An analysis of the shear strength of recycled aggregates

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ABSTRACT

Increasing the use of recycled aggregates in civil engineering applications has the potential to reduce the current natural aggregate consumption requirements of the construction industry. Current levels of natural aggregate production are under threat due to tighter restrictions on planning issues related to quarrying. This paper presents a study on the shear strength of recycled aggregates with relevance to the use of such materials as backfill to structures. The tests were conducted on crushed concrete and demolition debris in a 300 mm x 300 mm x 179 mm shear box, and the results were compared with those of a limestone aggregate. The paper also includes an investigation into the possible application of an analysis of dilatancy (originally developed for use in soil mechanics) to aggregates and in particular to those aggregates of a soft-grain type such as recycled aggregates.

1. INTRODUCTION

The use of recycled aggregate as fill to structures and as unbound layers in road pavements provides a means of consuming secondary materials in relatively less demanding situations resulting in a potential reduction in the quantity of natural aggregate required by the construction industry. Recycled aggregates, such as crushed concrete and demolition debris, have been the subject of much research in recent years, particularly for use as aggregate in new concrete and in road pavements. However, the tests conducted during the research, on which this paper reports, are more closely related to the potential use of recycled aggregates as fill to structures, although the shear strength of an aggregate has also been found to be a useful property in determining the quality of an aggregate for use as road sub-base [1].

This paper reports on the results of shear strength tests conducted on both crushed concrete and demolition debris using crushed limestone as a control. Also included is an investigation into the possibility of applying an analysis of shear strength parameters (developed originally for the analysis of soils) to the case of well graded aggregates. Finally, conclusions are made on the appropriateness of using such materials as alternatives to natural aggregates in situations where shear strength is an important factor.

2. MATERIALS AND METHODS

2.1 Materials

For the purposes of this paper, the term 'crushed concrete' refers to clean crushed concrete produced from the break-up and crushing of concrete slabs from road pavements. Any material containing other constituents, such as brick, glass, asphalt, wood or block, is referred to as 'demolition debris'. The limestone used as the control is a clean limestone, typically used for and passing the standard requirements for road sub-base in rhe U.K. (2]. All materials were well-graded and of similar grading; the grading envelope used is presented in Table 1 (2].

Table 1– Particle grading envelope for recycled aggregates		
Sieve size (mm)	% Passing - Upper limit	% Passing - Lower limit
0.075	10	0
0.6	22	8
5	45	25
10	40	70
37.5	85	100
75	100	100

2.2 Test apparatus

The shear box tests were conducted in a large direct shear box, the internal dimensions of which were 300 mm x 300 mm x 179 mm. The arrangement of the box was similar to that of the standard Casagrande 60 mm shear box used in the case of soil shear strength testing. The shearing force was generated by an electrical motor driving a mechanical screw jack via a 42 speed gear box with speeds ranging from 0.125 X 10-3 mm/min to 6.1 mm/min. The shear force was measured using a 10 tonne load cell and the vertical force was measured using a 20 kN load cell, with the vertical force having been exerted by a fully self-contained hydraulic pressure system. The shear was set at a slow rate of 0.117 mm/min to ensure that the peak shear stress would not be missed.

Table 2 – Test conditions for test series A								
Test series LA			Test series DA			Test series CA		
Test No.	ρ_d (kg/m^3)	I_d	Test No.	ρ_d (kg/m^3)	I_d	Test No.	ρ_d (kg/m^3)	I_d
L1	2103	0.9	D1	1700	0.76	C1	1805	0.9
L2	2063	0.85	D2	1674	0.72	C2	1729	0.78
L3	1991	0.75	D3	1645	0.68	C3	1714	0.76
L4	1768	0.44	D4	1550	0.54	C4	1665	0.68
L5	1705	0.35	D5	1480	0.44	C5	1528	0.47
-	-	-	D6	1418	0.35	C6	1450	0.35

Note: LA = Series A Limestone Samples; DA = Series A Demolition Debris Samples; CA = Series A Crushed Concrete Samples

2.3 Test sample preparation

The optimum moisture content (OMC) and the peak dry density (P_{dp}) were obtained using the compactibility test for graded aggregates, as specified in [3]. Each of the aggregates to be tested was mixed at a moisture content just below OMC since samples of low density would be difficult to obtain at OM C. By varying only one parameter, *i.e.* density, the data obtained from the tests could be analysed more easily. The moisture contents of limestone, demolition debris and crushed concrete, at which the tests were carried out, were 3%, 10% and 7%, respectively. This range of moisture contents can be attributed to the different water absorption properties of the aggregates; recycled aggregates tend to absorb more water due to their high mortar and debris content.

2.4 Aggregate test series

Two test series were conducted for each of the aggregates: the first, Series A, involved varying the density of the aggregate samples, and the second, Series B, involved varying the vertical stress. Preparation of samples at peak dry density was avoided to eliminate the possibility of crushing the aggregate particles; the maximum density targeted for the samples was thus 0.9 P_{dp} (kg/m^3) . Density is also expressed in terms of relative density which is defined as follows:

$$I_d = \frac{e_{max} - e}{e_{max} - e_{min}} \quad (1)$$

where e_{max} is the maximum voids ratio, e_{min} is the minimum voids ratio and e is the voids ratio of the material [4].

The data presented by [4] were from tests conducted on sands, whereas the tests conducted during this research were primarily on well-graded aggregates which contained particles in the range of 0.05 mm to 37.5 mm. A test for measuring the minimum density of gravelly soils is described by [5] where the sample is tipped quickly into a 152 mm mould, similar to that used in the CBR test [6]. The material in the mould is weighed and the density of material in this condition corresponds to the minimum density which can be obtained.

When this test was carried out on the aggregates, it was found that the densities were not as low as would be desirable for direct comparison with the tests on sand conducted by [4] and some assumptions were therefore required. It can be seen from results reported in [4] that the critical state plane strain angle of friction ($ø_{cv}$) is difficult to obtain at values of $I_d < 0.22$. Based on this information, an Id of0.35 was assumed for the loosest samples of aggregate in the shear box tests and an I_d of 0.9 was used for the densest samples.

The vertical stress, at which Series A was conducted, was 50 kN/m^2; the other test conditions including the dry density (p_d) and the relative density (I_d) are listed in Table 2. The tests in Series B were carried out at similar densities, but the vertical stress (σ_v) was varied from 50 kN/m^2 to 200 kN/m^2. The test conditions for this series are listed in Table 3.

Table 3 – Test conditions for test series B

σ_v	Test Series LB			Test Series DB			Test series CB		
(kN/m^2)	Test No.	ρ_d (kg/m^3)	I_d	Test No.	ρ_d (kg/m^3)	I_d	Test No.	ρ_d (kg/m^3)	I_d
50	L2	2063	0.85	D4	1550	0.54	C2	1729	0.78
50	-	-	-	D2	1674	0.72	-	-	-
75	L7	2081	0.87	D7	1591	0.6	C7	1662	0.68
100	L8	2104	0.9	D8	1573	0.58	C8	1731	0.78
150	L9	2106	0.9	D9	1651	0.69	C9	1708	0.75
200	L10	2097	0.89	D10	1640	0.67	C10	1650	0.66

Note: LB = Series B Limestone Samples; DB = Series B Demolition Debris Samples; CB = Series B Crushed Concrete Samples

Table 4 – Particle grading of sand	
Sieve size (mm)	% Passing
2.36	100
1.18	83.1
0.6	0.67
0.3	0.11

Table 5 – Test conditions for sand tests							
Test series SA				Test series SB			
σ_v (kN/m^2)	Test No.	ρ_d (kg/m^3)	I_d	σ_v (kN/m^2)	Test No.	ρ_d (kg/m^3)	I_d
50	S1	1682	0.7	50	S1	1682	0.7
50	S2	1621	0.5	100	S4	1707	0.78
50	S3	1520	0.12	200	S5	1737	0.88
-	-	-	-	200	S6	1679	0.7

2.5 Tests on sand

Shear box tests were also conducted on sand to confirm that the 300 mm shear box yielded results which were comparable with those obtained from other research. The particle grading of the sand used is presented in Table 4. It also proved useful to compare the results for sand with those from the tests on aggregates. The extreme voids ratios, e_{min} and e_{max} for the sand were 0.49 and 0.78, respectively. The conditions for the tests on sand are listed in Table 5. The dense samples of sand were prepared by raining the sand from a height of 400 mm above the top of the shear box. The loose samples were prepared by tipping the sand gently from a small container allowing no free fall and avoiding the sloping of the sand surface during placement.

3. SHEAR TESTS

Once the vertical force had been applied in each ofthe shear box tests, shearing was started. Failure was assumed to have occurred when a peak in the curve relating shear stress to the shear displacement of the box had been observed. The standard approach used to interpret the results from a shear box test is summarized in Fig. 1, where it is assumed for theoretical purposes that the shear stress (τ_{yx}) and the vertical stress (σ_{yy}) are measured on the central plane.

Using Fig. 1, the direct shear angle of friction can be defined as

$$\phi_{ds} = \tan^{-1}\left(\frac{\tau_{yx}}{\sigma_{yy}}\right) \qquad (2)$$

As is generally the case with shear box testing, it was not possible to make stress measurements on the central plane and thus the boundary measurements of shear and vertical stresses were used. For the purposes of this work, the shear stress is denoted by τ and the vertical stress by σ_v.

Palmeira [7] discussed another method for the interpretation of results from shear tests in relation to which [8] and [9] reported that the horizontal plane in the centre of a shear box is a direction of zero extension. This observation and the assumption that the axes of principal stress and principal strain increments coincide are used to generate the method of interpretation illustrated in Fig. 2.

The coincidence of the axes is fundamental to the theory of plasticity and was found to be true by [10] in a simple shear box and by [9] in a direct shear box. The direct shear angle of friction ($ø_{ds}$) is not measured on the plane of maximum stress ratio and therefore underestimates the maximum angle of friction that can be obtained. The plane strain angle of friction ($ø_{ps}$) is the angle measured on the plane of maximum stress ratio (see Fig. 2). The two angles of friction, that is ($ø_{ds}$) and ($ø_{ps}$), can be related by the angle of dilation (Ψ) (defined below).

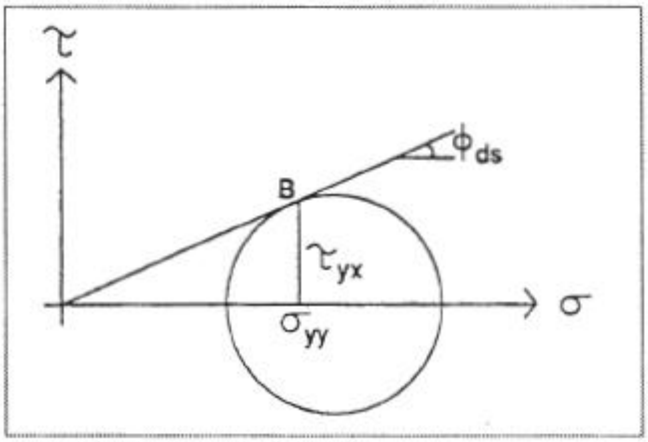

Fig. 1 – Direct shear angle of friction.

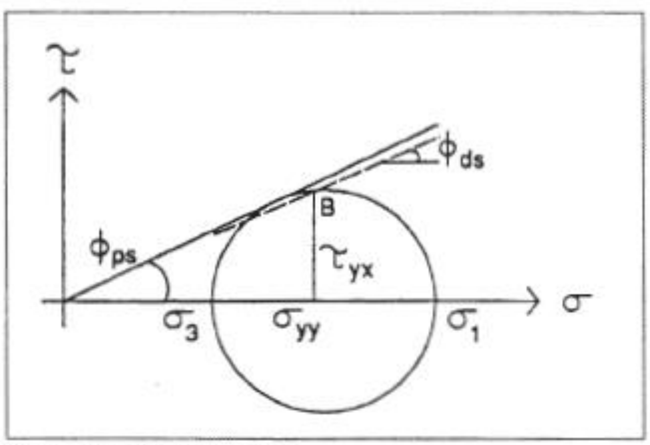

Fig. 2 – Shear test parameters.

During shearing of a dense sample of material, it can be seen in Fig. 3 that the horizontal band of material at the centre of the shear box dilates. To obtain the rate of dilation, the horizontal and vertical displacements are measured during a shear box test. A dense sample will dilate until it reaches a state where, during further shearing, the rate of dilation remains zero. This normally occurs towards the end of a test and is knovn as the critical state. If the sample is very loose, it will compress

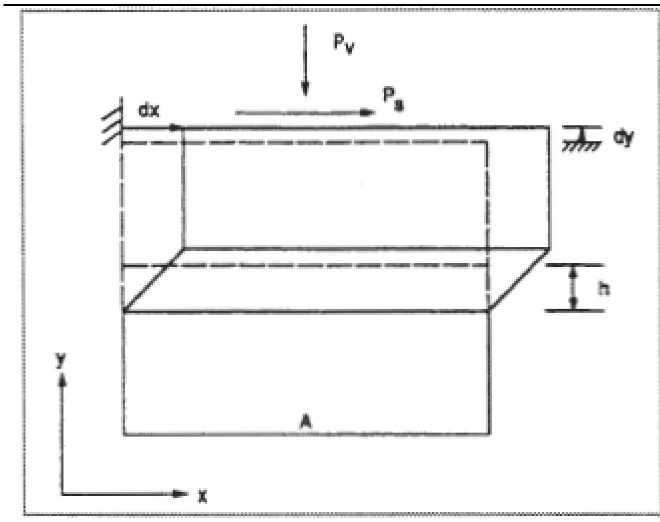

Fig. 3 – Definition of direct shear test [8].

until it also reaches a condition in which the rate of dilation is zero. The angle of friction of the material in this state is termed the critical state angle of friction.

With reference to Fig. 3, the vertical stress is:

$$\sigma_v = \frac{P_v}{A} \tag{3}$$

where P v is the vertical force and A is the area of the shear box.

The shear stress is defined as:

$$\tau = \frac{P_s}{A} \tag{4}$$

where P s is the shear force.

The vertical strain increment is:

$$\frac{dy}{h} = d\varepsilon_{yy} \tag{5}$$

and the shear strain increment is:

$$\frac{dx}{h} = d\gamma_{yx} \tag{6}$$

where h is the height of the deforming

zone of the sample.

The rate of dilation can therefore be defined as:

$$\frac{dy}{dx} = -\frac{d\varepsilon_{yy}}{d\gamma_{yx}} = \tan\psi \qquad (7)$$

4. RESULTS

The results for the peak direct shear angle of friction $(\phi_{ds})_p$ and ρ_{dp} were taken from each test and plotted for each of the four materials in Fig. 4 where it can be seen that the shear strength of limestone, crushed concrete and demolition debris are similar and increase with increasing density. The difference in densities between the materials can be attributed to the different properties of the particles and the fact that limestone particles are denser than recycled aggregate particles. As expected, the shear strength of the sand is lower than that of the other materials.

The peak direct shear angle of friction values, $(\phi_{ds})_p$, for series A are plotted against the peak rate of dilation (dy/dx) in Fig. 5 in order to obtain an estimate of the critical state direct shear angle of friction, $(\phi_{ds})_{cv}$, for each of the aggregates. A good approximation for $(\phi_{ds})_{cv}$ can be obtained where the regression lines cross the abscissa. Using this method, $(\phi_{ds})_{cv}$ for the aggregates ranges between 37.5^{O} and 40° with the limestone achieving the highest value and the crushed concrete the lowest. Also shown in Fig. 5 for purposes of comparison are the data from the tests performed on sand.

The relationships between $(\phi_{ds})_p$ and σ_v for the B series are plotted in Fig. 6. Similar relationships appear to exist for demolition debris and limestone where (~ds)p did not vary very much. The data for sand followed a similar trend. The variation in data in the case of each aggregate is quite small, and so it can be concluded that $(\phi_{ds})_p$ is more influenced by density than by vertical stress.

5. ANALYSIS

A flow rule analysis using Taylor's energy correction [11] for the data obtained from the tests performed on the aggregates and sand is presented hereafter. In addition, the data is interpreted and analysed using Bolton's dilatancy index [4].

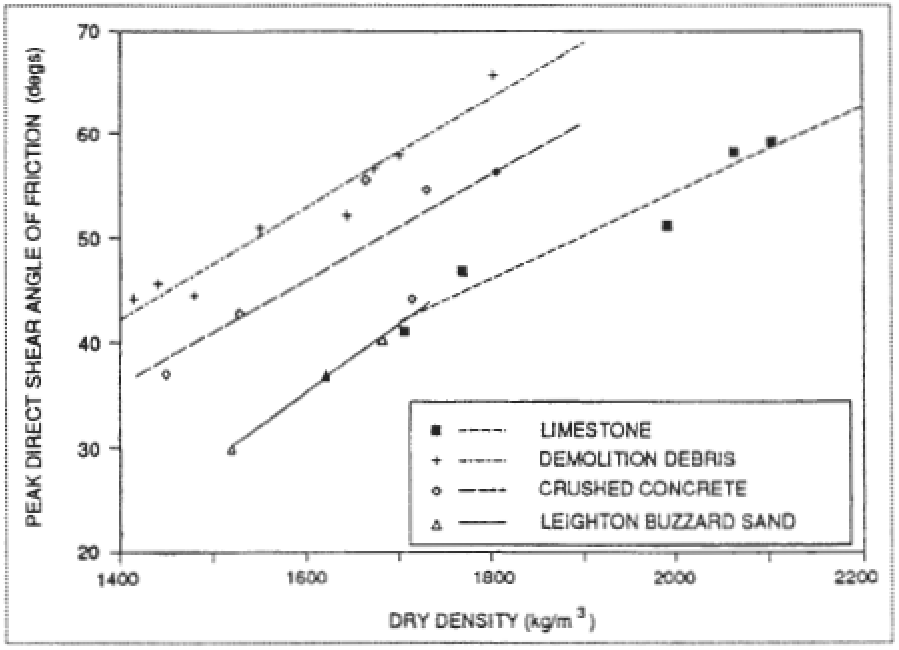

Fig. 4 – Influence of density on the peak direct shear angle of friction.

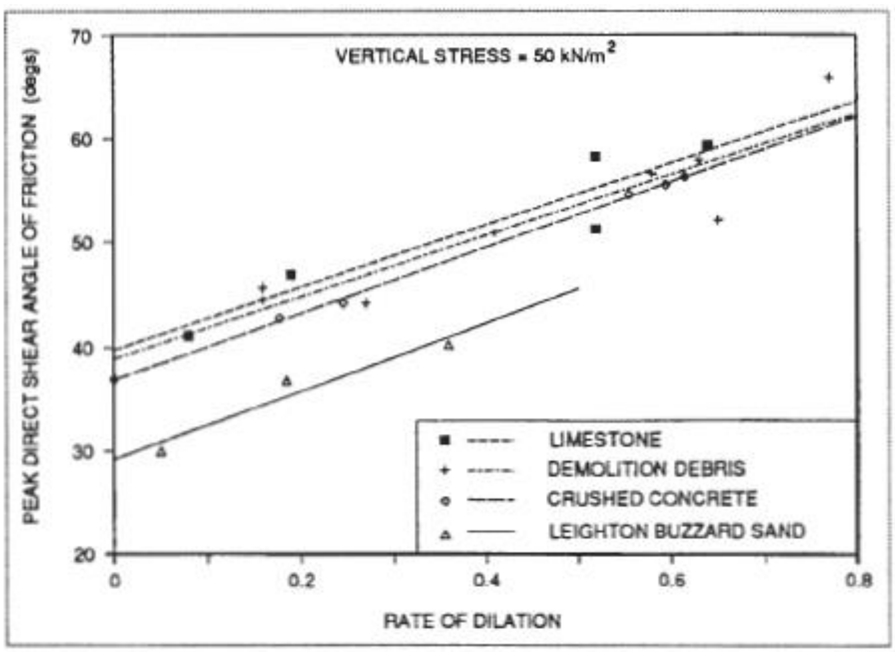

Fig. 5 – Peak rates of dilation for test series A.

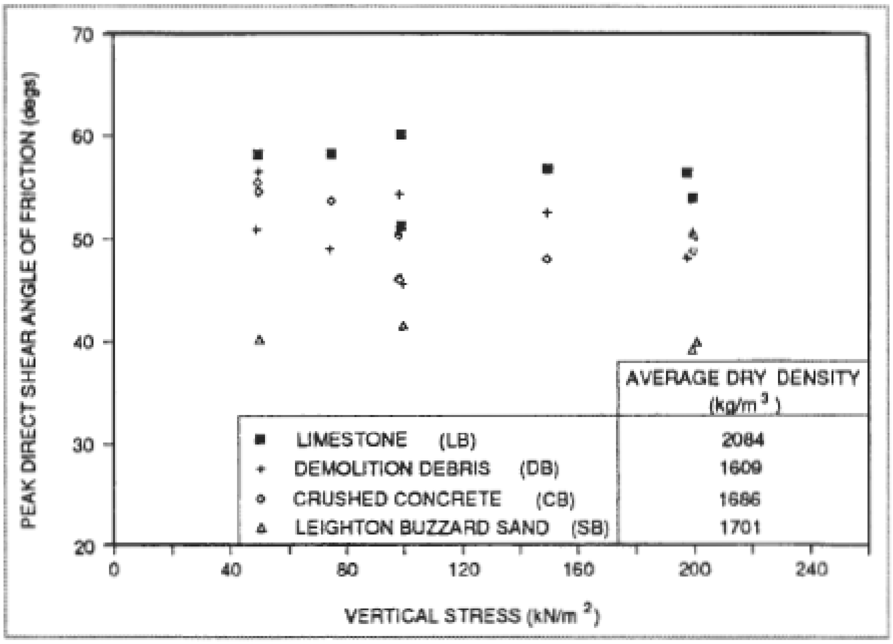

Fig. 6 – Influence of vertical stress on the peak direct shear angle of friction.

5.1 The flow rule analysis

A flow rule relates strain to stress during the plastic flow of a material and therefore the balance of energy in a direct shear test can be explained by a flow rule. A soil element in the central region of a shear box after an increment of shear strain has been applied is examined. By using the energy correction proposed by [11], the increment of energy per unit volume of this soil element can be defined as:

$$\frac{\tau_{yx}}{\sigma_{yy}} + \frac{d\varepsilon_{yy}}{d\gamma_{yx}} = m \qquad (8)$$

where τ_{yx}/σ_{yy} is the stress ratio measured on the central plane and m is a constant which equals the stress ratio when the rate of dilation is zero. Assuming that the horizontal is a direction of zero extension and that the axes of principal stress and principal strain increments coincide, equation (8) reduces to:

$$\frac{\tau_{yx}}{\sigma_{yy}} - \tan\psi = \sin\phi_{cv} \qquad (9)$$

where Ψ is the angle of dilation and ϕ_{cv} is the critical state angle of friction [12].

Since measurements could not be made on the central plane during testing, the boundary measurements of stress ratio (τ/σ_v) and rate of dilation (dy/dx) were used in the analysis. For each test, the values between $(\tau/\sigma_v)_p$ and (τ/σ_v) at the end of the test were plotted against dy/dx. Samples of these plots are presented as Figs. 7 and 8. If a uniform zone of deforming material existed in these tests, then the regression lines through the data should give 1:1 lines intersecting the abscissa at sin ϕ_{cv} [12]. This was not expected for the test data presented here because when a free top platen is used in a shear box test, rotation occurs due to the non-symmetrical arrangement of the shear box. It was also noted by [12] that measurement of the rate of dilation on the boundary of a shear box underestimates the rate of dilation on the central plane; it was also found that even when a symmetrical direct shear box is used, i.e. with a fixed top platen, the measured stress ratio is still higher than in a simple shear box arrangement. These conclusions by [12] would suggest that the regression lines for the data presented here should have higher slopes than 1:1.

In some cases, the regression lines on the plots had slopes of less than 1:1, but in most, higher slopes were evident as is the case in Figs. 7 and 8. Values of ¢cv had to be estimated from the plots to perform the following analysis. Upper and lower values of ϕ_{cv} for each material were obtained from plots similar to those presented in Figs. 7 and 8 for each test; these values are presented in Table 6.

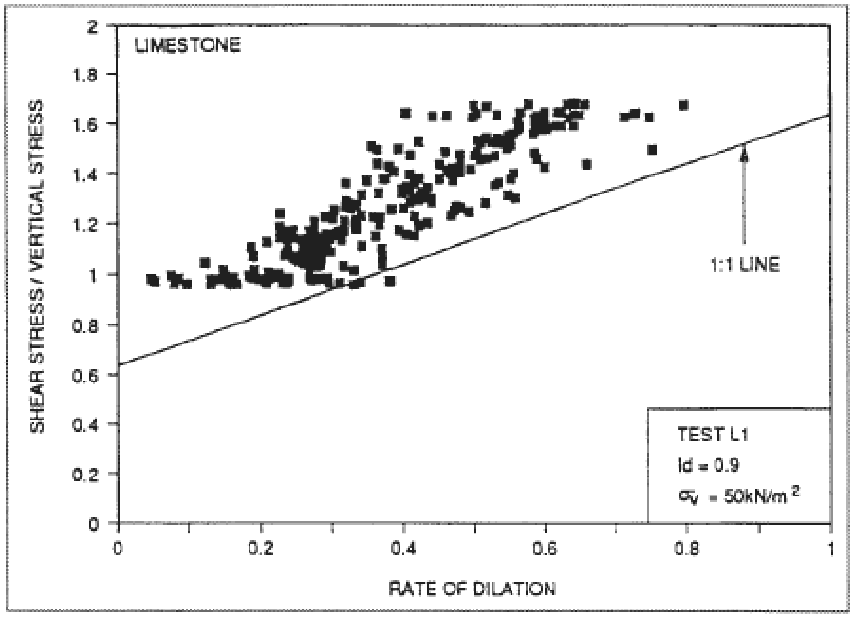

Fig. 7 – Relationship between stress ratio and rate of dilation for test L1 (limestone).

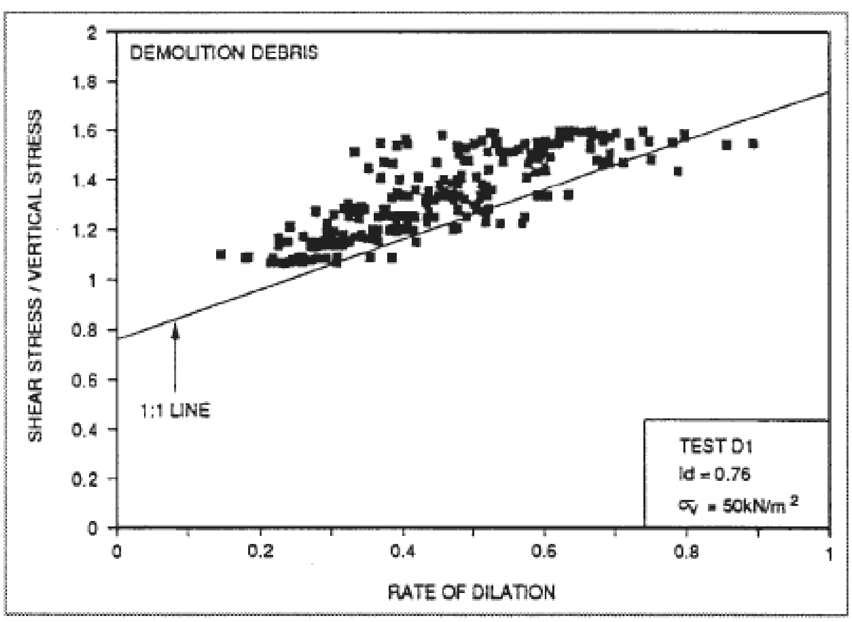

Fig. 8 – Relationship between stress ratio and rate of dilation for test D1 (demolition debris).

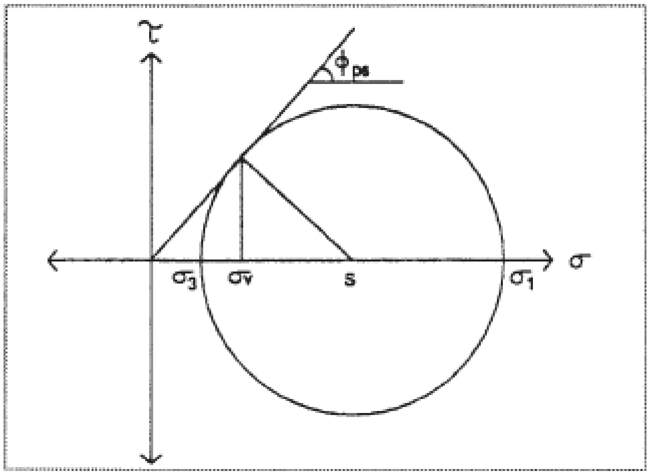

Fig. 9 – Mohr circle of stress.

5.2 Analysis using dilatancy index

Using the values of ϕ_{cv} presented in Table 6 and the $(\phi_{ds})_p$ for each of the tests, the peak plane strain angle of friction $(\phi_{ps})_p$ could be found using the following equation which was derived by [13]:

$$\tan \phi_{ds} = \tan \phi_{ps} \cos \phi_{cv} \qquad (10)$$

Bolton [4] developed a consistent treatment of both density and confining pressure in shear box tests by using a dilatancy index (I_r) which he defined as follows:

$$I_r = I_d(Q - \ln p') - R \qquad (11)$$

where Q = a constant depending on the material type, p' = the mean effective stress and R = a constant = 1. It was stated by [4] that Q is dependent on the compressibility and mineralogy of the particles of material; the value of Q for quartz and felspar sand was found to be 10. It was further suggested by [4] that for other materials, Q could range from 5. 5 for chalk to 8 for limestone. Estimates of Q for recycled aggregates are made below.

To find I_r, the mean effective stress, p', must be calculated. A formula for p' can be derived using the geometry of the Mohr circle of stress shown in Fig. 9. The mean principal stress is defined as:

$$s = \frac{\sigma_1 + \sigma_3}{2} \qquad (12)$$

and the maximum shear stress is:

$$t = \frac{\sigma_1 - \sigma_3}{2} \qquad (13)$$

where σ_1 and σ_3 are the major and minor principal stresses.

Table 6 – Upper and lower limits of ϕ_{cv} used in the dilatancy index analysis

Material type	ϕ_{cv} (degrees)	
	Upper Limit	Lower Limit
Sand	34.5	27
Limestone	45	35
Demolition Debris	47	36
Crushed Concrete	49	37

Table 7 – Results of analysis on shear box test data

Material type	ϕ_{cv} (degrees)	Q	ϕ_{cv} (degrees) Loose Heap Test
Sand	33	9.6	33
Limestone	40	12.5	42.5
Demolition Debris	41	13.2	37
Crushed Concrete	42	11.5	39.5

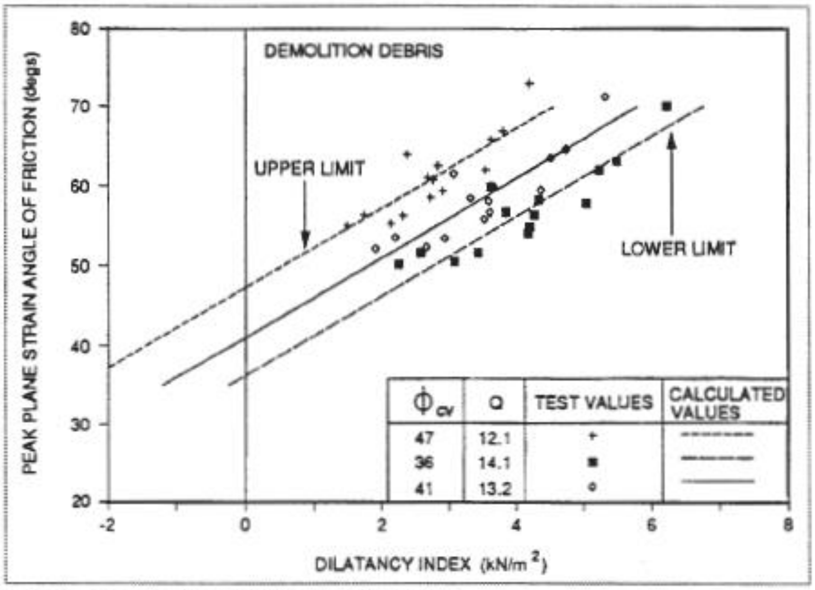

Fig. 10 – Determination of Q for demolition debris.

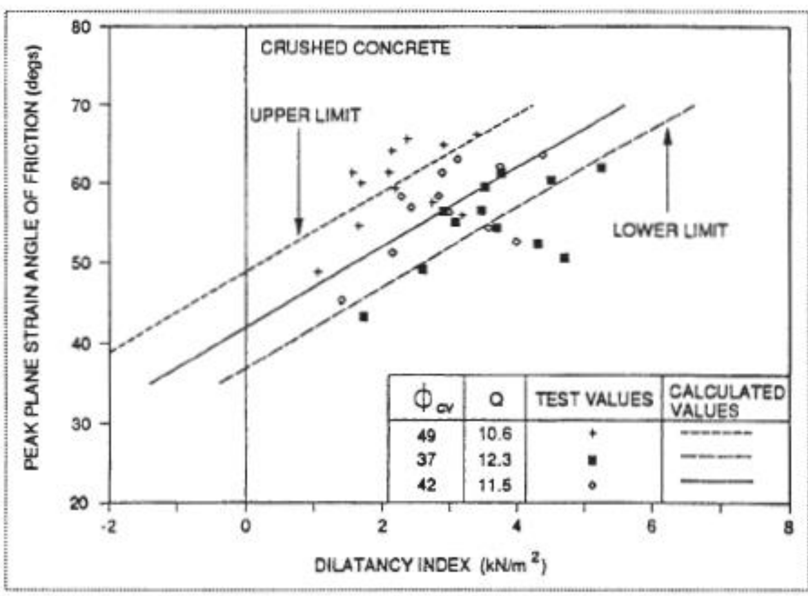

Fig. 11 – Determination of Q for crushed concrete.

It was found in [10] that in a simple shear box arrangement, the intermediate stress σ_2 was 0.74 s. By substituting this value into the equation for mean effective stress:

$$p' = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \qquad (14)$$

it can be concluded that s is a good approximation for p'. From the geometry described in Fig. 9, the mean effective stress can be defined as:

$$p' = \frac{\sigma_v}{Cos^2\ \phi_{ps}} \qquad (15)$$

Each of the aggregates was dealt with separately, but the results from both Series A and B for each aggregate were used in the analysis, regardless of vertical stress or density. Assuming a start value of Q = 10, I_r was calculated for each result using equation (11). The I_r values were then plotted against the corresponding $(\phi_{ps})_p$ values. To examine the accuracy of the chosen value of Q, the following calculations were made. Using the definition of plane strain [4]:

$$(\phi_{ps})_p - \phi_{cv} = 5\, I_r \qquad (16)$$

for a range of $(\phi_{ps})_p$ from 350 to 700, I_r values were calculated using equation (16) and the resulting linear relationship between $(\phi_{ps})_p$ and I_r was drawn on the same plot as the I_r obtained from the experimental data. By varying the value of Q in the calculations, the data points on the graph were made to have the best fit to the calculated line. The results of this analysis for demolition debris and crushed concrete can be seen in Figs. 10 and 11, respectively. An average value of ϕ_{cv} was obtained from these figures for each of the materials. The final values of Q and ϕ_{cv} from the analysis are listed in Table 7 along with ϕ_{cv} obtained by another method [14] which was used as a validation of the analysis.

The other means by which an approximation of ϕ_{cv} can be determined is by tipping the material quickly to form a loose heap and then excavating from the toe of the slope until a smooth slope is formed. The angle of this slope to the horizontal is ϕ_{cv}, and the accuracy to which it can be measured was stated in [14] to be 1°.

6. DISCUSSION

It is apparent from Fig. 4 that demolition debris and crushed concrete could be used successfully as backfill to structures on the basis of shear strength because ($ds)p for both materials was found to be quite high, particularly to the case of dense samples. These recycled aggregates could be considered as lightweight, and this property along with high shear strength could be useful in operations where high quality gravel is used at present. Naish [15] estimated a potential reduction from 9% to 5% in the total cost of the structure if lower quality aggregates were used as backfill to bridge abutments.

It was stated by [4] that Q depends on the mineralogy and compressibility of a material and suggested that Q should be lower for a material containing soft grains. It is not clear in his paper whether this recommendation was made specifically for small-grained, uniform materials. It appears from the results quoted earlier that Q may also be dependent on the coefficient of uniformity, the ratio of the shear box dimensions to the maximum particle size or angularity. It was also found by [4] that for tests on sands, equation (16) could be used between the limits of O < I_r < 4. All data from the tests on aggregates fitted the correlations well, but these data suggest that a limit of

$$(\phi_{ps})_p - \phi_{cv} = 20° \quad (17)$$

is too conservative for well-graded aggregates; so it is therefore suggested that it be raised to:

$$(\phi_{ps})_p - \phi_{cv} = 25° \quad (18)$$

i.e. the upper limit of I_r be raised to 5. It has been agreed, based on the results obtained, that the recommendation made by [4] should be adhered to in the case of sand.

When comparing the $ø_{cv}$ values (obtained by the two different methods) in Table 7, it can be seen that demolition debris reached a value of 37° in the loose heap test, which was 4 ° lower than that obtained in the analysis. However, the loose heap test result for limestone was 2.5° higher than the result from the analysis, and for crushed concrete it was 2.5° lower. The accuracy of the loose heap test on well-graded aggregates therefore was lower than the 1^0 accuracy which was suggested by (13) for tests on sands. However, it would be expected that the slope of a heap of material containing non-uniform and relatively large particles would not be as well-defined as that of a heap of sand where the particles are small and uniform. To offset some of this discrepancy, a mass of 15 kg was used in the case of the aggregates compared with a mass of 2 kg for the sand. Despite the difference in mass used, the results obtained would suggest that an even larger quantity of aggregate would be required to obtain an accuracy of 1°.

7. CONCLUSIONS

1. There appeared to be little difference between the shear strength of limestone, a standard natural aggregate, and that of the recycled aggregates for which friction angles between 54° and 58° were obtained at high densities.

2. Although it was suggested by [4] that materials containing grains which were softer than quartz or felspar should have Q values (constant in the formula for dilatancy index) of less than 10, the Q values determined for the aggregates in the research reported here ranged between 11.5 and 13.2.

3. The critical state angles of friction of the aggregates were found to be between 40° and 42°, but the difference between the calculated and the measured values using the loose heap test varied between 2.5^0 and 4°. It is concluded therefore that the 1^0 accuracy suggested by [14] could not be achieved for well-graded aggregates unless very large quantities of the material were used for the loose heap test.

ACKNOWLEDGEMENTS

The Institute of Demolition Engineers (U.K.), the Geotechnical Group at TRL (U.K.), George Milligan and Richard Jewell are thanked for their support and assistance.

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