

A hybrid approach for deriving probability distributions of rock mass discontinuity parameters for slope reliability analysis

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ABSTRACT: It is widely recognized that the rock mass discontinuity properties (e.g., dip angle and dip direction, trace length and aperture, cohesion and friction) can greatly affect the stability of rock slope. The discontinuity properties are often considered to be cross-correlated random variables and are modeled by standard probability distribution families. Reliability analyses of rock slopes accounting for uncertainties in the discontinuity properties and construction of discrete fracture network do not make best use of available field survey data of rock masses. In this paper, a hybrid approach, proposed by Ching and Phoon (2019), is employed for constructing the multivariate probability distribution of rock mass discontinuity properties based on field survey data. Then the discrete fracture network models and probabilities of failure of the jointed rock slopes are obtained accordingly. Copula theory is adopted to model the prior and site-specific joint probability distributions of the discontinuity parameters. A rock slope in China is taken as an example to demonstrate the effectiveness of the proposed method. The research outcomes can provide a new perspective to understand the distribution of rock mass discontinuities and slope failure mechanism.

KEYWORDS: rock slope; reliability analysis; rock mass discontinuity parameters; discrete fracture network; Copula; Hybrid analysis.

1. INTRODUCTION

Stability evaluation of jointed rock slopes has received increasing attentions in the engineering construction industry, for instance in hydropower plants, open-pit mining and highway construction. Rock masses are natural geological bodies that consist of rock blocks and various discontinuities. These discontinuities play an essential role in the stability of jointed rock slopes. To facilitate the slope stability evaluation, conventional geological mapping, drilling and physical prospecting methods are usually adopted to acquire geometric information (e.g., shape and location) and mechanical information (e.g., cohesion and friction angle) of rock mass discontinuity parameters. These traditional methods typically work well on structural planes at a large scale, but they are not suitable for

detecting small-scale joints or faults in fractured rock masses. As a result, very limited information of rock mass discontinuities (i.e., joints and fractures) can be collected from the natural outcrops, boreholes and/or flat cavities of rock masses (Xiao and He, 2022).

To accurately depict the spatial distribution characteristics of rock mass discontinuity properties, modeling of discrete fracture networks (DFN) of rock masses based on the probability theory and statistics has been substantially investigated (e.g., Yue et al., 2016; Alghalandis, 2017; Zhu et al., 2022). First, the distributions of discontinuity parameters are determined from field tests combined with engineering judgment. Then, the discontinuities parameters are randomly sampled according to specified distributions. Finally, the Monte-Carlo Simulation (MCS) method is adopted to generate

the DFN models and conduct reliability analysis of rock slopes. The distributions of discontinuities parameters identified in the first step can greatly affect the accuracy of the constructed DFN model and slope reliability analysis results.

It is usually assumed that the discontinuity parameters can be modeled by standard probability distribution families, including normal, lognormal and Beta distributions. The limited flexibility of these parametric models often results in unrealistic DFN model statistics and unreasonable slope reliability results. In practice, the distributions of geometric parameters and shear strength parameters of discontinuities are derived from the limited field survey data (e.g., outcrop measurement, borehole camera, and close-up photogrammetry) (Zhu et al., 2022). However, it is a technical challenge to derive the distributions of discontinuity parameters with a small amount of collected data. In this regard, the hybrid method, proposed by Ching and Phoon (2019), provides an effective tool to derive the hybrid multivariate distribution of discontinuity parameters based on sparse site-specific data and specified prior distributions. Significant advances also have been made in the inference of the distributions of geotechnical properties using classical Bayesian method (e.g., Papaioannou and Straub, 2017; Zhou et al., 2017, Jiang et al., 2018, 2020; Huang et al., 2021; Jiang et al., 2022). In this study, the hybrid approach is adopted for constructing the site-specific multivariate distribution purely based on site-specific data because it is premiant for tackling extremely sparse and incomplete site-specific data.

Characterization and modeling of the DFNs of rock masses also have been extensively investigated (e.g., Yue et al., 2016; Guo et al., 2021), but the effect of cross-correlation between geometric parameters (e.g., dip angle and dip direction, trace length and aperture) of discontinuities is often neglected in constructing the joint orientation or DFN model. Consequently, the constructed DFN models and

reliability analysis results of jointed rock slopes are inconsistent with the engineering reality. Therefore, it is of great necessity to construct the joint probability distribution based on the field survey data accounting for the cross-correlation between geometric parameters. Additionally, the influences of geometric parameters of discontinuities on the slope reliability are rarely considered. As reported in the literature (e.g., Zheng et al., 2014; Roy et al., 2019), the uncertainties of geometric parameters also have prominent influences on the slope reliability. The computational effort for generating a DFN model and conducting slope reliability analysis will be greatly increased once the uncertainties of the geometric parameters are considered. To alleviate high computational demands, a surrogate model of the factor of safety can be constructed prior to performing the reliability analysis.

This paper is organized as follows. In Section 2, a hybrid approach for deriving the joint probability distribution of discontinuity parameters is presented. In Section 3, the Copula theory is briefly introduced to construct joint probability distribution. In Section 4, a case study of rock slope in China is investigated to illustrate the proposed approach. Finally, several conclusions are drawn based on the reliability analysis.

2. HYBRID APPROACH

The hybrid approach can effectively incorporate sparse site-specific data and capture the correlation behaviors in site-specific data to make meaningful predictions (Ching and Phoon, 2019). According to Ching and Phoon (2019), the hybrid multivariate probability density function (PDF) of random variables $\mathbf{X}$ is proportional to the direct product of the prior and site-specific multivariate PDFs:

$$f(\mathbf{x}|hb) \propto f(\mathbf{x}|prior) f(\mathbf{x}|site) \quad (1)$$

where $f(\mathbf{x}|hb)$ is the hybrid multivariate PDF;

$f(\mathbf{x}|prior)$ is the prior multivariate PDF;

$f(\mathbf{x}|site)$ is the site-specific multivariate PDF.

In addition, the posterior PDF of random variables $\mathbf{X}$ can be obtained using traditional Bayesian method as follows (Jiang et al., 2018):

$$f''_x(\mathbf{x}) = aL(\mathbf{x})f'_x(\mathbf{x}) \quad (2)$$

where $f'_x(\mathbf{x})$ is the prior PDF; a is a normalization constant that ensures $f''_x(\mathbf{x})$ integrating to 1.0; and $L(\mathbf{x})$ is the likelihood function. $f''_x(\mathbf{x})$ can be derived analytically when $f'_x(\mathbf{x})$ and $L(\mathbf{x})$ satisfy the conjugate condition.

It is easily found that the hybridization process in Eq. (1) is similar to multiplying the prior PDF with the likelihood function together to produce a posterior PDF in Eq. (2). In the hybridization process, $f(\mathbf{x}|prior)$ is multiplied by $f(\mathbf{x}|site)$ to produce $f(\mathbf{x}|hb)$, while $f'_x(\mathbf{x})$ is multiplied by $L(\mathbf{x})$ to produce $f''_x(\mathbf{x})$ in the conventional Bayesian analysis. Because of the similar structures of Eqs. (1) and (2), the BUS approach (Bayesian Updating with Structural reliability methods) proposed by Straub and Papaioannou (2015) is employed to sample the hybrid distribution.

3. CONSTRUCTING THE MULTIVARIATE DISTRIBUTION VIA COPULA THEORY

Based on limited field survey data, univariate marginal distributions can be readily obtained, but it is not trivial to derive the multivariate distribution of discontinuity parameters. In this regard, Copula theory provides an effective and versatile way to construct the prior multivariate distribution of random variables (e.g., Tang et al., 2013; Han et al., 2020). According to the Copula theory, any type of multivariate distribution can be decomposed into univariate marginal PDFs and a Copula function. With the aid of the Copula theory, more realistic DFN models and stability analysis results of jointed rock slopes can be obtained. The implementation procedure for constructing the bivariate distribution of discontinuity orientations (e.g.,

dip angle and dip direction) is summarized as follows.

(1) Collect the field survey data of dip angle and dip direction of a rock mass discontinuity.

(2) Calculate the values of Akaike information criterion (AIC) for different marginal functions and Copula functions. Determine the most optimal marginal distribution and Copula function with the minimum AIC values.

(3) Construct a bivariate distribution of dip angle and dip direction using the most optimal marginal distribution and Copula function. The readers are referred to Tang et al. (2013) and Han et al. (2020) for details.

4. CASE STUDY

4.1 Engineering background

A discontinuity-controlled rock slope in the Xinjiang Uygur Autonomous Region, China is used to illustrate the proposed approach. The slope has a height of 175 m and an average slope angle of 40°. Through elaborate field geological investigations using the window method, it is found that the slope is dominated by IV and V structural planes. There are no large controlling structural planes. There are three sets of dominant joints identified in the rock masses. The orientation of the most developed joint set (J_1) is 280~355°∠60~84°. The orientation of the secondary developed joint set (J_2) is 140~190°∠40~70°. The orientation of the least developed joint set (J_3) is 300~340°∠8~25°.

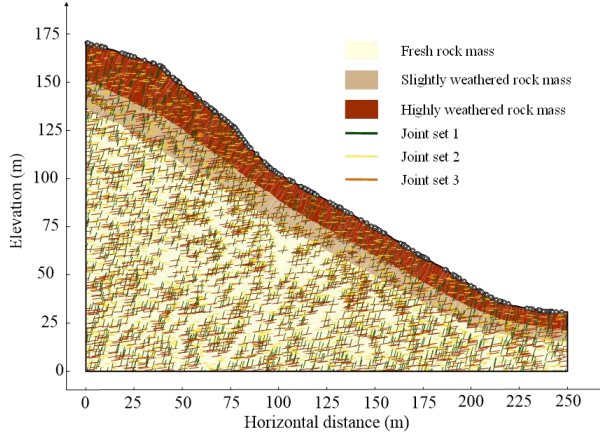


Figure 1: Finite element model of the slope.

The slope mainly consists of fresh, slightly weathered and highly weathered rock masses and three joint sets. A finite element model is established according to the geological profile of the slope, as shown in Figure 1.

Table 1: Prior distributions and statistics of discontinuity parameters for three joint sets.

Parameters	Probability distribution	Mean	Standard deviation
D_1 (°)	Normal	80.3	6.0
DD_1 (°)	Normal	321.9	9.0
D_2 (°)	Lognormal	58.2	9.0
DD_2 (°)	Lognormal	180.5	7.9
D_3 (°)	Normal	11.2	4.3
DD_3 (°)	Normal	302.0	26.1
c (MPa)	Normal	0.02	0.006
ϕ (°)	Normal	22.0	2.0

Table 2: Values of mechanical parameters of the rock masses.

Parameters	Fresh rock mass	Slightly weathered rock mass	Highly weathered rock mass
γ (kN/m ³)	28.0	27.5	27.0
σ_b (MPa)	0.7	0.5	0.3
E (GPa)	31000	8500	2000
μ	0.23	0.26	0.31
c (MPa)	1.62	1.01	0.36
ϕ (°)	53.6	48.1	38.5

Six geometric parameters (dip angle D_1 and dip direction DD_1 of joint set J_1 , dip angle D_2 and

dip direction DD_2 of joint set J_2 , dip angle D_3 and dip direction DD_3 of joint set J_3) and two shear strength parameters (cohesion c and friction angle ϕ) are considered as cross-correlated random variables in this study. Table 1 shows the prior distributions and statistics of eight discontinuity parameters. The cross-correlation coefficients between D_1 and DD_1 , D_2 and DD_2 , and D_3 and DD_3 are evaluated as -0.28, -0.21 and 0.14, respectively. The AIC values of the commonly used copula functions for each pair of parameters are calculated. The Gaussian copula is identified as the most optimal Copula function for each pair of parameters. Then, the prior bivariate PDF of dip angle and dip direction as well as that of cohesion and friction angle are constructed with the Gaussian copula. The other mechanical parameters of rock masses and joint sets are treated as constants and their values are summarized in Tables 2 and 3, respectively, according to Huang (2019).

Table 3: Values of mechanical parameters of three joint sets.

Parameters	J_1	J_2	J_3
Linear density	0.35	0.30	0.10
Mean trace length (m)	4	1	5
Normal stiffness (MPa/m)	160000	160000	210000
Shear stiffness (MPa/m)	11000	11000	13000

4.2 Slope reliability analysis based on the prior information

Before performing slope reliability analysis, the factorial design is adopted to determine the relationship between the factor of safety and discontinuity parameters. The analysis of variance (ANOVA) (Montgomery, 2017) in combination with the half-factorial design (Roy et al., 2019) are employed to identify the discontinuity parameters that have significant effects on the slope stability. The DFN model of the slope is generated in Phase² software (Rocscience, 2015), and the finite element shear

strength reduction method is used to evaluate the factor of safety.

Table 4 summarizes the ANOVA analysis results. It can be observed that the discontinuity parameters that have significant influences on the slope stability are D_1 , DD_1 , DD_2 , D_3 , DD_3 , c and ϕ . The influence of D_2 on the slope stability is insignificant because the corresponding p -value is greater than 0.05. Therefore, D_2 can be regarded as a constant in the subsequent reliability calculations. To improve the computational efficiency, a quadratic polynomial with cross-term-based surrogate model is constructed to approximate the relationship between the factor of safety and seven significant discontinuity parameters, in which 156 random samples are generated using the central composite design. After that, the prior probability of slope failure is evaluated using the MCS with 100, 000 samples, and it is 1.11×10^{-3} .

Table 4: ANOVA analysis results based on prior statistics.

Factor	p -value	Significance level
D_1 (°)	< 0.0001	Significant
DD_1 (°)	0.0003	Significant
D_2 (°)	0.6498	Insignificant
DD_2 (°)	0.0002	Significant
D_3 (°)	< 0.0001	Significant
DD_3 (°)	0.0003	Significant
c (MPa)	< 0.0001	Significant
ϕ (°)	< 0.0001	Significant

4.3 Hybrid analysis of the bivariate distribution of discontinuity properties

To ensure the constructed DFN model and reliability analysis results of the slope close to the engineering reality, the bivariate distribution of discontinuity parameters needs to be derived with the collected field survey data. The statistical windows, line measurements and close-up photogrammetry methods are often used to acquire the field survey data. Following Huang (2012), the outcrops data of three joint sets at different locations around this slope are collected. Table 5 summarizes the data of the dip angles and dip directions of three joint sets that

transformed from the outcrops data. For each pair of parameters, the corresponding data in Table 5 are then used in conjunction with Copula function to fit the site-specific multivariate PDF of x_1 and x_2 as follows (Zhang et al., 2017):

$$f(x|site) = f_1(x_1|\mu_{x_1}, \sigma_{x_1}) f_2(x_2|\mu_{x_2}, \sigma_{x_2}) D[F_1(x_1), F_2(x_2); \theta] \quad (3)$$

where $D[F_1(x_1), F_2(x_2); \theta]$ is the Copula density function, in which θ is the correlation parameter; $f_1(x_1)$ and $f_2(x_2)$ are the PDFs of x_1 and x_2 , respectively; μ_{x_1} and σ_{x_1} , μ_{x_2} and σ_{x_2} are the means and standard deviations of x_1 and x_2 , respectively; $F_1(x_1)$ and $F_2(x_2)$ are the cumulative distribution functions of x_1 and x_2 , respectively. Note that the Copula density function and μ_{x_1} , σ_{x_1} , μ_{x_2} , σ_{x_2} and θ are determined directly based on the outcrops data in Table 5.

Table 5: Data of dip angles and dip directions of three joint sets.

D_1 (°)	DD_1 (°)	D_2 (°)	DD_2 (°)	D_3 (°)	DD_3 (°)
76	277	62	190	9	315
72	275	61	214	26	320
69	341	66	177	8	308
80	280	60	180		

After the hybridization process is completed, the means, standard deviations and correlation parameters are updated and listed in Table 6.

Table 6: Updated statistics of discontinuity parameters for three joint sets.

Parameters	Mean	Standard deviation	Correlation parameter
D_1 (°)	73.82	3.13	-0.26
DD_1 (°)	319.98	8.66	
D_2 (°)	62.41	2.38	-0.22
DD_2 (°)	181.75	7.21	
D_3 (°)	11.89	3.93	0.52
DD_3 (°)	312.93	3.78	

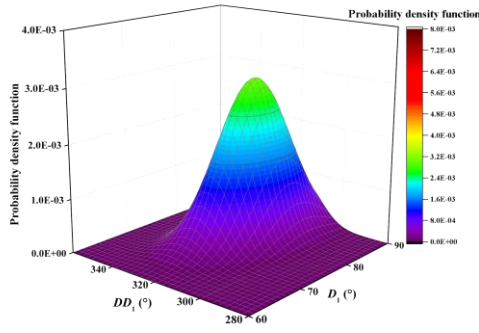
Table 7: Site-specific bivariate distribution of shear strength parameters.

Parameters	Distribution	Mean	Standard deviation
c (MPa)	Normal	0.0125	0.003
φ (°)	Normal	31.6	3.0

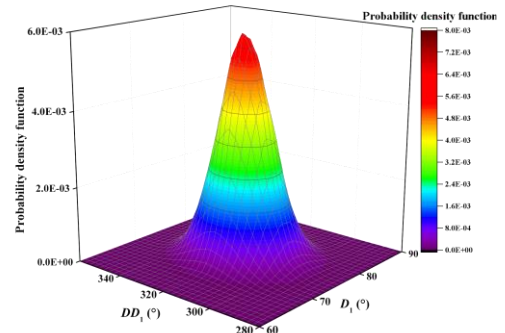
Table 7 summaries the site-specific bivariate distribution of c and φ that are independent with each other, which are used in conjunction with the prior bivariate distribution of c and φ to derive the hybrid bivariate distribution of c and φ . Because their prior and site-specific bivariate distributions are both bivariate normal distributions and satisfy the conjugate condition. Hence, the hybrid bivariate distribution has analytical solution and is bivariate normal distribution. The updated means and standard deviations of c and φ are 12.5 kPa

and 3 kPa, 31.6° and 3.0°, respectively.

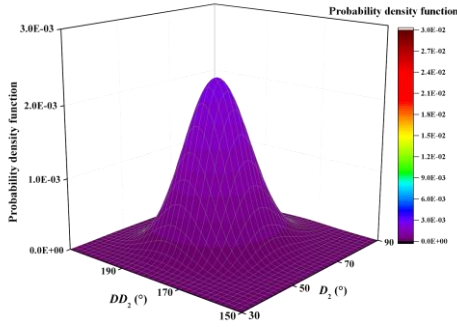
As for the six geometric parameters of three joint sets, the hybrid approach is employed to estimate their hybrid multivariate distributions. Figures 2(a)~(f) compare the three-dimensional prior and hybrid bivariate PDF plots of D_1 and DD_1 , D_2 and DD_2 , and D_3 and DD_3 , respectively. Note that the varying ranges of three axes are kept the same for the same joint set. As observed from Figure 2, the three-dimensional hybrid bivariate PDF plots become higher, thinner and more concentrated than the corresponding prior bivariate PDF plots. It implies that the uncertainties of discontinuity parameters are reduced, and more precise values of discontinuity parameters can be obtained and used in the slope reliability analysis.



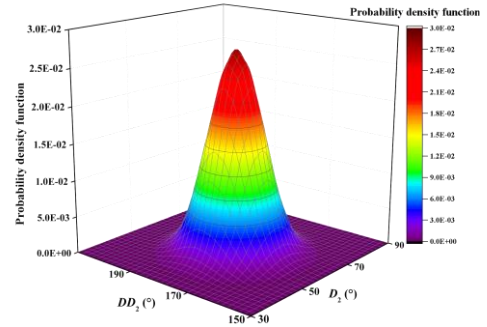
(a) Prior bivariate PDF for J₁



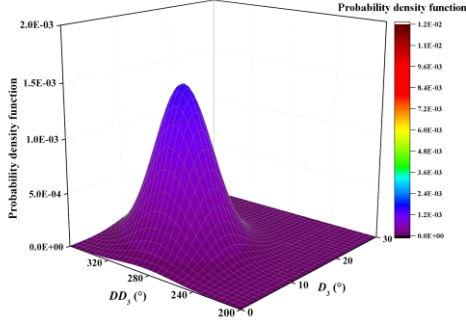
(b) Hybrid bivariate PDF for J₁



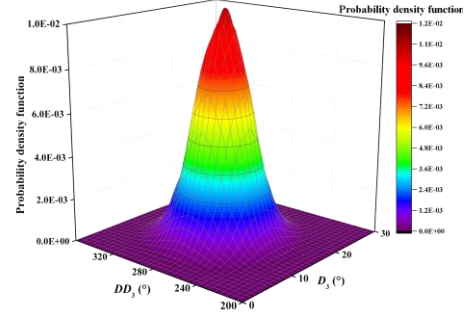
(c) Prior bivariate PDF for J₂



(d) Hybrid bivariate PDF for J₂



(e) Prior bivariate PDF for J3



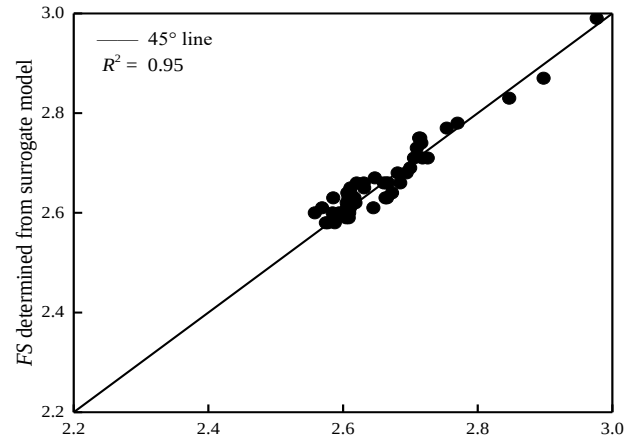
(f) Hybrid bivariate PDF for J3

Figure 2: Comparison of three-dimensional prior and hybrid bivariate PDF plots of dip angles and dip directions of three joint sets.

4.4 Slope reliability analysis based on the hybrid information

The slope reliability analysis is carried out based on the hybrid information of discontinuity parameters. The ANOVA analysis results indicate that the effects of D_1 , D_2 and DD_2 on the slope stability become insignificant. Therefore, only five parameters (DD_1 , D_3 , DD_3 , c and ϕ) are considered as random variables and other parameters are treated as constants in the subsequent slope reliability analysis. A quadratic polynomial with cross-term-based surrogate model is determined to fit the relationship between the factor of safety and five significant parameters as follows:

$$\begin{aligned}
 FS = & 0.000283 - 0.061510DD_1 + 0.015176D_3 + \\
 & 0.075208DD_3 - 0.000035c - 0.019543\phi + \\
 & 0.000146DD_1^2 + 0.001530D_3^2 - 0.000048DD_3^2 - \\
 & 0.000001c^2 - 0.000976\phi^2 + 0.000262DD_1D_3 - \\
 & 0.000122DD_1DD_3 + 0.029451DD_1c - \\
 & 0.000385D_3DD_3 - 0.001405D_3c - \\
 & 0.000297D_3\phi - 0.019270DD_3c - \\
 & 0.000082DD_3\phi - 0.001582c\phi
 \end{aligned} \quad (4)$$



FS determined from finite element method in Phase² software

Figure 3: Comparison of factors of safety determined from finite element method in Phase² software and surrogate model.

To validate the effectiveness of the surrogate model in Eq. (4), 50 sets of MCS samples are additionally generated. Figure 3 presents the factors of safety obtained from the finite element method versus those determined from Eq. (4) using these 50 samples. As observed from Figure 3, the values of FS obtained from the two approaches are in good agreement. The data clusters tightly along the 1:1 line with a coefficient of determination R^2 of 0.95. Based on these, the updated probability of failure is also evaluated using the MCS with 100,000 samples. It is 2.0×10^{-5} , which is two orders of magnitude less than the prior probability of failure $p_f = 1.11 \times 10^{-3}$.

5. CONCLUSIONS

The study employed a hybrid approach to derive the multivariate distribution of rock mass discontinuity parameters with limited field survey data, in which the cross-correlation between discontinuity parameters is considered. The DFN models and slope reliability analysis results considering the uncertainties of multiple parameters are obtained accordingly. The conclusions are summarized as follows:

(1) The sparse field survey data can be effectively integrated in a hybrid analysis to derive the multivariate distribution of rock mass discontinuity parameters. After the hybrid analysis, the uncertainties of discontinuity parameters are significantly reduced, more realistic DFN models and reliability analysis results of jointed rock slopes can be obtained.

(2) The ANOVA analysis results indicate the parameters that have significant influences on the slope stability are not the same before and after hybrid analyses. Construct a surrogate model of factor of safety in advance can greatly reduce the computational burden of slope reliability analysis.

(3) With the aid of the Copula theory, the cross-correlation between the discontinuity parameters can be well taken into account in constructing the prior site-specific multivariate distributions of discontinuity parameters.

6. ACKNOWLEDGMENTS

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