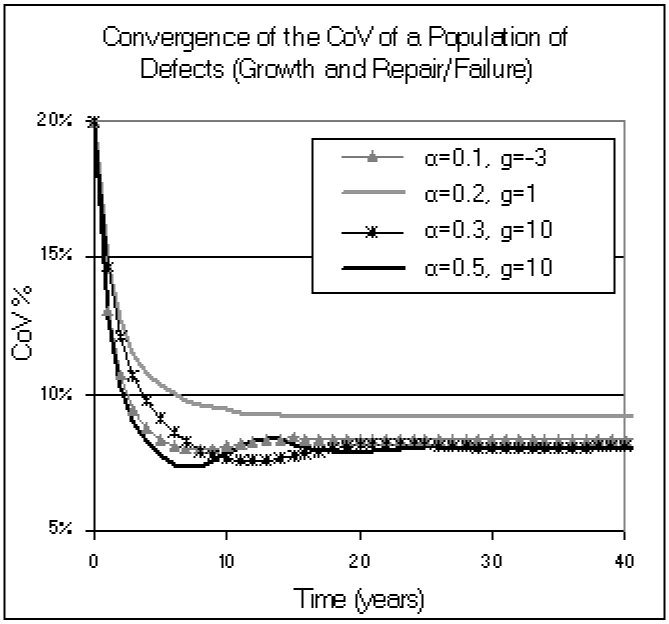

Convergence of the CoV of a Population of Defects (Growth and Repair/Failure)
20%
15%
10%
5%
CoV %
0
10
20
30
40
Time (years)
α=0.1, g=-3
α=0.2, g=1
α=0.3, g=10
α=0.5, g=10

Figure 10

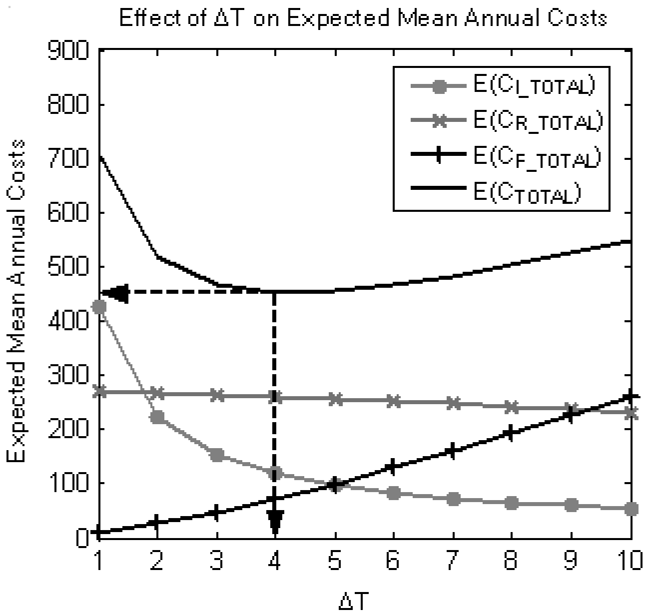

Effect of ΔT on Expected Mean Annual Costs
Expected Mean Annual Costs
ΔT
E(CI_TOTAL)
E(CR_TOTAL)
E(CF_TOTAL)
E(CTOTAL)
900
800
700
600
500
400
300
200
100
0
1
2
3
4
5
6
7
8
9
10

Figure 11

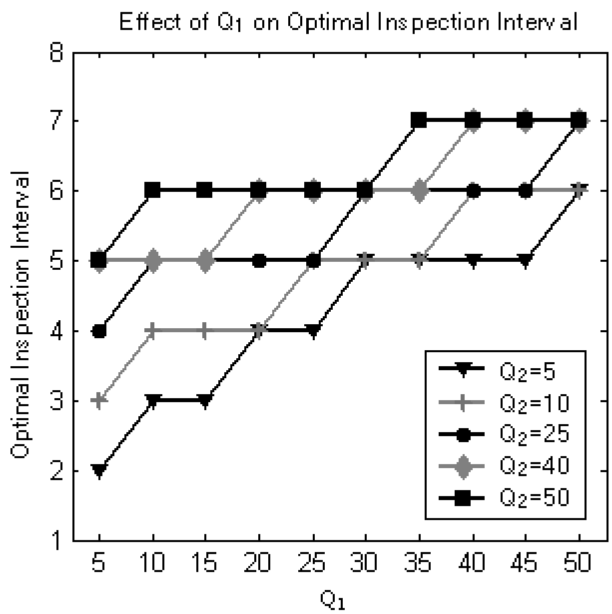

Effect of Q1 on Optimal Inspection Interval
Optimal Inspection Interval
Q1
8
7
6
5
4
3
2
1
5 10 15 20 25 30 35 40 45 50
Q2=5
Q2=10
Q2=25
Q2=40
Q2=50

Figure 12

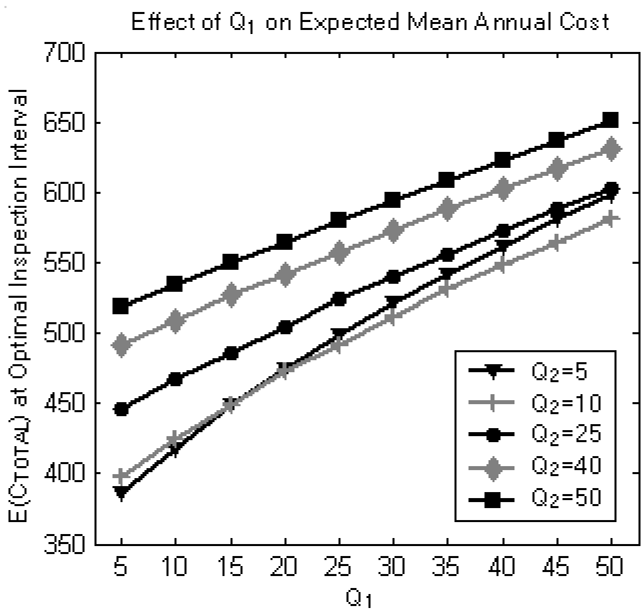

Effect of Q1 on Expected Mean Annual Cost
E(CTOTAL) at Optimal Inspection Interval
700
650
600
550
500
450
400
350
5 10 15 20 25 30 35 40 45 50
Q1
Q2=5
Q2=10
Q2=25
Q2=40
Q2=50

Figure 13

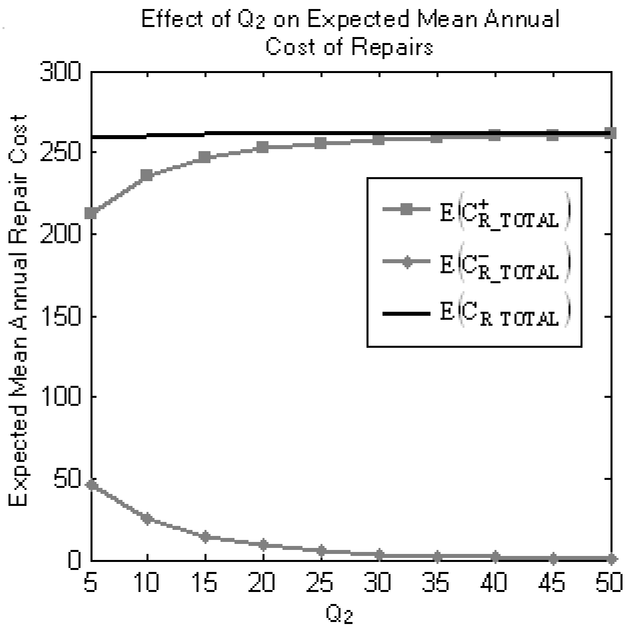

Effect of Q2 on Expected Mean Annual
Cost of Repairs
Expected Mean Annual Repair Cost
300
250
200
150
100
50
0
5
10
15
20
25
30
35
40
45
50
Q2
E(C+R_TOTAL)
E(C−R_TOTAL)
E(CR TOTAL)

Figure 14