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Evaluating the exact value of failure probability based on conditional outcrossing rate

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ABSTRACT: By taking the advantage of Poisson events, outcrossing method has been widely applied for time-variant reliability analysis considering continuous processes. Generally, the outcrossing method can only obtain an upper bound of failure probability, which will overestimate the risk and result in unnecessary maintenance costs. In this study, a general conditional outcrossing (GCO) method for time-variant reliability analysis is proposed, which can obtain a more accurate approximation of failure probability. The conditional outcrossing rate is defined as the outcrossing rate conditioned on fixing the values of time-invariant random variables, which is introduced to satisfy the assumption of independent outcrossing events. A numerical algorithm for GCO method is developed with the aid of Gauss-Legendre quadrature and point estimate method. The application of GCO method is demonstrated by three examples, including implicit LSF with finite element model and non-Gaussian non-stationary random processes. The failure probability obtained by the proposed GCO method is found to be in close agreement with that by Monte Carlo simulation, which demonstrates that the proposed GCO method can obtain a more accurate approximation of failure probability.

Structural reliability analysis aims to calculate the failure probability of a civil infrastructure with respect to its intended function throughout the lifetime. A realistic analysis needs to consider the deterioration of material properties and time-variant characteristics of external loads, making the structural reliability essentially time-variant in practice. By introducing the time dimension, time-variant reliability analysis is much more complex compared with classical time-invariant reliability analysis, and thus lots of methods have been proposed to procure a good compromise between the efficiency and accuracy.

Monte Carlo simulation (MCS) is recognized as the most reliable method for time-variant reliability analysis, as long as enough samples are generated. The biggest deficiency of MCS is that a huge number of samples are required when dealing with high reliability level problems. Subset simulation (SS) (Au and Wang 2014) was then developed as an efficient way to handle small failure probability problems, which replaces the small failure probability with the product of larger conditional failure probabilities and has been utilized in time-variant reliability analysis (Sonal et al. 2018; Du et al. 2019). Although SS can reduce the number of samples required, it is still sample-based and inevitably requires generating a relatively large sample. To improve the computational efficiency, many approximate methods have been proposed for time-variant reliability analysis, which can be roughly classified into four categories: i.e., extreme-value based method, series-system method, outcrossing method. The extreme-value based method defines the failure as the global extreme response of a forecast time interval being greater than a predefined threshold, and the focus is to construct the distribution of extreme response. Since analytical solution for the extreme response is generally intractable, surrogate models have been adopted to predict the extreme responses (Wang and Chen 2014; Hu and Mahadevan 2016; Wang et al. 2019). However, the construction of such surrogate model is complex, which generally involves time-consuming training processes (Hawchar et al. 2017; Wang et al. 2021). Recently, a series system method is proposed named as NEWREL (Gong and Frangopol 2019), which defines the time-variant failure as a series system problem considering time-invariant failure at a set of discretized time instants. With the series system defined, the time-variant problem is conducted by using an improved first order reliability method (FORM) (Zhou et al. 2017). Although this method is relatively efficient for general case, it may be inappropriate for strong nonlinear limit state functions (LSFs) as FORM is adopted, and its accuracy is sensitive to the prescribed time incremental intervals. The outcrossing method has been applied for problems considering discrete processes (Mori and Ellingwood 1993; Li et al. 2015; Lu et al. 2019), problems considering continuous processes with high reliability level (Hu and Du 2015; Li et al. 2016; Wang 2020; Qian et al. 2020), and problems considering continuous processes with relatively low reliability level. The outcrossing method for high-reliability level considering continuous processes is within the scope of this study.

Assuming that the outcrossings at different instants are independent, the outcrossing method considering continuous processes evaluates the failure probability based on the integration of outcrossing rate in time domain. The well-known outcrossing rate formula considering continuous processes was firstly proposed by Rice (1944), and then analytical expressions of outcrossing rate for some specific cases have been proposed (Breitung 1988; Guedes 1992; Hu and Du 2013). Inspired by the parallel system reliability formulation of outcrossing rate proposed by Hagen and Tvedt (1991), PHI2 method was proposed (Andrieu-Renaud et al. 2004) by combining a two-component parallel system model with the classical time-invariant reliability method. To improve the stability of PHI2 method, an analytical derivation of the outcrossing rate is proposed, which is named as PHI2+ method (Sudret 2008) and has been widely applied. To improve the computational efficiency for time-variant reliability analysis combined with finite element (FE) analysis, MPHI2 method (Zhang et al. 2021) was recently proposed, which can separate the time-consuming section in LSF from the time-reliability analysis cycle with the aid of method of moments. In all the existing methods to calculate outcrossing rate, the random variables at different instants are treated as independent values. However, being time-invariant, the values of random variables will remain the same for different instants, and existing methods may result in inappropriate evaluation of the outcrossing rate. Furthermore, although many improvements have been made to calculate outcrossing rate, the expression of failure probability remains the same for the last several decades, which can only evaluate the upper and lower bounds rather than the exact value of failure probability.

Therefore, to obtain a more appropriate evaluation, a general conditional outcrossing (GCO) method is proposed based on the conditional outcrossing rate, which is defined as the outcrossing rate conditioned on fixing the values of time-invariant random variables. The rest of this study is organized as follows: First, the upper and lower bounds of failure probability based on conditional outcrossing rate are investigated, and then the failure probability is formulated based on conditional outcrossing rate.

Second, a numerical algorithm to carry out GCO method is developed with the aid of Gauss-Legend quadrature and point estimate method (PEM). Third, the efficiency and accuracy of the proposed GCO method are demonstrated by three numerical examples, including implicit LSF with FE model and non-Gaussian non-stationary random processes. Finally, findings of the present study are summarized.

1. GCO METHOD FOR TIME-VARIANT RELIABILITY ANALYSIS

1.1. Bounds of failure probability based on conditional outcrossing rate

Denote time-variant LSF as $G(\mathbf{X}, \mathbf{Y}(t), t)$, where $\mathbf{X}$ and $\mathbf{Y}(t)$ are the vectors of random variables and random processes, respectively; and t represents the time instant. Then, the failure within a forecast time interval [0, T] is defined as $G(\mathbf{X}, \mathbf{Y}(t), t)$ being negative for at least one time instant. The corresponding failure event E_{f} can be mathematically expressed as

$$E_{\mathrm{f}} = \{\exists t \in [0,T] : G(\mathbf{X},\mathbf{Y}(t),t) \le 0\} \tag{1}$$

The outcrossing event is defined as $G(\mathbf{X}, \mathbf{Y}(t), t)$ crossing the zero-level from safe domain (i.e., $G(\mathbf{X}, \mathbf{Y}(t), t)>0$) to failure domain (i.e., $G(\mathbf{X}, \mathbf{Y}(t), t) \le 0$). From the perspective of outcrossing event, E_{f} can be equivalently interpreted as either $G(\mathbf{X}, \mathbf{Y}(0), 0) \le 0$ or at least one outcrossing happens within (0, T] conditioned on $G(\mathbf{X}, \mathbf{Y}(0), 0)>0$. Then, E_{f} is expressed as

$$E_{\mathrm{f}} = \{G(\mathbf{X},\mathbf{Y}(0),0) \le 0\} \cup \{N^+(0,T) > 0 | G(\mathbf{X},\mathbf{Y}(0),0) > 0\} \tag{2}$$

where $N^+(0,T)$ represents the number of outcrossings within (0, T]. Based on Eq. (2), the (cumulative) failure probability for [0, T], i.e., $P_{f,c}(0,T)$ is defined as

$$\begin{aligned} P_{f,c}(0,T) &= \mathrm{Prob}[\{G(\mathbf{X},\mathbf{Y}(0),0) \le 0\} \\ &\cup \{N^+(0,T) > 0 | G(\mathbf{X},\mathbf{Y}(0),0) > 0\}] \end{aligned} \tag{3}$$

Analytical calculation of $P_{f,c}(0,T)$ defined in Eq. (3) is relatively difficult. As an alternative, the upper and lower bounds of $P_{f,c}(0,T)$ will be investigated in the following.

1.1.1. Upper bound

Let $v^+(t|\mathbf{x})$ denote the conditional outcrossing rate at t conditioned on $\mathbf{X}$=$\mathbf{x}$, the upper bound of $P_{f,c}(0,T)$, i.e., $P_{\mathrm{upper}}(0,T)$, can be formulated as follows (Shinozuka 1964; Hagen and Tvedt 1991)

$$P_{\mathrm{upper}}(0,T) = P_{f,i}(0) + \int_{\Omega_{\mathrm{x}}} \left[\int_0^T v^+(t \mid \mathbf{x}) dt \right] f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x} \tag{4}$$

where $P_{f,i}(0)$ is the instantaneous failure probability at initial instant; Ω_{x} is the domain of $\mathbf{X}$; and $f_{\mathbf{x}}(\mathbf{x})$ denotes the joint probability density function (PDF) of $\mathbf{X}$. As $\mathbf{X}$ is time-invariant, it should remain the same for calculating outcrossing rate at different instants, and thus the conditional outcrossing rate $v^+(t|\mathbf{x})$ is introduced. It should be noted that Eq. (4) is assumed that $\mathbf{X}$ and $\mathbf{Y}$ are independent from each other. When $\mathbf{X}$ and $\mathbf{Y}(t)$ are correlated, normalization techniques, such as Nataf transformation (Liu and Der Kiureghian 1986) and moment-based normal transformation (Zhao and Lu 2021), can be firstly utilized for transforming them into independent vectors.

1.1.2. Lower bound

From the perspective of survival event, the failure probability $P_{f,c}(0,T)$ defined in Eq. (3) can also be expressed as

$$\begin{aligned} P_{f,c}(0,T) &= 1 - \mathrm{Prob}[\{\forall t \in [0,T] : G(\mathbf{X},\mathbf{Y}(t),t) > 0\}] \\ &= 1 - P_{s,c}(0,T) \end{aligned} \tag{5}$$

where $P_{s,c}(0,T)$ is the survival probability within [0, T], which can be expressed as (Ellingwood and Mori 1993):

$$P_{s,c}(0,T) = P_{s,i}(0) L_{s,c}(0,T) \tag{6}$$

where $P_{s,i}(0)$ is the instantaneous survival probability at initial instant; and $L_{s,c}(0,T)$ is the survival probability for (0, T] conditioned on $G(\mathbf{X},\mathbf{Y}(0),0)>0$. Let $h(t|\mathbf{x})$ denote the conditional hazard function at t, which is defined as the probability of failure within (t, dt+t] given that the structure survives through (0, t] and $\mathbf{X}=\mathbf{x}$. Then, based on the definition, $L_{s,c}(0,T)$ and $h(t|\mathbf{x})$ can be formulated as

$$L_{s,c}(0,T)=\int_{\Omega_{\mathbf{x}}}\exp\left[-\int_0^T h(t|\mathbf{x})dt\right]f_{\mathbf{x}}(\mathbf{x})d\mathbf{x} \quad (7)$$

$$h(t|\mathbf{x})=\lim_{dt\to 0}\frac{\text{Prob}[t<T_f\le t+dt]}{\text{Prob}[T_f>t]dt} \quad (8)$$

where T_f represents the instant at which $G(\mathbf{x}, \mathbf{Y}(T_f), T_f)$ becomes non-positive. Based on the definition of outcrossing event, $h(t|\mathbf{x})$ can be transformed to

$$h(t|\mathbf{x})=\lim_{dt\to 0}\frac{\text{Prob}[\{N_c^+(0,t]=0\}\cap\{N_c^+(0,t+dt]>0\}]}{\text{Prob}[N_c^+(0,t]=0]dt} \quad (9)$$

where $N_c^+(0,t]$ and $N_c^+(0,t+dt]$ are respectively the numbers of outcrossings within the time intervals (0, t] and (0, $t+dt$] conditioned on $\mathbf{X}=\mathbf{x}$. For $t\neq 0$, $\text{Prob}[N_c^+(0,t+dt]>0]$ > $\text{Prob}[N_c^+(t,t+dt]>0]$. Then, the lower bound of $h(t|\mathbf{x})$ is obtained as

$$h(t\,|\,\mathbf{x})>\lim_{dt\to 0}\frac{\text{Prob}[\{N_c^+(0,t]=0\}\cap\{N_c^+(t,t+dt]>0\}]}{\text{Prob}[N_c^+(0,t]=0]dt} \quad (10)$$

For practical engineering structures that are designed to reach a relatively high reliability level, the outcrossing event rarely occurs and thus can be assumed as independent with each other. Then, based on Eq. (11), the lower bound of $h(t|\mathbf{x})$ can be simplified as

$$\begin{aligned} h(t|\mathbf{x})&>\lim_{dt\to 0}\frac{\text{Prob}[N_c^+(0,t]=0]\times\text{Prob}[N_c^+(t,t+dt]>0]}{\text{Prob}[N_c^+(0,t]=0]dt}\\ &=\lim_{dt\to 0}\frac{\text{Prob}[N_c^+(t,t+dt]>0]}{dt} \quad (11)\\ &=v^+(t\,|\,\mathbf{x})\end{aligned}$$

Combining Eqs. (7), (8) and (12), the lower bound for $P_{f,c}(0,T)$, i.e., $P_{\text{lower}}(0,T)$, can be formulated as

$$P_{\text{lower}}(0,T)=1-P_{s,i}(0)\int_{\Omega_{\mathbf{x}}}\exp\left[-\int_0^T v^+(t\,|\,\mathbf{x})dt\right]f_{\mathbf{x}}(\mathbf{x})d\mathbf{x} \quad (12)$$

1.2. Proposed failure probability based on conditional outcrossing rate

In practical engineering, the value of $P_{f,c}(0,T)$ is of interest. As $P_{f,c}(0,T)$ is always within the range of $[P_{\text{lower}}(0,T), P_{\text{upper}}(0,T)]$, it can be expressed as $\alpha_1 P_{\text{lower}}(0,T)+\alpha_2 P_{\text{upper}}(0,T)$ with $\alpha_1+\alpha_2=1$. To obtain a determined value of $P_{f,c}(0,T)$ for practical application, α_1 and α_2 are set to be 0.5. Then, according to Eqs. (4) and (13), $P_{f,c}(0,T)$ is formulated as

$$P_{f,c}(0,T)=1-\frac{1}{2}P_{s,i}(0)+\frac{1}{2}E[P_c(T|\mathbf{x})] \quad (15)$$

$$E[P_c(T|\mathbf{x})]=\int_{\Omega_{\mathbf{x}}}P_c(T|\mathbf{x})f_{\mathbf{x}}(\mathbf{x})d\mathbf{x} \quad (16a)$$

$$P_c(T|\mathbf{x})=I_T(T|\mathbf{x})-P_{s,i}(0)\exp\left[-I_T(T|\mathbf{x})\right] \quad (16b)$$

$$I_T(T|\mathbf{x})=\int_0^T v^+(t\,|\,\mathbf{x})dt \quad (16c)$$

where $E[\cdot]$ denotes the expectation; $P_c(T|\mathbf{x})$ is referred to as the conditional failure probability within [0, T] conditioned on $\mathbf{X}=\mathbf{x}$; $I_T(T|\mathbf{x})$ denotes the integration of conditional outcrossing rate over time domain [0, T] given $\mathbf{X}=\mathbf{x}$. Since the failure probability as shown in Eqs.(15) and (16) is defined using conditional outcrossing rate, the time-variant reliability assessment based on Eqs.(14) and (15) is referred to as GCO method, which is short for general conditional outcrossing method.

It can be observed from Eqs. (15) and (16) that the calculation of $P_{f,c}(0,T)$ requires to solve a two-level integration of conditional outcrossing rate over time domain and random-variant space, i.e., the calculation of $I_T(T|\mathbf{x})$ and $E[P_c(T|\mathbf{x})]$, which is difficult to calculate analytically. As an alternative, a numerical integration method is proposed and will be discussed in the next section.

2. NUMERICAL ALGORITHM TO CARRY OUT GCO METHOD

2.1. Integration of conditional outcrossing rate over time domain

Letting $t=T\cdot\tau/2+T/2$, the integration interval of t for $v^+(t|\mathbf{x})$ defined in Eq. (16b) can be transformed into (−1, 1] of τ. Then, using the Gauss-Legendre quadrature (Abramowitz and

Stegun 1972), the integration of $v^+(t|\mathbf{x})$ over time domain, i.e., $I_T(T|\mathbf{x})$, can be approximated as

$$I_T[T \mid \mathbf{x}] \cong \tilde{I}_T[T \mid \mathbf{x}] = \frac{T}{2}\sum_{l=1}^{m_T} v^+(\boldsymbol{\tau}_\mathbf{T}(l)|\mathbf{x})\,\boldsymbol{\omega}_\mathbf{T}(l) \quad (17)$$

where m_T is the number of estimate points for Gauss-Legendre quadrature; $\boldsymbol{\tau}_\mathbf{T}(l)$ and $\boldsymbol{\omega}_\mathbf{T}(l)$are the lth abscissa and corresponding weight;$\boldsymbol{\tau}_\mathbf{T}=T\cdot\boldsymbol{\tau}_\mathbf{0}/2+T/2$ is the vector of selected instants; $\boldsymbol{\tau}_\mathbf{0}$ is the m_T-dimensional vector of original abscissa of Gauss-Legendre quadrature. Generally, the accuracy of Gauss-Legendre quadrature for time-variant reliability can be guaranteed with m_T set to be 4 (Zhang et al. 2022), and the corresponding values of $\boldsymbol{\tau}_\mathbf{T}$ and $\boldsymbol{\omega}_\mathbf{T}$ are respectively {±0.3400, ±0.8611} and {0.6521, 0.6521, 0.3479, 0.3479}.

The conditional outcrossing rate $v^+(t|\mathbf{x})$ can be readily calculated with the aid of existing models (Sudret 2008; Zhang et al. 2021). For general cases, PHI2+ method can be applied and $v^+(t|\mathbf{x})$ is formulated as

$$v^+(t|\mathbf{x})=\frac{\left\|\boldsymbol{\alpha}(t+\Delta t|\mathbf{x})-\boldsymbol{\alpha}(t|\mathbf{x})\right\|}{\Delta t}\phi[\beta(t|\mathbf{x})]\psi\left[\frac{\beta(t+\Delta t|\mathbf{x})-\beta(t|\mathbf{x})}{\left\|\boldsymbol{\alpha}(t+\Delta t|\mathbf{x})-\boldsymbol{\alpha}(t|\mathbf{x})\right\|}\right] \quad (18)$$

where $\|\cdot\|$ is the norm of a vector; $\boldsymbol{\alpha}(t|\mathbf{x})$ is the unit normal vectors in Gaussian space at t given $\mathbf{X}=\mathbf{x}$; $\phi(\cdot)$ is the probability density function (PDF) of standard normal distribution; $\beta(t|\mathbf{x})$ is the time-invariant reliability index corresponding to $G(\mathbf{x}, \mathbf{Y}(t), t)$; and $\psi(y)=\phi(y)-y\Phi(y)$ with $\Phi(\cdot)$ defined as the cumulative distribution function (CDF) of standard normal distribution. Generally, $v^+(t|\mathbf{x})$ can be easily calculated by PHI2+ method as given in Eq. (18). When the derivative of LSF is difficult to obtain (e.g., finite element model is included), MPHI2 method (Zhang et al. 2021) can be applied alternatively.

2.2. Integration of conditional failure probability in random-variate space

Substituting Eqs. (16) and (17) into Eq. (15), the failure probability $P_{f,c}(0, T)$ can be approximated as

$$P_{f,c}(0,T) \cong 1-\frac{1}{2}P_{s,i}(0)+\frac{1}{2}E[\tilde{P}_c(T|\mathbf{x})] \quad (19)$$

$$\tilde{P}_c(T|\mathbf{x})=\tilde{I}_T(T|\mathbf{x})-P_{s,i}(0)\exp\left[-\tilde{I}_T(T|\mathbf{x})\right] \quad (20)$$

Eq. (19) shows that one of the key issues of evaluating $P_{f,c}(0,T)$ is to determine the $E[\tilde{P}_c(T|\mathbf{x})]$, i.e., the mean value of $\tilde{P}_c(T|\mathbf{x})$. Point estimate method (PEM) (Zhao and Ono 2000) is applied as an alternative, where $E[\tilde{P}_c(T|\mathbf{x})]$ is approximated by the weighted summation of evaluated $\tilde{P}_c(T|\mathbf{x})$ at all distinct permutations of the input variables. When the number of estimate points for PEM is m_r and the dimension of $\mathbf{X}$ is n_x, $m_r^{n_x}$ times of the evaluation of $\tilde{P}_c(T|\mathbf{x})$ will be necessary in PEM, which will be time-consuming for large m_r and n_x. To reduce the computation burden, bivariate dimension-reduction method (BDRM) (Xu and Rahman 2010) is applied in combination with PEM, and then $E[\tilde{P}_c(T|\mathbf{x})]$ is calculated as

$$E[\tilde{P}_c(T|\mathbf{x})] \cong L_{ij}-(n_x-2)L_i+\frac{(n_x-1)(n_x-2)}{2}L_0 \quad (21)$$

$$L_0=\tilde{P}_c(T \mid \boldsymbol{\mu}) \quad (22)$$

$$L_i=\sum_{i=1}^{n_x}\sum_{l=1}^{m_r}\boldsymbol{\omega}_\mathbf{r}(l)\tilde{P}_c\{T \mid \mu_i[\mathbf{u}_\mathbf{r}(l)]\} \quad (23)$$

$$L_{ij}=\sum_{j=i}^{n_x-1}\sum_{j>i}^{n_x}\sum_{l=1}^{m_r}\sum_{q=1}^{m_r}\boldsymbol{\omega}_\mathbf{r}(l)\boldsymbol{\omega}_\mathbf{r}(q)\tilde{P}_c\{T \mid \mu_{ij}[\mathbf{u}_\mathbf{r}(l),\mathbf{u}_\mathbf{r}(q)]\} \quad (24)$$

where $\boldsymbol{\mu}$ is the vector of mean values of $\mathbf{X}$; $\mu_i[\mathbf{u}_\mathbf{r}(l)]$ is defined by replacing the ith element of $\boldsymbol{\mu}$ with $IN[\mathbf{u}_\mathbf{r}(l)]$; $\mu_{ij}[\mathbf{u}_\mathbf{r}(l),\mathbf{u}_\mathbf{r}(q)]$ is defined by replacing the ith and jth elements of $\boldsymbol{\mu}$ with $\{IN[\mathbf{u}_\mathbf{r}(l)], IN[\mathbf{u}_\mathbf{r}(q)]\}$; $\mathbf{u}_\mathbf{r}$ and $\boldsymbol{\omega}_\mathbf{r}$ are the vectors of abscissa and corresponding weight for PEM; and $IN(\cdot)$ is the inverse normal transformation function. When the marginal distributions of random variables are known, $IN(\cdot)$ can be realized by Nataf transformation (Liu and Der Kiureghian 1986), otherwise moment-based normal transformation (Zhao and Lu 2021) can be utilized.

3. EXAMPLES

The first example considers the time-variant reliability of a steel cantilever tube beam as shown in Fig. 1 (Hawchar et al. 2017). The beam is subjected to time-invariant random loads F_1, F_2 and P, and time-variant torsion $T_o(t)$. The beam may fail when the Von Mises stress exceeds the yield strength, and the corresponding LSF $G(\mathbf{X}, \mathbf{Y}(t), t)$ considering the deterioration in the yield strength is formulated as

$$G(\mathbf{X},\mathbf{Y}(t),t)=R_0(1-0.02t)-\sqrt{\sigma_b^2+3\tau_b^2(t)} \quad (26)$$

$$\sigma_b=\frac{F_1\sin(\theta_1)+F_2\sin(\theta_2)+P}{A}+\frac{F_1\cos(\theta_1)L_1+F_2\cos(\theta_2)L_2}{W} \quad (27)$$

$$\tau_b(t)=\frac{T_o(t)}{2W} \quad (28)$$

$$A=\frac{\pi}{4}\left[D^2-(D-2h)^2\right], W=\frac{\pi}{32D}\left[D^4-(D-2h)^4\right] \quad (29)$$

where R_0 is the initial yielding strength; L_1=60mm and L_2=120mm are the distance from the fixed end to F_1 and F_2; θ_1=10° and θ_2=5° are the vertical angles of F_1 and F_2, respectively; D is the outer diameter of the beam; and h is the thickness of the beam. The statistical information of random inputs is listed in Table 1. The random inputs are assumed to be independent.

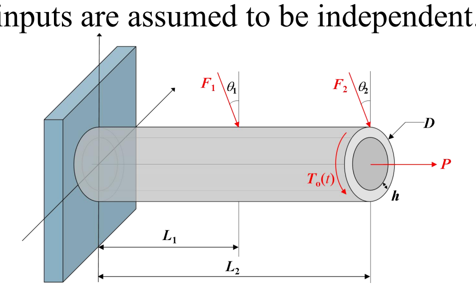

Fig. 1. Cantilever tube beam for Example 1

Table 1. Statistical information of random variables and processes for Example 1

Parameters	Distributions/processes	Mean	COV	ACF
Outer diameter D (mm)	Normal	42	0.0119	None
Thickness h (mm)	Normal	5	0.02	None
Initial yielding strength R_0 (MPa)	Normal	560	0.15	None
Load F_1 (kN)	Normal	1.0	0.15	None
LoadF_2 (kN)	Normal	1.0	0.15	None
Load P (kN)	Gumbel	0.8	0.1	None
Torsion $T_o(t)$ (kN·m)	Gaussian process	1.4	0.1	$\exp[-(\Delta t)^2]$

The forecast time interval under consideration is 5years. For comparison, the time-variant reliability is also analyzed using PHI2+ method and MCS. The time interval is uniformly divided into 100 and 600 intervals for PHI2+ method and MCS, while the number of simulations at each instant for the MCS is 10^6. The failure probabilities using different methods are compared in Fig. 2. To be more clearly, the upper and lower bounds based on conditional outcrossing rate are also plotted in Fig. 2. It can be found from Fig. 2 that:

- The failure probability $P_{f,c}(0,T)$ obtained by PHI2+ method is significantly larger than the upper bound based on conditional oucrossing rate. This is because PHI2+ method evaluates the upper bound using general outcrossing rate, which overestimates the uncertainty by assuming the time-invariant random variables at different instants are independent.
- $P_{f,c}(0,T)$ obtained by MCS falls into the range defined by the upper and lower bounds based on conditional outcrossing rate, which demonstrates that the bounds based on conditional outcrossing rate can help to estimate the possible value of failure probability.
- For all the time intervals considered, the failure probability obtained by the proposed GCO method is in close agreement with that by MCS, which indicates that the proposed method is reliable for this example.

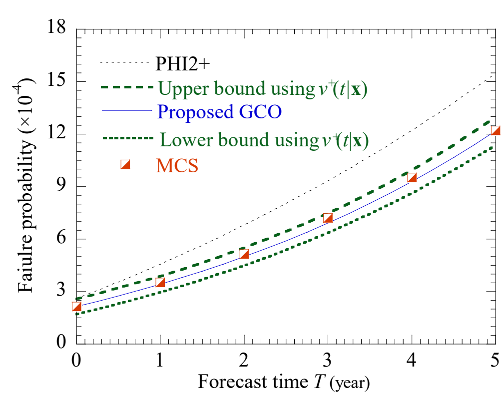

Fig. 2. Failure probability using different methods for Example 1

4. CONCLUSIONS

GCO method is proposed for time-variant reliability analysis in this study. The failure probability is formulated as a two-level integration of conditional outcrossing rate over time domain and random-variate space. An efficient numerical algorithm for GCO method is developed, where the Gauss-Legendre quadrature is employed to approximate the integration over time domain and the integration over random-variate space is computed using PEM combined with BDRM.

A numerical example is presented to illustrate the application of GCO method in time-variant reliability analysis. It is found that the upper and lower bounds based on conditional outcrossing rate can make a narrow bound for the possible values of failure probability, and the proposed GCO method can obtain failure probability in close agreement with that obtained using MCS. This indicates that the proposed GCO method makes it possible to obtain a more accurate approximation of failure probability, and can help to make cost-effective maintenance scheme for owners and asset managers.

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