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# **Visible light scattering from micron scale silicon nitride structures**

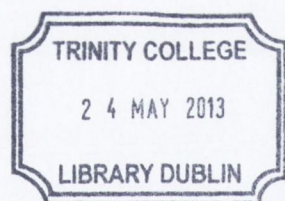
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by

David McCloskey

A thesis submitted for the degree of Doctor of Philosophy  
in the University of Dublin

Under the supervision of  
Prof. John Donegan  
School of Physics  
Trinity College Dublin



*Thesis 9977*

## Declaration

This thesis has not been submitted as an exercise for a degree at this or any other university. The work presented is entirely my own, apart from the assistance mentioned in the acknowledgements. I agree that Trinity College Dublin may lend or copy this thesis upon request.

David McCloskey

David McCloskey

October 2012





## Summary

Light scattering from complex and irregular shapes is responsible for many optical effects in nature. Butterfly wings change colour when viewed from different angles, due to a complex nanoscale scattering structure. Ice halos are formed by light scattering from hexagonal prisms formed from ice crystals. Radar scattering from leaves of trees is commonly modelled as scattering from thin disks. By reproducing and modifying these nature inspired structures numerous applications such as electronic displays, colour changing paint, and anti-counterfeiting have been found.

This thesis deals with light scattering from micron scale scatterers defined in a 400nm layer of silicon nitride  $\text{Si}_3\text{N}_4$ . With illumination from visible light these particles are in the Mie scattering regime and are referred to as “small particles”. Shapes with different cross sections are defined in the thin layer using electron beam lithography (EBL) and inductively coupled plasma etching (ICP). Visible light is scattered strongly both in plane and out of plane by the structures. The intensity distribution in the near-field (Fresnel region) can be controlled by controlling the cross sectional shape of the scatterer. A simple imaging technique is outlined to map the properties of the near-field intensity distribution from the far-field. This is a simple, non-invasive technique, which gives a lot of information about the scattering without the need for SNOM or more complicated near-field imaging techniques. The technique works particularly well in this case as the near-field intensity distribution of interest is formed primarily from propagating light. The experimental results are compared with a full 3D finite element method (FEM) model of the near-field intensity.

In Chapter-1 an introduction to light scattering and a historical overview is given. Current experiments in light scattering from particles in this size range are outlined, followed by an overview of the project objectives. Chapter-2 presents the analytical theory of light scattering for small particles, and the numerical techniques used throughout the project. Chapter-3 is a background of all the experimental techniques used throughout the project. It is a stand-alone chapter, but is referenced heavily throughout the subsequent chapters. Chapter-4 outlines the fabrication and optimisation used to create the scattering structures. Chapter -5 is the first chapter dealing with experimental results and details the light scattering from microdisk scatterers with diameters ranging from 1-10  $\mu\text{m}$ . Chapter-6 explores scattering from low symmetry shapes such as squares and triangles, which depend more sensitively on the in-plane incident angle and polarisation. Chapter-7 studies the out of plane scattering in the far-field using a back focal plane imaging method. Chapter-8 deals with larger structures known as Cartesian ovals. These structures are designed specifically to produce small focal spots with no



spherical aberration. Chapter-9 re-examines the microdisk structures as resonant cavities. High quality factor modes known as whispering gallery modes (WGM) exist in these cavities. The modal structure of the microdisks is investigated using a simple  $\mu$ PL technique. Finally, Chapter-10 is a general conclusion from the entire work and an outlook for future work.



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## Publications

**D. McCloskey**, J. J. Wang, and J. F. Donegan, "Low divergence photonic nanojets from Si<sub>3</sub>N<sub>4</sub> microdisks," *Opt. Express* **20**, 128-140 (2012).

Highlighted in *Nature photonics*: "Light scattering: Photonic Nanojets," *Nature Photonics* **6**, 138–139 (2012).)

And republished online: in *Virtual Journal for Biomedical Optics* **7** (3), A. Dunn, and A. Durkin, eds. (2012).

## Conference publications

Y. P. Rakovich, **D. McCloskey**, J. F. Donegan, and " Photonic Nanojets in Coupled Microcavities," in *CLEO/Europe and EQEC 2009 Conference Digest*(Optical Society of America, Munich, 2009).

**D. McCloskey**, Y. P. Rakovich, and J. F. Donegan, "Controlling the properties of photonic jets," in *12th International Conference on Transparent Optical Networks (ICTON)*, (Munich, 2010 ).

**D. McCloskey**, and J. F. Donegan, "Silicon nitride microdisks in the visible range," in *Transparent Optical Networks (ICTON), 2011 13th International Conference on*(2011), pp. 1-4.

**D. McCloskey**, and J. F. Donegan, "Light scattering from silicon nitride microdisks," in *Transparent Optical Networks (ICTON), 2012 14th International Conference on*(2012), pp. 1-4.

## Conference poster presentations

**D. McCloskey**, Y. Rakovich, and J. F. Donegan, "Properties of photonic jets" in *Photonics Ireland*(Cork, 2009).

**D. McCloskey**, and J. F. Donegan, "Light scattering from Silicon Nitride Microdisks," in *The 12th international conference on near-field optics, nanophotonics nad related techniques (NFO)*(San Sebastian, Spain 2012).





## ABBREVIATIONS AND ANNOTATIONS

|                    |                                          |
|--------------------|------------------------------------------|
| 2D                 | Two-dimensional                          |
| 3D                 | Three-dimensional                        |
| AFM                | Atomic force microscopy                  |
| Ar <sup>+</sup>    | Argon ion                                |
| BaTiO <sub>3</sub> | Barium titanate                          |
| CCD                | Charge coupled device                    |
| CHF <sub>3</sub>   | Trifluoromethane                         |
| c-QED              | Cavity quantum electrodynamics           |
| DDA                | Discrete dipole approximation            |
| DLS                | Dynamic Light Scattering                 |
| DOE                | Design of experiment                     |
| DPSS               | Diode pumped solid state                 |
| DRAM               | Dynamic random-access memory             |
| DWDM               | Dense wavelength division multiplexing   |
| EBL                | Electron beam lithography                |
| EBPVD              | Electron beam physical vapour deposition |
| EBPVD              | Electron beam Physical vapour deposition |
| EUVL               | Extreme ultra-violet lithography         |
| FCS                | Fluorescence correlation spectroscopy    |
| FDTD               | Finite difference time domain            |
| FEM                | Finite element method                    |
| FIB                | Focused ion beam                         |
| FOV                | Field of view                            |
| F <sub>p</sub>     | Purcell enhancement factor               |
| FSR                | Free spectral range                      |
| GIS                | Gas injection system                     |
| Hp                 | half pitch                               |
| ICP                | Inductively coupled plasma               |
| LPCVD              | Low pressure chemical vapour deposition  |
| MDR                | Morphologically dependent resonance      |
| MPU                | multi-core processing unit               |
| NA                 | Numerical aperture                       |
| OCT                | Optical coherence tomography             |

|                                |                                        |
|--------------------------------|----------------------------------------|
| ODEs                           | Ordinary differential equations        |
| OPL                            | Optical path length                    |
| PDEs                           | Partial differential equations         |
| PDMS                           | Polydimethylsiloxane                   |
| PL                             | Photoluminescence                      |
| PML                            | Perfectly matched layers               |
| PNJ                            | Photonic nanojet                       |
| Q                              | Quality factor                         |
| QCE                            | Quantum confinement effect             |
| Ra                             | Arithmetic average of absolute values  |
| RET                            | Resolution enhancement technology      |
| RF                             | Radio frequency                        |
| Rh6G                           | Rohdamine 6G laser dye                 |
| RMS                            | Root mean square                       |
| SEM                            | Scanning electron microscopy           |
| SF <sub>6</sub>                | Sulphur hexafluoride                   |
| Si <sub>3</sub> N <sub>4</sub> | Silicon Nitride                        |
| SiO <sub>2</sub>               | Silicon dioxide                        |
| SNOM                           | Scanning near field optical microscopy |
| TE                             | Transverse electric                    |
| TM                             | Transverse magnetic                    |
| TMAH                           | Tetramethylammonium hydroxide          |
| UV                             | Ultra violet                           |
| V                              | Mode volume                            |
| WGM                            | Whispering Gallery mode                |



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## Chapter 1 Introduction

All matter is made up of discrete electric charges; electrons and protons. When electromagnetic radiation is incident on these charges it causes them to oscillate. These oscillating charges in turn emit radiation in all directions. This secondary radiation is known as the scattered field. These oscillating charges may lose energy by other mechanisms such as heating, which is known as absorption. This is the basis for a classical description of the interaction of light and matter, and can be used to describe many optical effects. The total process of absorption and re-radiation is known as scattering.

For example when a plane wave of light is incident on a dielectric medium it changes direction (refraction) and the propagation velocity of the light is reduced. This can be thought of the electromagnetic field creating oscillating dipoles in the material which take energy from the incident field and re-radiate it. Depending on the material these dipoles will have a specific dipole moment (charge  $\times$  separation distance) and be arranged in a particular lattice. The exact form of the re-radiated field is determined by Maxwell's equations which govern the relationship between electric fields, magnetic fields, charge and current densities. The fields from each of these dipoles interfere with each other and as it happens create a plane wave which propagates at an angle defined by Snell's law with a phase velocity which is the speed of light in a vacuum divided by the refractive index of the material. The propagation of light in a homogeneous medium is therefore a multiple scattering process, and the resultant macroscopic effects observed are the average effects of many single scattering events.

The interaction of light with matter can be considered at different length scales. The length scale of interest in turn determines the approach to use. When the particle under consideration is much smaller than the wavelength of the incident radiation the approximation of Rayleigh scattering can be used. This approximates the particle as a single dipole with a polarisability which depends on the size and refractive index of the particle. Rayleigh scattering is applicable to gas molecules found in the atmosphere and is responsible for the blue colour of the sky (and the yellow colour of the sun) amongst other things. When the particle size is on the order of the wavelength of light this approximation is no longer valid and a physical optics approach must be used which involves solving Maxwell's with the specific boundary conditions of the particle. Light scattering in this size regime is responsible for a host of optical effects such as rainbows, halos and glories, light beams in forests, clouds and fog, smoke, the colour of



milk and opal and many other effects. Finally, when a particle is much larger than the wavelength of the incident light a geometrical optics approach can be used.

This thesis is primarily concerned with light scattering by particles which fall into the intermediate regime between Rayleigh scattering and geometrical optics. For visible light which has a wavelength of 500nm this refers to particles with characteristic dimensions ranging from 1-10 $\mu$ m. We refer to particles in this size regime as “small” particles. Current technology allows us to fabricate and manipulate single particles of this size with ease and accuracy. Combining this with intense monochromatic and coherent light sources such as lasers creates a playground to observe and understand the interaction of light and matter on this length scale.

## 1.1 Historical overview of light scattering by small particles

A complete analytical solution for light scattering by a sphere in a homogeneous medium was first derived in the early 1900s. The credit for the solution is divided between Lorentz, Debye, Mie and Rayleigh who published papers on the subject around that time [1-4]. The main paper was written by Gustav Mie in 1908 [1] and so the theory is generally known as Lorentz- Mie theory or just Mie theory [5].

This elegant solution can now be found in many modern textbooks [6, 7] and gives an expression for the scattered field from a plane wave incident on a sphere as an infinite sum of individual terms which contain Bessel functions and Legendre polynomials. The sum can be truncated after a finite number of terms which depends on the size of the sphere [8]. Although the theory applies to all size ranges it is in general more complex than the Rayleigh approximation for very small particles and computationally expensive for particles in the geometrical optics size regime. For these reasons the term Mie scattering has become synonymous with scattering in this intermediate size regime. The truncated infinite sum must be computed numerically. Despite the discovery of the analytical solution in 1908 it was not until the 1977 with a combination of higher computational power and monochromatic light sources (lasers) that the solution could be verified experimentally [9].

The theory predicts a strange behaviour of resonances in light scattering from a spherical particle which was recognised by Hulst [10]. The frequency of these resonances depends only on size of the sphere relative to the incident wavelength and refractive index contrast between the particle and its surroundings. The common parameters to use are the size parameter  $x_s = 2\pi R / \lambda$  where  $R$  is the radius of the sphere, and the refractive index contrast  $m = n / n_0$ , where  $n$  and  $n_0$  are the refractive index of the particle and the surrounding medium respectively. These resonances are known under many names such as, whispering gallery modes



(WGM), partial wave resonances, surface modes, and virtual modes, natural frequencies to name but a few [11]. They are part of a more general class of resonances which depend only on the shape, size, and refractive index contrast of the particle known as morphologically dependent resonances (MDRs) [12]. These resonances were first observed experimentally at optical frequencies by Ashkin and Dziedzic in their studies using radiation pressure to levitate dielectric spheres [9].

Early studies on light scattering by single small particles used mainly two techniques which we refer to as the resonance technique and the scattering technique [13]. Single droplets of fluid were held in position using electrostatic or electro-dynamic balances [14-16] and illuminated with a laser approximating a plane wave [17]. In the resonance technique light scattered in a specific direction is monitored while the size parameter is varied either using a tuneable laser source [18, 19] or an evaporating or condensing droplet [20-23]. The size and refractive index of the particle can then be calculated to a high precision based on the position and spacing of the resonances [24-26]. The second technique which we call the scattering technique moves the position of the detector with respect to the incident laser and records the intensity as a function of angle. This intensity as a function of angle is known as the differential scattering cross section or the phase function. The distribution of this phase function is very sensitive to  $x_s$  and  $m$  and can be fitted to Mie theory to accurately determine these particle size and refractive index [16, 27, 28].

Investigation of light scattering from small particles has led to a better understanding of the interaction of light and matter, however it is as a tool to study other systems where it comes into its own. Since the early experiments in 1980s many applications and commercial products based on light scattering have arisen across a plethora of disciplines. For example dynamic light scattering (DLS) is a large field of study which gives information properties of solids, liquids and gases based on the temporal fluctuations of scattered light [29, 30]. Surface charge or Zeta potential of colloidal particles can be measured using DLS by applying an electric field and using the Doppler effect [31, 32]. Optical coherence tomography (OCT) uses coherence of scattered light from different planes in a scattering medium (not optically transparent) such as biological tissue to construct a 3D image of the sample. It is commonly used as a non-invasive diagnostic tool in ophthalmology [33, 34] and cardiology [35, 36]. The field of optical trapping (optical tweezers) grew from the early experiments of Ashkin and co-workers who were originally concerned with light scattering from micron scale dielectric particles [37, 38]. Optical trapping has now become an invaluable tool to study displacements and forces in the nanometer and piconewton regimes respectively [39, 40]. This opened a new field of physical biology where forces and interactions between biological organisms can be accurately



measured [41-44]. Random lasing has been observed in dense scattering medium and attributed to Mie scattering [45, 46].

Morphologically dependent resonances (MDR) in spherical and cylindrical geometries are an ideal system to study fundamental and applied physics across many fields. These resonators can take many forms[47] including microspheres [48], microdisks [49], microrings[50, 51], microtoroids[52], microtubes[53], micropillars[54, 55] and microbottles [56, 57]. The combination of high intensity and small mode volume of light trapped in whispering gallery modes allows non-linear optical effects to be observed at low input intensities[58], lasing can exist at extremely low threshold [59, 60], and single photon emission can be enhanced and controlled [61-64]. These resonators have found applications in bio-sensing, as the position and width of the resonances are extremely sensitive to the refractive index of the surrounding medium [65-67]. By functionalising the surfaces of microspheres single protein and virus binding events have been observed [68, 69]. Arrays of microrings or microdisks are used as ultra-compact filters for optical communications with application in areas like dense wavelength division multiplexing (DWDM) [50, 51, 70]. They are ideal for studying fundamental physics such as cavity quantum electrodynamics (c-QED) [71-73].

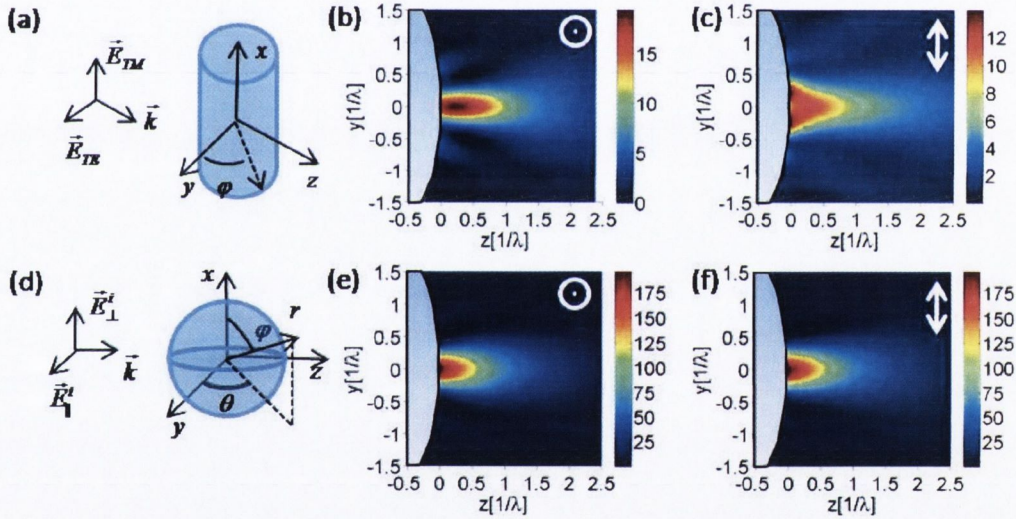
Investigation of light scattering by small metallic particles has led to the field of plasmonics which promises to marry optical and electronic functionalities at a nanometer scale[74]. Small metallic particles composed of gold, silver or platinum can have scattering cross sections which are much larger than the physical size of the particle[75]. This makes them ideal tools for trapping light and they are currently being used to improve efficiency in photovoltaic devices [76]. Large field enhancements close to these particles are being used to test fundamental physics [77] and non-linear optics[78, 79]. These particles can effectively focus the energy of the incident light beyond the diffraction limit[80-84].

## 1.2 Motivation / state of the art

Recently light scattering in the near-field of particles in the intermediate size regime has been revisited. It was predicted numerically from finite difference time domain (FDTD) simulations values of  $x_s$  and  $m$  that the scattered fields from infinite cylinders create a near-field intensity distribution with a long narrow intensity lobe on the shadow side of the cylinder [85-87]. The unique aspect of this main lobe was that it had a lateral extent that was slightly smaller than the diffraction limit for a Gaussian beam and a much lower divergence angle. This results in a sub-wavelength beam which stays sub wavelength over 10s of wavelengths [88-90]. This main lobe was termed a photonic nanojet (PNJ) by Chen, Taflov, and Backman in 2004 [85] due to the



jet like near field intensity distribution from analogy with fluidics. The effect was quickly observed in spheres in a similar size regime and shown to produce a narrower more intense PNJ[91]. Figure 1.1 shows an example of the near field total intensity distribution outside an infinite cylinder and a sphere calculated using Mie theory. The co-ordinate systems and incident  $\mathbf{k}$  vector and polarisations are shown in Fig. 1.1 (a) and (d). The incident wavelength is 532nm on a cylinder and sphere of  $5\mu\text{m}$  diameter ( $x_s=30$ ) and refractive index of 1.68. The intensity of the incoming wave is 1, thus the scale directly represents enhancement over the background intensity. Distance on the x and y axis is scaled in units of the wavelength of incident light. For this particular size parameter and refractive index contrast we see an intensity enhancement of 18 times for a cylinder and 180 times for a sphere. The intensity distribution is more dependent on incident polarisation for the case of a cylinder.



**Figure 1.1(a)** Co-ordinate system and the two cases of incident light on a cylinder considered. Light is a linearly polarised plane wave and propagates in the direction of the z-axis. In the TM case the electric field is in the z-direction. In the TE case the electric field is in the y-z plane only. **(b)** Intensity distribution of light scattered from an infinite cylinder for TM case with  $m=1.68$  and  $x_s=30$ . **(c)** Relative intensity for TE case. **(d)** The two cases of incident light on a sphere. **(e)** Intensity from a sphere with  $m=1.68$  and  $x_s=30$  and perpendicular polarised incident light **(f)** Relative intensity for parallel case.

These computational techniques combined with the commercial availability of high quality mono-disperse microspheres has led to a substantial amount of literature since 2004 on both the intrinsic properties of the PNJ and applications [92, 93]. The effect has been explained analytically using the angular spectrum representation [94], and Debye series[95]. The low



divergence of the main lobe was shown to be a direct result of a small proportion of the light being scattered into higher angle lobes [94]. The PNJ scattering distribution has been experimentally observed directly with centimetre scale spheres at microwave frequencies (30 GHz)[96], at optical frequencies using a scanning con-focal microscopy technique [97], and indirectly using reflection from a substrate [98]. Numerical simulations show that the photonic jet can exist for a range of size parameters as long as the refractive index contrast is close to the optimum of 1.7 [99, 100].

The importance of the PNJ is evidenced from the numerous applications which have been proposed and experimentally verified. For example a metallic nanoparticle passing through the PNJ creates a large backscattering perturbation, which depends on the third power of the nanoparticle diameter as opposed to the sixth power in standard Rayleigh scattering [85, 101-103]. This effect has been exploited to detect a 50nm gold particle using a BaTiO<sub>3</sub> microsphere embedded in a PDMS matrix [104]. The PNJ effect has been used in direct write lithography, producing features with minimum sizes ~100nm with light of wavelength 355nm [105]. PNJs have been proposed for optical data storage, with resolution higher than BluRay™ [106, 107], single molecule detection and fluorescence enhancement [108, 109], and super-resolution optical microscopy [110]. Regular arrays of spheres can be used to create patterns in an underlying substrate [111]. PNJs have also been used for low loss optical transmission [112], and as a precise cutting tool for medical applications [113]. Non-linear effects such as two photon fluorescence enhancement have been observed using the PNJ. With focused Gaussian illumination a shorter main lobe is formed. This provides 3D confinement and can be used to further reduce the size of a Gaussian focal spot. This effect was employed in order to improve fluorescence correlation spectroscopy (FCS) measurements [114]. The intensity gradient of the PNJ allows metallic nanoparticles to be trapped. The trapping force depends very sensitively on the size of the particles and the trapping position can be changed by changing the polarisation [115]. Layered microspheres have also been investigated to produce highly elongated PNJs at the cost of a lower intensity peak [116]. Recently atto-litre detection volumes have been created using PNJ and fluorescence depletion (2 orders of magnitude smaller than confocal). This greatly simplifies single molecule experiments allowing use of higher concentrations of molecules [117].

### 1.3 Project objective and aims

Much of the experimental work thus far has focused on PNJs from dielectric spheres, despite the original predictions being in cylindrical geometries. This is due to the fact that the intensity enhancement for a sphere is greater than that of a cylinder [86].



This project studies the scattering of visible light with micron scale planar dielectric structures on a substrate. By planar we mean that the height of the structures is much smaller in height than their lateral extent. These structures are 400nm in height and between 1 and 10 $\mu$ m in lateral extent and scatter visible light strongly. The idea is to control the intensity distribution in the region close to the particle (near field) by controlling the shape and refractive index of the particle. Although the intensity enhancement close to these structures is not expected to be as large as the case of a sphere there are many advantages to using a planar system. The particles size and shape and position can be easily controlled to high precision in the fabrication process. Higher refractive index materials can be used. The particles can be made cheaply in large volumes, and in arrays that could then be integrated on chip. Integration with microfluidics could produce on chip solutions for single molecule fluorescence detection, nano-particle detection, or bio-sensing applications. Ideally these structures could be integrated with light delivered via optical rib waveguides and use the diffractive optical effect to produce unique focal volumes. This thesis is a first step towards achieving devices for these applications.

Analytical solutions only exist for a few simple geometries in a homogeneous medium such as spheres and infinite cylinders [10]. Approximate analytical solutions can be found for disks of finite height [118, 119] by applying certain assumptions such as approximating the internal field with that of an infinite slab [120], by approximating the disk as thin [121], or by placing restrictions on the incident beam [122]. Rayleigh-Gans theory is used for a finite height cylinder of low refractive index contrast [123-125]. All of these techniques have a limited range of applicability due to the assumptions of the approximations used. Modern numerical techniques such as discrete dipole approximation (DDA) [126, 127], finite difference time domain (FDTD) [128] and finite element method (FEM) allow us to numerically solve Maxwell's equations to calculate the scattering from arbitrary shapes. The finite element method is employed in this project as it is particularly suited to studying complex shapes as the mesh is not confined to a rectangular domain [129, 130]. These methods are limited only by computational power and available memory.

Light scattering from irregular shaped metallic nanoparticles is currently being explored by many groups [131]. The far field properties like the position of the resonances can be changed by changing the shape of the particle[132]. The near field intensity distribution also depends strongly on shape and novel structures can be used such as bow-tie antennae to maximise the intensity enhancement [133, 134]. Much less work has been done on light scattering from dielectric particles of irregular shape [135]. The objective of this project is thus to examine light scattering from different shaped scatterers in the Mie regime, and to control the near and far field scattering intensity distributions by controlling the shape of the scatterer.



Scattering from complex and irregular shapes is responsible for many optical effects in nature. Butterfly wings change colour when viewed from different angles. This is due to a complex nanoscale scattering structure [136, 137]. Ice halos are formed by light scattering from hexagonal prisms [138]. Radar scattering from leaves of trees is commonly modelled as scattering from thin disks [139]. By reproducing and modifying these nature inspired structures applications in electronic displays, colour changing paint, and anti-counterfeiting have been found.

MDRs in irregular shapes have also been studied by many groups. Usually the modes are of lesser quality than the WGM case. Numerical [140, 141] and experimental work [142, 143] show these cavities can support a complex modal pattern which in some cases can have advantages over WGMs. Chaotic modes can be observed in deformed microcavities [144-146] and microdisks with asymmetrically positioned cut outs [147].

## 1.4 Thesis layout

The layout of the thesis is as follows. In Chapter-2 the analytical and numerical techniques used to study light scattering are outlined. The analytical formulae for light scattering from an infinite circular cylinder are given and quantities of interest are calculated. These quantities are then explored for a full variation of size parameter and refractive index contrast. The numerical technique used is the finite element method (FEM). The principle behind the technique is outlined briefly and elucidated with a simple 1D example. The Stratton-Chu formula is then used to calculate the far field scattering distributions from the FEM calculated near fields for 2D and 3D cases and compared with the analytical solution for an infinite cylinder and sphere as a check of the accuracy of the numerical method.

The purpose of Chapter-3 is to explain in detail the various techniques used in sample preparation, sample analysis and measurement throughout the project. It can be read as a stand-alone chapter, however is used as a reference throughout the rest of the thesis. The fabrication techniques used include: electron beam lithography (EBL), UV lithography, inductively coupled plasma etching (ICP). Analysis techniques include: electron beam physical vapour deposition (EBPVD), scanning electron Microscopy (SEM), focused ion beam milling (FIB), profilometry, atomic force microscopy (AFM), electron diffraction, and ellipsometry. Measurement techniques include: micro-photoluminescence ( $\mu$ PL), scanning confocal microscopy, scanning near field optical microscopy (SNOM), charged coupled device (CCD) imaging. A brief background of these techniques is provided.

In Chapter-4 the material system used is introduced, the particular fabrication process used to create the scattering particles is outlined (EBL and ICP etch) and the optimisation



experiments are shown. The result is a process to create structures with smooth vertical sidewalls optimised for photonic applications.

Chapter-5 is the first of the results chapters. A direct imaging technique used to map the in plane scattering of microdisks is introduced. The calibration of the system is outlined and the image interpretation is discussed. Results are for the dependence of the near-field scattered intensity from microdisks on the size parameter, the incident angle of the light, and the polarisation of the incident and collected light. The effect of the substrate on the scattering from the structures is also investigated. The results are compared with 2D and 3D FEM results. The effect of Gaussian focused illumination and change of refractive index contrast are briefly explored.

In Chapter-6 the direct imaging method is applied to scattering particles with square and equilateral triangle cross sections. The scattered field from these shapes is very sensitive to the orientation of the particle with respect to the incident beam. Four symmetric cases are examined in detail as well as the polarisation dependence.

In Chapter-7 an experimental technique for mapping the out of plane far field distribution is introduced. This works by imaging the back focal plane of the objective onto the CCD and gives a map of the far field intensity as a function of angle. These experimental results are compared with those calculated using 3D FEM and the Stratton-Chu formula. The agreement between them is reasonable and provides insight into the image formation in the direct imaging method introduced in the previous chapter.

In Chapter-8 slightly larger structures in the geometrical optics regime are considered. These structures known as Cartesian ovals have an elliptical cross section optimised to focus light without any spherical aberration. Some variations on these structures are proposed which allow us to create tight focal volumes outside the structures.

Then in Chapter-9 we show results relating to whispering gallery modes (WGM) in the microdisks obtained in parallel with the scattering results. The material used ( $\text{Si}_3\text{N}_4$ ) emitted a broad defect photoluminescence (PL) when pumped with a 488nm  $\text{Ar}^+$  laser. This PL was modulated with the resonance structure of the cavity. This is a common technique for exploring the properties of resonators, and was exploited in this project to investigate the modal structure of the microdisks. The modes of the cavity are fully characterised and compared with an Effective index analytical theory. The transition between microdisk and microring resonators is explored by comparing size matched disks and rings of different thicknesses.

Finally Chapter-10 is a general conclusion on the entire work carried out in the project and a suggestion for future work based on the techniques developed in this thesis.



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## Chapter 2 Light Scattering from microdisks: theory

### 2.1 Introduction

Although the scattering of light by small particles has been an intense area of study for many years, analytically calculating the near-field scattered from arbitrarily shaped objects in the size parameter range  $1 < x_s < 100$  still remains a major issue. This range is slightly too big for approximations of Rayleigh scattering to hold and slightly too small for geometrical optics. It is sometimes referred to as the anomalous diffraction regime [1]. In this regime Maxwell's equations must be solved fully without any simplifications in order to accurately capture the scattering effects. Unfortunately full analytical solutions exist for only a few shapes. Spheres and infinite circular cylinders in homogeneous mediums are well understood and give rise to intriguing resonance structures in the scattering parameters [2]. Variations on these such as ellipsoids, thin disks and wires can also be solved for by extension of the theories [1].

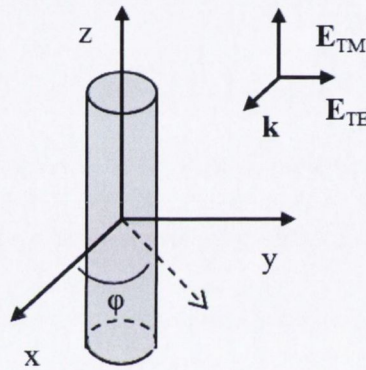
In this chapter the analytical theory for scattering of a plane wave by an infinite circular cylinder is outlined as a first approximation to scattering from a disk. From this the dependence of features in the near field intensity distribution on cylinder diameter and refractive index are explored. Optimum parameters to create a PNJ on the shadow side of the cylinder are identified.

A lot can be understood about the dependence of the main scattering lobe on the physical parameters using the 2D analytical solution of an infinite cylinder. This project however deals with structures of a finite height which is on the order of the wavelength of the incident light. This means that not only do these structures scatter light in the plane of the incident beam, but they also strongly scatter light out of plane. In order to calculate the near field of arbitrarily shaped 3D scatters numerical techniques are required. The finite element method [FEM] is one of the commonly used numerical methods in electromagnetism [3]. It is a powerful numerical tool for solving partial differential equations, and is ideally suited to scattering problems of this type. The method is introduced, and its specific application in this project is outlined. The far-field distribution can then be calculated from the FEM results using the analytical Stratton-Chu integral equation [4]. This approach is outlined and its accuracy is validated by comparison with scattering from analytically solvable shapes.



## 2.2 Analytical solution for an infinite circular cylinder

A complete analytical solution for the electric and magnetic fields can be found for the case of an infinite cylinder in a homogeneous medium, using a separation of variables approach to solve Maxwell's Equations in cylindrical co-ordinates. In this method, the field is split into three parts, the incident field ( $E^i$ ), the internal field ( $E^{int}$ ), and the scattered field ( $E^s$ ). The total field outside the scatterer is given by vector addition of  $E^i$  and  $E^s$ . The geometry and co-ordinate system is shown in Fig.2.1. The incident field is a time harmonic plane wave propagating in the positive x direction, which is given in cylindrical co-ordinates by Eq.2.1 for the TM case.



**Figure 2.1** Geometry of infinite circular cylinder of radius  $a$ , illuminated by a plane wave propagating in the  $x$  direction. TE and TM polarisations as shown.

The internal and scattered fields are obtained by expanding the electric field in terms of circular cylindrical functions (Bessel and Hankel functions) and then imposing boundary conditions to find the expansion coefficients. Arbitrary incident polarisation can be described by the basis cases transverse electric (TE) and transverse magnetic (TM). For the TM case the incident light is polarised along the axis of the cylinder, whereas for TE it is polarised orthogonal to the axis as shown in Fig.2.1. In the TM case, the electric field has components only in the  $z$  direction along the axis of the cylinder. The analytical expressions for the incident, scattered and internal electric field are then given by:

$$E_z^i(kr, \phi) = E_0 \sum_{n=-\infty}^{\infty} i^n J_n(kr) e^{in\phi} \quad (2.1)$$

$$E_z^s(kr, \phi) = -E_0 \sum_{n=-\infty}^{\infty} i^n b_n H_n^{(1)}(kr) e^{in\phi} \quad (2.2)$$

$$E_z^{\text{int}}(mkr, \phi) = E_0 \sum_{n=-\infty}^{\infty} i^n d_n J_n(mkr) e^{in\phi} \quad (2.3)$$

$E_0$  is the incident electric field amplitude. The refractive index contrast  $m$  is the ratio of the refractive index of the cylinder to the surrounding medium. The size parameter  $x = 2\pi R / \lambda = kR$  is fixed for a particular case by the ratio of the cylinder radius  $R$  to the incident wavelength  $\lambda$ . This is the natural unit to use as the analytical solution does not depend on the wavelength and radius independently, but on their ratio.  $J_n$  is the Bessel function of order  $n$ , and  $H_n^{(1)}$  is the Hankel function of the first kind or order  $n$ . The expansion coefficients  $b_n$  and  $d_n$  are complex scalars given by Eq.2.4 and Eq.2.5. They are fixed by satisfying the boundary conditions at the cylinder surface and depend only on the size parameter  $x$  and the refractive index contrast  $m$ .

$$b_n = \frac{mJ'_n(mx)J_n(x) - J_n(mx)J'_n(x)}{mJ'_n(mx)H_n^{(1)}(x) - J_n(mx)H_n^{(1)'}(x)} \quad (2.4)$$

$$d_n = \frac{1}{J_n(mx)} [J_n(x) - b_n H_n^{(1)}(x)] \quad (2.5)$$

The prime refers to the derivative of these functions with respect to their argument. For the TE case, the electric field has both radial and azimuthal components. The expressions for the incident, scattered and internal field components are:

$$E_r^i(kr, \phi) = \frac{E_0}{kr} \sum_{n=-\infty}^{\infty} i^n J_n(kr) e^{in\phi} \quad (2.6)$$

$$E_\phi^i(kr, \phi) = -iE_0 \sum_{n=-\infty}^{\infty} i^n J'_n(kr) e^{in\phi} \quad (2.7)$$

$$E_r^s(kr, \phi) = \frac{E_0}{kr} \sum_{n=-\infty}^{\infty} i^n a_n n H_n^{(1)}(kr) e^{in\phi} \quad (2.8)$$

$$E_\phi^s(kr, \phi) = iE_0 \sum_{n=-\infty}^{\infty} i^n a_n H_n^{(1)'}(kr) e^{in\phi} \quad (2.9)$$

$$E_r^{\text{int}}(mkr, \phi) = -\frac{E_0}{m^2 kr} \sum_{n=-\infty}^{\infty} i^n c_n n J_n(mkr) e^{in\phi} \quad (2.10)$$

$$E_\phi^{\text{int}}(mkr, \phi) = -\frac{E_0}{m} \sum_{n=-\infty}^{\infty} i^n c_n J'_n(mkr) e^{in\phi} \quad (2.11)$$



The expansion coefficients in this case are:

$$a_n = \frac{J'_n(mx)J_n(x) - mJ_n(mx)J'_n(x)}{J'_n(mx)H_n^{(1)}(x) - mJ_n(mx)H_n^{(1)'}(x)} \quad (2.12)$$

$$c_n = \frac{1}{J_n(mx)} [J_n(x) - a_n H_n^{(1)}(x)] \quad (2.13)$$

A complete derivation can be found in the standard scattering text books by P.W. Barber and S.C. Hill [5], C.F. Bohren and D.R. Huffman [2] and H.C. van de Hulst [1].

### 2.2.1 Useful quantities

The solutions for the electric field components can then be used to calculate experimentally observable quantities. The total intensity outside the cylinder for TM and TE is proportional to:

$$I_{ext}^{TM} = |E_z^i|^2 + |E_z^s|^2 \quad (2.14)$$

$$I_{ext}^{TE} = |E_r^s + E_r^i|^2 + |E_\phi^s + E_\phi^i|^2 \quad (2.15)$$

Whereas the intensity inside the cylinder is given by:

$$I_{int}^{TE} = |E_z^{int}|^2 \quad (2.16)$$

$$I_{int}^{TM} = |E_r^{int}|^2 + |E_\phi^{int}|^2 \quad (2.17)$$

The far field scattered intensity as a function of angle for the TM case can be simplified to [1]:

$$I_s(\phi, x, m) = \frac{2}{\pi k} \left| b_0 + 2 \sum_{n=1}^{\infty} b_n \cos(n\phi) \right|^2 I_0 \quad (2.18)$$

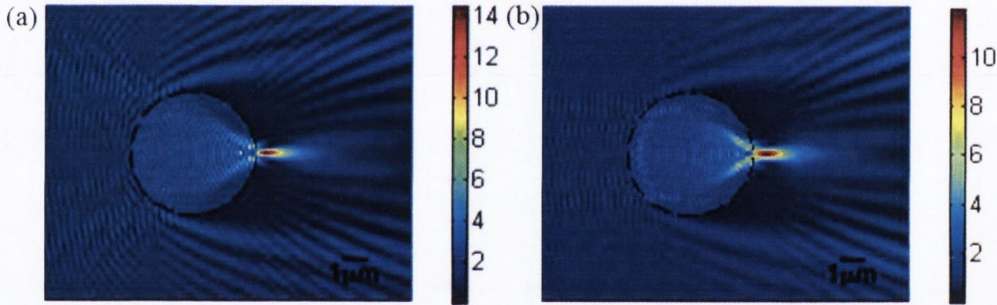
where  $I_0$  is the incident intensity. The expression for the TE case is found by replacing  $b_n$  with  $a_n$ . Since for infinite cylinders the light is diffracted in only in the plane perpendicular to the cylinder's axis, the intensity decrease as a function of  $1/r$  and not the usual  $1/r^2$ . The angular dependence is contained in the terms in the norm.

### 2.2.2 Numerical evaluation

The infinite sums in Eq. 2.1-2.3 and Eq. 2.6-2.11 must be calculated numerically. Only a finite number of terms in the sum contribute to the scattered field and this increases with the size parameter  $x$ . Empirically it is found that the sum can be truncated at a finite value which depends on the size parameter:

$$n_c = x + 4.05x^{1/3} + 2 \quad (2.19)$$

This is known as the Wiscombe condition [6]. The electric fields can be calculated by numerically summing Eq. 2.1-2.3 for the TM case and Eq. 2.6-2.11 for the TE case. The near field intensity distribution can then be calculated using Eq. 2.14-2.15 for TM and Eq.2.16-2.17 for TE. This is implemented through MATLAB and the code is given in Appendix A.



**Figure 2.2** (a) Intensity map of the near-field of an infinite cylinder of  $5\mu\text{m}$  diameter and refractive index 1.5. TM polarised plane wave of wavelength 532nm is incident from the left. (b) TE polarised case. Colour scale in units of incident intensity

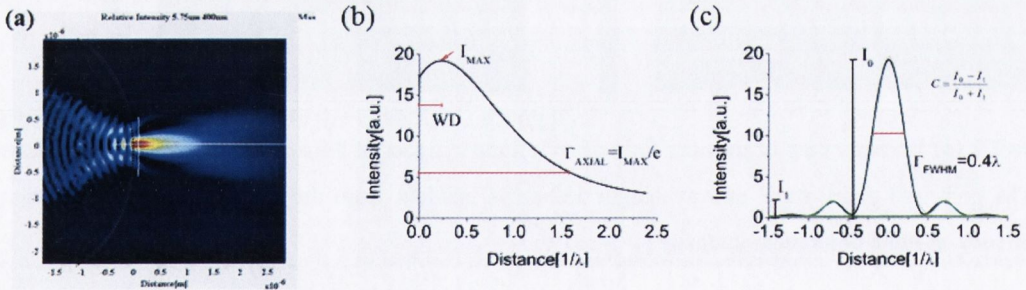
Figure 2.2 shows the total intensity distribution for infinite circular cylinders of  $5\mu\text{m}$  diameter. The incident wavelength is 532nm and light propagates from left to right. The light is polarised out of the plane of the page i.e. TM case in (a) and TE polarised in (b). The false colour map shows the intensity relative to the input intensity  $I_0$ . The refractive index contrast in this case is 1.5.

The main characteristics of interest from the near field scattering distribution are a long main scattering lobe on the shadow side of the cylinder and lower intensity higher order lobes scattering at higher angles. We see from Fig.2.2 that the TM case gives a narrower main lobe. From Eq. 2.1-2.3 and Eq.2.6-2.11 it is evident that the radial dependence of the scattered field is a combination of different orders of weighted Bessel functions, whereas the angular dependence is contained in a weighted sum of  $\cos(n\theta)$  terms. Only a finite number of terms in the sum are non-zero and the main contribution comes from values of  $n$  less than  $x$ . Thus the scattered field



is dominated mainly by these components. This results in a fixed number of lobes in the scattered field which increases with the size of the disk. These vary from even to odd numbers of lobes, which results in either a single central lobe or two symmetric ones either side of the centre of the disk respectively in the scattered field  $\mathbf{E}_s$ . The near field intensity distribution of the central lobe has been termed a photonic nanojet (PNJ) [7].

The  $1/e^2$  waist, length and maximum intensity of this lobe depend in a complex manner on the size parameter and the refractive index contrast. These near-field maps require on the order of twenty seconds to be computed. This means we can systematically vary the parameters of the system and observe the effects on the near field intensity distribution. The scattered field depends strongly on the diameter of the cylinder  $D$ , the incident wavelength  $\lambda$  and the refractive index contrast of the cylinder to the surrounding medium  $m$ . To see the effect of varying these parameters we characterise the near-field intensity distribution by defining six parameters. Fig.2.3 shows the scheme in which we extract information from the numerical simulations. We take two linear profiles of the intensity distribution. The first is an axial profile shown as a horizontal line in Fig. 2.3 (a), and plotted in Fig. 2.3 (b). The second is a transverse profile (vertical line in Fig. 2.3 (a)), and plotted in Fig. 2.3 (c).



**Figure 2.3:** (a) Close up of the relative intensity distribution of scattering from a cylinder. The white lines indicate the axes over which we took one dimensional profiles of the intensity. (b) Axial intensity profile. We define  $I_{MAX}$  as the maximum Intensity in the axial profile. By fitting a Lorentzian distribution to the profile we define  $\Gamma_{AXIAL}$  as the  $1/e^2$  distance. The vertical dashed line defines the surface of the cylinder (c) Transverse intensity profile taken at 50nm from the spheres surface. We fit each lobe with a Gaussian and define  $\Gamma_{FWHM}$  as the FWHM of the central peak. The contrast  $C$  is defined as shown

The interesting properties of the PNJ are its maximum intensity  $I_{max}$ , the FWHM at the maximum  $\Gamma$ , the  $1/e^2$  falloff of the jet, the divergence angle of the main lobe  $\theta$ , the distance between the maximum intensity and the surface of the cylinder WD, and finally the contrast  $C$  which defines the ratio between the intensity in the jet and the next nearest lobes. The following table explains the variable parameters and dependent parameters we use to characterise the system, as illustrated in the plots of Fig.2.3(b) and (c).



### Variable Parameters

**M:** Refractive index contrast ( $n/n_0$ )  
**D:** The diameter of the scatterer  
 **$\lambda_0$ :** The wavelength of the incident light in free space

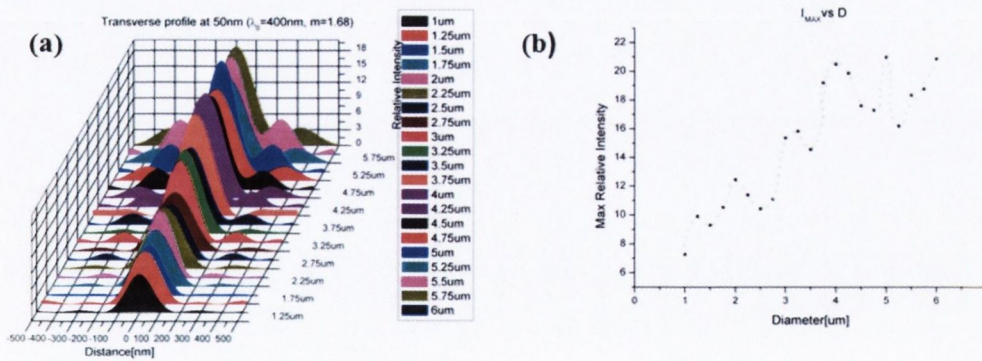
### Dependent Parameters

**$I_{MAX}$ :** Maximum intensity along the axis  
 **$\Gamma_{FWHM}$ :** Full Width Half maximum of Intensity of the central lobe transverse to propagation at the position of  $I_{max}$   
 **$\theta$ :** The divergence angle of the main lobe . Defined as the angle subtended by the line joining points on the FWHM of the jet.  
**WD:** The distance from the surface of the sphere to the point of highest intensity in the axial plane.  
 **$\Gamma_{AXIAL}$ :** Refractive index contrast ( $n/n_0$ )

$\lambda_0$  and D are connected through the size parameter  $x_s = \pi D / \lambda_0$ . In our case however we are interested in calculating the actual intensity in the focal area for different incident wavelengths and cylinder diameters. The intensity will increase with the diameter, so we vary D and  $\lambda_0$  independently. We begin by varying one variable parameter while holding the other two constant and observing the effect on the dependent parameters.

### 2.2.3 Dependence on cylinder diameter

In order to investigate the dependence of  $I_{MAX}$  on D, we perform a numerical experiment, where we hold the wavelength constant while we vary the diameter of the cylinder in steps of 250nm from 1 $\mu$ m to 5 $\mu$ m. We choose a refractive index of 1.68 at 400nm, which corresponds to Melamine Formaldehyde (MF), a relatively high refractive index polymer.

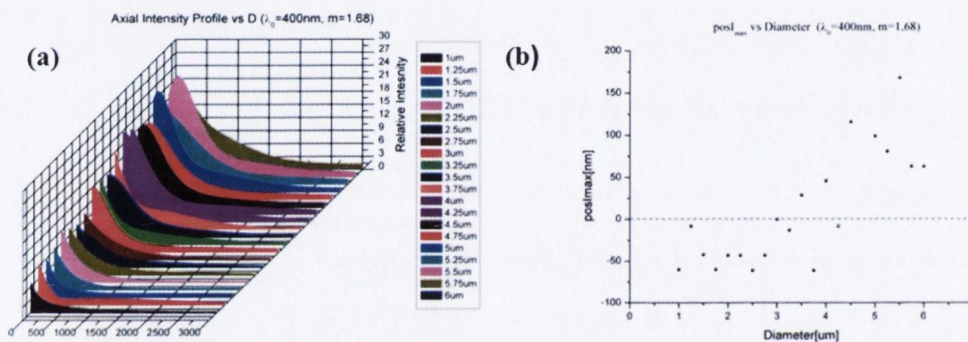


**Figure 2.4 :** (a) Waterfall plot of the transverse intensity profile for increasing cylinder diameter. The profile is taken 50nm from the surface of the sphere (vertical line in Fig.2.3(a)). There are 200 points linearly distributed symmetrically about the axis over 1 $\mu$ m. We see the variation in size and position of the lobes. (b) The maximum intensity ( $I_{MAX}$ ) at a fixed position 50nm from the cylinder surface is plotted against diameter. We see a gradual increase with a superimposed ripple structure increasing frequency with increasing diameter.



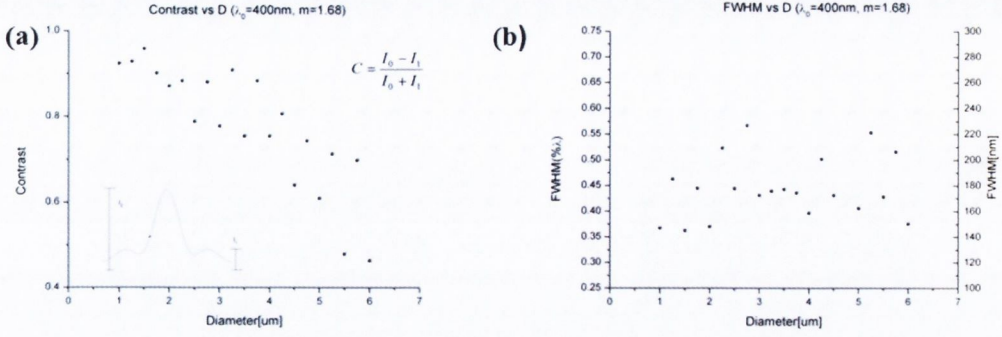
Figure 2.4 shows that as the diameter increases the intensity of the main lobe and first order lobes increase, but not monotonically. The increase is gradual with a superimposed "ripple" structure with increasing "ripple frequency" with increasing diameter, as shown in Fig.2.4(b). It is well known that the overall extinction, scattering and absorption efficiencies contain this structure [5], but it is not obvious that the near-field intensity distribution ( and PNJ) would contain the same structure. This suggests optimum sizes of the cylinders to obtain high intensity and high contrast between the central and side lobes.

As expected there is also a strong dependence of the axial intensity distribution on the variable parameters. The shape of the decay is not trivial and depends on the size parameter as shown in Fig. 2.5 (a).



**Figure 2.5 :** (a) Axial intensity distribution versus the distance from the cylinder surface. The intensity oscillates inside the cylinder and then produces a peak close to the surface, with a long tail extending out to 3  $\mu\text{m}$ . (b) The position of the point of highest intensity versus the diameter of the cylinder. The field maximum is inside the cylinder for diameters under 3  $\mu\text{m}$  for a refractive index contrast of 1.68.

From the waterfall plots it is clear that there is an important dependence of both the axial and transverse intensity profiles on the cylinder diameter. Thus it is very important to optimise the cylinder size for particular applications. The contrast between the main lobe and the side lobes decreases with increasing diameter as shown in Fig. 2.6 (a). The transverse profile of the main lobe can be approximated by a Gaussian (Fig. 2.3 (c)) and fitted to obtain the FWHM as a function of diameter. The FWHM, plotted in Fig. 2.6 (b), varies around  $0.45 \lambda_0$ . By choosing carefully the cylinder diameter we can reduce the main lobe as far as  $\lambda_0/3$ . This is possible as the main lobe does not contain all of the scattered power, and some is scattered at high angles [8].

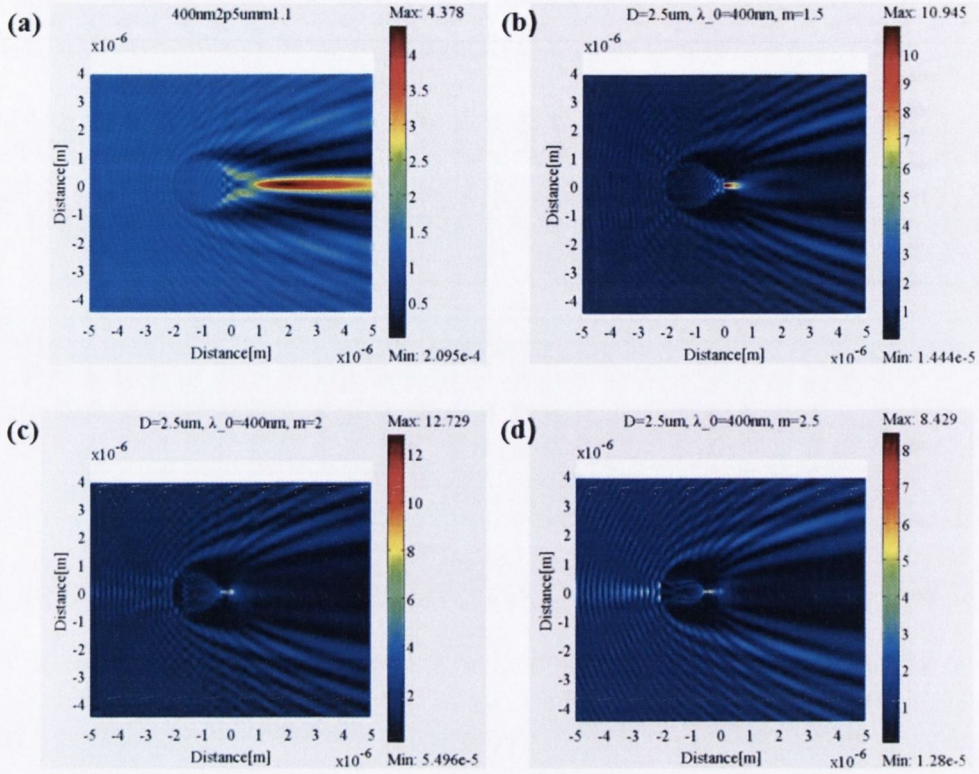


**Figure 2.6 :** (a) There is a general decrease in contrast with increasing cylinder diameter. The secondary lobes become more significant with increasing diameter, however the absolute value of intensity increases with cylinder size. (b) The FWHM of the central lobe varies around  $0.45 \lambda_0$ .

#### 2.2.4 Dependence on refractive index contrast $m$

For this study we choose a cylinder of diameter  $2.5\mu\text{m}$  and vary the refractive index contrast while keeping the wavelength fixed at  $400\text{nm}$ . We choose a  $2.5\mu\text{m}$  cylinder on the basis that the scattering resonances will be quite broad, yet observable and should become narrower as the refractive index increases. We find that the shape and intensity of the beam depends greatly on the refractive index. This is to be expected as the Mie scattering spectrum is very sensitive to changes in refractive index. This variation makes it more difficult to accurately compare the distributions. We look at the max intensity ( $I_{\text{MAX}}$ ), the intensity position from the cylinder surface (WD) and the FWHM in the transverse plane at the position of maximum axial intensity ( $\Gamma_{\text{FWHM}}$ ) as a function of refractive index contrast. Near-field intensity profiles are shown at values of  $m=1.1$ ,  $m=1.5$ ,  $m=2$ , and  $m=2.5$  in Fig. 2.7.

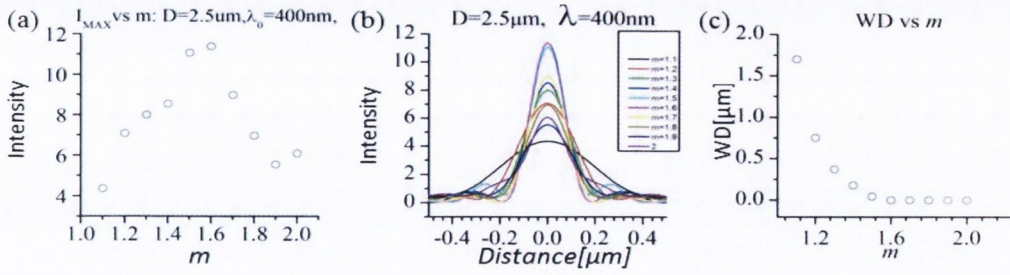




**Figure 2.7** : Sample images of the intensity distribution as a function of refractive index contrast. **(a)**  $m=1.1$ , note the low intensity contrast and elongated shape **(b)**  $m=1.5$  closer to the optimum regime **(c)**  $m=2.0$ , PNJ is still high intensity, but maximum is now inside the surface. **(d)**  $m=2.5$ , PNJ completely destroyed, large backscattering.

Intuitively, one might think that since the width of the main lobe is defined by half the wavelength in the medium, in order to decrease the spot size one should increase the refractive index. We show that this is true however there is a limit to how far you can go. For a refractive index contrast of over 2 (Fig 2.7 (c) and (d)), the light becomes highly confined in the modes of the cylinder and is trapped by total internal reflection (TIR). Most of the light is backscattered and the intensity in the main lobe quickly drops off.





**Figure 2.8 :** (a) The intensity in the jet shows a peak at a refractive index contrast of around 1.6. This shows that material is an important consideration in (b) The transverse intensity profile is taken at the position of maximum intensity for varying refractive indices. (c) The variation of the position of maximum intensity in the axial direction is shown to vary exponentially with refractive index.

Again, there is a non-trivial dependence of the jet properties on refractive index contrast. We see that the FWHM maximum of the jet depends on  $m$  as shown in Fig. 2.8 (b) . The position of maximum intensity in the axial direction depends exponentially on the refractive index (Fig.2.8(c)). Another important result is that there is an optimum refractive index contrast to achieve the highest intensity in the PNJ. Fig. 2.8 (a) shows this is around  $m=1.6$ .

## 2.3 The finite element method

### 2.3.1 Introduction

A lot of insight into light scattering by microdisks can be gained by studying the dependence of scattering from an infinite cylinder in the previous section. The microdisks however are 3D structures and scatter light strongly out of plane as-well as in-plane. We use a 3D FEM approach to model these structures fully.

In general FEM works by converting a PDE into its weak form. The geometry is then divided into a finite number of discrete elements. The solution is assumed to be a linear combination of shape functions which approximate the solution over each domain. This linear combination is put into the weak form on the PDE and the same shape functions are used as test functions to create a system of linear equations for the expansion coefficients of the linear combination. The result is a large symmetric matrix which can be solved either directly using standard techniques such as Gauss Jordan elimination or iterative methods.

One main advantage of FEM over other numerical techniques is that the size and shape of the elements can be chosen arbitrarily. Arbitrarily shaped geometries can be accurately modelled and areas of interest can be solved with higher resolution using a variable mesh size. The finite element method is one of the commonly used numerical techniques in electromagnetism. It is used for scattering problems in two and three dimensions [9]. Examples



of its application include calculating curvature loss [10] and solving non-linear problems [11] in waveguides. It has been used to calculate mode volumes in microdisks with different sidewall profiles [12-15]. It is commonly used to calculate near field profiles in metallic particles exhibiting surface plasmon resonances [16].

### 2.3.2 Example: Heat conduction in 1D

FEM can be best understood by application to a simple 1D PDE problem [17]. Consider a rod of length  $L$  whose left end is exposed to a constant heat flux  $q$  and with the right end held at a constant temperature  $T=T_L$ , as shown in Fig.2.9. Assume also that the wire is insulated and that it has an internal heat source of magnitude  $Q$  (say from a current passing through the wire for example). We wish to find the steady state temperature distribution as a function of position of the rod  $T(x)$ .

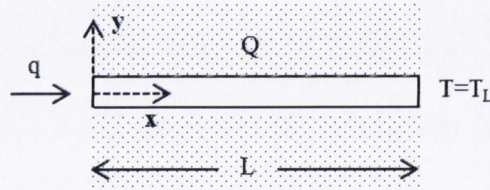


Figure 2.9 Conducting rod of length  $L$ .

The equation governing this system is the 1D heat equation which is easily found by applying Fourier's law. The equation and boundary conditions for this case are

$$\begin{aligned}
 -K \left[ \frac{d^2 T}{dx^2} \right] &= Q & \text{for } x \in [0, L] \\
 -K \left[ \frac{d^2 T}{dx^2} \right] &= q & \text{for } x = 0 \\
 T &= T_L & \text{for } x = L
 \end{aligned} \tag{2.20}$$

Since the thermal conductivity  $K$  in this case is constant the analytical solution of this equation is easily found by integration to be

$$T_A(x) = T_L - \frac{q}{K}(L-x) + \frac{Q}{2K}(L^2 - x^2) \tag{2.21}$$

Now we can find the solution of Eq. 2.20 numerically using FEM and compare with Eq. 2.21. We begin by dividing the domain into sub intervals known as finite elements  $e_k$  (Fig.2.10).

$$e_k = \{x : x_k \leq x \leq x_{k+1}\} \quad (2.22)$$



**Figure 2.10:** Discretisation of domain into elements  $e_k$  and node points  $x_k$ .

The end points  $x_k$  of each interval are known as nodes. The temperature distribution will then be approximated using known predetermined functions of  $x$ , denoted  $\phi_j(x)$ . Over the whole domain  $0 \leq x \leq L$

$$T(x) \cong a_1\phi_1(x) + a_2\phi_2(x) + \dots + a_{n+1}\phi_{n+1}(x) \quad (2.23)$$

The functions  $\phi_i$  are called the shape functions.  $T(x)$  is the approximate solution and  $T_A(x)$  is the true solution. We define a residual functional  $R(T,x)$  such that

$$R(T,x) = -K \frac{d^2T}{dx^2} - Q \quad (2.24)$$

For the approximate solution  $T(x)$  this will be non-zero and we know from Eq.2.21 that

$$R(T_A,x) = -K \frac{d^2T}{dx^2} - Q = 0 \quad (2.25)$$

So for any  $T \neq T_A$  we cannot force the residual to vanish at every point, instead we multiply the residual  $R$  by a weighting function  $W$ , and force the integral of the weighted expression to vanish. This gives us weak solutions

$$\int_0^L W(x)R(T,x)dx = 0 \quad (2.26)$$

By choosing different weight functions and replacing them in Eq. 2.14 we can generate a system of linear equations for the unknown parameters  $a_i$  which will then determine the approximate solution  $T(x)$  given by Eq. 2.23. The type of weighting function is arbitrary. We will use the Galerkin method which chooses the shape functions in order to create symmetric matrices and reduce computational effort

$$W_i(x) = \phi_i(x) \quad (2.27)$$

Subbing Eq.2.24 and 2.27 into Eq.2.26 gives



$$\int_0^L \phi(x) \left[ -K \frac{d^2 T}{dx^2} - Q \right] dx = 0 \quad (2.28)$$

Integrating by parts to the second derivative term results in

$$\int_0^L \epsilon \frac{d\phi}{dx} \frac{dT}{dx} dx - \int_0^L \phi Q dx - K \phi \frac{dT}{dx} \Big|_{x=0}^{x=L} = 0 \quad (2.29)$$

Writing T as a sum of weight functions given by Eq. 2.23 gives us an equation for each weighting function  $\phi_i$

$$\sum_{j=1}^{n+1} \left[ -\int_0^L \epsilon \frac{d\phi_i}{dx} \frac{d\phi_j}{dx} dx \right] a_j - \int_0^L \phi_i Q dx - K \phi_i \frac{dT}{dx} \Big|_{x=0}^{x=L} = 0 \quad (2.30)$$

$$i = 1, 2, \dots, n+1$$

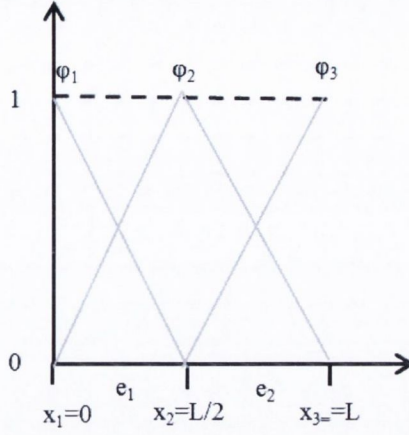
Now we need to choose our node points, and the shape functions. For the simplest case we divide our domain into two equally spaced segments  $e_1$  and  $e_2$ , so that we have 3 nodal points  $x_1 = 0$ ,  $x_2 = L/2$  and  $x_3 = L$ . We choose linear shape functions  $\phi_i$  such that  $\phi_i(x_i) = 1$  and  $\phi_i(x_{i+1}) = 0$  and vice versa:  $\phi_{i+1}(x_i) = 0$  and  $\phi_{i+1}(x_{i+1}) = 1$ :

$$\begin{aligned} \phi_i(x) &= \frac{x_{i+1} - x}{x_{i+1} - x_i} \\ \phi_{i+1}(x) &= \frac{x - x_i}{x_{i+1} - x_i} \end{aligned} \quad (2.31)$$

With this basis the parameters  $a_i$  become the approximations for the temperature T at each node. The derivatives of the shape functions are

$$\begin{aligned} \frac{d\phi_i}{dx} &= \frac{-1}{x_{i+1} - x_i} \\ \frac{d\phi_{i+1}}{dx} &= \frac{1}{x_{i+1} - x_i} \end{aligned} \quad (2.32)$$

The nodes, elements and linear shape function are depicted in Fig.2.11



**Figure 2.11:** Nodes  $x_k$ , elements  $e_k$  and shape functions  $\phi_k$ .

We can now evaluate Eq.2.30 on each the three domains individually. If we introduce the

matrix notation  $\phi = [\phi_i \ \phi_i]$  and  $\mathbf{a} = \begin{bmatrix} a_i \\ a_{i+1} \end{bmatrix}$  then Eq. 2.30 on  $e_1$  becomes

$$K \left[ -\int_0^{L/2} \varepsilon \left[ \frac{d}{dx} \phi \right]^T \left[ \frac{d}{dx} \phi \right] dx \right] \mathbf{a} - Q \int_0^L \phi^T dx - \begin{bmatrix} q \\ 0 \end{bmatrix} = 0 \quad (2.33)$$

But on domain  $e_1$

$$\phi = \begin{bmatrix} 1-2x/L \\ 2x/L \end{bmatrix} \text{ and } \frac{d}{dx} \phi = \begin{bmatrix} -2/L \\ 2/L \end{bmatrix}$$

Subbing these into Eq. 2.33 and integrating gives

$$\frac{2K}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} - \frac{QL}{4} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} q \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (2.34)$$

The same procedure for  $e_2$  results in

$$\frac{2K}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix} - \frac{QL}{4} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (2.35)$$

Combining Eq. 2.34 and 2.35 gives

$$\frac{2K}{L} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} - \begin{bmatrix} q \\ 0 \\ 0 \end{bmatrix} - \frac{QL}{4} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (2.36)$$

Since we know  $a_3 = T_L$  from the boundary conditions equation 2.36 reduces to a solvable system of two linear equations with two unknowns  $a_1$  and  $a_2$

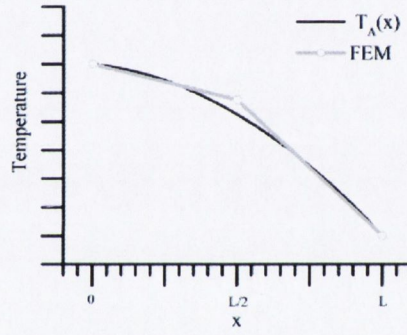


$$\begin{aligned}
 a_1 - a_2 &= \frac{qL}{2K} + \frac{QL^2}{8K} \\
 -a_1 + a_2 &= \frac{QL^2}{4K} + T_L
 \end{aligned}
 \tag{2.37}$$

which has solutions

$$\begin{aligned}
 a_1 &= \frac{qL}{K} + \frac{QL^2}{2K} + T_L \\
 a_2 &= \frac{qL}{2K} + \frac{3QL^2}{8K} + T_L
 \end{aligned}
 \tag{2.38}$$

This approximate solution can then be compared with the analytical one derived in Eq. 2.21.



**Figure 2.12** Comparison between analytical solution  $T_A$  and values obtained through FEM

Figure 2.12 shows the plot of the analytical solution from Eq.2.21 compared with the piecewise linear approximation calculated from FEM. The solution is not exact, and can be improved by dividing the domain into more elements. This solution used linear interpolation, but higher order polynomials can be used to increase accuracy. The extension to higher dimensions is straight forward, however higher dimensions require larger matrices and the computational demands rise rapidly. In two dimensions the space is usually discretised using triangular elements in 2D and tetrahedrons in 3D, but higher order polygons are also possible.

The FEM allows us to accurately calculate the EM field in the vicinity of an arbitrarily shaped scatterer, however the size regime we are interested in is difficult as it requires large matrices and a lot of computer memory. Recent advances in computing power mean that we can push these numerical techniques into the small particle regime and get near-field distributions for arbitrary shapes. The simulations in this project are performed on a workstation with 8 processors (two intel quad cores) and 64GB of RAM.

### 2.3.3 Implementation in COMSOL

COMOSL MULTIPHYSICS™ is a commercially available finite element method package. It provides the capability to utilise FEM efficiently in large problems, and couple different physical effects such as EM and heat transfer and mechanical deformation. The PDEs which govern the interaction of electric and magnetic fields and matter, (and ultimately the propagation of EM radiation in media) are the macroscopic Maxwell equations. They are an elegant collection of physical laws which were discovered separately and amalgamated by Maxwell in 1861[18]. In differential form they are

$$\vec{\nabla} \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (2.39)$$

$$\vec{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.40)$$

$$\vec{\nabla} \cdot \mathbf{D} = \rho \quad (2.41)$$

$$\vec{\nabla} \cdot \mathbf{B} = 0 \quad (2.42)$$

where  $\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$  is the magnetic field strength (with  $\mathbf{M}$  the magnetisation),  $\mathbf{J}$  is a current density,  $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$  is the displacement field (with  $\mathbf{P}$  the polarisation field),  $\mathbf{E}$  is the electric field and  $\mathbf{B}$  is the magnetic flux density. Equation 2.39 is known as the Maxwell Ampere law and states that a current density  $\mathbf{J}$  or a time varying electric field  $\mathbf{D}$  can act as a source of magnetic field  $\mathbf{H}$ . Equation 2.40 is Faradays law that a time varying magnetic field produces an electric field. Equations 2.41 and 2.42 are Gauss' laws for electric and magnetic fields respectively. Equation 2.41 states that the electric flux across a closed surface is equal to the charge it encloses (in integral form). Equation 1.42 states that there are no magnetic monopoles.

The displacement field and the magnetic field strength are related to the electric field and the magnetic flux density via the constitutive equations, which for a linear isotropic material are given by

$$\mathbf{D} = \epsilon \mathbf{E} \quad \epsilon = \epsilon_r \epsilon_0 = (1 + \chi) \epsilon_0 \quad (2.43)$$

$$\mathbf{H} = \frac{1}{\mu} \mathbf{B} \quad \mu = \mu_r \mu_0 \quad (2.44)$$

$$\mathbf{J} = \sigma \mathbf{E} \quad (2.45)$$



where  $\sigma$  is the conductivity. Taking the curl of Eq. 2.39 and using Eq.2.40 and the constitutive relations result in

$$\vec{\nabla} \times \mu_r^{-1} (\vec{\nabla} \times \mathbf{E}) - k_0^2 (\epsilon_r - \frac{j\sigma}{\omega\epsilon_0}) \mathbf{E} = 0 \quad (2.46)$$

which using the vector identity

$$\vec{\nabla} \times (\vec{\nabla} \times \mathbf{A}) = \vec{\nabla} (\vec{\nabla} \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \quad (2.47)$$

can be written as

$$\nabla^2 \mathbf{E} - k_0^2 n^2 \mathbf{E} = 0 \quad (2.48)$$

The FEM method is then used to solve equation 2.48 over the domain of interest.

### 2.3.4 Boundary conditions

Any PDE system needs boundary conditions in order to model the specific physical problem. At the boundary between two different materials we have the following conditions:

$$\begin{aligned} \mathbf{n} \times (\mathbf{E}_1 - \mathbf{E}_2) &= \mathbf{0} \\ \mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) &= \mathbf{0} \end{aligned} \quad (2.49)$$

where  $\mathbf{n}$  is the outward normal unit vector, and the subscripts 1, 2 denote two different materials. It is these boundary conditions that result in reflection and refraction of light at an interface. Boundary conditions can be used to take advantage of symmetry in the model. For example we use the perfect electric conductor (PEC) boundary condition to halve the size of the simulation domain needed:

$$\mathbf{n} \times \mathbf{E} = \mathbf{0} \quad (2.50)$$

This is a special case of the electric field boundary condition that sets the tangential component of the electric field to zero. It is used for the modelling of a lossless metallic surface, for example a ground plane or as a symmetry type boundary condition. It imposes symmetry for magnetic fields and “magnetic currents” and anti-symmetry for electric fields and electric currents.

Another boundary condition used is the Perfect Magnetic Conductor (PMC) boundary condition;

$$\mathbf{n} \times \mathbf{H} = \mathbf{0} \quad (2.51)$$

It is a special case of the surface current boundary condition that sets the tangential component of the magnetic field and thus also the surface current density to zero.

It imposes symmetry for electric fields and electric currents.

### 2.3.5 Stratton-Chu equations

Often it is interesting to look at the EM field distribution far away from the scatterer. In free space propagating radiation will have electric and magnetic fields transverse to the direction of propagation. The intensity will fall off as  $1/r^2$ . It is common to take the limit as  $r \rightarrow \infty$ , and look at the angular distribution of the light. This is known as the far field angular distribution. The FEM method works over a finite domain and calculates the electric and magnetic fields close to the scatterer in the near field zone. The far field angular distribution can be extracted from the near field using a surface integral equation, known as the Stratton-Chu equation [4]. The Stratton Chu formula in COMSOL in 2D is given by:

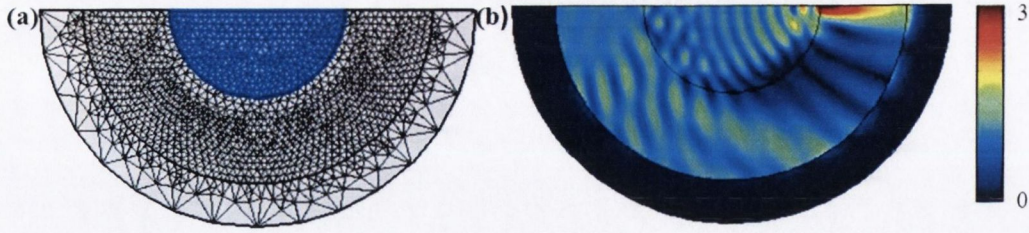
$$\mathbf{E}_p(\theta, \phi) = \sqrt{\lambda} \frac{jk}{4\pi} \mathbf{r}_0 \times \int [\mathbf{n} \times \mathbf{E} - \eta \mathbf{r}_0 \times (\mathbf{n} \times \mathbf{H})] \exp(jk\mathbf{r} \cdot \mathbf{r}_0) dS \quad (2.36)$$

Where  $\mathbf{r}_0$  is the unit vector pointing from the origin to the field point p. If the field points lie on a spherical surface  $S'$ ,  $\mathbf{r}_0$  is the unit normal to  $S'$ .  $\mathbf{n}$  is the unit normal to the surface  $S$ ,  $\eta$  is the impedance,  $k$  is the wave number,  $\lambda$  is the wavelength,  $\mathbf{r}$  is the radius vector (not a unit vector) of the surface  $S$  and  $\mathbf{E}_p$  is the calculated far field at point p.

### 2.3.6 2D modelling of light scattering

The 2D geometry of interest is defined using a graphical user interface (GUI). The properties of the different domains are assigned by specifying the complex refractive index on the domain. The geometry used to model light scattering from an infinite cylinder is shown in Fig. 2.13. The domain is divided into smaller triangular elements as shown in Fig. 2.13(a). The grey regions have a refractive index of  $n_1 = 1$  to simulate air and the blue region has an index of  $n_2 = 2.1 - 6 \times 10^{-4}i$  to simulate  $\text{Si}_3\text{N}_4$  (measured from ellipsometry). The mesh size must be small in order to get accurate results and convergence in iterative techniques. We found that in 2D a mesh size of  $\lambda/6$  was sufficient, where  $\lambda$  is the wavelength in the medium. This particular mesh consisted of 2236 elements which resulted in solving for 16000 degrees of freedom. The norm of the electric field is shown in Fig.2.13(b). The magnitude of the false colour map is relative to the norm of the incident electric field  $|E_0^i|$ .



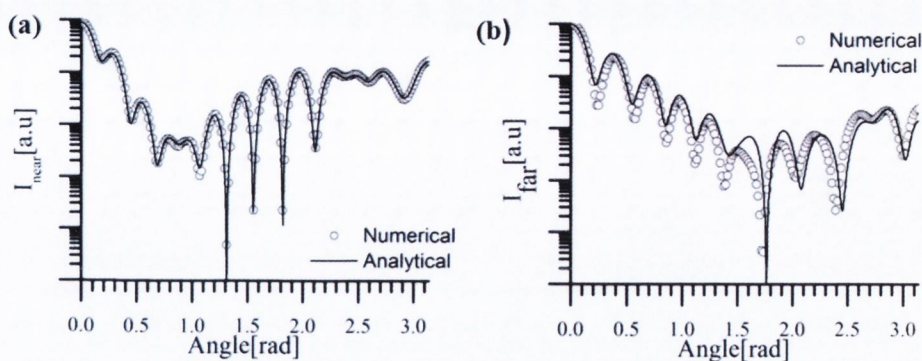


**Figure 2.13:** 2D geometry used to calculate near field and far field scattering from an infinite circular cylinder in Comsol. (a) The mesh pattern distribution used. (b) The norm of the total electric field normalised to the incident electric field norm.

Due to the symmetry of the shape only half the domain is needed. We use a perfect magnetic conductor boundary condition on the top edge in order to induce symmetry in the electric field distribution. In order to truncate the simulation domain we need to introduce perfectly matched layers (PMLs). This is the outer most domain in Fig.2.13. It simulates an absorbing material around the domain which is specifically formulated to minimise reflections from the boundaries [19]. This method can then be applied to arbitrary shapes (with at least one symmetry plane). The near field intensity distribution is calculated as  $I = |\mathbf{E} \cdot \mathbf{E}^*|$ .

We first need to check that the FEM solutions are accurate for known shapes. We use the infinite cylinder as a model example. Fig. 2.14 (a) shows a comparison of the intensity on the surface of the cylinder as a function of angle, with 0 corresponding to the forward scattering direction, calculated using the analytical and FEM methods. The log scale plot shows that the FEM method accurately predicts the near field distribution. Figure 2.14 (b) shows the far field Intensity as a function of angle. The analytical solution is calculated using Eq. 2.18 and the numerical solution is calculated by first solving for the near-field distribution using FEM and extrapolating to the far field by integrating over a circle enclosing the cylinder and using the Stratton-Chu equations.

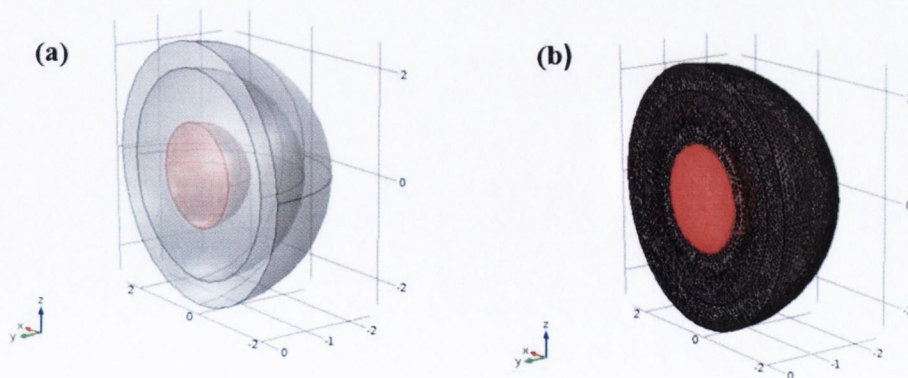




**Figure 2.14 (a)** Near-field intensity profile on the surface of a cylinder of diameter  $2\mu\text{m}$  and linearly TM polarised input at  $532\text{nm}$ , comparison between the analytical solution (solid line) and numerical 2D FEM. **(b)** Far-field scattering distribution for an infinite cylinder of radius  $1\mu\text{m}$ . The solid line is the analytical solution and the scatterplot is the numerical solution using FEM to calculate near-field and Stratton-Chu to extract far-field.

### 2.3.7 3D modelling of light scattering

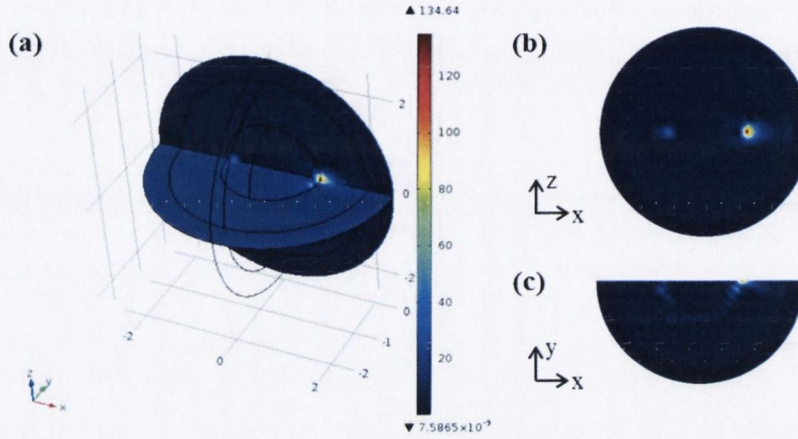
The domain used to model light scattering by 3D objects is shown in Figure.2.15(a). The domain is divided into 546,970 tetrahedral elements of maximum size  $100\text{nm}$  as shown in Fig. 2.15(b). Again we have a PML region surrounding the domain. Light is incident in the  $+x$  direction and can be simulated at various angles. When the light is polarised parallel to the  $y=0$  plane (p polarised case) we used perfect magnetic conductor (PMC) boundary conditions across the plane to induce symmetry. When the light is polarised perpendicular to the  $y=0$  plane (s polarised case) we used perfect electric conductor (PEC) boundary conditions on the  $y=0$  plane.



**Figure 2.15 (a)** 3D domain used for modelling of light scattering. **(b)** The mesh used has 546,970 mesh elements with a maximum element size  $100\text{nm}$ , resulting in approximately 1.5 million degrees of freedom.

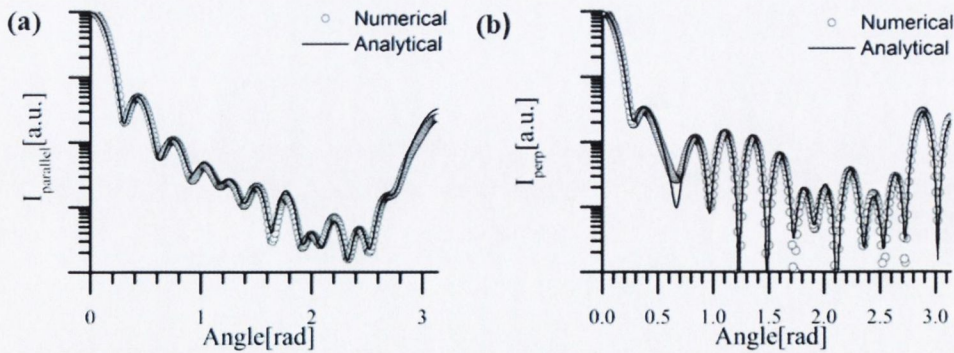


The intensity distribution for the case of incident light of wavelength 532nm and propagating in the +x direction, linearly polarised in the +z direction is shown in Fig.2.16.



**Figure 2.16:** Relative intensity for light scattering from a microsphere of diameter  $2\mu\text{m}$  and refractive index 2.(a) Shows a 3D representation with a cut plane through the  $y=0$  plane and the  $z=0$  plane. (b) x-z plane (c) x-y plane. The intensity distribution is symmetric around the  $y=0$  plane.

The intensity distribution shows a small sub-wavelength spot of high intensity on the surface of the sphere. This is 136 times more intense than the incident radiation. To check the validity of the solution we compare it with the analytical solution with the same parameters. The analytical solution for light scattering from a sphere can be found using a similar separation of variables solution as the case of an infinite cylinder [2]. Figure. 2.17 (a) shows the far field scattering distribution in the x-z plane, and Fig. 2.17 (b) shows the same in the x-y plane. These are calculated by applying the Stratton-Chu equations along the boundary of the domain and PML layer.



**Figure 2.17 (a)** Far field scattering profile for a sphere of diameter  $2\mu\text{m}$  with incident wavelength of 532nm

## 2.4 Conclusions

The analytical solution for light scattering from an infinite circular cylinder was outlined. By numerically evaluating this solution the dependence of the PNJ scattering distribution on size parameter and refractive index contrast has been studied. The TM case is shown to give a narrower PNJ than the TE case. The maximum intensity in the PNJ increases in general as the diameter (size parameter) increases. The increase is not monotonic however, and depends in a complex way on the scattering expansion coefficients. This ripple structure was known to exist in the far-field scattering cross sections. We have shown that the near-field also follows this dependence. The position of maximum intensity depends on both the size parameter and the refractive index contrast and can be varied from inside the cylinder to outside. The FWHM of the PNJ is shown to vary around a value of  $0.45\lambda$  for a refractive index of 1.68, and depends very sensitively on the size parameter. An optimum refractive index contrast for the highest intensity PNJ was shown to exist at 1.6.

The finite element method was introduced using a simple example of heat conduction in 1D. The implementation of the method in COMSOL was discussed. The procedure for numerically finding the far-field scattering distribution of an arbitrarily shaped scatterer was introduced. This uses FEM to calculate the near-field and then the Stratton Chu equation to calculate the far-field. The accuracy of this method was evaluated in both 2D and 3D by comparison with the analytical solution of a cylinder and a sphere respectively. The theory and numerical approaches outlined in this chapter will be used for comparison with experimental results in later chapters.



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## Chapter 3 Experimental Methods

### 3.1 Introduction

The purpose of this chapter is to explain in detail the various techniques used in sample preparation, analysis and measurement throughout the project. It can be read as a stand-alone chapter, however it will be used as a reference throughout the rest of the thesis. Particular emphasis is placed on electron beam lithography (EBL) and inductively coupled plasma etching (ICP) as they are used extensively throughout the project.

This project involves the fabrication of micron scaled optical structures using standard techniques from the microelectronics industry. The fabrication techniques used include: electron beam lithography (EBL), UV lithography, inductively coupled plasma etching (ICP). Analysis techniques include: electron beam physical vapour deposition (EBPVD), scanning electron Microscopy (SEM), focused ion beam milling (FIB), profilometry, atomic force microscopy (AFM), electron diffraction, and ellipsometry. Measurement techniques include: micro-photoluminescence ( $\mu$ PL), scanning confocal microscopy, scanning near field optical microscopy (SNOM), and charged coupled device (CCD) imaging. A brief background of these techniques will be provided. More detail of the specific fabrication processes and experimental setups will be left until later chapters.

### 3.2 Fabrication techniques

Microlithography/Nanolithography is the dominant means of fabrication in the microelectronics, micro-electro-mechanical systems (MEMS), and integrated photonics industry. It refers to a complete process by which a material is shaped from the “top down” by using masking and etching techniques. The process can be broken down into three basic stages; exposure, development and etching. Complex structures require many process steps.



### 3.2.1 Photolithography

The most prolific form of nanolithography is photolithography (or UV lithography). In this process a photosensitive polymer known as a resist is spin coated onto the substrate. The thickness of this resist layer is determined by the rpm and time of the spin coat, and can be accurately controlled from hundreds of nanometers to tens of microns if the viscosity of the resist is known.

The first form of photolithography was known as contact lithography and was used to produce integrated circuits (ICs) in the early 1960s [1]. A glass plate coated with copper known as a mask was placed on the surface of the substrate. A pattern defined in the mask allowed light through in certain areas. The mask was exposed using a homogeneous intensity of UV radiation inducing a chemical change in the areas of the polymer which have been exposed to UV radiation. There are two common categories of resist known as positive and negative tone resists. For positive tone resist the areas which are exposed are rendered susceptible to the development step, whereas, for a negative tone resist the exposed areas are rendered resistant to the development. The size of realisable features with this method was limited to  $\sim 10\ \mu\text{m}$ .

Modern Photolithography systems use projection printing lithography such as the TWINSCAN NXT:1950i from AMSL [2]. These use a complex and expensive ( $\sim \$30,000,000$ ) collection of lenses which can form an image of the mask with  $\text{NA} \sim 1.35$  using immersion optics [3]. Combined with Eximer lasers with wavelength of 193 nm and new resists 32 nm pitch can currently be achieved.

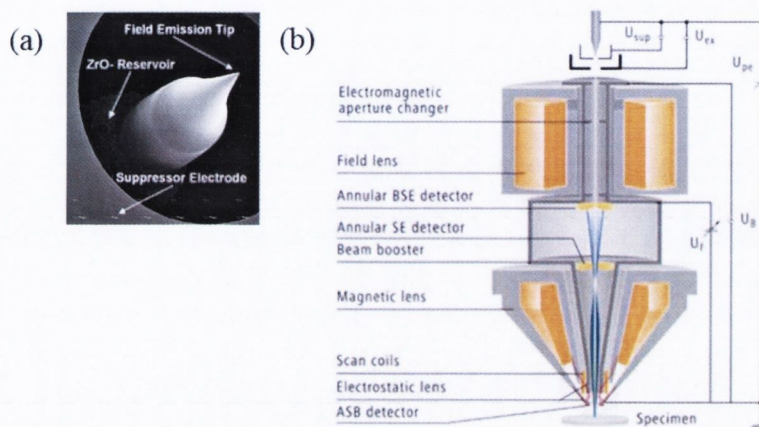
The exposed resist is then “developed” by immersion in a solvent which will dissolve only the exposed (unexposed) areas for positive tone (negative tone). After development the remaining resist can be used as a mask for the etching process. If the resist is not resilient enough to the etching process a metal deposition step may be needed followed by a “lift off” step.

Etching refers to removal of the actual material of the substrate. Until now no modification of the substrate has actually occurred. The resist could be removed chemically without any modification to the original substrate. Etching transfers the pattern of the resist into the underlying material. It can be done chemically using acids (wet etching) or physically using accelerated ions (dry etching), or using a combination of the two. Wet etching can produce an isotropic etch or etch along one particular crystallographic direction. Dry etching allows more control over the etching process and can produce highly anisotropic etches. Energetic ions are accelerated towards the substrate and depending on the gas composition and ion velocities both physical and chemical etching can be achieved.



### 3.2.2 Electron beam lithography

This process differs from photolithography only in the exposure step [4]. Instead of flood illumination by a UV source, electrons are accelerated across a high voltage and focused electromagnetically to a small spot on the surface of the resist.



**Figure 3.1(a)** Tungsten tip for thermoelectric generation of electrons. **(b)** Schematic of modern electron beam column with electromagnetic lenses adapted from SEM manual [5]

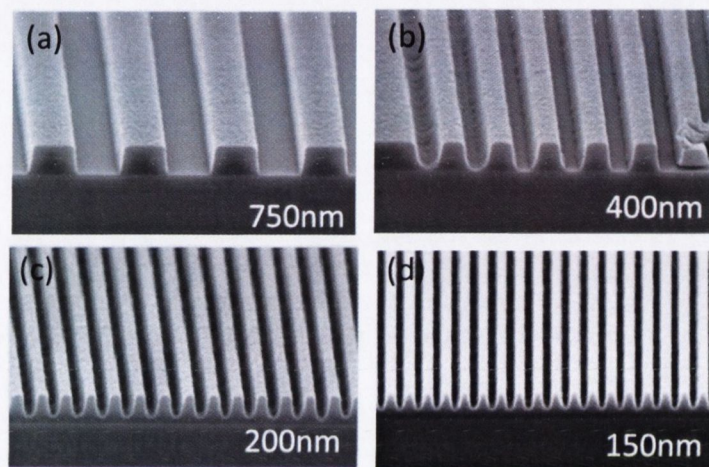
Electrons are generated from a heated tungsten tip source (Fig. 3.1 (a)) and accelerated across a distance with a high voltage (5-100 keV). This distance is known as the electron column. Ultra high vacuum is required to avoid scattering of electrons in the column

( $<9 \times 10^{-9}$  mbar), whereas high vacuum ( $10^{-6}$ - $10^{-7}$ ) is sufficient in the sample chamber. The electron beam is passed through a physical aperture of fixed size (10-30  $\mu\text{m}$ ), controlling the current in the beam. The beam is focused to a diffraction limited spot ( $\sim 8$  nm) using the magnetic and electrostatic lenses. Standard electron optics are depicted in Fig. 3.1 (b). Fine tuning of the electromagnetic lenses can change the focal position and adjust for astigmatism to give the best results.

For EBL the substrate is coated with a polymer which is sensitive to electrons. Similarly to UV lithography this can be positive or negative tone. The sample is then raster scanned with respect to the spot. A beam blanker controls which areas are exposed and which are not. This is an inherently slow process as each pixel must be individually exposed for a fixed amount of time. This precludes its use in large scale manufacturing processes. The effective wavelength of the beam of electrons is much smaller than that of UV radiation meaning that a much higher resolution can be obtained. Structures with dimensions on the order of tens of nanometres can be produced using e-beam lithography. For example line gratings



with pitches (smallest distance between features) of 750 nm down to 150 nm are shown in Fig. 3.2. These are adapted from the datasheet of common EBL resist NLOF 2070 produced by Microchemicals [6]. After the development step, the SEM images are taken showing the resist features. We see an overlapping as the pitch decreases due to secondary electron scattering known as the “proximity effect”. Using EBL structures with features less than 10 nm can be created [7, 8]



**Figure 3.2** Resist after development all with exposure dose of  $42 \mu\text{C}$ . (a) Pitch of 750 nm, (b) pitch of 400 nm, (c) pitch of 200 nm (d) pitch of 150 nm. Images adapted from AZ NLOF 2070 datasheet from Microchemicals GmbH [6].

### 3.2.3 Emerging lithographic techniques

The demand for smaller feature size integrated circuits is pushing the limits of current UV lithography techniques. New techniques are needed in order to create devices with the necessary dimensions. Today modern projection techniques can reach 32 nm half-pitch (hp) with an incident wavelength of 193 nm. This is because of “resolution enhancement technology” (RET), which uses prior information about the structure to be fabricated to use tricks to get around the diffraction limit. “Wavefront engineering” allows one to take advantage of the phase of the light to produce interference fringes and increase contrast [9]. Complex computer software now designs the optimum mask design and compensates for the features being so close that they tend to overlap (“proximity effect”). The simple idea of immersing the sample in a liquid has also been used in order to increase the Numerical Aperture (NA) to 1.35 [10]. Standard UV lithography is expected to continue to the 22 nm node, after which more exotic solutions will be needed [11].



Double exposure is a technique capable of doubling the spatial frequency of the lithographic technique. Two lithographic processes are preformed sequentially thus the minimum feature size is determined by the stage resolution. This technique is currently being used to push dynamic random-access memory (DRAM) and multi-core processing units (MPU) down to 22 nm half pitch [11].

The most likely successor of UV lithography at the moment is Extreme UV lithography (EUVL) [12]. This involves changing the wavelength of the light used to extremely short wavelength in the soft X-Ray regime (13 nm), thus reducing the diffraction limit. This introduces numerous problems, some of which include : (1) EUV sources are not widely available and are low power [13, 14]. (2) conventional transmissive optics cannot be used as most materials strongly absorb light in this regime. Innovative solutions employing reflective optics must be used resulting in a complete redesign of the illumination optics [12]. (3) Accurate mask design becomes very difficult and expensive. (4) High vacuum and very clean environments must be employed, making this a slow and expensive technology at presently. Prototype devices have been produced by ASML since 2010 and are currently in testing [2].

Another approach is arrayed EBL. EBL is already capable of producing sub 10nm features. The main issue is making it commercially viable to produce on mass scale. A number of solutions have been suggested for this including imaging square apertures [15], and using multiple electron beams [16].

A relatively new form of lithography that doesn't follow the optical trend is nano imprint lithography [17, 18]. A mould can be made once using a high resolution technique such as EBL. This mould is then pressed into a resist to create the desired resist profile. This circumvents any limitation caused by the diffraction limit and is a very promising solution for IC manufacture in the future [11]. One immediate issue of concern is this is a contact method of lithography and thus the mould would be prone to damage resulting in poor throughput. Self-cleaning moulds have been demonstrated [19]. So far 6 nm features have been demonstrated using this technique and it is expected to continue to 3 nm [20].

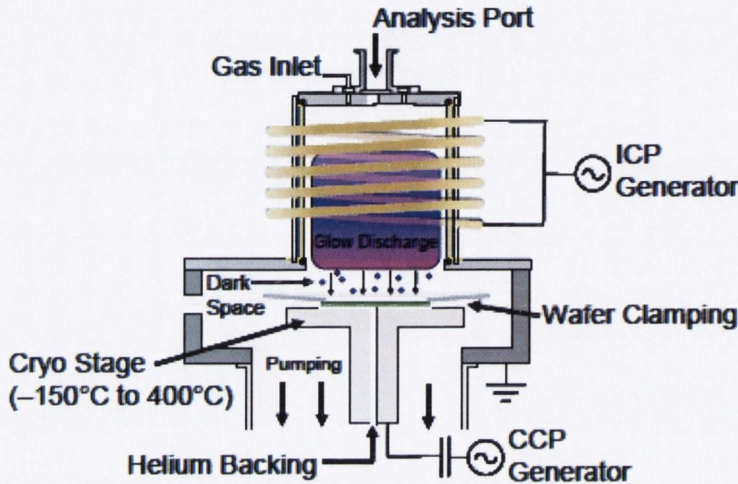
Finally self-assembly is a bottom up approach which promised to build the structures using chemical means as opposed to etching. Periodic features with nanoscale resolution can be formed by heating of block co-polymers, however controlling the patterns is still an issue [21]

### **3.2.4 Inductively Coupled Plasma Etching**

Inductively coupled plasma reactive ion etching (ICPRIE) is a particular form of plasma etching, which allows independent control over the density and energy of ions in the plasma. It



allows the use of both physical and chemical etching to create highly anisotropic etches which are essential for photonic applications. A typical ICP RIE setup is depicted in Fig. 3.3 which is adapted from the thesis of Micheal David Henry [22].



**Figure 3.3:** Schematic of standard ICP tool setup (adapted from [22])

In an ICP process the plasma is generated inductively using a radio frequency (RF) source connected to a large coil around a vacuum chamber. Gases are injected into the chamber and the magnetic field generated circulates electrons around causing collisions and ionising the gases. A characteristic glow is observed from the plasma which is indicative of the gases used. This results from inelastic collisions transferring energy to bound electrons. These electrons may return to the ground state by emitting a photon. An RF frequency is also applied capacitively to the plasma (CCP). The RF frequency allows for efficient ionisation of the gases due to the mismatch in mass between the electrons and the ions. The massive ions cannot track the change in electric field, thus the electrons get multiple passes to ionise the gas. Electrons are absorbed by the anode and build up a negative charge on a capacitor placed between the anode and the RF source. This creates a bias ( $V_b$ ) between the neutral plasma and the negatively charged plate which accelerates the ions towards the substrate. Thus the density of the plasma can be controlled via the ICP power and the momentum of the ions can be controlled via the CCP power (Forward power). High bias results in ions with large momentum and increases the physical aspect of the etch, decreasing selectivity, whereas low bias results in a more chemical etch.



Choice of gases and gas ratios can have a large effect on the properties of the etch [23]. Etch chemistries can vary widely, even for the same substrate. For silicon etching fluorine chemistry is most common and gases such as sulphur hexafluorine ( $\text{SF}_6$ ) and tetrafluoromethane ( $\text{CF}_4$ ) are used to provide a high density of F ions. Three basic etching mechanisms for Si exist with this chemistry: 1) *Chemical etching*: F ions combine with Si on the surface to create volatile  $\text{SiF}_4$  and this is pumped away, 2) *Physical etching*: F ions knock away an Si ion by imparting momentum, 3) *Ion assisted etching*: F ions chemically bind with the Si on the substrate and remain.

Plasma etching is a crucial enabler for the microelectronics industry. Many studies have been conducted particularly with Si and the etching properties of these substances are well known and easily controlled [24].

### 3.2.5 Wet etching

It is worth mentioning that wet etching is also a common method of etching, although it is not used in this project. It is purely a chemical etch. Parameters such as the chemical makeup of the solvent and the temperature are manipulated to control the etch [25]. Wet etching tends to be more isotropic, or can be manipulated to etch preferentially along one crystallographic direction preferentially. For example V shaped grooves can be created in Si using a KOH etch [26].

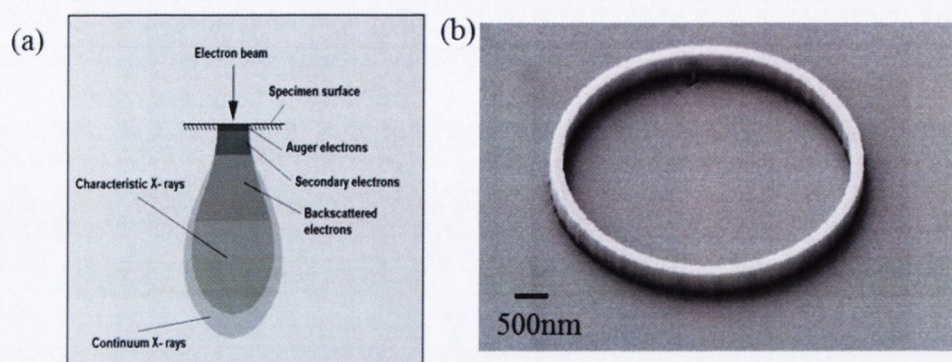
## 3.3 Sample Analysis techniques

Once samples are fabricated it is imperative the dimensions are accurately verified for optimisation of the fabrication process and for comparison of the device with theory. A brief outline of each of the techniques used to analyse the structures is given below.

### 3.3.1 Scanning electron microscopy

Scanning electron microscopy is one of the most commonly used techniques for analysing structures with dimensions below the diffraction limit of light [27]. Electrons are accelerated to a surface using the same setup as in Fig. 3.1 (b). These electrons interact with the surface of the sample in a finite bulb shaped region (Fig. 2.4 (a)), the size of which depends on the incident current and acceleration voltage. The electrons then interact with the substrate and are either scattered, or absorbed resulting in the emission of X-rays. An image of the substrate can be reconstructed using these scattered electrons. Two types are commonly used to create images; secondary electrons (SE) and backscattered electrons (BSE).





**Figure 3.4 (a)** Electron interaction area. **(b)** Typical image of a microring resonator taken with an SEM.

Secondary electrons are generated by multiple inelastic scattering close to the surface of the substrate and typically have energies  $< 50$  eV. Back scattered electrons (BSE) have energies  $> 50$  eV and are generated by elastic scattering further into the sample and contain depth information. The X rays generated give information on the composition of the materials. The secondary electrons are collected either using the in-lens detector or the secondary electron detector (SED) placed outside the column to produce an image of the sample surface. Information can be gathered from the energy of the electrons and their angular distribution.

### 3.3.2 Profilometry

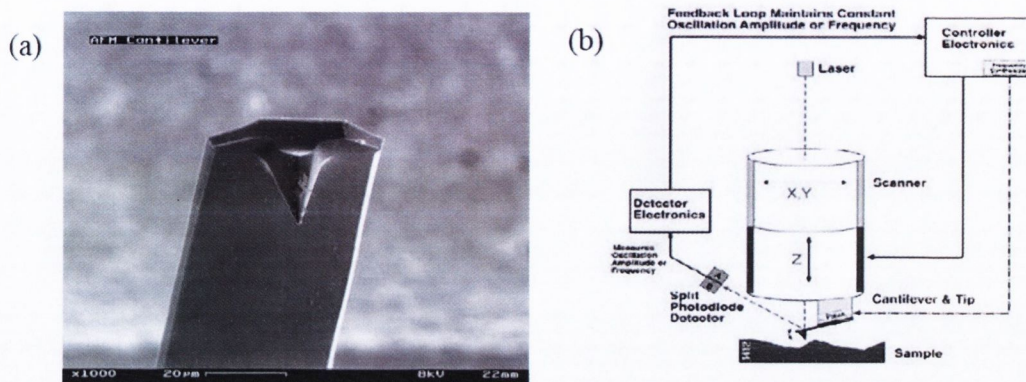
Profilometers measure surface roughness by scanning a sharp diamond tip in contact with the surface of the sample. The change in height is converted to a voltage using a piezoelectric element. This voltage is then digitised and displayed as a height. In this project a Dektack profilometer by Bruker [28] is used to measure resist heights. This provides a quick and reliable method for measuring etch rates and structure heights to nanometer resolution. The tip had diameter of  $25\text{ }\mu\text{m}$  which defines the maximum resolution in the x-y plane.

### 3.3.3 Atomic Force Microscopy (AFM).

Atomic force microscopy (AFM) provides accurate surface topology and roughness measurements [29]. It operates using a micron scale cantilever that has a small tip (nanometer scale radius of curvature) at the end which interacts with the substrate (Fig. 3.5 (a)). When the tip is brought to within a few nanometers of the surface, forces between the tip and the substrate result in a bending of the cantilever in accordance with Hooke's law. The deformation is measured by detecting the deflection of a laser reflected from the surface of the cantilever using



a split photodiode. A schematic of a typical setup is shown in Fig.3.5 (b). AFM is usually operated in one of three modes; contact mode, non-contact mode and tapping mode.



**Figure3.5: (a)** Typical geometry for an AFM cantilever. **(b)** Schematic of optical detection and electronic feedback system.

In contact mode the cantilever is lowered until it makes physical contact with the substrate and the deflections hits a predefined set point. The tip is then dragged across the surface and an electronic feedback system is used to keep the cantilever deflection constant. The surface topology can then be extracted from the voltage needed to keep the deflection constant.

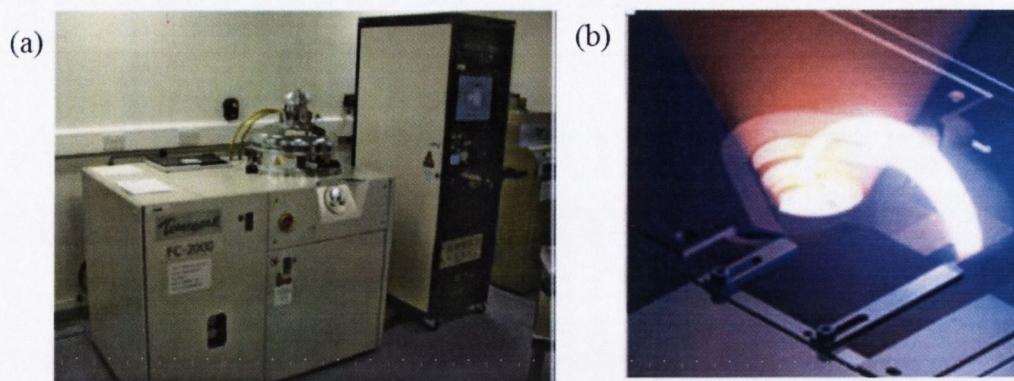
In non-contact mode the cantilever is brought in close proximity to the surface and driven at a frequency close to a resonance. The amplitude of oscillation is usually  $\sim 10$  nm, but depends on the cantilever properties such as length, shape and material. When the tip is in close proximity to the surface, Van der Waals interactions between the tip and the surface cause a change in the resonance frequency. The tip is then scanned across the surface of the sample keeping the oscillation frequency fixed by adjusting the height of the cantilever. This method produces a similar image to contact mode, but with less damage to the tip and the sample.

Tapping mode is a combination of contact and non-contact methods. The tip is driven at a specific frequency and brought close enough to make contact with the surface intermittently. This technique is useful in situations where there is a layer of liquid on the sample surface preventing the tip getting close enough to interact via van der Waals forces.

### 3.3.4 Electron beam physical vapour deposition (EBPVD)

Good SEM measurements require a conductive substrate. For this reason it is common to coat dielectric samples with a thin layer (5-10 nm) of gold for better SEM analysis and FIB measurements. This project uses electron beam physical vapour deposition to coat the samples with a 2 nm layer of Ti for adhesion followed by a 5 nm layer of gold (Fig. 3.6).



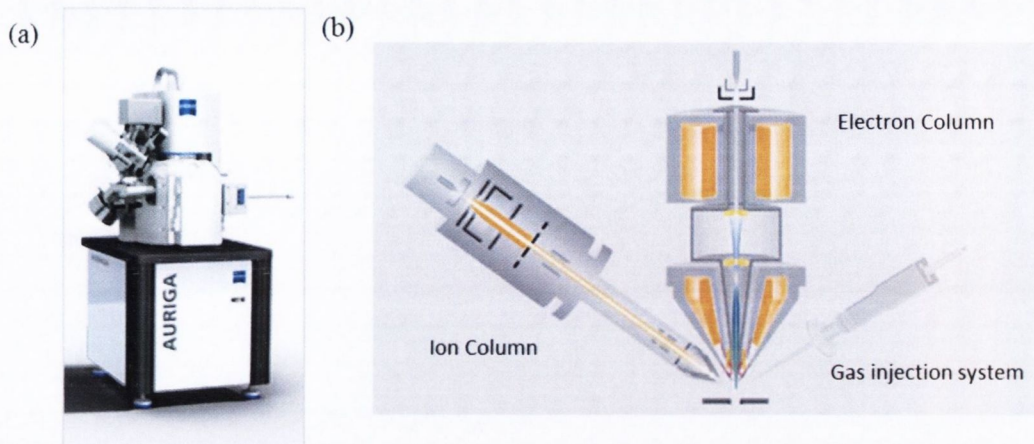


**Figure 3.6:** (a) Temescal EBPVD system. (b) Electrons are emitted from a tungsten source and controlled using a magnetic field to hit a target material.

The process takes place in ultra-high vacuum ( $\sim 2 \times 10^{-8}$  Torr). Electrons are emitted from a heated tungsten source in a similar fashion to SEM and are accelerated across 10 kV. The current is much higher than in the case of SEM (0.6 A). The electrons are directed towards a crucible containing the desired metal for deposition. The beam of electrons is swept across the crucible using the magnetic field to achieve uniform heating until evaporation. The material then disperses into the vacuum and coats all the surfaces evenly with the material. The deposition rate is monitored by observing the change in frequency of a quartz resonator in the chamber. This process results in a very homogeneous coating of material that can be controlled to the angstrom level [30].

### 3.3.5 Focused Ion beam milling (FIB)

Focused ion beam milling operates on a similar principle to scanning electron microscopy, in fact, it is common to have both SEM and FIB milling capabilities in the same tool [31]. Ions are generated and accelerated across a column. These ions have more mass than electrons and can physically etch the substrate by knocking out the existing material (ion milling). Again these ions are not subject to the optical diffraction limit and can be focused to very small spots ( $< 2.5$  nm). This allows a method to accurately shape materials on the nanoscale.



**Figure 3.7** (a) Tool used in fabrication (b) Schematic of ion column, electron column and gas injection system in Carl Zeiss Auriga.

The tool used in this project was a combined FIB and SEM tool called Auriga from Carl Zeiss (Fig. 3.7 (a)). Gallium ions are created using field emission from a heated liquid metal ion source (LMIS). The ions are then accelerated and focused using electrostatic lenses in the ion column (Fig 2.5 (b)). The ion column is at an angle of  $54^\circ$  to the electron column. Localised ion assisted deposition can be performed using the gas injection system (GIS). Metals can be deposited to protect structures and assist in the milling process. This tool allowed us to mill our structures and then examine the sidewall angle using the SEM.

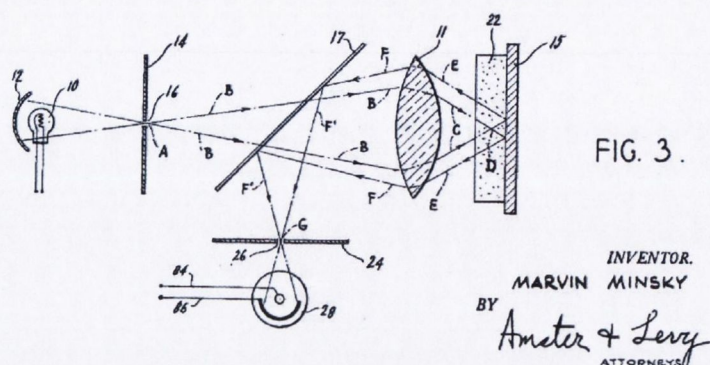
### 3.4 Sample measurement techniques

This project deals with optical properties of micro scale devices, so most of the measurements were performed optically.

#### 3.4.1 Confocal Microscopy

Confocal microscopy employs a micron scale pinhole in a conjugate plane to reject out of focus light from a sample [32]. The method was originally proposed by Minsky in 1957. This results in slightly higher resolution than in a standard white light microscope, particularly along the axis. Fig. 3.8 shows the basic principle.





**Figure 3.8** Original Sketch of principle of confocal microscopy proposed by Minsky in 1957.

Light from a source is spatially filtered by passing it through an aperture (14). This light is then focused onto the specimen using an objective lens. Light is collected using the same lens and redirected using a beam splitter. This light is focused at the same distance from the back of the objective as the source. The out of focus components of the light coming from the sample will be brought to focus at slightly different planes. These are then cut off by the aperture as they will produce bigger spot sizes at the aperture plane. This results in a thin elliptical point spread function. Three dimensional images of samples can then be reconstructed by scanning the sample through the focal region using a piezoelectric scanner. Many variations of this technique exist.

### 3.4.2 Micro-Photoluminescence measurements. ( $\mu$ PL)

Light of high energy (short wavelength) is focused onto the sample. This light is absorbed by the sample and re-emitted with a longer wavelength as some energy is lost to the sample through processes such as phonon emission. The excitation wavelength is filtered out and the remaining light is passed to a spectrometer. The spectral composition of the photoluminescence contains information about the specific material being measured. Micro photoluminescence simply uses a microscope objective to localise the excitation and collection region in space. The sample can then be scanned through this region to reconstruct a 2D map of the optical properties of the material. In this project we used  $\mu$ PL to measure the WGM structure of a cavity as the spectral content of the  $\mu$ PL is modified due to the presence of the cavity.

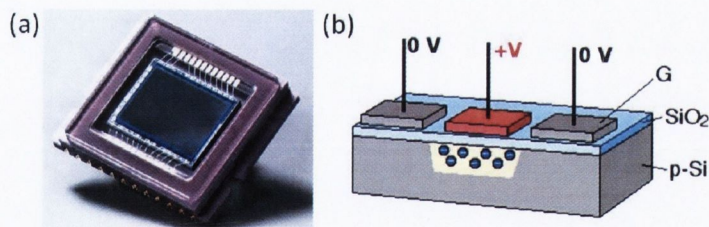


### 3.4.3 Ellipsometry

Ellipsometry is a common technique for measuring the complex refractive index of a slab of materials [33]. It measures the amplitude and phase of light after specular reflection from the substrate. Circularly polarised input light will result in elliptically polarised light after reflection hence the name. When the substrate consists of a small number of well-defined layers of optically homogenous and isotropic materials, fitting procedures can be used to extract the height and complex refractive index of the layers. The input light can be split into components parallel and perpendicular to the substrate and the Fresnel reflection equations can be used. In this project we used ellipsometry to verify the heights and refractive indices of a  $\text{Si}_3\text{N}_4/\text{SiO}_2/\text{Si}$  layer substrate.

### 3.4.4 Charge Coupled Device (CCD)

CCD measurement is used extensively in this project to quantify light scattered and emitted from structures. It is therefore important to fully understand the functionality and error introduced by using a CCD [34]. The CCD is a semiconductor device which converts photons to electrons. A typically CCD is shown in Fig. 3.9.



**Figure 3.9** (a) Typical design of CCD including a hermetically sealed casing. (b) Schematic of readout technique. Photoelectrons are generated by exposure to light over a fixed time and then translated to a signal amplifier by sequentially changing the voltage on each pixel.

Although individual CCDs can differ in design the basic operation involves using a photosensitive material to generate photoelectrons. These are generated linearly in proportion to the number of incident photons. The photoelectrons are then manipulated to be read off sequentially and by recording the time at which they arrive, you can work out the spatial position of the pixel from which the electrons came. This current is then converted to a voltage using a capacitor and then sampled and converted to a digital signal. Thus you are left with a digital signal which represents a spatial map of photon flux or intensity.



#### 3.4.4.1 *CCD Device parameters.*

There are various physical parameters than can vary between CCDs that make them good for different jobs:

**Electron well depth:** is the number of photoelectrons which can be generated in a single pixel. This limits the usable shutter time and dynamic range of a CCD.

**Physical pixel size:** the physical pixel size is actual size of each pixel. This varies and is usually between 4-10  $\mu\text{m}$ . Small pixel size means higher spatial sampling at fixed magnification, but lower dynamic range due to limited electron well depth. In our case the physical pixel size is 4.6  $\mu\text{m}$  in both directions.

**Sensor size and pixel number:** The overall size of the sensor limits the usable field of view at a particular magnification. There are standard sizes which are usually measured in terms of the length of their diagonal. We used a 1/3 inch Sony CCD. This had 1024×768 pixels with physical dimensions of 5.80 mm×4.92 mm.

**Quantum Efficiency:** The quantum efficiency is the ratio of the average number of electrons generated per photon and varies from 0-1. This is wavelength dependent and usually is above 0.7 (70 %), which is much more efficient than photographic film which only responds to 2 % of the light.

#### 3.4.4.2 *Sources of noise*

The total noise in a CCD system comes from a variety of sources. The main contributors are: **photon noise, dark noise, and read noise.**

**Photon Noise:** Light interacts with matter in quantised packets known as photons. Photon noise is associated with the variation in the rate of arrival of the individual photons. It follows a Poisson distribution which means that the noise is proportional to the square root of the total signal.

**Dark noise:** This is due to thermal generated electrons in the device. It is independent of the intensity of light on the device, but highly dependent on temperature.

**Read noise:** This is a combination of noise that occurs from reading the electrons from the individual pixels and transforming the current into a voltage signal, and subsequent processing and analog-to-digital (A/D) conversion. Most of this noise comes from the on chip preamplifier, and thus is added uniformly to each pixel value. The Signal to noise ratio is defined as:

$$SNR = \frac{PQ_e t}{\sqrt{PQ_e t + Dt + Nr^2}}$$

where P is the incident photon flux (photons/pixel/second),

Q(e) represents the CCD quantum efficiency, t is the integration time (seconds), D is the dark current value (electrons/pixel/second), and N(r) represents read noise (electrons rms/pixel).

In the CCD used, Sony employs hole accumulation diode sensors (HAD) to reduce the dark current providing low noise even at room temperature. The signal to noise ratio for this is greater than 60 dB. So the signal is 1000 times larger than the noise. This is what limits the dynamic range of the measurement.

#### 3.4.4.2 Variable parameters and their effects

With this device we have control over the shutter speed/ exposure time and the gain. The exposure time is literally the amount of time the CCD is exposed to light for each image.

The gain is defined as:

$$g = -20 \log_{10} \left( \frac{\# \text{ of electrons per pixel}}{\# \text{ of counts per pixel}} \right)$$

Increasing the gain will increase the number of counts per electron, thus allowing observation of smaller signals. This also amplifies the noise.



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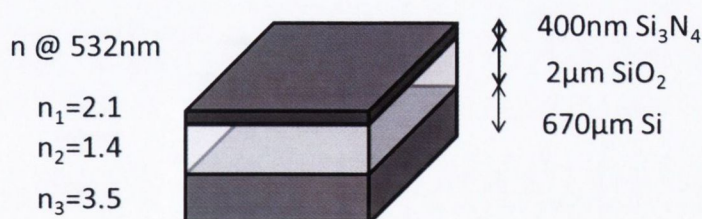
## Chapter 4 Fabrication of planar structures

### 4.1 Introduction

In this chapter the material system used is introduced, the fabrication process for planar structures is outlined (EBL and ICP etch) and the optimisation experiments are shown. The result is a process to create structures with smooth vertical sidewalls optimised for photonic applications.

### 4.2 Material

Silicon nitride is a material which has been widely used in the microelectronics industry, as oxidation and diffusion masks, as non-volatile metal/nitride/oxide/semiconductor memory elements and more generally for passivation [1]. Its excellent strength in bulk over a large range of temperatures has led it to be used as a niche material in high stress environments, such as ball bearings in jet engines [2, 3]. Its mechanical properties also make it useful for membranes, cantilever beams and other structures associated with MEMS devices [4]. Recently it has been considered for optical devices [5-9]. It has a large band-gap which means it is transparent throughout the visible, near infrared and infra-red range from 300 nm to several micrometers. It has a relatively high refractive index in the visible of 2.1, which allows structures with strong confinement and small size. It has no free carrier absorption and two photon absorption is not an issue at wavelengths above 600 nm [10]. For this reason it has been proposed as a key material to develop integrated photonics components in the visible range.

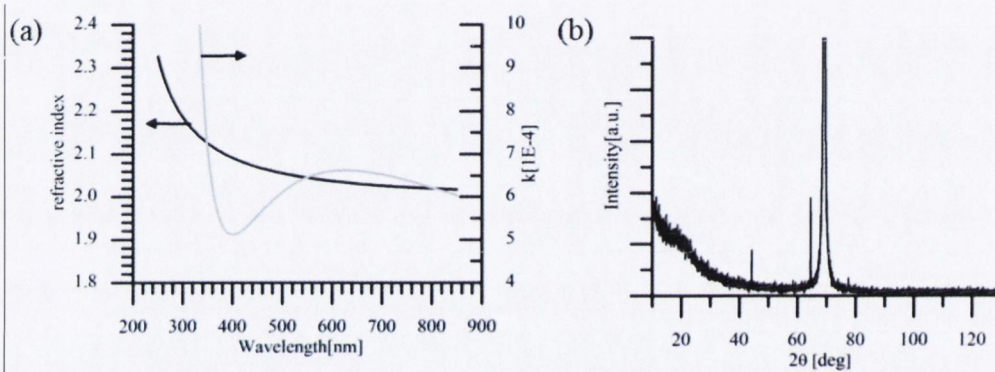


**Figure 4.1** Depiction of layered structure. The real component of refractive index of each layer at 532 nm is shown on the left and the layer thickness on the right.

This project used commercially purchased wafers from Si-Mat [11] with the designed layer structure in Fig. 4.1. A P/Boron doped Si wafer with  $\langle 100 \rangle$  orientation was used as a substrate. A 2  $\mu\text{m}$  thermal oxide was grown on top and a 400 nm layer of stoichiometric  $\text{Si}_3\text{N}_4$  was

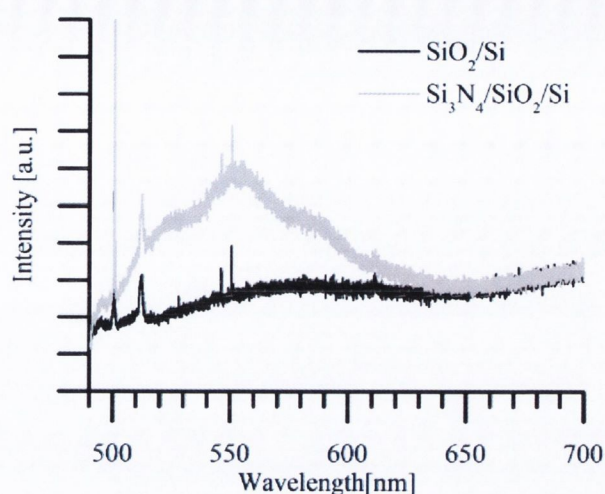


deposited on the oxide using LPCVD. A dichlorosilane and ammonia gas mixture was used in a horizontal vacuum furnace at a temperature of 830°C. This resulted in a low stress layer of  $\text{Si}_3\text{N}_4$  with less than 100 MPa of tensile stress. The refractive index and extinction co-efficient were measured over the visible range using ellipsometry (Fig. 4.2 (a)). At 532 nm the refractive index of the  $\text{Si}_3\text{N}_4$  was 2.1 and the  $\text{SiO}_2$  layer was 1.45. The oxide layer provides an optical spacer necessary for WGM experiments. This  $\text{Si}_3\text{N}_4$  layer was an amorphous film as confirmed by X-ray diffraction (Fig. 4.2 (b)). The X-ray diffraction pattern shows no features expected from structure in the  $\text{Si}_3\text{N}_4$  layer. The most intense peak at  $68^\circ$  is due to the  $\langle 100 \rangle$  plane of the underlying Si substrate. The other peaks can be accounted for from either the substrate or the  $\text{SiO}_2$  layer.



**Figure 4.2** (a) Ellipsometry measurements showing the real part of the refractive index on the left hand axis and the imaginary part (extinction co-efficient) on the right hand side. (b) Results from X-ray diffraction showing no crystalline structure in the  $\text{Si}_3\text{N}_4$  layer. (Peaks are from the Si base wafer.)

The film exhibited a weak Photoluminescence (PL) when pumped with a 488 nm or 532 nm laser shown in Fig. 4.3. The black line represents the PL from a  $\text{SiO}_2/\text{Si}$  layer where the  $\text{Si}_3\text{N}_4$  has been removed by etching, whereas the grey line shows the full structure. Three broad peaks are observed. The origin of this PL is still under debate in the literature. The main proposed mechanism for PL is attributed to the formation of small Si nanocrystals in the  $\text{Si}_3\text{N}_4$  layer [10, 12-14]. This PL is a combination of emission from the Quantum Confinement Effect (QCE) in these nanocrystals and emission from the interface between the Si nanocrystals and the silicon nitride matrix. The effect of changing the Si content on formation of nano-crystals is well documented [15]. Defect emission from the presence of hydrogen in the crystal also contributes to the PL [16]. Both the PL strength and shape depend strongly on the deposition method used. Recently it has been suggested that the PL is not due to nanoparticle formation but in fact due to band tail luminescence from the Si matrix [17]. Regardless, this PL is useful for probing the cavity structure of resonators fabricated in the  $\text{Si}_3\text{N}_4$  layer.

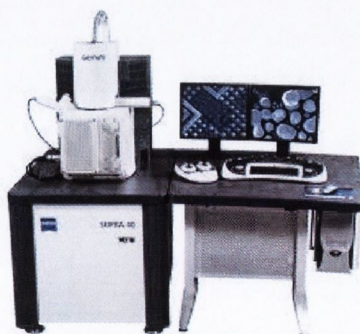


**Figure 4.3** Photoluminescence spectrum from un-patterned sample. The black line is  $\text{SiO}_2/\text{Si}$  where the  $\text{Si}_3\text{N}_4$  layer has been removed by etching. The grey is the total PL from  $\text{Si}_3\text{N}_4/\text{SiO}_2/\text{Si}$  when excited by 488 nm  $\text{Ar}^+$  laser. The sharp background peaks are Raman lines from the  $\text{Si}/\text{SiO}_2$  substrate.

### 4.3 EBL resist patterning

The planar structures in this project are fabricated using a combination of electron beam lithography (EBL) and inductively coupled plasma etching (ICP). The basics of these processes are outlined in Chapter 3.

The tool utilised in this project was a Carl Zeiss Supra 40 SEM with Raith Elphy Quantum software for EBL (Fig. 4.4). This tool is cable of patterning with voltages from 0.1 kV to 30 kV. The ultimate resolution obtainable in any EBL process depends not only on the beam focusing quality, but also on the resist used.



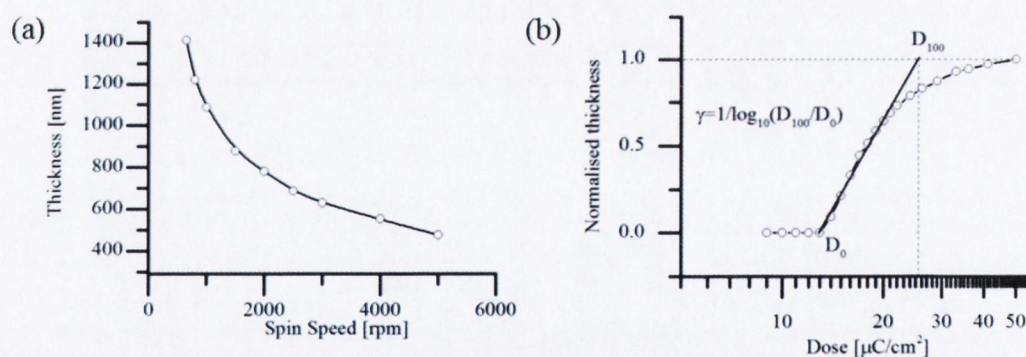
**Figure 4.4:** Carl Zeiss Supra 40 system used for EBL.



Many different EBL resists are available and careful choice should be made with a specific project in mind. We chose a resist called NLOF 2070 from Microchemicals [18]. This is a negative tone resist which is suitable to making small isolated structures. The spin curve and contrast curve for this resist are shown in Fig. 4.5. The spin curve plots resist thickness (after soft-bake) as a function of spin speed for a fixed time (45 s). Using this calibration we then chose a spin speed of 5000 rpm for 45 s in order to get a resist height of 500 nm. The contrast curve of a resist plots the ratio of the height of the remaining resist after development to the initial height ( $d'/d$ ) versus the exposure dose. The slope of this curve is known as the contrast  $\gamma$  and is given by the expression

$$\gamma = \frac{1}{\log_{10}(D_{100}/D_0)} \quad (2.6)$$

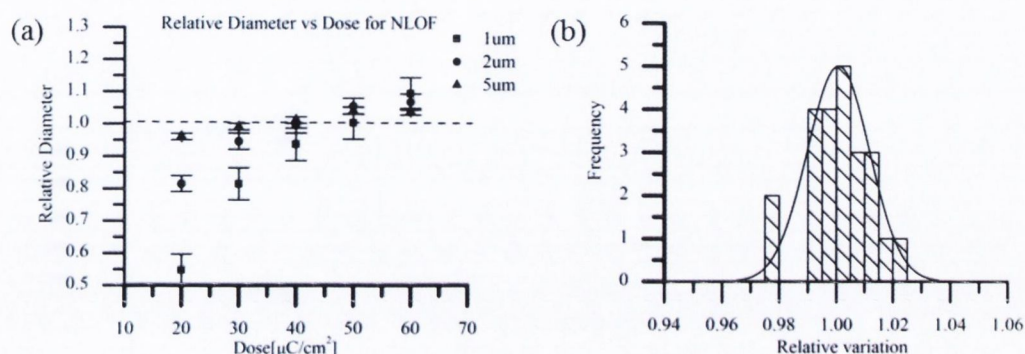
where doses  $D_{100}$  is the saturation dose and  $D_0$  is the minimum dose needed to start crosslinking as shown in Fig.4.5 (b). In UV lithography the gamma value ( $\gamma$ ) is a simple measure of how well the resist can convert the distorted patterns of a blurred projected image into a sharp binary stencil. High contrast is desirable and increases the resolution of the entire lithography process. For NLOF the gamma value is 3.5. In EBL the higher the gamma value the better as it reduces the effect of secondary electrons exposing the resist and proximity effects.



**Figure 4.5(a)** Spin Curve for NLOF 2070 **(b)** Contrast curve from NLOF 2070.

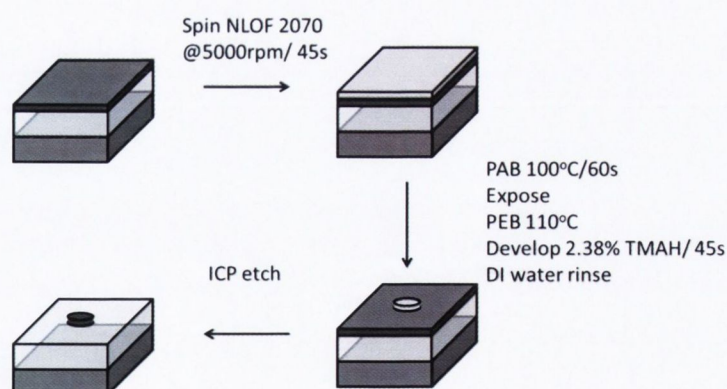
From Fig.4.5 (b) we see that the dose saturates the resist. For best results it is advised to use a dose just before saturation, i.e. in the range of 30-50  $\mu\text{C}/\text{cm}^2$  in this case. A dose test was performed by exposing a pattern of disks of different diameter ranging from 1-10  $\mu\text{m}$ , developing them and then measuring the resist features using SEM. Fig. 4.6 (a) shows the diameter of the resist feature normalised to the designed diameter. The optimum dose to

achieve the expected diameter differs depending on the diameter of the disk, however we see that for the three sizes  $40 \mu\text{C}/\text{cm}^2$  gives structure with diameter within 10 % of the design. This is used as a base dose and fine tuning is done by multiplying each structure by a dose factor at a later stage.



**Figure. 4.6** (a) Dose Test: 5  $\mu\text{m}$ , 3  $\mu\text{m}$  and 1  $\mu\text{m}$  disks are patterned using different dose settings. (b) Variation test: The same disk diameter is patterned 24 times and the diameter is measured using SEM. They show a STD of 1 %.

For micro-resonators it is important to be able to construct disks of the same size reproducibly. Fig.4.6 (b) shows the SEM measured diameters of 24 different disks with the same designed diameter, using  $40 \mu\text{C}/\text{cm}^2$  dose. It shows that disks can be reproduced with a 1% standard deviation in diameter using this method. After these calibration experiments it is possible to create resist patterns reliably and reproducibly. The process flow chart is shown in Fig. 4.7.



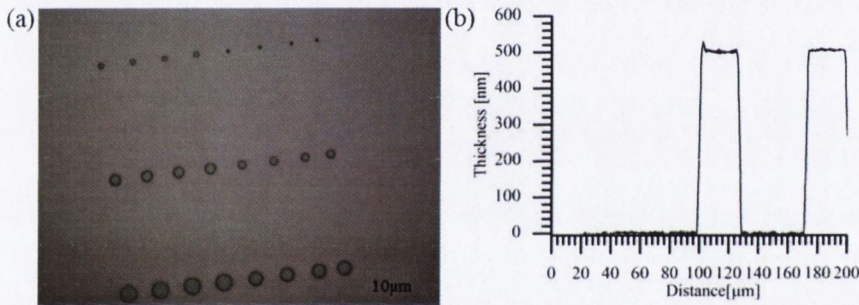
**Figure 4.7:** Process flow chart starting in top left hand corner.

Samples are diced into 10 mm x 10 mm square chips using a Disco diamond blade dicing tool for ease of processing [19]. The chip is first cleaned using sonication in acetone for 10 mins followed by sonication in IPA for 10 mins and dried using an  $\text{N}_2$  gun in a cleanroom



environment. The chip is spin coated for 45 s at 5000 rpm with NLOF 2070 resulting in a 500 nm thick resist layer. The resist is then soft-baked at 100 °C for 60 s to remove excess solvent, improve adhesion of the resist to the wafer and anneal any shear stress induced during the spin coating. Then the chip is placed into the sample chamber and contacted to the substrate using carbon tape. The pattern is exposed using the software controlled beam blanker and scanning stage.

The resist is chemically activated so requires a post exposure bake (PAB) at 110 °C for 60 s to cross-link the exposed polymer. The samples were then developed by immersion in 2.38 % Tetramethylammonium hydroxide (TMAH) for 30 s and then a DI water rinse, and spin dried. The structures are checked for defects optically before the etching process. A typical optical image of a resist pattern is shown in Fig. 4.8 (a). The resist height is checked using the DEKTAK profilometer (Chapter 3), and a sample profile is plotted in Fig. 4.8 (b).



**Figure 4.8:** (a) Typical pattern after resist development: Microdisks with diameters of 10 μm, 8 μm, 6 μm, 4 μm, 2 μm, and 1 μm with 4 repeats of each size. (b) Profile of resist height taken using Dektack profilometer, with resist height on the y axis and distance along the sample in the x axis..

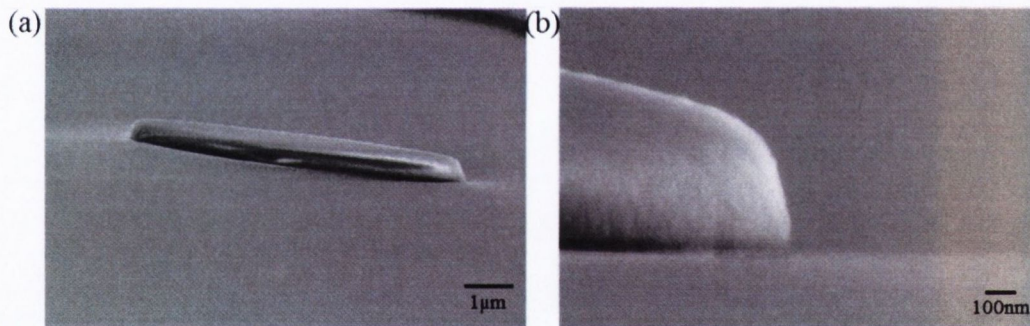
#### 4.4 Etching Optimisation

An Oxford Instruments Plasmalab System100 ICP180 etch tool was used in this project. This has a 3 kW ICP source and is capable of 300 W RF RIE bias power.



**Figure 4.9:** Oxford Instruments Plasmalab System100 ICP180

The basic processes involved in ICP etching are outlined in Chapter 3. Etching is used to transfer the patterned resist into the underlying substrate. For optical applications it is desirable to have sharp  $90^\circ$  sidewalls which are smooth to minimise scattering losses. The etching recipe needs to be optimised in order to achieve this. The resist profile after development was examined using SEM and is shown in Fig. 4.10.



**Figure 4.10:** (a) Resist profile imaged with high angle SEM. (b) Closeup

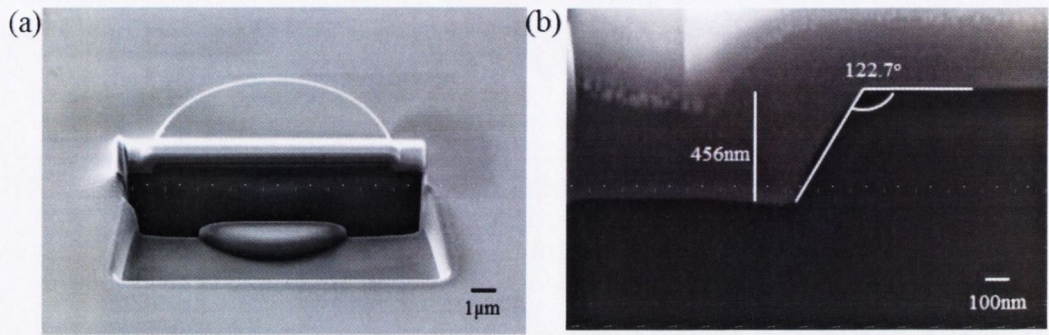
The SEM shows that the resist sidewall is angled with some intrinsic roughness in the resist. Two different etching recipes were used in this project. Starting parameters were used which were known to work well with  $\text{Si}_3\text{N}_4$  from etches used for microelectronics. A FIB cross sectioning technique was used to examine the sidewall angle, and AFM was used to examine the surface roughness.

#### 4.4.1 Focused ion beam cross sectioning

The samples are coated with a 5 nm layer of gold using an EBPVD process as detailed in Chapter 3. Then using a focused ion beam (FIB) with a gas injection system (GIS) “bootstraps” of platinum are deposited to retain the shape of the structure during milling. This is the rectangular layer in Fig. 4.11 (a). The Structure is then bombarded with focused Gallium ions



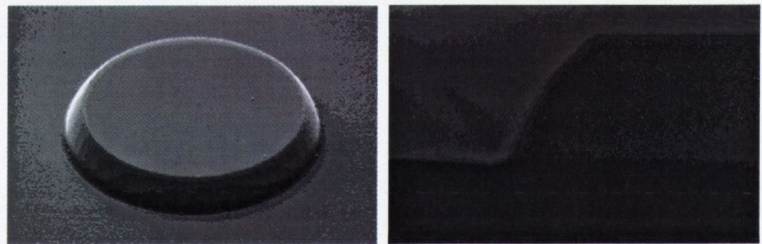
which mill away the surface. A high angle SEM image then allows us to see the sidewall angle. The height of the disk and the angle of the sidewall can then be calculated by taking into account the SEM angle.



**Figure 4.11**(a) SEM image of milled microdisk. (b) Close-up with height and angle measurement.

#### 4.4.2 SF<sub>6</sub> CHF<sub>3</sub> etch

Initially etching was performed using a mix of SF<sub>6</sub> and CHF<sub>3</sub> gasses. The ratios were 40 sccm of SF<sub>6</sub> and 10 sccm of CHF<sub>3</sub>. An RF forward power of 50 W was used and an ICP forward power of 1500 W. Typical results of etched microdisks are shown in Fig. 4.12



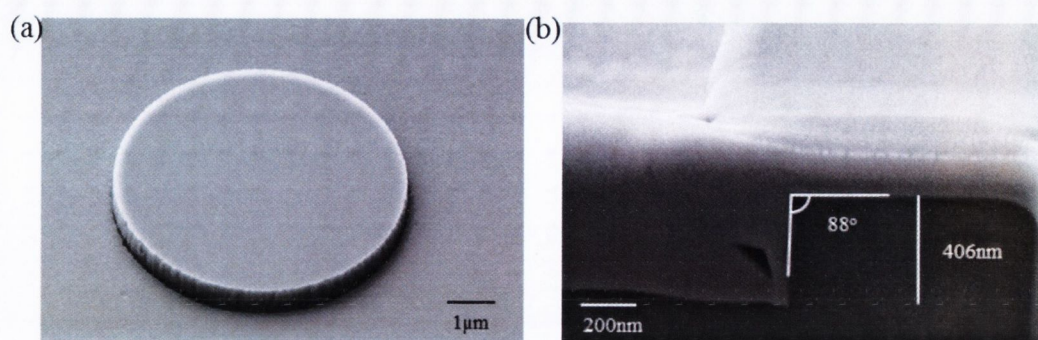
**Figure 4.12** (a)SEM image of microdisk after SF<sub>6</sub>CHF<sub>3</sub> etch (b) SEM image of sidewall profile using the FIB technique.

The etch resulted in very smooth side walls (low surface roughness), however the sidewall angle was large at 57.3° from normal. The sidewall also displayed significant curvature as shown in Fig.4.12 (b).

#### 4.4.3 Ar CHF<sub>3</sub> etch

To achieve a more vertical sidewall a mixture of 40 sccm of Ar and 10 sccm of CHF<sub>3</sub> was used. A RF forward power of 100 W and an ICP power of 1500 W. Typical results for this etch are shown in Fig. 4.13.





**Figure 4.13** (a) SEM image of microdisk after  $\text{ArCHF}_3$  etch (b) Closeup SEM images of sidewall angle after FIB processing.

A design of experiment (DOE) was carried out with 9 different etch recipes. The  $\text{ArCHF}_3$  etch was used as it was more reliable, and produced more vertical sidewalls. The  $\text{SF}_6\text{CHF}_3$  etch would not strike all of the time for the different parameters. The two parameters varied were the ICP forward power [For] and the RIE Forward power [RIE]. The ICP power controls the density of the plasma, whereas the RIE power controls the acceleration of the ions. These two parameters combined affect the bias voltage which is what accelerates the ions across the dark space. For high bias we would expect a more aggressive etch, and for lower bias a more isotropic etch. The parameters used in the DOE and the resulting effect on etch rate are shown in table 4.1.

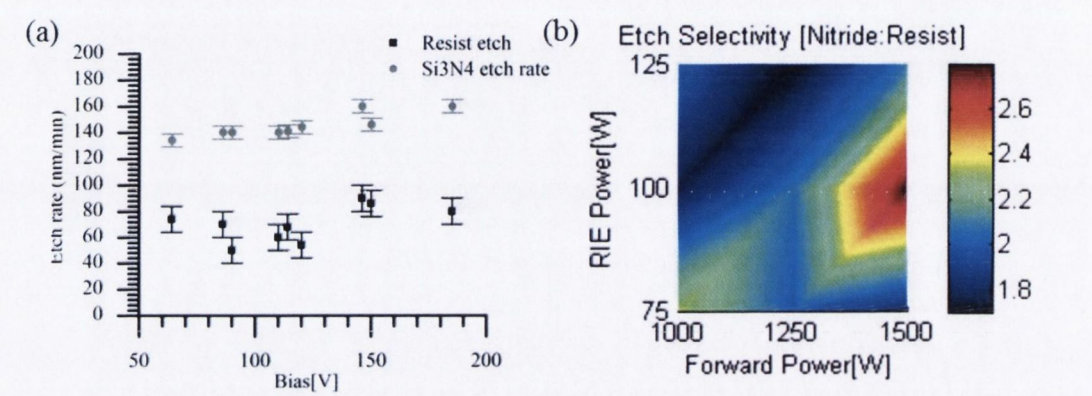
**Table 4.1**

| Sample | For[W] | RIE[W] | Bias[V] | Resist<br>etch<br>rate | $\text{Si}_3\text{N}_4$<br>Etch<br>rate. | Etch<br>Selectivity<br>$\text{Si}_3\text{N}_4$ :resist |
|--------|--------|--------|---------|------------------------|------------------------------------------|--------------------------------------------------------|
| S31    | 1000   | 75     | 110     | 60                     | 140                                      | 2.33:1                                                 |
| S32    | 1250   | 75     | 86      | 70                     | 140                                      | 2:1                                                    |
| S33    | 1500   | 75     | 64      | 74                     | 134                                      | 1.81:1                                                 |
| S34    | 1000   | 100    | 146     | 90                     | 160                                      | 1.78:1                                                 |
| S35    | 1250   | 100    | 114     | 68                     | 141                                      | 2.07:1                                                 |
| S36    | 1500   | 100    | 90      | 50                     | 140                                      | 2.8:1                                                  |
| S37    | 1000   | 125    | 185     | 80                     | 160                                      | 2:1                                                    |
| S38    | 1250   | 125    | 150     | 86                     | 146                                      | 1.7:1                                                  |
| S39    | 1500   | 125    | 120     | 54                     | 144                                      | 2.67:1                                                 |

An  $\text{O}_2$  plasma etch was used to remove the resist in a controlled fashion. The  $\text{O}_2$  molecules react with the carbon in the resist in order to create  $\text{CO}_2$  resulting in the removal of the resist. A plot



of the etch rate of  $\text{Si}_3\text{N}_4$  and the resist as a function of increasing bias is shown in Fig.4.14 (a). The change in etch selectivity as a function of the RIE and forward powers are shown in Fig.4.14 (b).

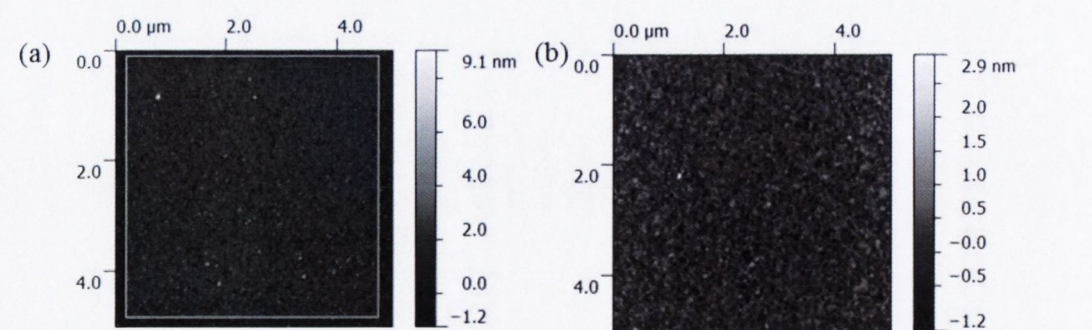


**Figure 4.14** (a) Increasing etch rate of  $\text{Si}_3\text{N}_4$  and resist with increasing bias. (b) Etch selectivity between  $\text{Si}_3\text{N}_4$  and resist for different combinations of RIE and Forward powers.

The RIE and Forward power were chosen in order to maximise the selectivity between the resist and the  $\text{Si}_3\text{N}_4$  layer.

4.5 AFM measurements

For optical applications it is important to get an idea of the surface roughness induced by the etching step. The smoothness of the surface after the etching process was checked using AFM. Initially the surface roughness of an un-patterned  $\text{Si}_3\text{N}_4$  wafer was examined (Fig.4.15 (a)). This was after the dicing process so is representative of all the samples used before patterning. Since the resist remains after the etching process and is removed via  $\text{O}_2$  etching, we would expect the top of the microdisks to have this roughness.



**Figure 4.15** (a) Surface Roughness measured with AFM of un-patterned wafer. (b) Surface roughness of  $\text{SiO}_2$  outside of disk after patterning and etching.

AFM of un-pattered  $\text{Si}_3\text{N}_4$  shows and RMS surface roughness of 0.44 nm, with a Ra of 0.32 nm. The range is 10.3 nm. Figure, 4.15(b) shows an AFM of the  $\text{SiO}_2$  surface just outside the structure. It has an RMS roughness of 0.28nm and range of 4 nm .

## 4.6 Conclusions

Overall this provides a relatively simple method of producing microdisks. Using a negative tone resist as a mask in the etching process minimises the number of process steps and removes the need for a lift off step. Microdisks of various diameters can be produced reliably and reproducibly with a small variation in disk diameter as shown in Fig. 4.6. The  $\text{SF}_6$   $\text{CHF}_3$  etching process results in very low surface roughness on the edge of the disk which is desirable for high Quality whispering gallery modes. The Ar  $\text{CHF}_3$  at results in vertical sidewalls at the cost of more surface roughness. The results obtained here compare well with surface roughness of similar etching processes in the literature [20-23].



## 4.7 References

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## Chapter 5 Light scattering from microdisks

### 5.1 Introduction

Experimental studies of light scattering from water droplets suspended in vacuum either by laser trapping or electromagnetic balance, show a complex pattern of light in the image plane. For spheres bright spots appear just inside the surface of the sphere in the forward and backward scattering directions. These were referred to as glare spots or glare points [1, 2]. These glare points were explained in part for large spheres through geometrical optics, however as the diameter of the sphere gets smaller a full physical optics approach is needed. Mie theory provides the full analytical solution for a sphere in a homogeneous medium and can predict the formation of this glare point image as the Fourier transform of the scattering amplitude over the finite range of angles collected by the lens [3].

Early experiments performed by Ashkin and Dziedzic after discovering resonances in the trapping force required to levitate a water droplet [4] used direct imaging of the droplet to study the scattering properties as a function of incident wavelength. They found that the intensity of the spots in the near field varied with the same dependence as the resonances in the trapping force [5]. This technique was then applied to non-spherical particles formed by accumulations of single spheres [6]. The distance between the glare points on a sphere was shown to be related to the diameter [1] and this technique has been used to size spheres in aerosols and solution [7].

In this chapter direct imaging has been used to study light scattering from  $\text{Si}_3\text{N}_4$  microdisks on a dielectric substrate. To our knowledge this is the first application of this method to complex scatterers on a substrate where an analytical solution does not exist. The recorded images are compared with the analytical solution for the near-field intensity of light scattering for an infinite cylinder and with full 3D FEM numerical simulations. The effect of changing the incident angle, polarisation and polarisation axis of an analyser in front of the CCD are studied. Using this technique we can indirectly map regions of the near-field by examining the scattered light in the far field.

An interesting case arises when the disks are immersed in water. For this case the refractive index contrast is such that photonic nanojets (PNJ) are formed. This is a specific near-field intensity distribution, where the product of the divergence angle of the main scattering lobe and its  $1/e^2$  waist is smaller than achievable with a focused Gaussian beam [8]. In the case of scattering in water the image formed is a combination of the light scattered from the disk and



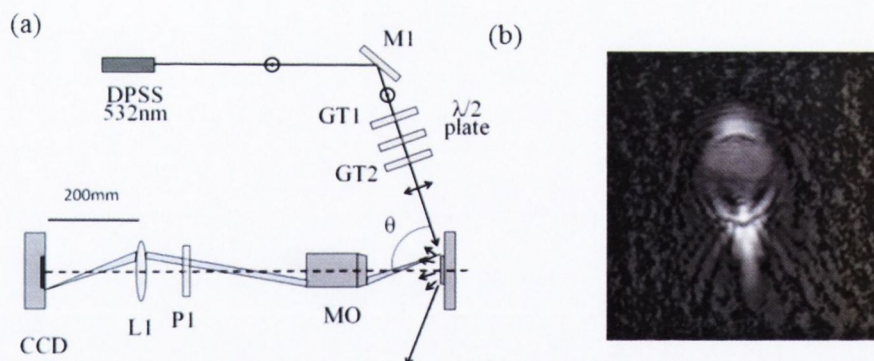
intrinsic scattering due to density fluctuations in water. The order of magnitude for Rayleigh scattering in air is too small to contribute to the image, however in water the Rayleigh scattering is  $\sim 1000$  times stronger and falls into the range of the signals we are observing [9].

From Mie theory it is known that for a sphere or infinite cylinder the light scattering distribution depends only on two parameters: the size parameter  $x_s = 2\pi R / \lambda$  and the refractive index contrast  $m = n/n_0$ . In this chapter, the effect of varying diameter on the scattering distribution while holding the wavelength fixed (size parameter dependence) is investigated. The dependence of the light scattering on the polarisation of incident and collected light is investigated. The dependence of the near-field images on incident angle for a single disk diameter are studied (results are representative of the other disk sizes.). The effect of the substrate layer used in the experiment is discussed as it strongly affects the scattering properties of the disks. The experimental results are then compared to the analytical solution for an infinite cylinder (equivalent to 2D FEM) and full 3D Finite element methods results.

## 5.2 Experimental measurements

### 5.2.1 Experimental setup

The experimental setup is depicted in Fig. 5.1 (a). Linearly polarised light is incident from a 532 nm wavelength continuous wave (CW) diode pumped solid laser (DPSS), with a power of 3.18 mW and a  $1/e^2$  beam diameter of 0.54 mm. The light is introduced with an angle of incidence  $\theta_i$  from the surface normal. This angle is controlled by linearly translating mirror M1, and adjusting the position of the polarisation optics. Two Glan-Thompson polarisers (GT1 and GT2) were used in order to get a high extinction polarisation ratio. A half wave plate was inserted to rotate the polarisation. The scattered light is collected from above using a long working distance (6 mm), flat field corrected, 100x microscope objective with an NA of 0.7 (Mitutoyo). The objective was infinity corrected, and a tube lens of focal length 200mm was used to form the primary image directly onto a CCD. The result is a gray-scale pixel image of a typical scattered light distribution as shown in Fig. 5.1 (b).



**Figure 5.1 (a)** Laser light is introduced with an angle of incidence  $\theta$ . Most of the light is reflected specularly, but a small percentage is scattered at large angles by the finite height of the disk. The scattered light from the structure is collected using the microscope objective (MO) and imaged onto a CCD as indicated by the marginal ray shaded in blue. The input beam is a Gaussian with a large beam diameter of 0.54 mm, making the excitation effectively a plane wave on the scale of the microdisk. **(b)** Typical gray scale image taken with a single exposure. The disk diameter was 6  $\mu\text{m}$ .

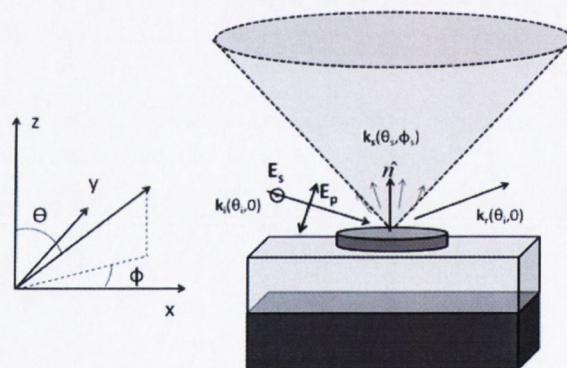
The CCD used was a 1/3 inch Sony ICX204AL with a resolution of  $1024 \times 768$  and 4.65  $\mu\text{m}$  physical pixel size. With a magnification of 100x each image pixel was 46.5 nm, resulting in a field of view of  $36 \mu\text{m} \times 47 \mu\text{m}$ . The long working distance objective with a relatively low NA of 0.7 allows us to input beams up to an angle of  $45^\circ$  without the reflected beam entering the cone of the objective directly; however this means that the images recorded are diffraction



limited to 463 nm using the Rayleigh criterion,  $R = 0.61\lambda/NA$ . An analyser can be inserted at position P1 to examine the polarisation of the scattered light.

### 5.2.2 Coordinate system definition

The standard problem of light scattering by an arbitrarily shaped object in free space is usually defined in terms of angles with respect to a scattering plane composed of the incident  $\mathbf{k}$  vector and the direction of observation. Since this experiment contains a planar interface and we are collecting the scattered light over a range of angles simultaneously using a microscope objective (MO) we dispense with the usual nomenclature of parallel and perpendicular components to the scattering plane, and instead use the familiar idea of s and p polarised light. The p polarised light has electric field ( $\mathbf{E}_p$ ) parallel to the plane containing the incident  $\mathbf{k}$  vector and the normal to the surface, and s polarised light has electric field  $\mathbf{E}_s$  perpendicular to this as depicted in Fig. 5.2.



**Figure 5.2** Coordinate system used in experimental system is displayed displaced on the left. The origin is taken at the centre of the disk. The electric field vector  $\mathbf{E}_s$  for s polarised light and  $\mathbf{E}_p$  for p polarised light are depicted. The reflected light leaves at the same angle as the incident light along  $\mathbf{k}_r$  and is not collected by the MO. Only scattered light with  $\mathbf{k}_s(\theta, \phi)$  in the range of angles defined by the NA of the objective is collected.

To fully characterise the scattering from the structure we need a Cartesian and spherical system of coordinates. The origin is taken at the centre of the disk at the interface of the  $\text{Si}_3\text{N}_4$  and  $\text{SiO}_2$  layer. The unit vectors are shown displaced from the origin on the left of Fig. 5.2 for clarity. The x-y plane is coplanar with the substrate with the z axis normal to the substrate. The angles  $\theta$  and  $\phi$  are used to describe the angular dependence of the out of plane and in-plane scattering respectively. Note that  $\theta=0$  corresponds to the direction normal to the substrate. All incident angles are described with respect to the normal using  $\theta$ . In all experimental measurements, save the study of dependence of scattering on incident angle, the incident light is at a fixed angle to



the normal  $\theta_i$  with either s or p polarisation as shown in the diagram. The light is always incident in the positive x direction, from the left of the image.

### 5.2.3 Image interpretation

The observed images such as that of Fig .5.1 (b) are a combination of two mechanisms. In the first mechanism bright spots are formed inside and at the edges of the microdisk due to out of plane light scattering from the finite height of the  $\text{Si}_3\text{N}_4$  structures similar to glare points in spheres [3]. Due to the high angle of incidence and low surface roughness of the substrate the input light is almost completely specularly reflected. This results in a negligible background far from the disks. Only light which is scattered into the cone of the MO is observed in the image. Each pixel in the image can be thought of the angle averaged differential scattering cross section over the collection angle of the microscope objective of light coming from that position in the object plane. The bright spots formed inside and on the edges of the disk thus correspond to positions on the disk where a lot of light is scattered out of plane and into the cone of the objective.

In the second mechanism light which is scattered in-plane from the disks is then scattered again from the rough  $\text{SiO}_2$  surface. This gives a map of the in plane angular dependence of the near-field intensity which shares many of the characteristics of the near-field calculated from numerical and analytical work. As this is a second order effect the pattern of lobes formed outside the scatterer is an order of magnitude less intense than the spots formed on the edges of the disk. The image observed experimentally therefore does not correspond exactly to a map of the total near-field intensity distribution however it contains a lot of information about the near-field intensity distribution which can be used to confirm numerical models.

### 5.2.4 Imaging procedure and reproducibility

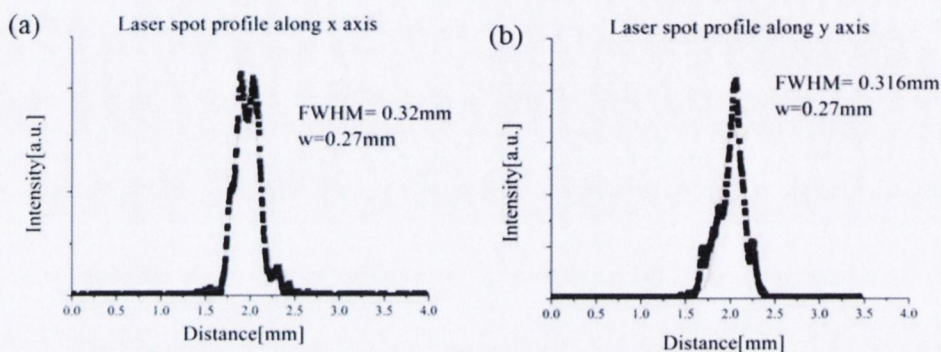
Experimentally we wish to reproduce as accurately as possible a plane wave incident on a single microdisk. In order to do this we use a laser with a Gaussian mode which has a spot size much greater than the size of an individual disk. The laser spot size was measured experimentally using normal incidence directly on the CCD, with ND filters to reduce the intensity and avoid damage. In the standard nomenclature for ND filters the optical density  $d$  is given as the negative log of the ratio of input intensity  $I_0$  and transmitted intensity  $I$ :

$$d = -\log_{10}(I / I_0) \quad 5.1$$

For an exposure time of 5.5 ms and a gain of 7.410 dB the image of the laser spot was recorded. The incident power was 3.2 mW and a combination of ND filters were used giving  $d=6.062$ . i.e.

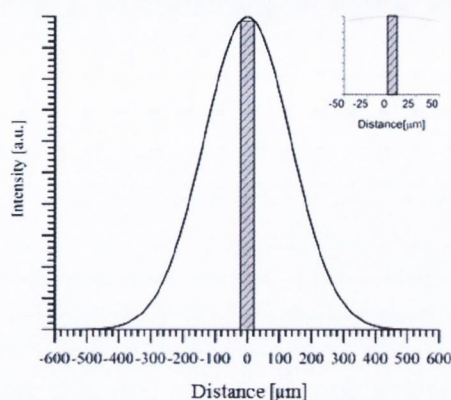


the light intensity was six orders of magnitude smaller than that of the incident intensity. The exposure time and gain settings were chosen so as not to over expose the CCD. Cross sections of the spot in the x and y directions were taken, with x corresponding to the long axis of the CCD. These were fitted with a Gaussian to find the FWHM and the  $1/e^2$  half width of the beam  $w$ . Knowing that each pixel is  $4.65\mu\text{m}$  from the physical pixel size and the fact that the image is not magnified we were able to convert the units into millimetres. The results are shown in Fig. 5.3.



**Figure 5.3** (a) Intensity distribution of the incident beam along x-direction. (b) Along y direction.

The spot distribution here subject to variation due to scratches on the many ND filters used. The distribution used in the scattering experiments was more Gaussian. Ideally the noise can be removed by spatial filtering; however we found experimentally that it had little effect on the consistency of the experimental results. This shows that our laser spot is roughly symmetric with a FWHM of 0.32 mm (320  $\mu\text{m}$ ). The  $1/e^2$  diameter is  $2w_x \sim 2w_y = 0.54$  mm. This spot size is big enough to approximate a plane wave of constant intensity incident on a single microdisk which is less than 10  $\mu\text{m}$  in diameter. The intensity distribution compared with the size of the total field of view of the microscope objective and a single 10  $\mu\text{m}$  diameter disk is shown in Fig. 5.4.



**Figure 5.4** Gaussian intensity distribution of the incident laser with  $w=0.54$  mm. The shaded region is the  $46\text{ }\mu\text{m}$  field of view of the MO. The inset shows a close-up of the intensity distribution  $50\text{ }\mu\text{m}$  either side of the maximum. The shaded region in the inset is the size of an individual  $10\text{ }\mu\text{m}$  diameter microdisk.

Figure 5.4 is a scaled image showing the relative size of the structures compared to the waist of the laser beam. The shaded grey rectangle is the size of the  $46\text{ }\mu\text{m}$  field of view (FOV) of the microscope objective. The grey rectangle in the inset on the right shows the size of a  $10\text{ }\mu\text{m}$  disk compared to the incident intensity. This shows that the laser intensity can be taken as constant over this size range. We would expect only a 2% drop in intensity between two disks of the same size with one in the centre of the FOV and one on the periphery purely due to the Gaussian illumination. This was consistent with experimental observations which showed that the same intensity is scattered from a disk regardless of position in the field of view (FOV). For consistency all measurements were made with the disk in the centre of the FOV.

### 5.2.5 Dynamic range extension and intensity calibration

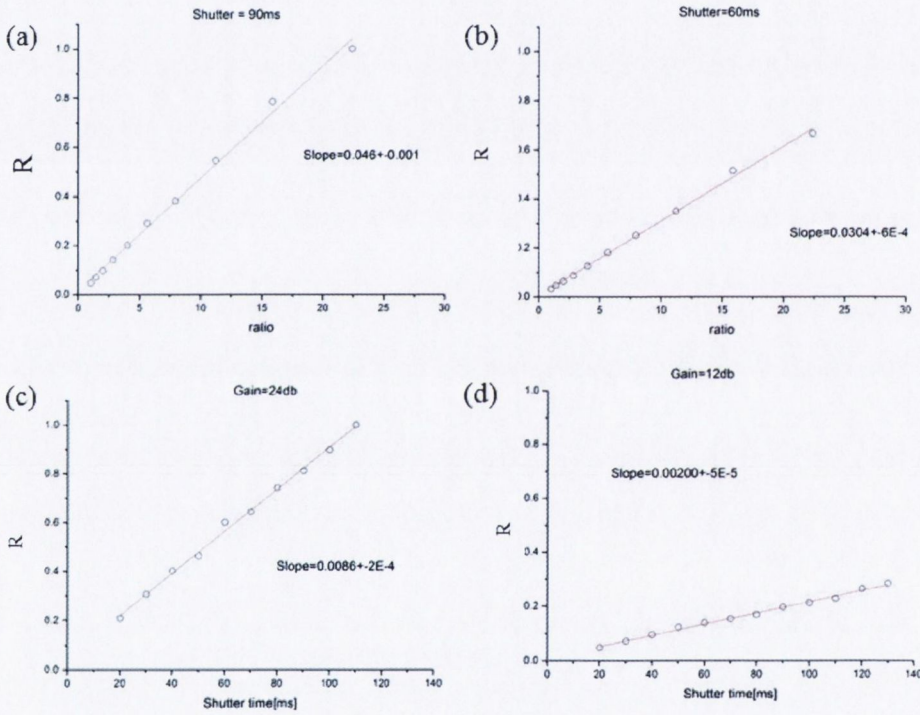
The dynamic range of a CCD is the difference between the brightest and dimmest values which can be resolved in an image. It is limited in a single shot image by the electron well depth of the camera which is related to the pixel size (see Chapter-3.4.4). The images recorded in bitmap format from the CCD are displayed scaled as pixel values from 0-256. The scaling can be changed linearly by changing the gain value of the camera and the exposure time. The CCD used was an 8-bit Sony ICX204AL 1/3 inch with a pixel size of  $4.65\text{ }\mu\text{m}$ . The dynamic range was relatively small due to the small pixel size. By changing the exposure time and gain settings we can increase the dynamic range of the final image by taking multiple images, scaling them and adding them back together. This is a standard procedure. This particular CCD was ideally suited to this procedure as it employs Sony's Hole-Accumulation Diode (HAD) technology. This allows the CCD to be over exposed without adverse effects such as blooming. The



calibration curves are shown in Fig. 5.5. The image of a microdisk is taken under constant illumination, varying the exposure time  $s$  and the value gain  $g$  independently. Since the gain here is conventionally defined as

$$g = 20 \log_{10}(I/I_0) \quad 5.2$$

we define  $ratio = 10^{-g/20}$ . The CCD response is linear to this factor. We also define the parameter  $R = p/256$  which is the normalised pixel value for ease of calculation. Extracting the slope of the plots  $R$  versus  $ratio$  and  $R$  versus  $s$ , a calibration formula can be generated.



**Figure 5.5:** Calibration curves. (a) Normalised pixel value versus ratio for fixed shutter time of 90 ms (b) 60 ms. (c) Normalised pixel value versus shutter time for fixed gain of 24 dB (d) 12 dB

The resulting calibration formula is

$$R(s, g) = (5.4 \times 10^{-4}) s 10^{g/20} \quad 5.3$$

where  $s$  is the exposure time in ms and  $g$  is the gain in dB. Using this equation we can compare two images taken with different exposure times and gain settings using

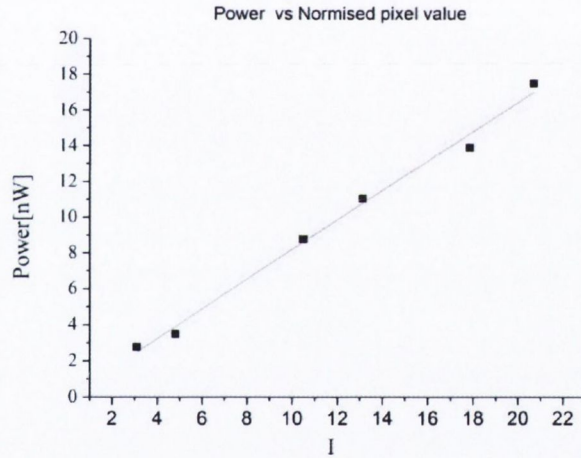
$$\frac{R(s_2, g_2)}{R(s_1, g_1)} = \left( \frac{s_2}{s_1} \right) 10^{(g_2 - g_1)/20} \quad 5.4$$

A standard shutter time of  $s_0 = 90$  ms (mid-range) and gain of  $g_0 = 0$  dB (easy calculation) is chosen as a reference scale which all measurements are normalised to. Setting  $R(s_0, g_0) = 1$  and using  $s_0 = 90$  ms and  $g_0 = 0$  dB results in

$$I(s, g, p) = \left( \frac{s}{90} \right) 10^{(g/20)} R \quad 5.5$$

where  $I$  is now the calibrated normalised pixel value. Equation 5.5 gives a calibrated pixel value so two images taken with different exposure time and gain settings can be compared.

The amount of incident power getting from the laser to the objective is also of interest. Using ND filters of known optical density, the normalized pixel value  $I$  can be calibrated to the incident power on the CCD  $P$  (Fig. 5.6).



**Figure 5.6** Calibration curve for normalised pixel value  $I$  to power  $P$  [nW].

CCDs are very sensitive, so care was taken and enough ND filters were used to reduce the incident laser power to within the range of the CCD. This was on the nW scale.

$$P(s, g, p) = 8.3 \times 10^{-10} I(s, g, p) \quad 5.6$$

Substituting back in the expression for  $I$  from Eq. 5.5 results in

$$P(s, g, p) = \frac{(p/256)}{(s/90)(10^{g/20})} 8.3 \times 10^{-10} \quad 5.7$$



This formula is the result of two calibrations. An image can now be taken with the camera at any exposure time  $s$ , and any gain value  $g$ . The pixel value  $p$  can be related directly to power onto the CCD. This calibration of course introduces error. The calibration error is calculated using the propagation of uncertainty formula:

$$\frac{\Delta P_{cal}}{P} = \sqrt{\left(\frac{\Delta m_p}{m_p}\right)^2 + \left(\frac{\Delta m_s}{m_s}\right)^2 + \left(\frac{\Delta m_g}{m_g}\right)^2} \quad 5.8$$

where  $m_p$  is the slope of the calibration curve between power  $P$  and calibrated normalised pixel value  $I$ ,  $m_s$  is the slope between shutter time  $s$  and normalised pixel value  $R$ , and  $m_g$  is the slope between ratio and the normalised pixel value. The parameters  $\Delta m_p$ ,  $\Delta m_s$  and  $\Delta m_g$  are the regression errors extracted from least square fittings of the slope.

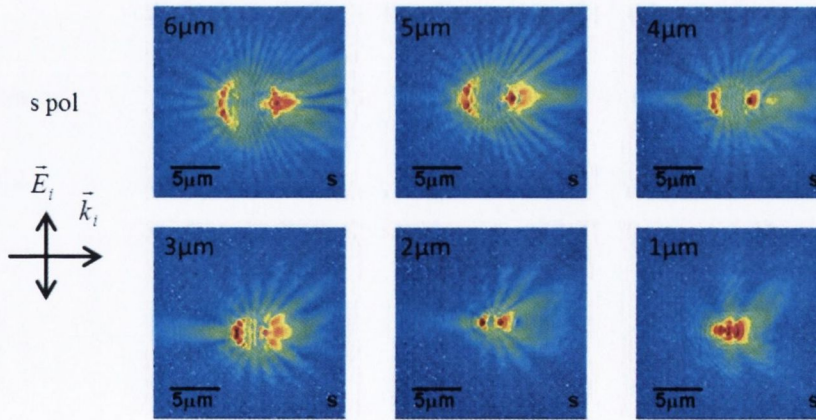
Assuming the shutter speed and the gain setting errors are negligible, the calibration error between converting pixel value to power is  $\Delta P_{cal} = 5\%$ . This means that more than a 5% change in intensity is necessary in order to observe an experimental change. Not converting to power and comparing values of calibrated normalised intensity  $I$  instead, the error is only 3%, which is a combination of the shutter time and gain calibration. Thus for light scattering experiments in this project only calibrated normalised intensity  $I$  is used to compare the differences in intensity between two samples.

## 5.3 Results

### 5.3.1 Dependence of light scattering on microdisk diameter

From Mie theory it is known that for a sphere or infinite cylinder in a homogeneous medium the scattering distribution depends on only two parameters; the size parameter  $x_s = 2\pi R / \lambda$  and the refractive index contrast  $m$ , which is the ratio of the index of refraction of the scatterer to the surrounding medium [10].

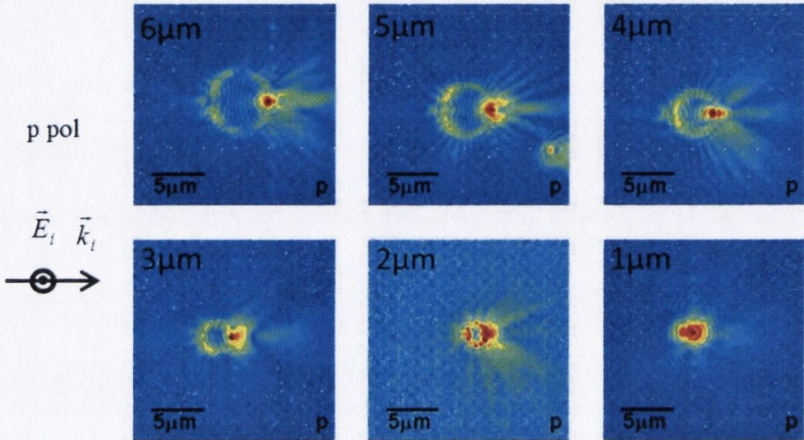
In order to probe the dependence of the scattering of these finite cylinders on substrates with size parameter it is necessary to vary either the diameter of the disk or the incident wavelength. A tuneable CW laser in the visible range would be ideal however in the absence of such a laser varying the diameter of the disk will suffice. For a refractive index contrast of 2.05, ( $\text{Si}_3\text{N}_4$  in air) incident light is focused to a sub-wavelength spot which is just inside the surface of the disk. Scattering distributions were recorded for disks ranging in diameter from  $1\text{ }\mu\text{m}$  to  $6\text{ }\mu\text{m}$ , with an incident wavelength of  $532\text{ nm}$ . This probes a size parameter range from  $x_s=6$  to  $x_s=35$ . The images are displayed in log scale in Fig. 5.7. The log scale allows us to see the full dynamic range of the image, and observe simultaneously the high intensity spots and the low intensity higher angle lobes. These lobes are an order of magnitude lower in intensity than the spots on the edge of the disk, but are the most important element in creating this unique intensity distribution. There is no polariser at position P1 in this case so all the light is collected to form the image.



**Figure 5.7:** Experimental images on log scale for scattering from individual disks. The light is incident at a fixed angle of incident wavelength is  $532\text{ nm}$  from the left of the image with a fixed input angle of  $\theta_i=80^\circ$  with a wavelength of  $532\text{ nm}$ . The incident light is the s polarised and propagates from left to right as shown.

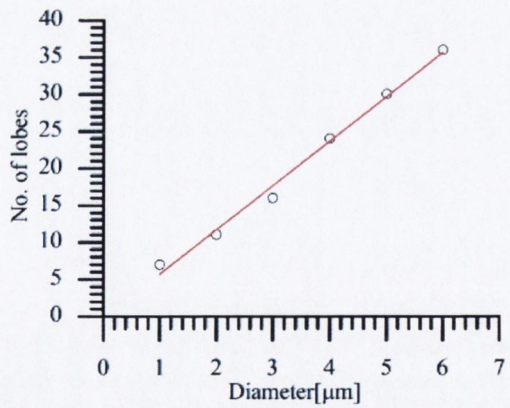


Figure 5.8 shows the same size variation for the p polarised case. The number and angular distribution of lobes in the p polarised case is similar but more difficult to observe. The lobes scattering in the forward direction are more pronounced in the image.



**Figure 5.8:** Experimental images on log scale for scattering from individual disks. The incident wavelength is 532nm from the left of the image with a fixed input angle of 80°. This is the p polarised case.

The number of lobes can be plotted as a function of disk diameter. This is displayed in Fig. 5.9 and is seen to increase linearly with the disk diameter (and thus size parameter) as expected.



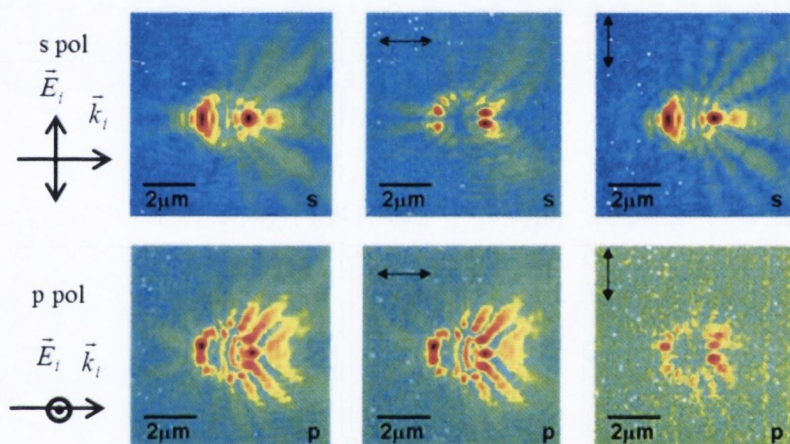
**Figure 5.9:** Number of in-plane scattering lobes observed experimentally for s polarised case as a function of the disk diameter .

The number of lobes is approximately equal to the size parameter of the disk  $x_s = 2\pi R / \lambda$  . This is in contrast to the case of an infinite cylinder in free space where the number of lobes is approximately  $2x_s$ .



### 5.3.2 Polarisation dependence

By placing an analyser at position P1 in Fig. 5.1 (a) we can examine the different polarisation components of the images. Fig. 5. 10 shows the images recorded from a 2  $\mu\text{m}$  diameter disk with 532 nm light incident in the positive x direction (from left) with an angle of  $80^\circ$  to the normal. The top row shows the results for s polarised incident light and the bottom for p polarised. The second column the axis of the polariser is along the x direction and in the third column the axis of the polariser is along the y axis as depicted by the arrow in the top left hand corner of the images.



**Figure 5.10** Light scattering from a 2  $\mu\text{m}$  diameter disk. Light is incident at  $80^\circ$  from normal along the positive x axis. The top row is s polarised and the bottom is p polarised. The first column is the log scale image without any analyser at position P1. The second is with the analyser along the x axis and the third with the analyser along the y axis as shown in top left hand corner of image.

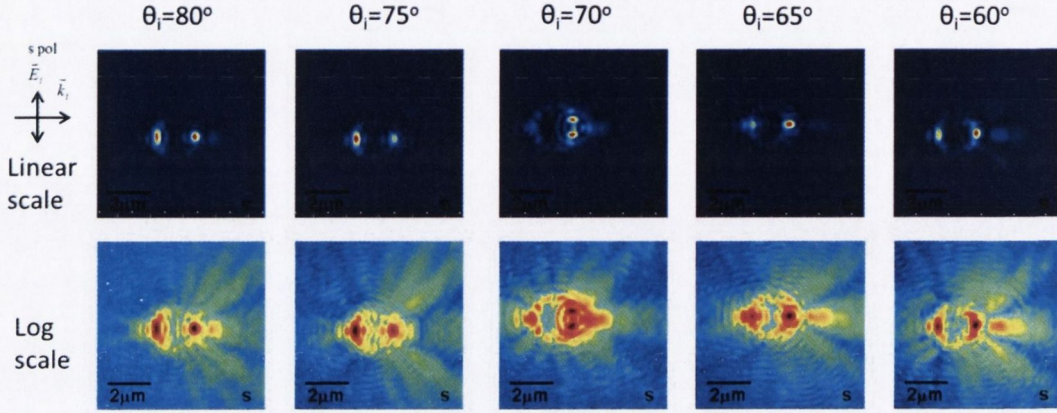
For the s polarised case, the dominant polarisation is along the y axis which is the direction of the incident polarisation. The x polarised component is much weaker but due to the high dynamic range of the measurement we can still observe it. This shows an intensity distribution which is symmetric around the centre of the disk, but zero along the axis, suggesting an anti-symmetric electric field distribution.

### 5.3.3 Dependence on incidence angle

The structures are 3D scatterers and thus we would expect a large dependence on the incident angle. This is indeed the case. The field distributions for light scattering as a function of incident angle and are shown in Fig. 5.11 for s polarised incidence on a linear and log scale. The measurements were performed on the same disk which had a diameter of 2  $\mu\text{m}$ . This relatively

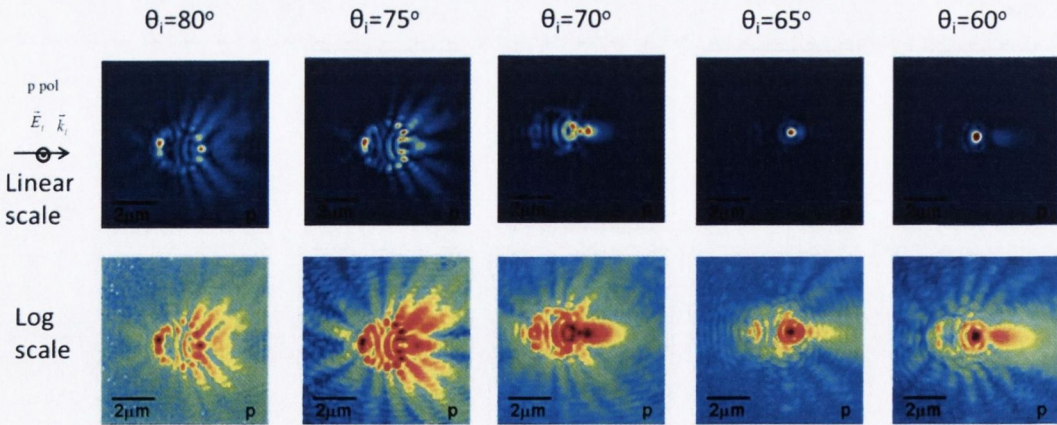


small diameter was chosen for ease of comparison with the full 3D FEM simulations, which are difficult to perform on larger disks. The results are representative of other microdisk sizes.



**Figure 5.11** Experimentally measured light scattering intensity distributions from a microdisk of diameter  $2\ \mu\text{m}$  as a function of incident angle  $\theta_i$ . The wavelength of the light is  $532\text{nm}$  and it is propagating in the positive  $x$  direction (from left of image.). The top row is a linear scale showing the image formed from out of plane scattering, and the bottom row is a log scale showing the in plane scattering pattern.

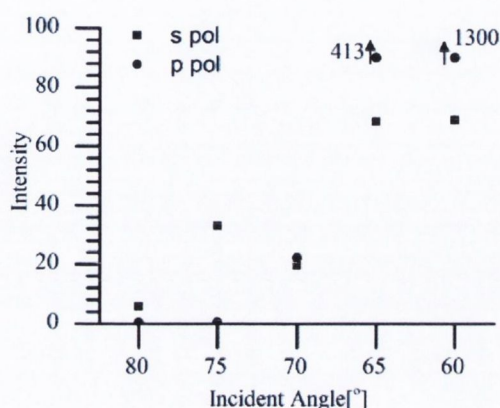
For the  $s$  polarised case, for most incident angles there are small changes in the field distribution. The in plane distribution has the same number of lobes at the same angle. However there is a large change at an angle of  $70^\circ$ . The  $p$  polarised case is shown in Fig. 5.12. This shows a more complex dependence on angle as we would expect as for the  $p$  polarised case changing the angle results in a change of the relative magnitude of the field components  $E_x$  and  $E_z$ .



**Figure 5.12** Experimentally measured light scattering intensity distributions from a microdisk of diameter  $2\ \mu\text{m}$  as a function of incident angle  $\theta_i$ . The wavelength of the light is  $532\text{nm}$  and it is propagating in the positive  $x$  direction (from left of image.). The top row is a linear scale showing the image formed from out of plane scattering, and the bottom row is a log scale showing the in plane scattering pattern.

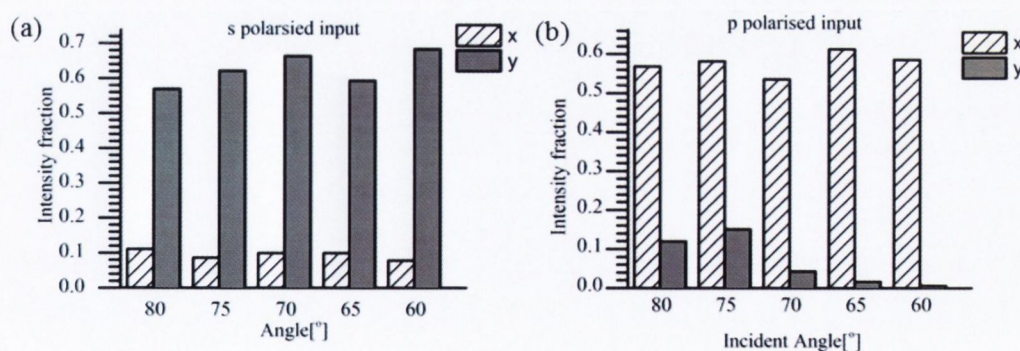


The maximum intensity collected as a function of angle for s and p polarised cases is shown in Fig. 5.13. The intensity is the normalized intensity calculated using the calibration technique outlined in section-5.2.5. There is a peak for the s polarised case at  $75^\circ$  which corresponds with the change in field pattern. This is not evident in the p polarised case.



**Figure 5.13** Maximum image intensity as a function of incident angle for s and p polarised input. The s polarised input uses the square markers and p polarised the circular markers.

By placing an analyser at position P1 the relative intensity in each polarisation as a function of input angle can be examined. For the s polarised case the light scattered light remains polarised mostly in the direction of the input polarisation and has no dependence on incident angle. For the p polarised case the scattered light is primarily polarised along the x axis with a y component that falls off with decreasing input angle. Here the input angle is with respect to the normal as depicted in Fig. 5.2.



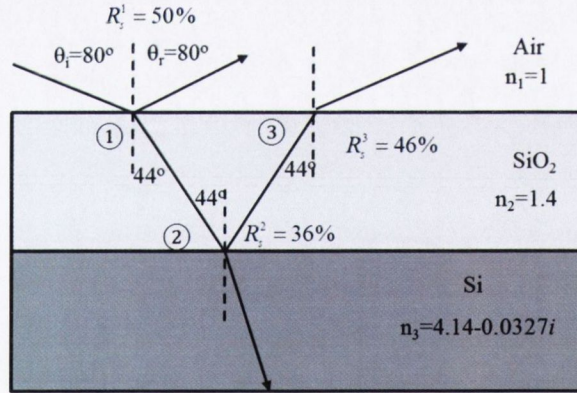
**Figure 5.14** Bar plot of the relative intensity in each polarisation direction of the analyser. The intensity is normalised relative to the intensity without an analyser. The patterned column represents the case with the polarisation axis in the x direction, and the grey column the case with the axis in the y direction. Results are shown for (a) s polarised input and (b) p polarised input.



The spatial distribution of the scattering is closer to the case of a disk in a homogeneous medium when the incident angle  $\theta_i$  is at larger angles. Thus for the rest of the experiments the input angle is fixed at  $80^\circ$ . The  $10^\circ$  offset from the surface avoids any of the beam being diffracted by the edge of the sample or contaminating particles on the substrate.

### 5.3.4 Effect of the substrate

The substrate has a large effect on both the near-field and far-field distributions. The basic properties of the scattered field can be modelled using 2D and the 3D numerical simulations of a microdisk in free space, but in order to match with experimental results it is necessary to include the substrate. With the presence of a substrate the field can no longer be described by a single plane wave. It is instead given by the sum of many planes waves with amplitudes defined by the Fresnel reflection and transmission coefficients and phase shifts depending on the thickness of the layers. Figure 5.15 shows the first 3 reflections and transmissions for s polarised light incident at  $80^\circ$  at a wavelength of 532 nm on a  $2\text{ }\mu\text{m}$  thick layer of  $\text{SiO}_2$  ( $n_2=1.4$ ) on top of a semi-infinite layer of Si ( $n_3=4.14-0.0327i$ ).



**Figure 5.15:** Ray diagram of the first three reflections and transmissions at the air- $\text{SiO}_2$ ,  $\text{SiO}_2$ -Si, and  $\text{SiO}_2$ -air interfaces respectively. The interfaces are labelled ①=> air- $\text{SiO}_2$ , ②=>  $\text{SiO}_2$ -Si, and ③=>  $\text{SiO}_2$ -air. R is the percentage of light reflected at the respective interfaces.

If we consider light reflection from the layered interface of Fig. 5.15 without the scatterer, the simple Fresnel reflection equations can be used to calculate the fields. The electric field in the air domain above the substrate are given by:

$$\vec{E} = \begin{pmatrix} 0 \\ E_0 \\ 0 \end{pmatrix} \exp(i\vec{k}_i \cdot \hat{r}) + \begin{pmatrix} 0 \\ r_1 E_0 \\ 0 \end{pmatrix} \exp(i\vec{k}_r \cdot \hat{r}) + \begin{pmatrix} 0 \\ t_1 r_2 t_3 E_0 \\ 0 \end{pmatrix} \exp(i\vec{k}_t \cdot \hat{r} + \varphi) + \dots \quad 5.9$$

where  $r_1, t_1, r_2, t_3$ , are the amplitude reflection and transmission coefficients at the interfaces denoted ①, ②, ③ in Fig.5.15. These coefficients are angle dependent and depend also on input polarisation. Their expressions are easily found by using the electromagnetic boundary conditions at the interface and are given by [11]:

$$r^s = \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \quad 5.10$$

$$t^s = \frac{2n_1 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_t} \quad 5.11$$

for the s polarised case, and

$$r^p = \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \quad 5.12$$

$$t^p = \frac{2n_1 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_t} \quad 5.13$$

for the p polarised case.

The phase difference  $\varphi$  in Eq. 5.9 is determined by the optical path travelled by the ray which goes through the SiO<sub>2</sub> layer and is given by

$$\varphi = \frac{15\pi n_2}{\cos(a \sin((n_1/n_2) \sin \theta))} \quad 5.14$$

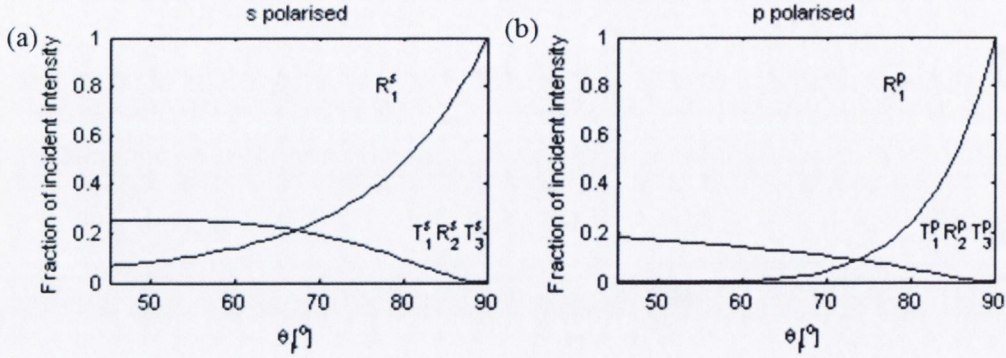
taking the incident wavelength as 532 nm and the height of the SiO<sub>2</sub> layer as 2μm which is the case in all of our experimental measurements. The reflectance R and transmittance T are the ratios of the intensities of the reflected and transmitted wave to the incident wave and are given by

$$R = |r^2| \quad 5.15$$

$$T = \frac{n_2 \cos \theta_t}{n_1 \cos \theta_i} |t^2| \quad 5.16$$

The reflectance of the air-SiO<sub>2</sub> interface is 50% of the incident intensity at 80° and the value of  $T_1 \cdot R_2 \cdot T_3$  is 10%. The next contributing term would have a coefficient of  $T_1 \cdot R_2 \cdot R_3 \cdot R_2 \cdot T_3$  and is less than 1% at 80° for both s and p polarisations. Thus only the first 3 terms for the field in the air just above the substrate are considered. Figure 5.16 plots R1 and  $T_1 \cdot R_2 \cdot T_3$  as a function of incident angle with (a) showing the s polarised case and (b) showing the p polarised case.

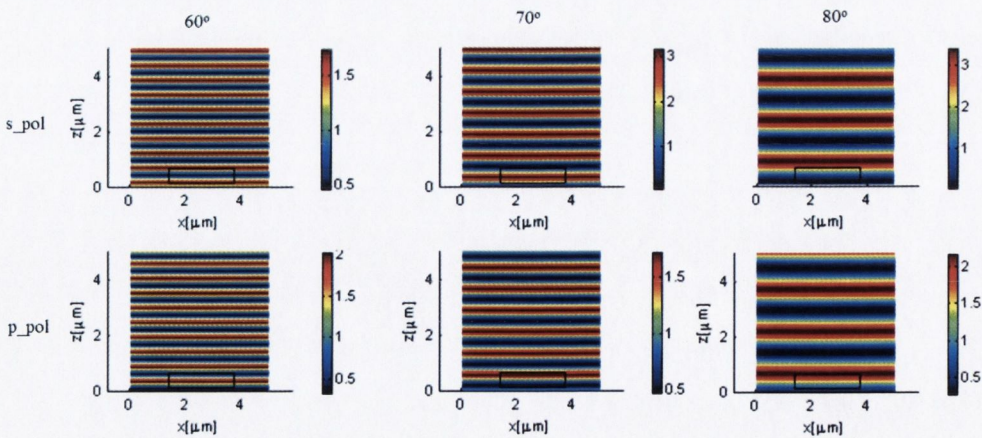




**Figure 5.16** Coefficients for the reflected wave and the transmitted-reflected-transmitted wave in the air domain above the  $\text{SiO}_2$  layer. The two rays considered are the one that reflects from the air- $\text{SiO}_2$  interface with probability  $R_1$ , and the ray when it crosses the air- $\text{SiO}_2$  interface, reflects from the  $\text{SiO}_2$ -Si interface and passes through the  $\text{SiO}_2$ -air interface. The percentage of incident light when it makes it through this path is given by  $T_1 R_1 T_3$ .

For an input beam of finite extent such as the Gaussian beam used in the experiment, the background field will be the sum of these three fields in the region close to the scatterer and then separate into the incident beam and the reflected beams further away.

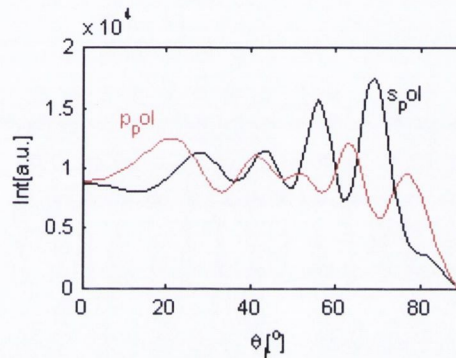
Figure 5.17 shows the intensity ( $|\vec{E} \cdot \vec{E}|^2$ ) for the s and p polarised cases for incident angles of  $60^\circ$ ,  $70^\circ$  and  $80^\circ$ , with  $\vec{E}$  calculated from Eq. 5.9. The outline of a  $2 \mu\text{m}$  wide and  $400 \text{ nm}$  high rectangle is shown to represent the position of a typical scatterer. The intensity is always a minimum at the interface with a sinusoidal dependence in the  $z$  direction with a period and amplitude which depends on the incident angle.



**Figure 5.17** Background intensity for input angles of  $60^\circ$ ,  $70^\circ$  and  $80^\circ$  for s polarised and p polarised incident fields. The black rectangles indicate the position of a  $400 \text{ nm}$  high microdisk.



Consider the background field as the field driving dipoles in the microdisk shows why this case is very different from that of a single plane wave. Integrating the intensity over the rectangular domain gives a figure of merit for how much the field will interact with the scatterer. This integrated value is shown for s and p polarisation in Fig. 5.18. This may account for the angle dependence of the scattered intensity of the scattered light seen in the experimental measurements in Fig. 5.13.



**Figure 5.18:** Intensity integrated over the  $2\mu\text{m} \times 400\text{nm}$  box in Fig.4.17 for s polarised and p polarised cases as a function of input angle.

There is a peak at  $70^\circ$  for the s polarised case which corresponds to a local minimum for the p polarised case.

## 5.4 Comparison with FEM simulations of the near-field

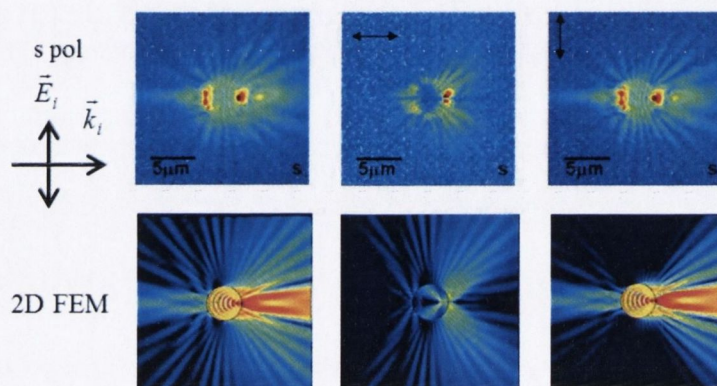
Care must be taken when comparing experimental results with FEM results. As explained in section 5.2.3, the images observed experimentally are a combination of light scattered out of plane by the microdisk, and light scattered in plane, then scattered again from the substrate. It is this secondary scattered light that we are interested in using to observe the intensity distribution in the near-field. The out of plane scattering shows up in the image as a high intensity pattern above and close to the edges of the scatter. This is explained in more detail in Chapter-7.

### 5.4.1 2D FEM

To begin with we compare the experimental images with the near-field intensity calculated through the finite element method (FEM) for a 2D case. This is effectively modelling scattering from an infinite cylinder, and gives results which agree with the analytical model outlined in Chapter-2. The 2D simulations are much less computationally expensive than the 3D



simulations. This allows use of fine meshes resulting in high resolution results; however the case being modelled does not correspond exactly to actual 3D scatterer. Figure 5.19 shows the image plane results for a disk of diameter  $4\text{ }\mu\text{m}$ . The second row of the figure shows the 2D FEM results for the case of an infinite cylinder with normal illumination at  $532\text{ nm}$ . The refractive index is taken as  $2.05$  (i.e. no effective index approximation is used.)



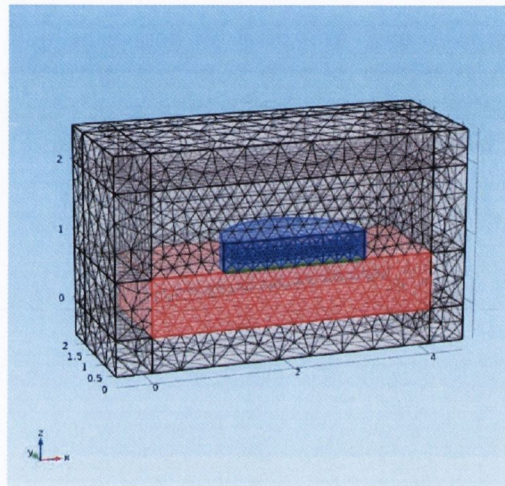
**Figure. 5.19:** Light scattering from a  $4\text{ }\mu\text{m}$  diameter disk compared to 2D FEM for an infinite cylinder for refractive index  $n=2.05$ . The incident light is s polarised and propagates in the positive x direction. The first row shows the experimental results, the second row the 2D FEM simulations. The first column is the total field, the second and third columns show the x-polarised and y-polarised components of the scattered field respectively.

Only the experimental image outside the microdisk should correspond to the intensity distribution. These results show that there is a qualitative correspondence between the experimental images and the near-field intensities calculated using 2D FEM. Specifically examining the polarisation response of the structures shows that for s polarised input (polarised along the y axis), most of the scattered light is polarised along the same axis, but a small amount is converted to the orthogonal polarisation along the x axis. The second and third columns of the second row show the squared norms of the x and y components of scattered electric field respectively. Close inspection shows that the angles and number of scattered lobes calculated from 2D FEM does not correspond exactly with the experimental results. In fact the FEM results predict twice as many lobes as are observed experimentally. This expected these structures cannot be described completely using a 2D FEM method. A full 3D method is needed which can take into account the incident angle, and the effect of the substrate.



### 5.4.2 3D FEM with substrate

Full 3D simulations were performed including the substrate. A rectangular domain surrounded by PMLs was used as shown in Fig. 5.20. The scattered field is symmetric around the  $y=0$  plane so only half the domain need be simulated. A PEC boundary condition is used on the closest face to induce the symmetry in the s polarised case and a PMC condition is used in the p polarised case.

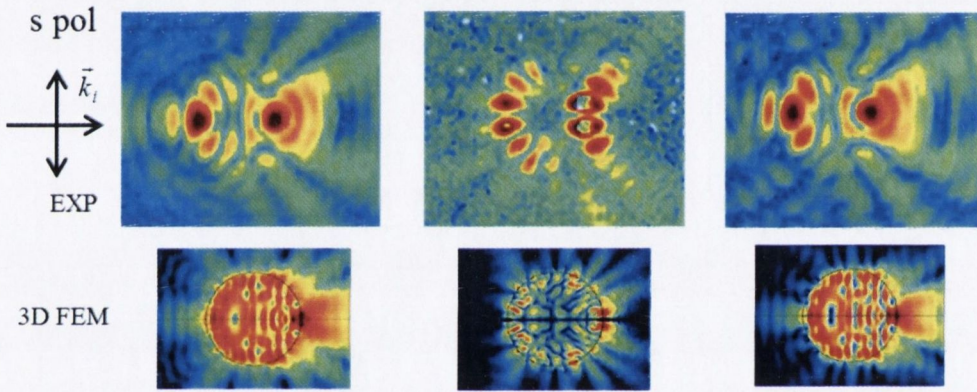


**Figure 5.20** Simulation domain and mesh used for near field simulations. Case shown is a microdisk of size  $2\ \mu\text{m}$ . The blue shaded region is the microdisk. The shaded red region is a  $\text{SiO}_2$  substrate. The domain is surrounded by index matched PMLs to absorb the scattered field. A PEC boundary condition is used on the closest face to induce symmetry for the s polarised case, and a PMC boundary condition is used for the p polarised case.

The blue shaded region in Fig.5.20 corresponds to the microdisk and the red shaded region is a substrate layer of  $\text{SiO}_2$  of index 1.4. This is taken as  $1\ \mu\text{m}$  thick, with absorbing PML layers surrounding the entire domain. Firstly the background field is calculated using the same domain with the index of the scatterer set to 1. This background field is equivalent to the incident and reflected fields from the  $\text{SiO}_2$  layer. We were not able to simulate the reflected field from the  $\text{SiO}_2$  interface, as then the domain was too large to solve, however this third term would modulate the result by only 10% maximum as seen from the analytical calculations of the reflection and transmission coefficients shown in section 5.3.4. This background field is then used to calculate the scattered field. The near-field includes the effect of a higher index substrate on the scattered field and the reflection of the scattered field from the substrate. The mesh size used is 100 nm in the air and substrate, 200 nm in the PML region and 50 nm in the scatterer.



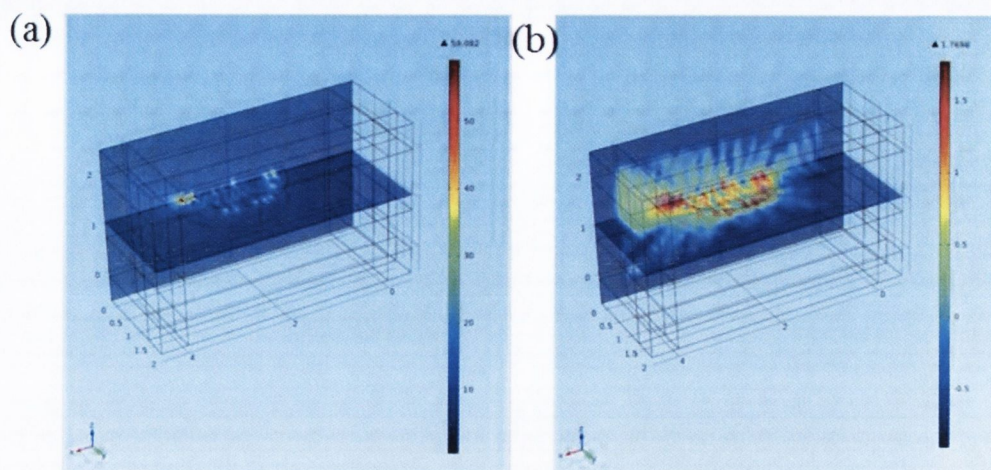
The 3D FEM results are compared to the experimental results for a  $2\mu\text{m}$  diameter microdisk and are displayed in Fig. 5.21. The simulation domain is limited to  $3\mu\text{m} \times 1.5\mu\text{m} \times 2\mu\text{m}$  due to the restrictions placed by the memory needed to solve it. This makes it difficult to see the pattern of the lobes. The experimental results are on the same scale but displayed over a slightly larger region to clearly see the lobes. The pattern inside the scatterer in the experimental measurements is not intended to map the near-field. The first row shows the experimental results, the second row the 3D FEM simulations. The first column is the total field, the second and third columns show the norms of the x polarised and y polarised components of the scattered field respectively.



**Figure 5.21** Experimental results for s polarised light scattering from a  $2\mu\text{m}$  diameter microdisk compared to a slice in the x-y plane through the centre of a 3D FEM simulation. The first row shows the experimental results, the second row the 3D FEM simulations. The first column is the total field, the second and third columns show the norms of the x polarised and y polarised components of the scattered field respectively.

Include the substrate results a scattered field distribution which correlates well with the experimental results. The number of lobes is now roughly equal to the size parameter. An interference pattern is formed due to the optical path length difference between light scattered from the disk into the air domain, and light which scatters into the substrate and is reflected back. This modulation is seen in both the 3D FEM results and the experimental results. A 3D representation of the scattered field is shown in a linear scale in Fig. 5.22 (a) and a log scale in Fig. 5.22 (b). Note that the light in Fig. 5.22 is coming from the right hand side of the image in contrast with previous images. This is to show the best angle for the specific simulation domain used.





**Figure 5.22** 3D FEM for a  $2\mu\text{m}$  diameter disk. Light is incident in the positive  $x$  direction which is from the right in this image. The incident light is  $s$  polarised at  $532\text{nm}$  and at an angle of  $80^\circ$  to the normal

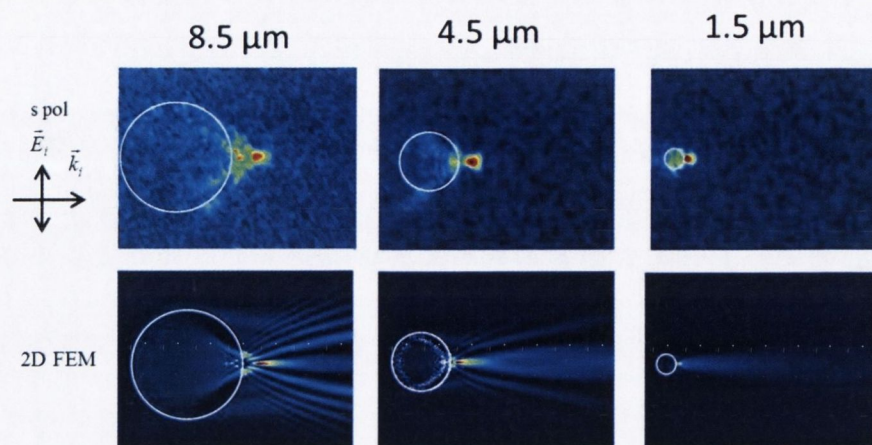
The 3D FEM method shows a rather complicated near-field when the disk is on the substrate which is distinct from the ideal case of a disk in free space (Fig. 5.19). There is still however a small sub wavelength spot which is 58 times the intensity of the background formed on the interface at the shadow side of the disk.

## 5.5 Additional experiments

### 5.5.1 Refractive index contrast dependence

When the disks are immersed in water this changes the refractive index contrast  $m$  to 1.5. In this case conditions are right to observe a scattering distribution termed a photonic jet where the maximum intensity is outside the disk, and the main lobe is long and narrow with a low divergence. Fig. 5.23 shows the results of scattering for disks of diameter  $1.5\mu\text{m}$ ,  $4.5\mu\text{m}$  and  $8.5\mu\text{m}$  with an incident plane wave illumination of  $633\text{ nm}$ . The incident wavelength is effectively reduced to  $\lambda/1.33$  ( $475\text{ nm}$ ) in water. Images over a  $20\mu\text{m} \times 14\mu\text{m}$  area are shown, and tri-linear interpolation is used.



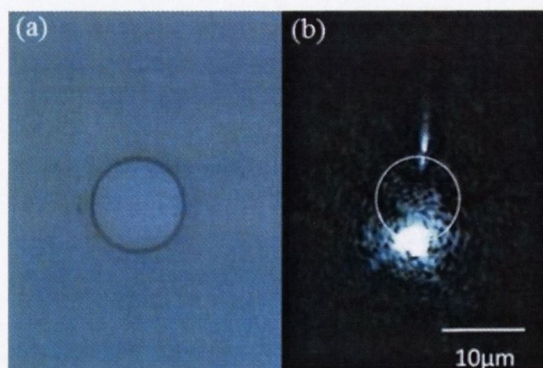


**Figure 5.23** Scattering distribution from microdisks in water. The incident wavelength is 633 nm which is effectively reduced to 475 nm in water. The refractive index contrast is now 1.5. In this case the maximum of the scattered light is outside of the disk and we see the photonic nanojet effect. The normalised intensity plots are shown for 3 different disk diameters (a) 1.5  $\mu\text{m}$  (b) 3  $\mu\text{m}$  (c) 6  $\mu\text{m}$  on a linear scale.

The intensity maximum is now outside the disk, and the main lobe is termed a PNJ. This configuration could have applications for biological sensing or nanoparticle detection in solution.

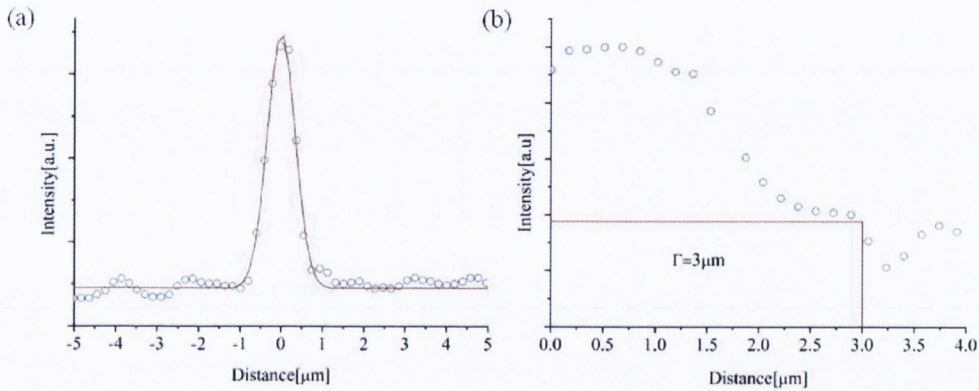
### 5.5.2 Focused Gaussian illumination.

Another interesting case is with Gaussian illumination perpendicular to the substrate. Focusing the incident light on the edge of the disk produces a low divergence beam on the opposite side of the disk as shown for the case of a 9  $\mu\text{m}$  diameter disk and 488 nm incident illumination in Fig. 5.24.



**Figure 5.24** (a) White light image of the 9  $\mu\text{m}$  diameter microdisk before illumination. (b) Image with 488nm CW laser illumination. The PNJ forms on the opposite side of the disk to the illumination.

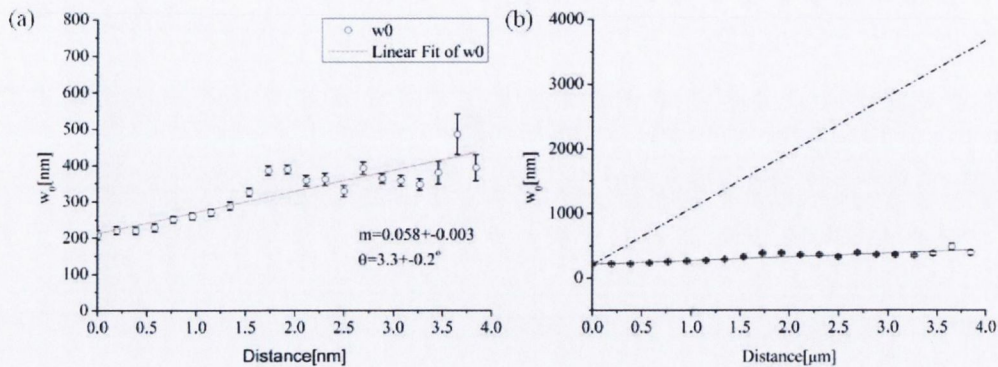
Due to the high NA of the MO, light incident at high angles is scattered by the disk and focused into the jet. Only a small portion of the total intensity is focused into the PNJ. The exposure time of the image in Fig. 5.24 (b) is chosen such that the jet is clearly visible, and the incident spot is over exposed. The transverse profile of the PNJ at the surface of the disk is plotted in Fig. 5.25 (a) and fitted with a Gaussian. The main lobe only is fitted as this corresponds to the PNJ. The FWHM of the jet at the point of maximum intensity is 510 nm. This corresponds to a  $w_0$  value of 205 nm ( $w_0$ =half the  $1/e^2$  waist).



**Figure 5.25** (a) Standard fitting of the PNJ to extract FWHM. The FWHM in this case is 510 nm corresponding to a  $w_0$  value of 205 nm. (b) The profile of the intensity along the  $\theta=0$  axis. The axial  $1/e$  falloff value is  $3 \mu\text{m}$ .

The profile along the  $\theta=0$  line is shown in Fig. 5.25 (b). The  $1/e^2$  falloff is long at  $3 \mu\text{m}$ . In any focused beam there is a relationship between the minimum beam waist and the divergence angle due to diffraction. The smaller the beam waist the larger the angle of divergence will be. The figure of merit is then the product of the beam waist and the divergence. In free space this is smallest for a focused beam with a Gaussian distribution. In Fig. 5.26  $w_0$  is plotted as a function of distance with 0 corresponding to the position of the point of maximum intensity outside of the disk. The straight line fit defines the angle of divergence of the main beam. There is a very low angle of divergence of only  $3.55^\circ$ .





**Figure 5.26 (a)** The transverse FWHM is plotted as a function of distance, with 0 corresponding to the point of highest intensity (smallest FWHM). The divergence angle is  $\theta = 3.3 \pm 0.2$  **(b)** Comparison between the divergence angle of the experimentally measured Jet and that of a Gaussian with the same waist. The dashed line represents the Gaussian divergence with an angle of  $42^\circ$ . This is 14 times the divergence angle of the PNJ.

Comparing this angle with the paraxial approximation for focusing of a Gaussian beam gives good insight into the unique properties of the PNJ. For a focused Gaussian the beam waist describes a hyperboloid with an angle of  $\theta$ . The relationship between the beam waist and the divergence angle is given by  $\theta \approx \lambda / \pi \omega_0$ . The paraxial approximation is only valid for small  $\theta$ , however it has been shown to be correct to within 10% for the high angle case [12]. This is useful as it allows us to use this analytical expression to compare the divergence of a Gaussian in free space to the photonic jet in Fig. 5.26 (b). The divergence angle of a focused Gaussian in free space with the same waist as the PNJ is  $44^\circ$ . Thus the divergence angle is over 10 times smaller than a classically focused Gaussian beam with the same waist. This low divergence is possibly the most distinguishing feature of a PNJ and is one of the properties which exhibits the most potential to be exploited for novel applications.



## 5.6 Conclusions

A simple imaging technique is demonstrated where light is collected using a microscope objective normal to a substrate. This technique was previously applied to spherical scatterers in a homogenous medium [5], but to our knowledge it has not been applied to more complex shaped scatterers on a substrate. The previous experiments used photographic film with limited dynamic range to record the images and focused on bright “glare spots”, which were explained as the Fourier transform of the angular distribution of the light scattered into the cone of the objective [3]. By using a calibrated CCD camera and a high dynamic range imaging method borrowed from photography, both the bright spots due to out of plane scattering and a secondary image formed from light which scatters in plane, then from the substrate could be observed. It is this low intensity part of the image that contains the information about the near-field intensity distribution. The CCD allows quantitative measurements to be made of the intensity in the image, which allows the dependence of the scattering distribution on microdisk diameter to be examined. By calibrating the CCD to incident power, the mechanisms of the light scattering can be confirmed. The power in the secondary lobes is a small fraction of the incident power which matches well with the expected power scattered from a rough surface [13, 14] with rms roughness given by the AFM measurements in Chapter-4.

The images in Fig. 5.7 and 5.8 show that there is an increasing number of scattering lobes with increasing microdisk diameter. The number of lobes is roughly equal to the size parameter as shown in Fig. 5.9. This sort of dependence is expected from comparison with light scattering from an infinite cylinder. However for an infinite cylinder the number of lobes would be roughly twice the size parameter. 3D FEM simulations in Fig. 5.20 show that this change in scattering distribution is due to the presence of the substrate. The higher index of the substrate may act to decrease the effective index of the scatterer leading to less lobes. Close inspection of the cross sections of the 3D FEM results in Fig. 5.20 (a)-(c) show that close to the microdisk boundary there are twice as many lobes which then merge to form one lobe further from the boundary.

The polarisation dependence experiments show that the light remains polarised primarily in the same direction as the incident polarisation for the s polarised case (along the y axis). For the p polarised case the incident light is polarised out of plane and will have a small component along the x axis due to the  $80^\circ$  incident angle. The scattered field is shown to be primarily polarised along the x axis. A small amount of the scattered light is in the orthogonal polarisation, which results in a symmetric image with a zero in intensity along at the centreline of the microdisk.



Experimentally the incident angle and substrate are shown to have a large effect on the scattering distribution. A large change in the intensity of the collected light was observed when varying the incident angle as shown in Fig. 5.13. The distribution of the scattered field remains unchanged except at  $\theta_i=70^\circ$  in the s polarised case and  $\theta_i=75^\circ$  in the p polarised case. These are shown to correspond with angles when there is a large intensity in of the background field in the region of the microdisk due to Fresnel reflection (Fig. 5.18).

Regardless of the input angle there is always a minimum of field at the interface between air and the  $\text{SiO}_2$ . This is not the optimum case for using these structures as focusing elements. By using a planar waveguide as the excitation source these structures could be more efficient as the excitation field would be a maximum at the centre of the scatterer.

When the microdisks were immersed in water the refractive index contrast changed to 1.5 and photonic nanojets were observed (Fig. 5.23). Rayleigh scattering in water allows scatters light from the main lobe strongly resulting in it being clearly visible in the experimental images [15]. It is difficult to see the lower intensity lobes in water as the water is constantly moving blurring the image. The dimensions of the PNJ will also be broadened to the diffraction limit due to convolution with the point spread function of the microscope objective. The microdisks in this configuration could be used to detect nanoparticles in solution [8] or as optical gradient traps [16]. The advantage of microdisks over sphere for this application is that they can be made from higher index materials so when immersed in water they are closer to the optimum refractive index to create a sharp PNJ [17]. The fabrication process also allows accurate postioning of the microdisks either before or after the sample of interest has been deposited, which is difficult with microspheres.

From the 3D FEM simulations the intensity contrast of the small spot outside the microdisk is shown to 56 times the background intensity. This is smaller than the case of a microsphere which is 100-150 times, but larger than the case of an infinite cylinder which is 20-30 times (see Chapter-2). This shows that the vertical confinement of the microdisks helps to increase the intensity of the spot from the case of an infinite cylinder.

Finally a brief observation of light scattering from a microdisk with Gaussian focused illumination is shown. This result was the first observed during WGM experiments and prompted the research into light scattering from microdisks. Using focused Gaussian illumination with microspheres results in a shorter PNJ than with a plane wave. The 3D confinement was used to enhance single molecule and FCS experiments by placing the sphere in the focus of an already tightly focused beam [18]. In this case the PNJ is much longer than expected for the given intensity contrast. The result is not surprising as the illumination is far from a plane wave. The Gaussian beam is in-fact perpendicular to the plane of the substrate.

The microdisk in Fig. 5.24 was etched using the  $\text{SF}_6/\text{CHF}_3$  recipe resulting in curve sidewalls as shown in Chapter-4. This may have contributed to the formation of the PNJ. Recently it has been shown that the scattering distributions from spheres can be modified greatly by controlling the amplitude and phase profiles of the incident beam [19]. Many different scattering distributions were created by modifying the input beam, and this could be applied to these microdisks.

Overall we have given the first results of light scattering by plane wave illumination from a microdisk structure on a substrate. We have shown experimentally that the near-field scattering distribution corresponds well with 3D numerical FEM simulations. The size parameter and refractive index contrast dependence was shown to vary in a similar way to the 2D analytical solution for an infinite cylinder. Immersed in water the high index of the microdisks allowed the formation of PNJs which have been shown to have many uses in scattering from spheres. Under focused Gaussian illumination a PNJ with a divergence 10 times smaller than that of a focused Gaussian beam with the same waist was observed. More research is needed to investigate the scattering properties of microdisks under different illumination conditions.



## 5.7 References

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15. M. J. a. G. R. Fournier, *Light scattering by particles in water: theoretical and experimental foundations* (Academic Press, 2007).
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## Chapter 6    Light scattering from low symmetry shapes.

### 6.1 Introduction

The EBL process by which the structures are fabricated allows us to vary the shape of the scatterer with nanoscale resolution and observe the effect on the scattered intensity distribution. This is not readily achievable with microspheres as they are generally produced using surface tension [1] or chemical means [2]. Light scattering from complex and irregular shapes is however responsible for many optical effects in nature. From rainbows to butterfly wings, clouds and ice halos, light scattering from complex shapes results in interesting far-field effects. It is logical then to assume that interesting effects can also be seen in the near-field intensity distributions of irregular shaped scatterers.

Microdisks are cylindrically symmetric so the scattering distribution does not depend on the in-plane incident angle  $\phi_i$ . The scattered field simply rotates such that the main lobe is always on the opposite side of the microdisk to the incident beam. By contrast low symmetry shapes such as equilateral triangles with 3-fold symmetry, and squares with 4-fold symmetry, will have scattering distributions which depend strongly on the in-plane incident angle. This study is restricted to these two cases of regular polygons with low symmetry.

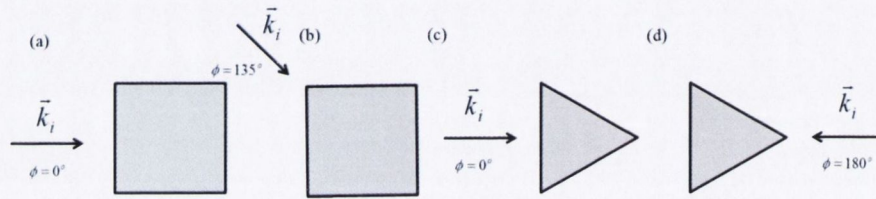
The imaging method introduced in the previous chapter was used to investigate scattering from squares and equilateral triangles with dimensions ranging from 1-6  $\mu\text{m}$ . These structures are of interest as the in-plane scattered field depends highly on orientation of the scatterer with respect to the incident light, and the incident and collected polarisation directions.

Shape effects have been widely explored in plasmonics, where the position of the surface plasmon peak and the near-field intensity distribution can be controlled by changing the particle shape [3, 4]. MDRs in dielectric cavities with square [5-9], triangular [10, 11] and other shapes is also an area of active study [12]. To our knowledge however, this is the first time the dependence of shape on the near-field scattering intensity in the Mie regime has been observed experimentally.



## 6.2 Experimental setup

The experimental setup and procedure described in Chapter-5.2 was used to image the in-plane scattering of the structures. We refer to this as the image plane method (IPM). The samples were mounted on a rotation stage in order to accurately change the in plane incident angle. Squares and equilateral triangles were fabricated using the EBL technique with the AR  $\text{CHF}_3$  etch outlined in Chapter-4. When the direction of the incident beam is taken into account there are two interesting symmetric cases for both triangles and squares. Figure. 6.1 shows the four cases studied.



**Figure 6.1** Four different cases of in-plane incident angle  $\phi_i$  are examined. They are labelled according to their orientation with respect to the incident light as (a) square\_flat, (b) square\_vertex, (c) triangle\_flat (d) triangle\_vertex.

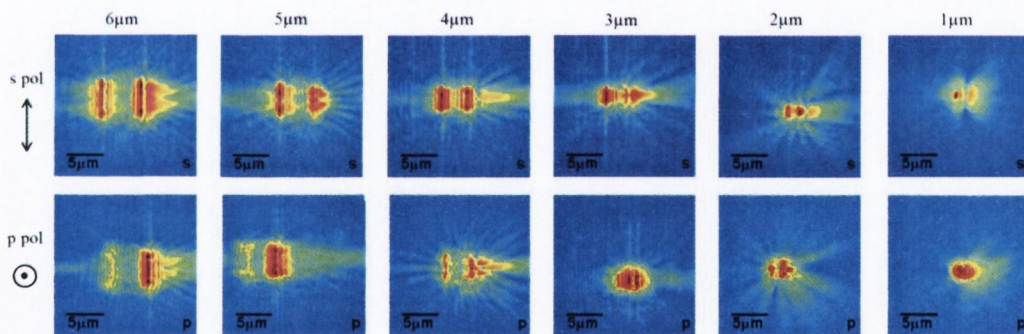
The same co-ordinate system as outlined in Fig. 5.2 is used, with  $\phi$  representing the in-plane angle and  $\theta$  representing the out of plane angle. These cases will be referred to repeatedly in the results section, so were labelled according to the shape orientation with respect to the incident light. Each case can be obtained at multiple in plane angles  $\phi$  (4 for a square and 3 for a triangle), so  $\phi$  cannot be used to label the cases. Instead the orientations displayed in Fig. 6.1 are labelled (a) square\_face, (b) square\_vertex (c) triangle\_face, and (d) triangle\_vertex. The term face or vertex refers to the first surface that the incident light crosses. These cases all have a mirror symmetry around the axis defined by the incident  $k$  vector, which produces a symmetric scattering distribution.

### 6.3 Experimental results

Multiple samples were fabricated and the dependence of the experimental images on size, incident polarisation and collection polarisation was examined. Particular attention was paid to the four cases of in-plane incident angle outlined in the previous section. These images were then compared to 2D and 3D FEM simulations of the near-field. In all the following experiments the out of plane incidence angle  $\theta_i$  was held constant at  $\theta=80^\circ$ .

#### 6.3.1 Size dependence for square

The parameter chosen to characterise the size of the scatterer was defined as the length of a side. Squares with sides ranging from 1-6  $\mu\text{m}$  in 1  $\mu\text{m}$  steps were fabricated. The size dependence of a square orientated with a flat edge in the direction of the incoming light (square\_face) is shown in Fig. 6.2 for both cases of s and p polarised incident light.

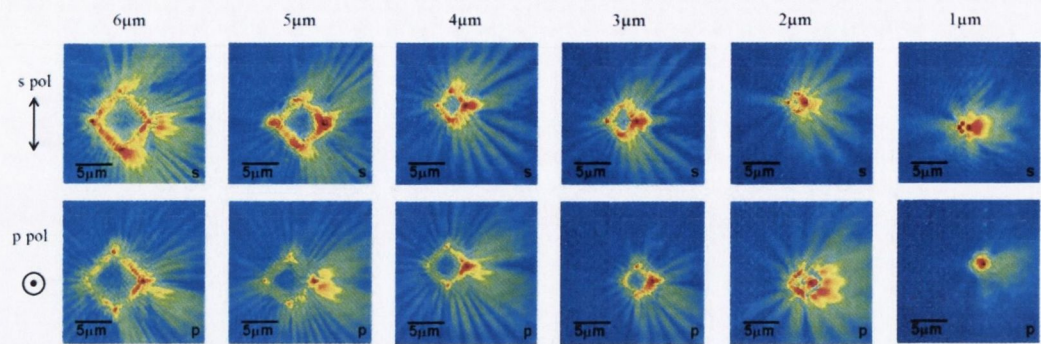


**Figure 6.2** Size dependence of light scattering from a square for s and p polarised incident light.

The incident angle is fixed at  $\theta=80^\circ$  and the incident wavelength is 532 nm. Light is incident from the left and the discontinuity in refractive index at the upper and lower boundaries causes the light to bend in towards the centre. This light interferes to form the near field intensity distribution on the right hand side of the image. For a square of side 4  $\mu\text{m}$  light is focused onto the interface and an intensity distribution similar to that of a photonic jet is formed, for both s and p polarised cases.

When the square is orientated with a vertex facing the incoming light (square\_vertex) a very different near-field intensity distribution is observed. Multiple lobes are formed and light is strongly scattered to high angles as shown in Fig.6.3 .



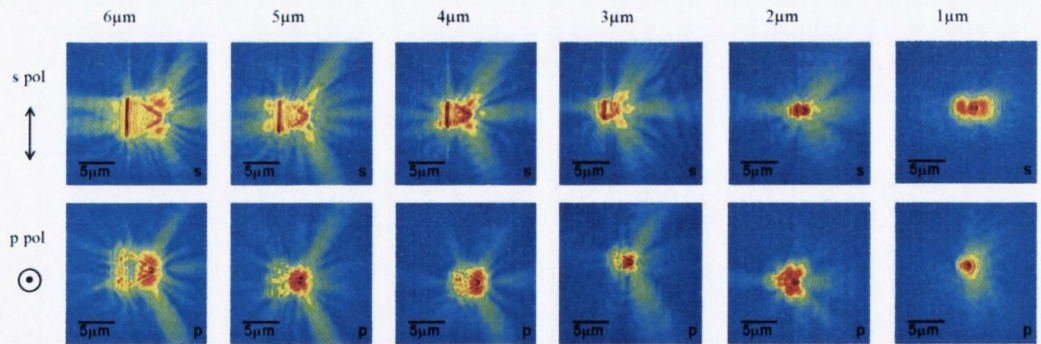


**Figure .6.3** Size dependence of light scattering from a square in the diamond orientation for s and p polarised incident light.

The intensity distribution for the square\_vertex case was extremely sensitive to the orientation of the square with respect to the incident light. When the correct orientation was achieved we could see a symmetric pattern with a single diffraction lobe in the centre. There is a noticeable difference in the scattering distributions for the s and p polarised cases.

**6.3.2 Size dependence for triangle**

Equilateral triangles with side of length 1-10  $\mu\text{m}$  were examined. For the triangle\_face case two main lobes are formed emerging perpendicular to the other two faces as shown in Fig 6.4. Interestingly the ratio of intensity in each of the two main lobes depends on incident angle (not shown here).

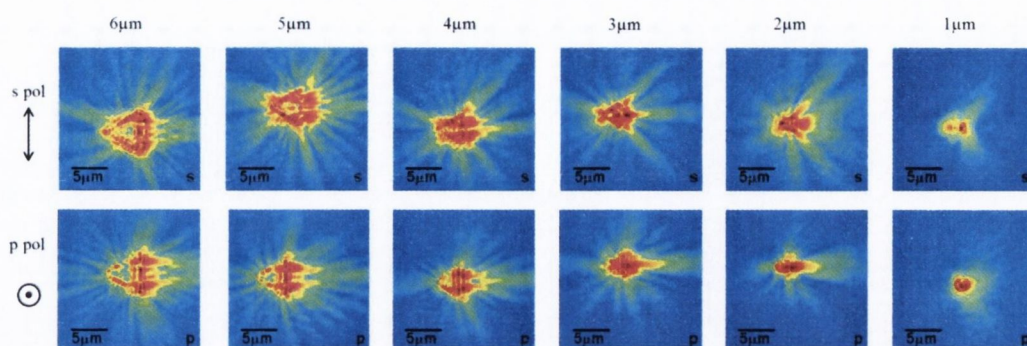


**Figure 6.4** Size dependence of light scattering from a square in the diamond orientation for s and p polarised incident light.

When the triangle is orientated with a vertex in the direction of the incoming light (triangle\_vertex) we get another symmetric pattern. For the 3  $\mu\text{m}$  and 4  $\mu\text{m}$  case this can also produce a long main lobe on the right hand side of the structure similar to a PNJ. For the 3  $\mu\text{m}$  s



polarised case 2 more strong lobes are also formed at the vertices of the triangle on the shadow side.

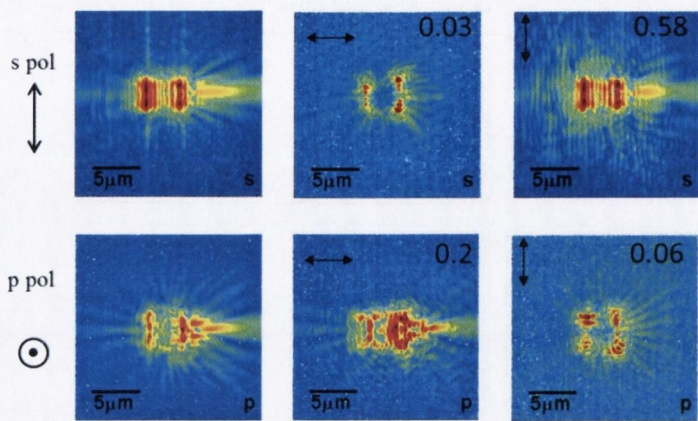


**Figure 6.5** Size dependence of light scattering from a square in the diamond orientation for s and p polarised incident light.

### 6.3.3 Polarisation dependence for a square

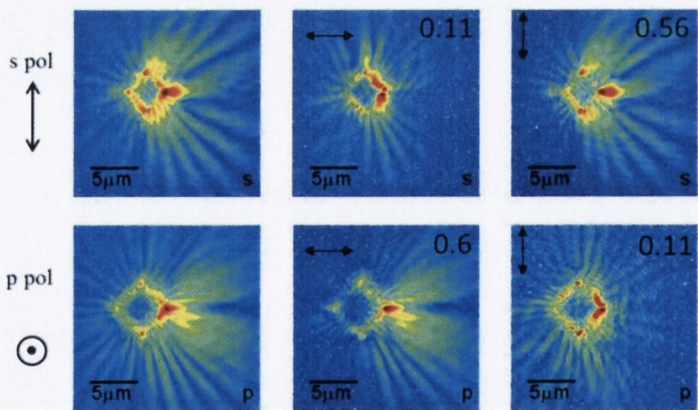
The squares and triangles show a greater dependence on incident and collected polarisation than the symmetric case of a disk. In Fig. 6.6 the polarisation dependence for the square\_flat case is shown. The first row is for s polarised incidence and the second for p polarised. The second column has an analyser in position P1 with its polarisation axis along the x direction as indicated by the arrow in the top left hand corner of the image. The second column has its axis in the y direction. The x and y direction are defined in the same manner as before (Fig. 5.2) with positive x horizontal from left to right and positive y vertical from bottom to top of the page. The number in the top right hand corner of the second and third columns is the relative intensity of the maximum in the image compared to the case without an analyser (first column.). This shows that for the square light is primarily polarised in the same direction as the incident light and a small amount is converted to the orthogonal polarisation. All polarisation results are laid out in the same format.





**Figure 6.6** Polarisation dependence for a square of side 4  $\mu\text{m}$ . Light is incident from the left with a wavelength of 532 nm. Top row is s polarised input and bottom row is p polarised input. The second column has an analyser in position P1 with axis along the x direction and the third column has axis along the y direction.

The case of square\_vertex has a more significant contribution from the orthogonal polarisation component, but the intensity distribution is still dominated by the same polarisation direction as the incident light. The first column which has no polariser at position P1 is the weighted addition of the second and third columns in Fig. 6.7.



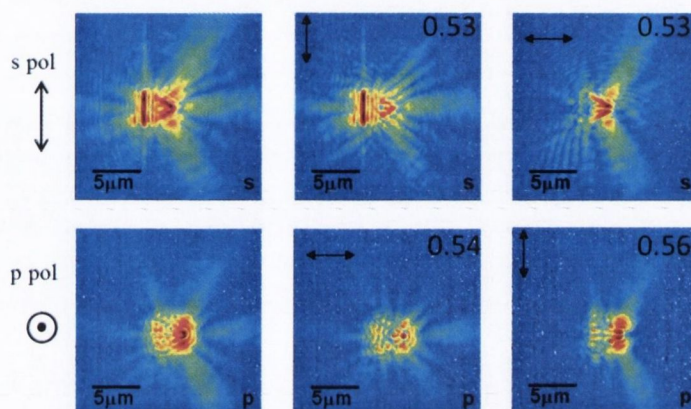
**Figure 6.7** Polarisation dependence for a square of side 4  $\mu\text{m}$  originated with a vertex towards the incident direction. Light is incident from the left with a wavelength of 532 nm. Top row is s polarised input and bottom row is p polarised input. The second column has an analyser in position P1 with axis along the x direction and the third column has axis along the y direction.

The field outside the scatterer in the forward direction is polarised in the same orientation as the incident light, whereas the back scattered components consist primarily of light polarised in the orthogonal direction.



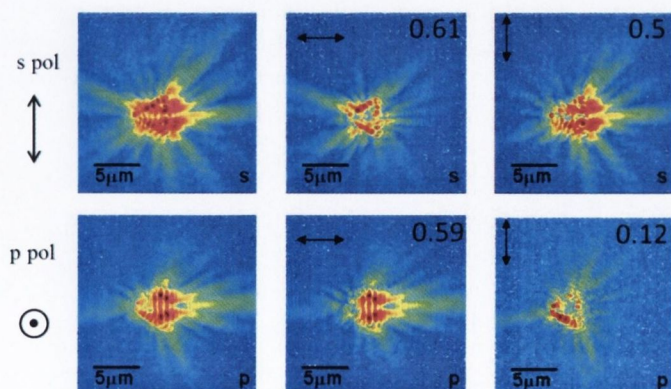
### 6.3.4 Polarisation dependence for a triangle

For a triangle orientated with a face towards the incident light (triangle\_face) the split between polarisations is even, with the first column in Fig. 6.8 being the addition of the second two. We see a number of lobes forming with the same polarisation direction as the incident light, and one large lobe forming with the orthogonal polarisation.



**Figure 6.8** Polarisation dependence for a triangle of side  $4\ \mu\text{m}$  orientated with a face towards the incident direction. Light is incident from the left with a wavelength of  $532\ \text{nm}$ . Top row is s polarised input and bottom row is p polarised input. The second column has an analyser in position P1 with axis along the x direction and the third column has axis along the y direction.

Finally for the case of a triangle with a vertex in the direction of the incident beam we get the results in Fig. 6.9. Again light is evenly split between polarisations but in this case the field distribution outside the triangle is dominated by the same polarisation direction as the incident light.



**Figure 6.9** Polarisation dependence for a triangle of side  $4\ \mu\text{m}$  orientated with a vertex towards the incident direction. Light is incident from the left with a wavelength of  $532\ \text{nm}$ . Top row is s polarised

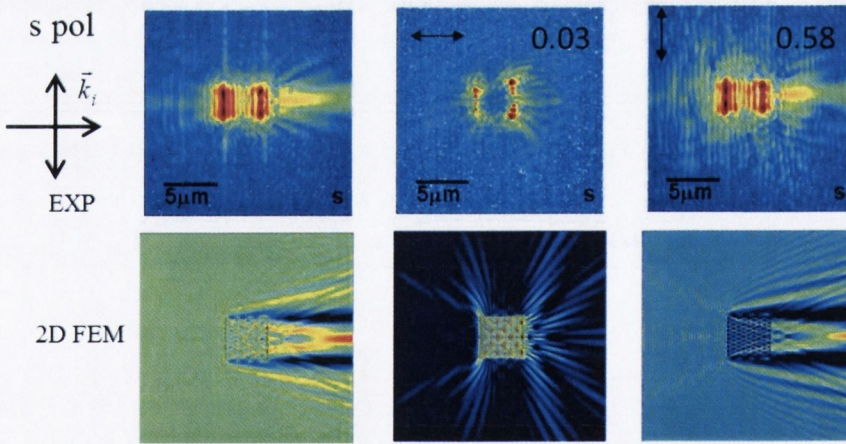


input and bottom row is p polarised input. The second column has an analyser in position P1 with axis along the x direction and the third column has axis along the y direction.

## 6.4 Comparison with FEM results

### 6.4.1 2D FEM

Similarly to Chapter 5 we start by comparing the experimental images to the 2D FEM simulations for the intensity of the scattered field close to the disk. Recall that only the low intensity pattern outside of the scatterer in the experimental images is expected to match with the numerical simulations. The case of a square of side  $4\text{ }\mu\text{m}$  with s polarised incident light is shown in Fig. 6.10. The figure is laid out in the same way as the comparisons in Chapter 5. The refractive index of the square is taken as 2.05. The incident light is s polarised and propagates in the positive x direction. The first row of Fig. 6.10 shows the experimental results, the second row the 2D FEM simulations. The first column is the total field, the second and third columns show the norms of the x polarised and y polarised components of the scattered field respectively.

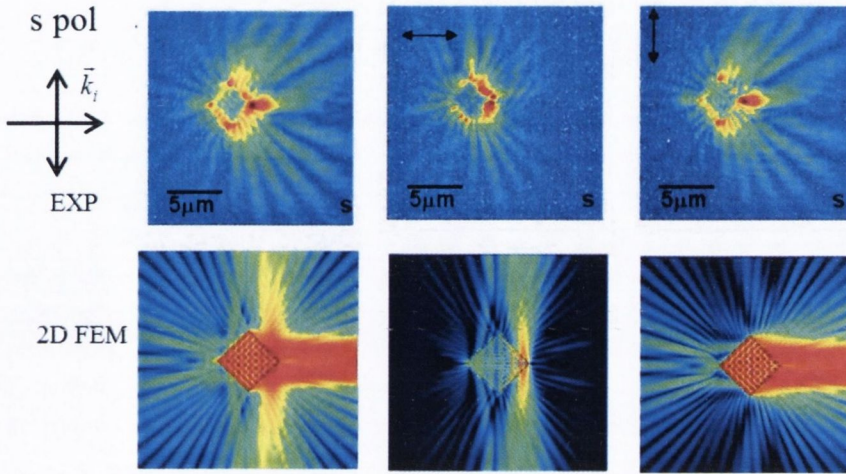


**Figure 6.10** Light scattering from a  $4\text{ }\mu\text{m}$  side square compared to 2D FEM. The refractive index of the square is taken as 2.05. The incident light is s polarised and propagates in the positive x direction. The first row shows the experimental results, the second row the 2D FEM simulations. The first column is the total field, the second and third columns show the norms of the x polarised and y polarised components of the scattered field respectively.

The 2D FEM results are in fact for the scattered field of an infinite rod with square cross section and refractive index of 2.05.



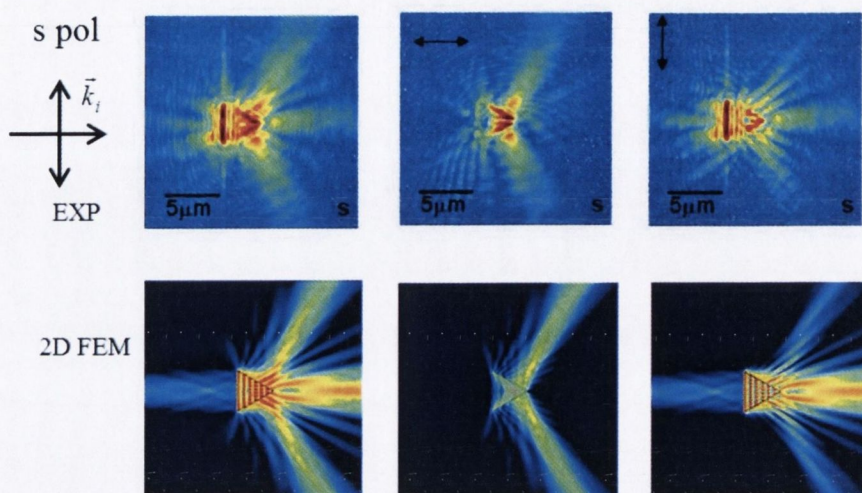
The scattering distribution changes dramatically when the orientation of the square with respect to the input beam is changed. The experimental results for s polarised incident light for the square\_vertex case are compared with the 2D FEM results in Fig 6.11.



**Figure 6.11** Light scattering from a 4  $\mu\text{m}$  side square orientated with a vertex in the direction of the incident light, compared to 2D FEM. The refractive index of the square is taken as 2.05. The incident light is s polarised and propagates in the positive x direction. The first row shows the experimental results, the second row the 2D FEM simulations. The first column is the total field, the second and third columns show the norms of the x polarised and y polarised components of the scattered field respectively.

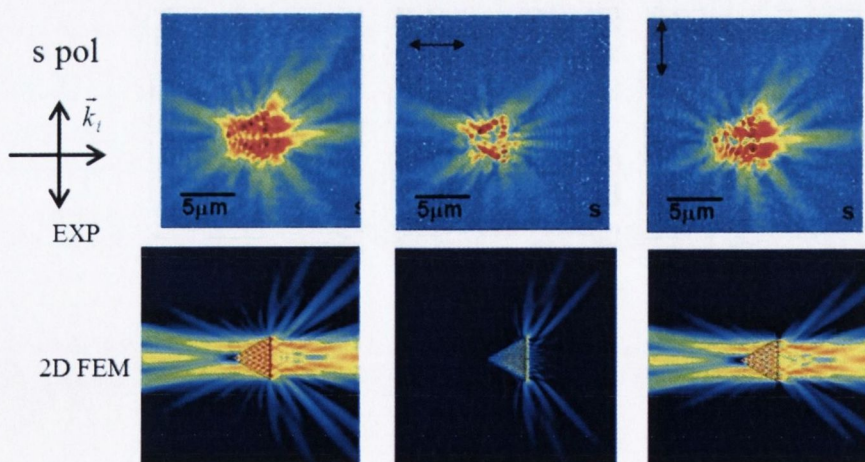
The experimental results for s polarised input for the case of a triangle with a flat face facing the input light are compared to the 2D FEM results in Fig.6.12. Experimentally the scattering in the forward direction is shown to be mainly composed of the same polarisation as the incident beam and the two side lobes are composed of the other polarisation. This behaviour is also observed in the 2D FEM simulations. The angles of the main lobes are in agreement with the 2D FEM results. The angles of the lobes measure experimentally match well with the 2D FEM.





**Figure 6.12** Light scattering from a 4  $\mu\text{m}$  side triangle orientated with a face in the direction of the incident light, compared to 2D FEM. The refractive index of the triangle is taken as 2.05. The incident light is s polarised and propagates in the positive x direction. The first row shows the experimental results, the second row the 2D FEM simulations. The first column is the total field, the second and third columns show the norms of the x polarised and y polarised components of the scattered field respectively.

Finally we show the comparison between the experimental and 2D FEM results for a triangle with a vertex facing the incoming light. The experimental results show that the lobe forms closer to the surface of the scatterer than in the 2D FEM cases and the number of lobes are different. The polarisation dependence correlates well between experiment and 2D FEM.

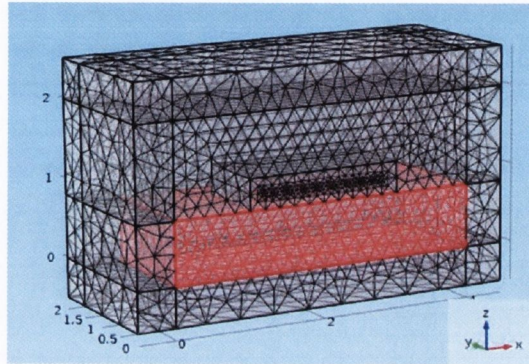


**Figure 6.13** Light scattering from a 4  $\mu\text{m}$  side triangle orientated with a vertex in the direction of the incident light, compared to 2D FEM. The refractive index of the triangle is taken as 2.05. The incident light is s polarised and propagates in the positive x direction. The first row shows the experimental results, the second row the 2D FEM simulations. The first column is the total field, the second and third columns show the norms of the x polarised and y polarised components of the scattered field respectively.



### 6.4.2 3D FEM with substrate

Full 3D simulations were performed including the substrate. The same rectangular domain as outlined in Chapter-5.4.2 was used depicted in Fig. 6.15. Again the domain was surrounded by PMLs and the symmetric around  $y=0$  allowed us to reduce the size of the simulation domain.

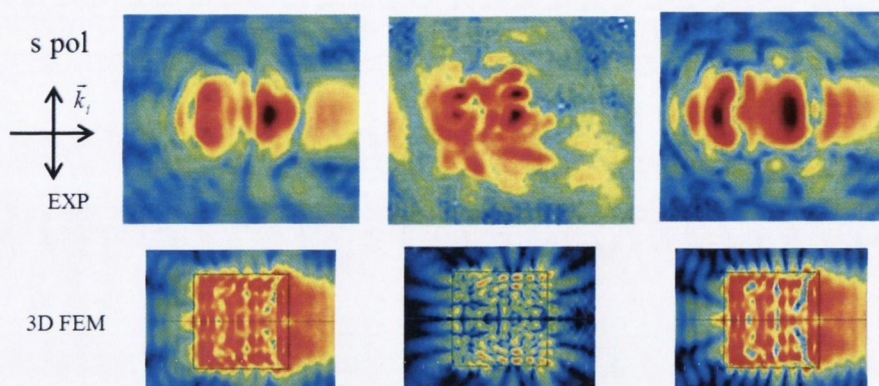


**Figure 6.14** Simulation domain and mesh used for near field simulations. Case shown is a square of size  $2\ \mu\text{m}$ . The shaded red region is a  $\text{SiO}_2$  substrate. The domain is surrounded by index matched PMLs to absorb the scattered field.

The red shaded region in Fig. 6.14 is a substrate layer of  $\text{SiO}_2$  of index 1.4. This is taken as  $1\ \mu\text{m}$  thick, with absorbing PML layers surrounding the entire domain. The same procedure as outlined in Chapter-5 is used. The background field is calculated using the same domain with the index of the scatterer set to 1, then this background field is used to calculate the scattered field. The near-field includes the effect of a higher index substrate on the scattered field and the reflection of the scattered field from the substrate. The mesh size used is  $100\ \text{nm}$  in the air and substrate,  $200\ \text{nm}$  in the PML region and  $50\ \text{nm}$  in the scatterer.

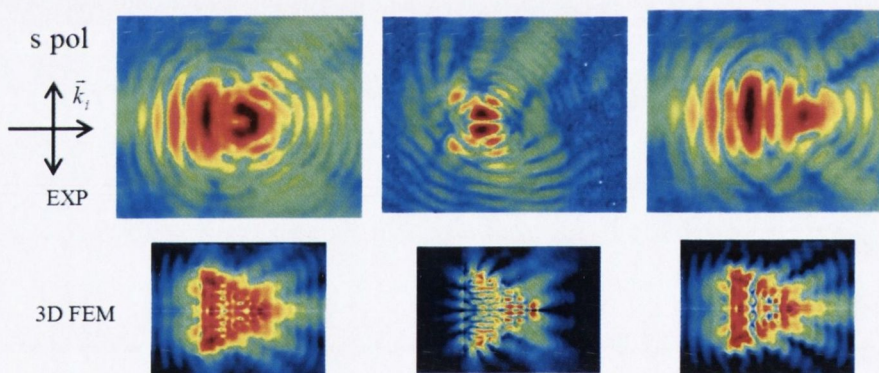
The 3D FEM results compared to the experimental results for a  $2\ \mu\text{m}$  side square are displayed in Fig. 6.15. The simulation domain is limited to  $3\ \mu\text{m} \times 1.5\ \mu\text{m} \times 2\ \mu\text{m}$  due to the restrictions placed by the memory needed to solve it. Again, the experimental results are on the same scale but displayed over a slightly larger region to clearly see the lobes. The 3D simulations give a smaller number of lobes than the 2D simulations which match better with the experimental results. In these results we are comparing the pattern outside the scatterer. The pattern inside the scatterer in the experimental measurements is not intended to map the near-field.





**Figure 6.15** Experimental results for s polarised light scattering from a 2  $\mu\text{m}$  side square compared to a slice in the x-y plane through the centre of a 3D FEM simulation. The first row shows the experimental results, the second row the 3D FEM simulations. The first column is the total field, the second and third columns show the norms of the x polarised and y polarised components of the scattered field respectively.

The 3D FEM results for the scattering from a 2  $\mu\text{m}$  triangle are also displayed in and compared with the experimental results in Fig. 6.16.



**Figure 6.16** Experimental results for s polarised light scattering from a 2  $\mu\text{m}$  side triangle compared to a slice in the x-y plane through the centre of a 3D FEM simulation. The first row shows the experimental results, the second row the 3D FEM simulations. The first column is the total field, the second and third columns show the norms of the x polarised and y polarised components of the scattered field respectively.

The interference pattern is seen in the experimental results due to the 2  $\mu\text{m}$  thick  $\text{SiO}_2$  layer which is reproduced in the 3D FEM results.



## 6.5 Conclusions

The in-plane scattered near-field distributions of squares and equilateral triangles on a substrate have been measured experimentally and compared with 2D and 3D FEM simulations. The image plane method shows that the intensity distribution of light in the near field region can be controlled by changing the shape of the scatter. The scattering distribution from these shapes depends very sensitively on the in-plane incident angle  $\phi_i$ . Four specific cases were examined in detail and labelled `square_face`, `square_vertex`, `triangle_face`, `triangle_vertex` as shown in Fig.6.1. These cases had a mirror symmetry with respect to the incident  $k$  vector, which resulted in a symmetric scattered field distribution.

Initially the size dependence for these cases was examined. For the `square_face` case shown on Fig. 6.2, light was diffracted by the discontinuity in refractive index at the edges of the square toward the centre on the shadow side of the scatterer. For the 4  $\mu\text{m}$  side case light is focused onto the boundary of the square and a PNJ like intensity distribution is formed. The position of the maximum intensity outside of the square can be controlled by changing the incident polarisation from s-polarised to p-polarised. This effect could be used in optical trapping and manipulation of nanoscale particles. For the 4  $\mu\text{m}$  case with p polarised illumination there is a region of low intensity surrounded by high intensity just outside the surface of the scatterer. This could be used for fluorescence depletion microscopy [13]. The p polarised case could be used to bleach the sample surrounding the low intensity region and then by changing the incident polarisation to s polarised the small un-beached sample could be investigated with high resolution. The 2  $\mu\text{m}$  and 3  $\mu\text{m}$  square-face case also showed an “on-off” dependence of the main lobe with incident polarisation which could be useful for switching.

The `square_vertex` case showed a great sensitivity to the in-plane incident angle. Precise control of incident angle to  $\pm 0.1^\circ$  was necessary in order to form the symmetric scattering distribution. For example the p polarised case in Fig. 6.3 was not perfectly aligned and we can see that the intensity is large towards the bottom of the image than at the top. This property could be used as a sensitive alignment tool on the microscale.

The `triangle_face` case in Fig.6.4 showed three main lobes on the shadow side of the scatterer for all the side lengths; one lobe in the centre and two propagating perpendicularly from the triangles faces. The relative intensity in the two higher angle lobes depends linearly on the in-plane incident angle. These scattering structures could be used as micron scale beam splitters with a splitting ratio which depends on the in-plane incident angle.

The `triangle_vertex` scattering distribution for a 3  $\mu\text{m}$  side triangle in Fig. 6.5 shows three main scattering lobes for the s polarised case and only one for the p polarised case. The ability



to turn on and off the higher angle lobes by changing incident polarisation may be useful for optical switching.

The polarisation dependence of the squares and triangles was examined. For the square\_face case the scattering distribution consisted mainly of light polarised in the same direction as the incident light. For the square\_vertex case light scattered in the forward direction had the same polarisation direction as the incident light, whereas backscattered light showed the orthogonal polarisation. The triangle cases were shown to have a roughly equal distribution in intensity between the two polarisations.

Experimental results show shorter intensity distributions with a maximum closer to the scatterer than the 2D FEM results. This is because the 2D FEM does not take into account the finite height of the scatterer and the out of plane divergence. The experimentally observed number of in-plane lobes was smaller than the number expected from the 2D FEM simulations as in the case of the microdisk in Chapter-5. When the substrate was added in the 3D simulations the number of lobes was halved and was consistent with the experimental results. The interference pattern is seen in the experimental results due to the 2  $\mu\text{m}$  thick  $\text{SiO}_2$  layer which is reproduced in the 3D FEM results.

Overall in this chapter we have studied to in-plane scattering distributions of squares and triangles of  $\text{Si}_3\text{N}_4$  on a  $\text{SiO}_2$  substrate. We have shown that unique intensity distributions can be created which are sensitive to both the incident angle and the incident polarisation. These structures could find many applications as on chip optical elements based on scattering.

## 6.6 References

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## Chapter 7 Out of plane scattering

### 7.1 Introduction

The angular elastic light scattering intensity distribution in the far field (often called phase function) is highly dependent on the shape and refractive index of the particle. Measurement of this phase function thus allows precision size and refractive index measurement of particles down to 0.1% error in size [1, 2]. Particle sizing is important in colloidal chemistry for sol and aerosols [3-5], biology [6, 7], metrology [8, 9] and meteorology [10-13]. Micron scale scatterers can be used to probe the surrounding environment due to the extreme sensitivity of the scattering distributions to changes in local refractive index [14, 15]. Spherical and cylindrical particles exhibit sharp resonances in the total intensity of light scattered [16]. Measurement of the position of these resonances provide a higher accuracy measurement of the diameter, however these measurements are slow and are dependent on the angle at which the light is collected [17].

Phase functions are commonly measured by rotating a detector with narrow angular acceptance with respect to the input laser direction, or equivalently using a lens to collect the scattered light over a range of angles [18]. This chapter displays the results of phase function measurements for particles on a planar substrate. The effect of the substrate is two-fold. Firstly the incident field is no longer a plane wave, but a combination of incident and reflected waves from the substrate. Secondly the scattered field will reflect from the substrate and interfere with itself. The measurements are performed by imaging the back focal plane of the objective (Fourier plane) onto the CCD in order to get a map of the far field scattering distribution as a function of angle. This method has been previously proposed as a high speed method of measuring microdroplet size and refractive index [1]. In that case light scattering from a microdroplet levitated in vacuum using an electromagnetic balance was measured and compared with the expected far field scattering distribution from Mie theory. This resulted in high speed measurements of size and refractive index with precision of 3 parts in  $10^4$  and 1 part in  $10^3$  respectively. To our knowledge this technique has not been applied to measure the phase functions of more complex scattering distributions. Various similar techniques for particle sizing were suggested based on interference in defocused images which could be related to the particle size [19-21]. Fourier plane imaging is receiving attention recently for mapping the

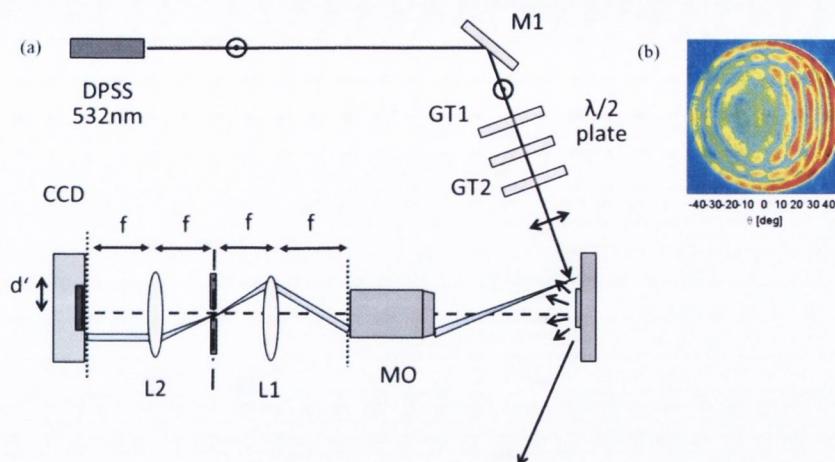


orientation of single molecule emitters [22], analysing cell morphology [23, 24], studying scattering distributions from plasmonic nanostructures structures [25, 26].

## 7.2 Experimental setup

### 7.2.1 Back focal plane imaging (Fourier plane imaging)

In order to further understand the image formation and out of plane scattering by the structures the back focal plane of the microscope objective is imaged onto a CCD. The setup used is depicted in Fig.7.1 below. Light is directed onto the scatterer at a fixed angle using M1 in the same way as the image plane method (IPM) described in Chapter-5. For these measurements the incident angle is fixed at  $80^\circ$ . The polarisation is controlled to be either s or p polarised with respect to the substrate using the two Glan-Thompson polarisers (GT1 and GT2) and the  $\lambda/2$  plate. Light is collected over a solid angle of  $90^\circ$  using the microscope objective. The cone of light collected is defined by the NA of the objective. The microscope objective is infinity and flat field corrected, meaning that a tube lens L1 is needed to form an intermediate image plane. The lens L1 is chosen with a focal length  $f=10$  mm meaning that an image with 50x magnification is formed 10 mm behind L1. Using a field iris at this intermediate image plane allows us to define the area in the sample from which the light is collected. The iris can go down to  $500\text{ }\mu\text{m}$ , which allows us to collect light from a  $10\text{ }\mu\text{m}$  diameter circle in the object plane. This means that a single scatterer can be isolated. Light passing through the iris is then collected with the lens L2 which has the same focal length as L1. The CCD is placed 10mm behind L2 where the back focal plane is formed again. There is no magnification as the focal lengths of L1 and L2 are the same. The diameter of the back aperture of the objective is 34 mm, whereas the height of the CCD active area is 35.7 mm meaning that the BFP image will fit optimally on the CCD without any magnification.



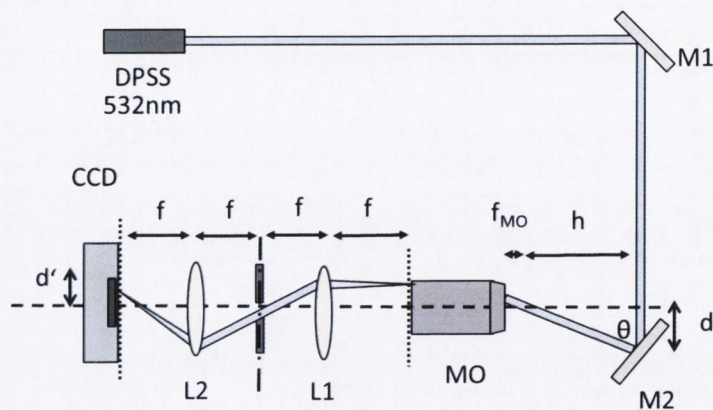
**Figure 7.1** (a) Experimental setup for back focal plane method. Incident light from the DPSS laser is directed onto the sample at a fixed input angle. The polarisation is controlled using GT1, GT2 and the  $\lambda/2$  plate. (b) Typical result, scattering from a  $3.62\ \mu\text{m}$  diameter MF microsphere on a  $\text{Si}_3\text{N}_4$  substrate. Displayed on log scale.

This is a standard imaging technique. The lens L2 is sometimes called a Bertrand lens [22] and a microscope in this configuration is called a conoscope. This configuration is widely used to study the alignment of liquid crystal phases [27-29]. Modern conoscopic devices are commercially available and are used for rapid measurement and evaluation of the electro-optical properties of LCD-screens (e.g., variation of luminance, contrast and chromaticity with viewing direction) [22].

### 7.2.2 Image calibration

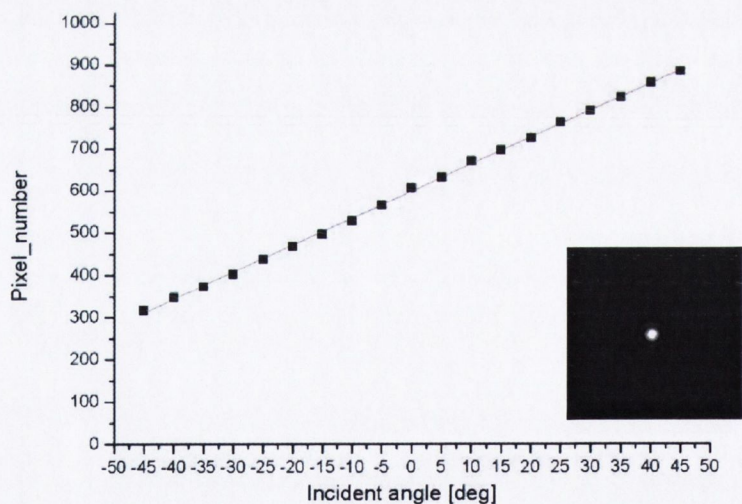
The intensity calibration and dynamic range extension described in Chapter-5.2.5 were needed to record all the detail in the image. It is necessary to calibrate the angle to pixel number on the CCD. This is easily done by translating a mirror M2 across the face of the objective at a known distance in order to input collimated light at a fixed angle as shown in Fig. 7.3. This is focused onto an Airy disk on the CCD at the image plane.





**Figure 7.2:** Experimental setup used for calibration of incident angle to pixel number on the CCD.

The microscope objective converts a collimated beam into a point at its back focal plane with distance from the optical axis which depends on the incident angle of the beam. In the setup above  $f_{MO}=2$  mm, and  $h=12$  cm  $\pm 0.5$  cm. The lateral position of M2 could be controlled to within 1 mm precision. The angle  $\theta$  is given by the inverse tan of  $d/(h+f_{MO})$ . Measurements were taken at  $5^\circ$  intervals resulting in the calibration graph shown in Fig.7.3.



**Figure 7.3:** Calibration graph from incident angle to the microscope objective to radial distance from the optical axis at the CCD. The graph shows that there are 6.4 pixels per degree

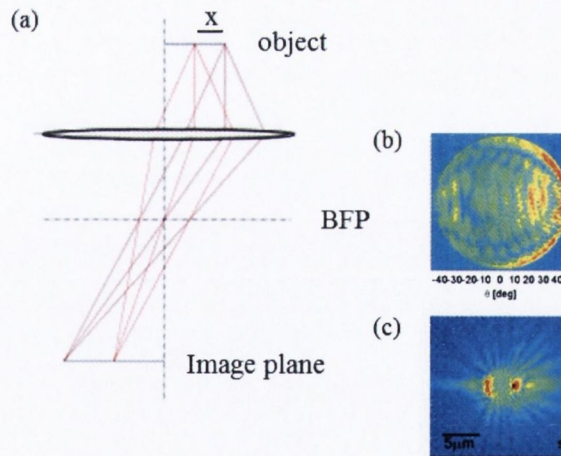
The microscope objective used is plano-corrected [30] meaning that the transformation between input angle before the objective and radial distance from the optical axis at the back focal plane is linear. For a scatterer an interference pattern is formed at the BFP which shows the magnitude of the far-field scattering as a function of angle. Specifically the transformation is

$$x=m\theta+c_1 \quad (6.1)$$

with  $m=6.4$  with angle measured in degrees, and  $c_1=280$  pixels. The slope is the same for both directions as the lens is rotationally symmetric, however as the CCD is rectangular with  $1024 \times 768$  pixels the axis crossing points  $c_1$  depends on angle  $\phi$ . The angle  $\phi$  remains the same in the image.

### 7.2.3 Image interpretation

This method is only applicable when used to collect light from a small region in the object plane. If it is used to collect light over region in the object plane which is too big, interference will occur from light which originates from different parts of the sample. This will affect the image at the back focal plane of the objective. A ray diagram of a simple lens is shown in Fig. 7.4 (a). Light from two points in the object plane which are separated by a distance  $x$  will pick up a phase difference at the BFP due to the different path lengths travelled after the lens. Light incident at the same angle on the lens is focused to the same point in the BFP regardless of its origin in the object plane.



**Figure 7.4** (a) Ray optics diagram of light propagation from the object plane, through the back focal plane to the image plane. Lines of the same angle are focused to a point at the back focal plane and interfere to produce the pattern shown in (b) The pattern formed in the image plane is shown in (c). The BFP and IPM images are shown to the right of their respective positions in the ray diagram.

By putting a  $500 \mu\text{m}$  aperture in the image plane with  $50\times$  magnification we are effectively defining an area of  $10 \mu\text{m}$  from which the light comes from in the object plane. This stops interference from light coming from different scatterers. The structures are large enough to cause interference from light scattered from different points in the structure. If two points are



separated by  $2\text{ }\mu\text{m}$  in the object plane, and emit light at  $15^\circ$  from the normal, then this light will interfere at the back focal plane with a phase shift of  $\sim 0.3\pi$ . This phase difference could be reduced by using lenses L1 and L2 with a longer focal length. A typical BFP image is shown on log scale for a  $4\text{ }\mu\text{m}$  diameter disk, with the corresponding image plane results shown below it. The BFP image is the Fourier transform on the image plane [31].

### 7.3 Out of plane scattering results

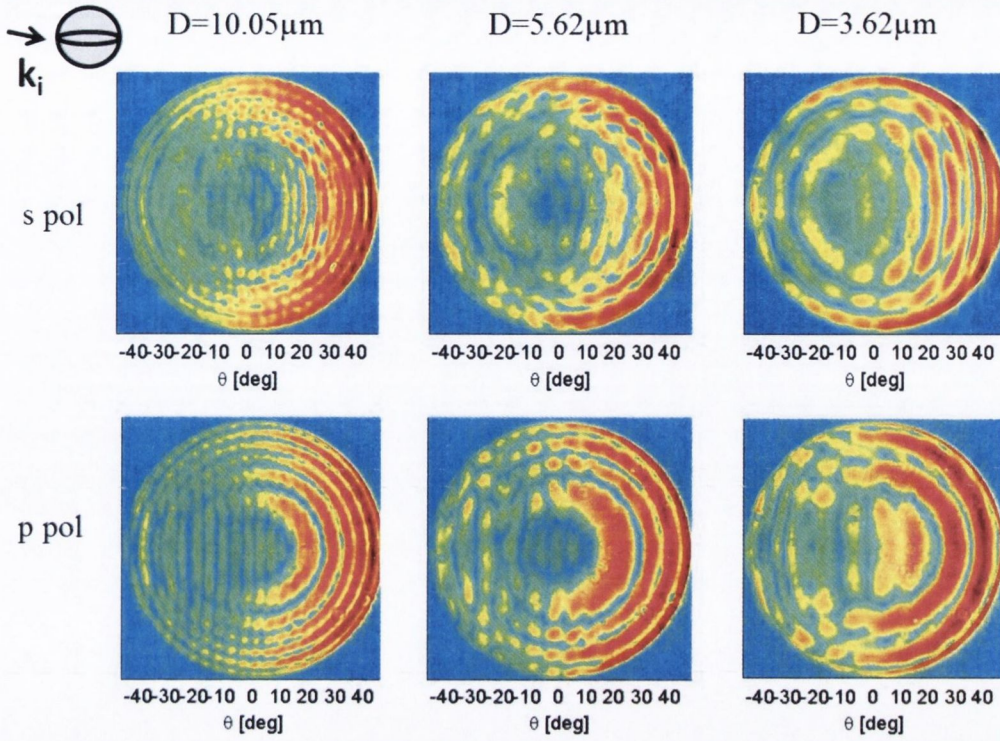
The back focal plane method allows us to map the out of plane scattering from structures defined by EBL. This explains the origin of the bright spots in the image plane as positions in the object plane where a lot of light is scattered into the cone of the objective. We start by measuring the out of plane scattering distribution using the BFP method with dielectric spheres on a  $\text{Si}_3\text{N}_4$  substrate.

#### 7.3.1 Calibration with spheres on a $\text{Si}_3\text{N}_4$ substrate

Dielectric microspheres made from Melamine Formaldehyde (MF) with a refractive index of  $n=1.68$  were used. A commercially available solution of mono-disperse spheres from Microparticles GmbH was drop cast onto a  $\text{Si}_3\text{N}_4$  wafer and diluted to have approximately 1 sphere in every  $50\text{ }\mu\text{m} \times 50\text{ }\mu\text{m}$  area. The particles had a standard deviation in size of  $50\text{ nm}$ .

The MO used had an NA of 0.7 which means a collection angle of  $90^\circ$ . This results in an angular range of  $-45^\circ$  to  $+45^\circ$  in  $\theta$ , and all  $360^\circ$  in  $\phi$ . Spheres with diameters  $D=10.05\pm 0.05\text{ }\mu\text{m}$ ,  $D=5.62\pm 0.05\text{ }\mu\text{m}$  and  $D=3.62\pm 0.05\text{ }\mu\text{m}$  were used. Again the incident wavelength was  $532\text{ nm}$  and the incident angle was fixed at  $80^\circ$ . The results for s pol and p pol are shown in Fig. 7.5 on a log scale.



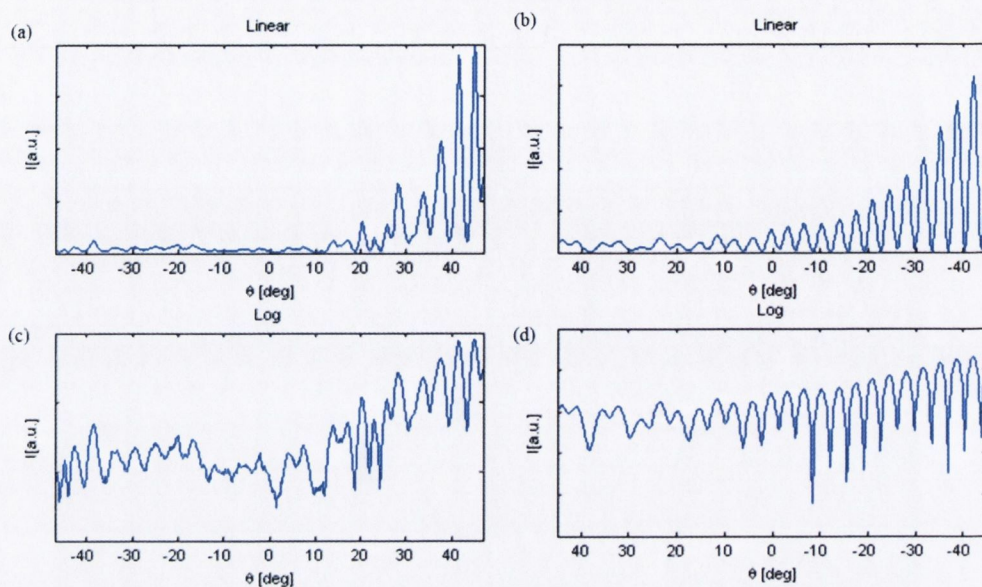


**Figure 7.5** Scattergrams from 3 different sized spheres on an  $\text{Si}_3\text{N}_4$  substrate. The images are on a log scale over 3 orders of magnitude. The red areas represent angles at which there is a lot of light scattered whereas the blue areas are angles where little light is scattered.

Light is incident from the left. These images show one of the main characteristics of Mie scattering, that light is scattered predominantly in the forward direction. For spheres light is also scattered strongly though high angles. The number of lobes increases with the size of the sphere as expected. Taking an average of profile of the scattering over a finite number of pixels close to the  $\phi=0$  plane is effectively like integrating the signal over a finite aperture. This gives a profile which we can compare with the analytical results from Mie theory.

The experimentally measured scattering profile along the  $\phi=0^\circ$  line averaged over an effective aperture of  $5^\circ$  is shown in Fig. 7.6 (a) and in log scale in (c). The analytical far-field scattering distribution for light scattered from a sphere in free space is shown on a linear scale in Fig. 7.6 (b) and in log scale in Fig. 7.6 (d). The distribution is calculated using the approach outlined in Borhren and Huffmann [16], and the code used to calculate this plot is given in Appendix. A. All plots are taken over the range of collection angles of the microscope objective.



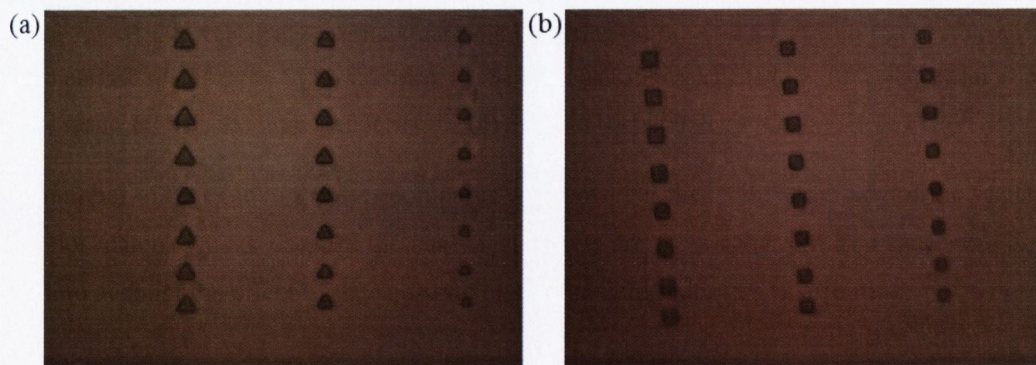


**Figure 7.6** (a) Experimentally measured far field scattering profile along  $\phi=0$  with  $5^\circ$  effective aperture for a dielectric sphere on an  $\text{Si}_3\text{N}_4$  substrate. (b) Scattering profile calculated from Mie theory for a sphere in free space. (c) Same as (a) but on log scale. (d) Same as (b) but in log scale.

The number of lobes in both cases compares well however the exact amplitude of the scattering does not match. This is because the analytical results are for a sphere in free space and do not take into account the effect of the substrate on the far field scattering distribution.

### 7.3.2 Effect of shape on the out-of-plane scattering

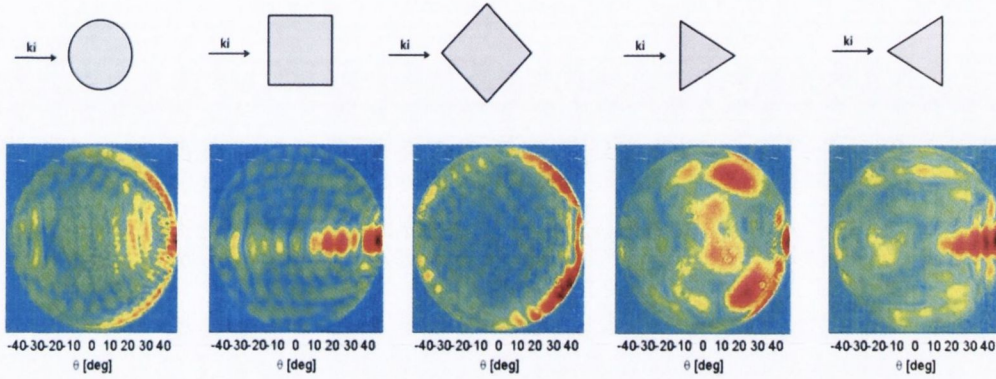
Again we restrict our experiments to light scattering from squares and equilateral triangles. An optical image of a typical sample is shown in Fig. 7.7.



**Figure 7.7** (a) Optical image of typical array of triangles with  $15\ \mu\text{m}$  spacing and size variation from  $1\text{-}6\ \mu\text{m}$  with 4 repeats. (b) Typical array of squares with same spacing, size variation and number of repeats as in (a).



These structures were etched using the Ar CHF<sub>3</sub> etching process resulting in nearly vertical (85°) sidewall angle as shown in the ICP etching process in Chapter-4. The four cases outlined in Chapter-6.2 were studied in detail. Out of plane light scattering from the microdisks studied in Chapter-5 was also examined. The results of the BFP measurements for s polarised incident light and structures of size 4 μm are shown in Fig. 7.8. The incident light k vector and the orientation of the shape are shown above the BFP image.



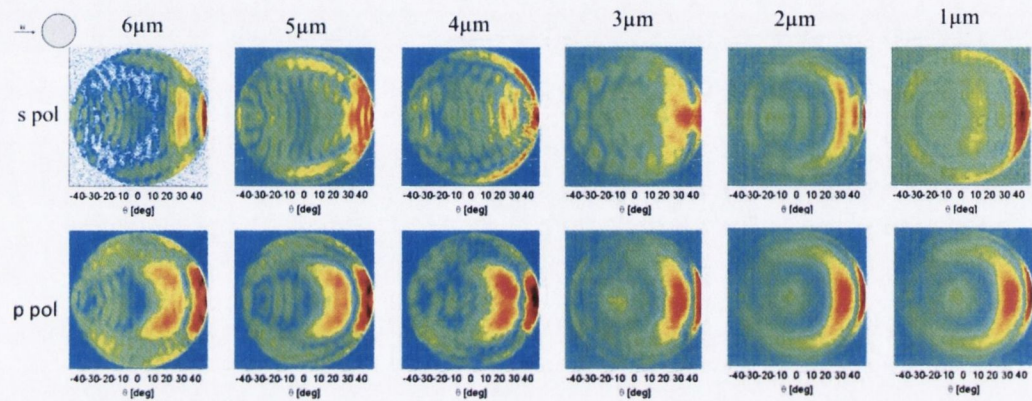
**Figure 7.8** Far-field scattergram for all the examined shapes and orientations. Incident light is s polarised and from the left. The incident k vector and the orientation of the shape are shown above the BFP image.

The light scattering distribution out of plane and hence the BFP image is very sensitive to the scatterer shape. As expected the planar structures don't scatter as much light out of plane as the spheres. Light is predominantly scattered in the forward direction. We are only observing a cone of 90° perpendicular to the substrate. The disks still scatter light at high  $\phi$  angles as expected from the image plane results of Chapter-5. The squares however scatter light predominantly out of plane along the  $y=0$  line. The out of plane scattering distribution is extremely sensitive to the orientation of the triangles and square with respect to the incident light, whereas this is not the case for disks due to their symmetry. We look at two cases for the square and two for the triangle, where the orientation of the scatterer with respect to the incident light is shown above the BFP image in Fig. 7.8. When the square is rotated by 45° we refer to it as a diamond. The diamond shape scatters light strongly and predominantly at high angles of  $\phi$ . For the triangle orientated with a flat side oriented towards the incident light (referred to as flat\_triangle) there is an interesting effect where light is split, and scattered predominantly in two directions of  $\phi$  normal to the two other faces. When the triangle is orientated with the point facing the incident light (point\_triangle) the out of plane scattering is significantly different with a narrow emission along the  $\phi=0$  direction as in the case of the square.



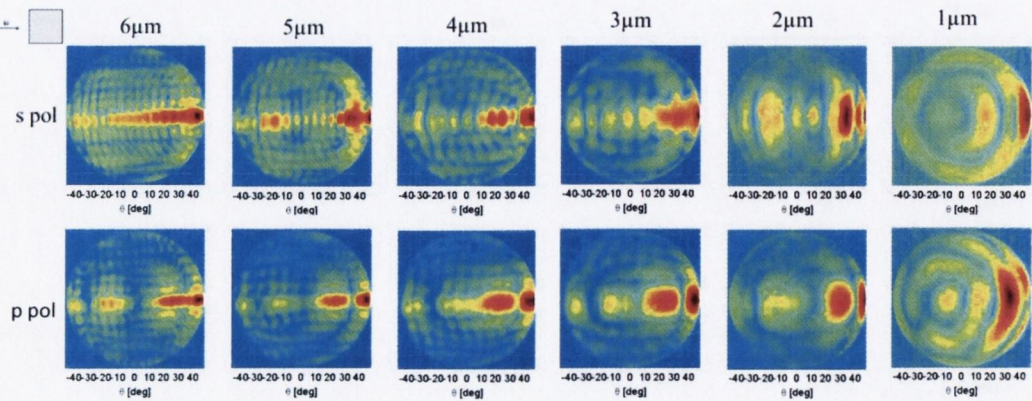
7.3.3 Effect of size on out of plane scattering

In Chapters-5 and 6 we saw that there was a linear increase in the number of in-plane lobes with the size of the scatterer. We find a similar result for the number of out of plane lobes. The out of plane scattering for a disk is displayed in Fig. 6.9 on a log scale for disks of diameter ranging from 1-6 $\mu\text{m}$  with s polarised incident light in the top row and p polarised light in the bottom row.



**Figure 7.9** Log scale BFP images for microdisks with sizes ranging from 6  $\mu\text{m}$  to 1  $\mu\text{m}$  for both s and p polarised incident light. Orientated as shown in the top left hand corner.

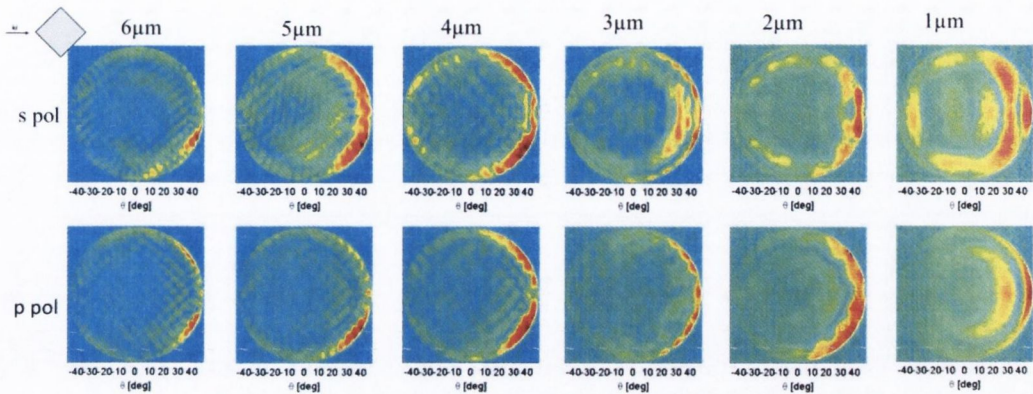
The disks scatter primarily in the forward direction. The number of out of plane lobes increases linearly with the size of the disk. The p polarised case shows that there is a very different out of plane scattering distribution than s polarised, with a dependence that is not so sensitive to disk diameter. The BFP images for squares with side ranging from 1-6 $\mu\text{m}$  are shown in Fig.7.10. Again the number of lobes increases with increasing size and the s and p polarised images are quite different.



**Figure 7.10** Log scale BFP images the case square\_face with sizes ranging from 6  $\mu\text{m}$  to 1  $\mu\text{m}$  for both s and p polarised incident light. Orientated as shown in the top left hand corner.

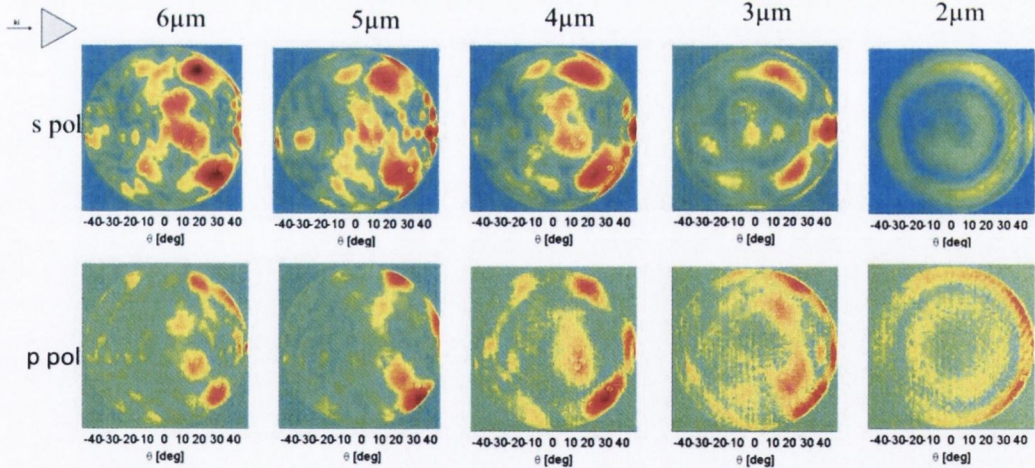


Regardless of the size the light is scattered primarily along the  $y=0$  axis. The same result for squares orientated in the diamond position as shown in the top left hand corner of Fig.7. 11



**Figure 7.11** Log scale BFP images for the case square\_vertex with sizes ranging from 6  $\mu\text{m}$  to 1  $\mu\text{m}$  for both s and p polarised incident light. Orientated as shown in the top left hand corner.

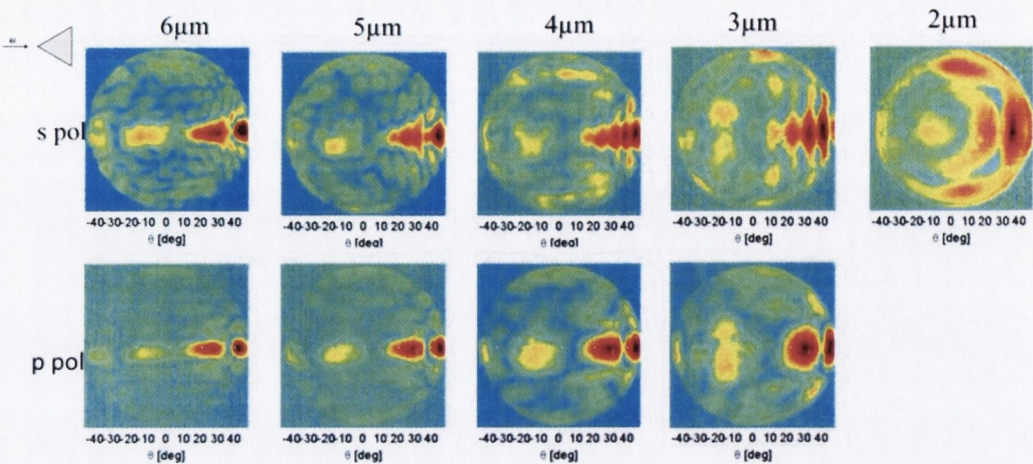
Light is scattered strongly at high angles with the number of lobes depending on the size of the structure. The results for triangles orientated with a flat face towards the incoming light are displayed in Fig. 7.12. Only sizes from 2-6 $\mu\text{m}$  are shown as the 1 $\mu\text{m}$  triangles were damaged.



**Figure 7.12** Log scale BFP images for the case triangle\_face with sizes ranging from 6  $\mu\text{m}$  to 2  $\mu\text{m}$  for both s and p polarised incident light. Orientated as shown in the top left hand corner.

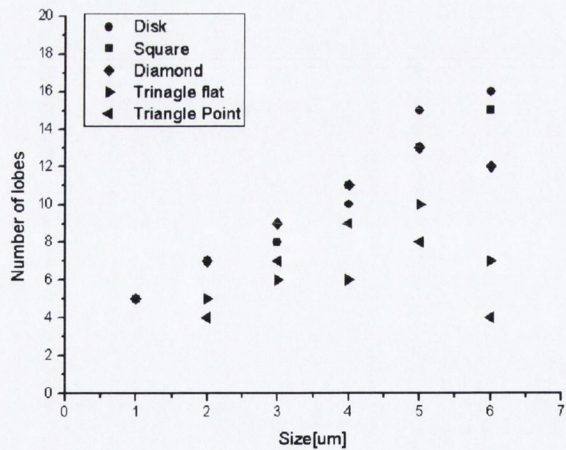
The light is scattered primarily in two directions of  $\phi$  with a number of lobes that decreases with decreasing size. The scattering becomes more uniform as the triangles go below 2 $\mu\text{m}$ . And finally the results for a triangles orientated with a vertex in the direction of the incoming light are displayed in Fig. 7.13





**Figure 7.13** Log scale BFP images the case triangle\_vertex with sizes ranging from 6  $\mu\text{m}$  to 2  $\mu\text{m}$  for both s and p polarised incident light. Orientated as shown in the top left hand corner.

Light is scattered in the forward direction along the  $y=0$  axis ( $\phi=0^\circ, 180^\circ$ ). The number of lobes decreases with decreasing size. The size dependence of the different cases examined is summarised in Fig. 7.14 where the number of out of plane lobes are plotted as a function of scatterer size. The lobes were counted along the  $\phi=0^\circ(\phi=180^\circ)$  plane.



**Figure 7.14** Number of lobes as a function of size for the different scatterer shapes examined under s polarised illumination.

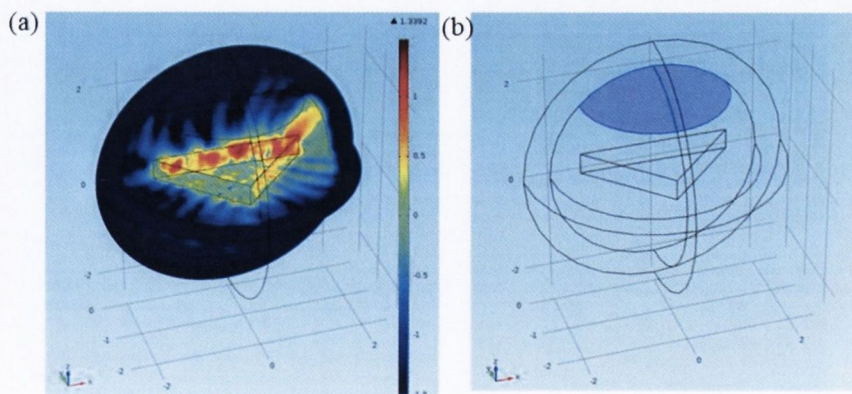
Features common to all shapes are a discrete number of scattering lobes, which increase with increasing size. The spheres and disks show a more even distribution over  $\theta$ , whereas the squares show that scattering is predominantly close to the  $\phi=0$  plane. The triangles show an interesting symmetric pattern with one lobe either side of the tip when illuminated with the flat of the triangle facing the incoming light. A relatively small amount of incident intensity ( $\sim 10^{-2}$ )



is scattered into this cone. This BFP experiment shows the origin of the light forming the image in the image plane. A linear increase in the number of out of plane lobes is seen for the disk square and diamond scatterers, however since the triangles scatter light predominantly out of the  $\varphi=0$  plane it is difficult to characterise the no of lobes from them.

## 7.4 Comparison with FEM

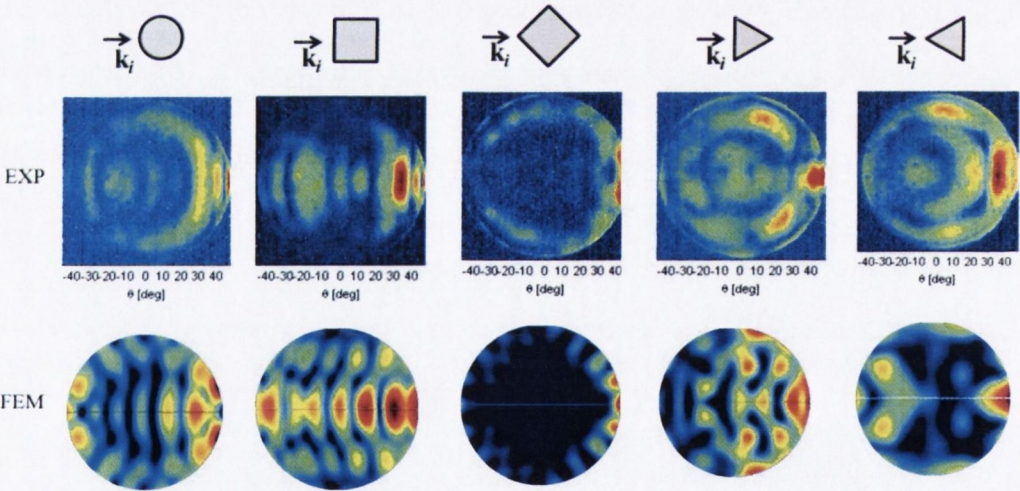
The far field scattering distribution for arbitrary shapes can be calculated using the Finite element method to calculate the near fields and the Stratton Chu formula to extrapolate to the far field as explained in Chapter-2. Unfortunately with the particular FEM software (COMSOL MULTIPHYSICS v.4.2) used in this project we were unable to use the Stratton Chu extrapolation with a substrate. Instead we solved for the scattered field using the input field calculated analytically using the first three terms of the Fresnel reflection given in Chapter 5.3.4. This works well and gives FEM results which are similar to the experimental results for s and p polarisation, however it does not take into account scattered light reflected from the substrate so some differences are expected. The simulation method is outlined in Fig.7.15. Initially the scattered near fields are calculated using the 3D FEM method. The incident field is a sum of plane waves calculated using the Fresnel reflection and transmission co-efficient. The scattered field is symmetric around the  $y=0$  plane, so only half of the domain is simulated to save memory. Perfect electric conductor (perfect magnetic conductor) boundary conditions are used for the s polarised (p polarised) case. The Stratton-Chu equation is used to extrapolate to the near fields in the sphere surrounding the scatterer to the far field as explained in Chapter-2.



**Figure 7.15 (a)** Near field intensity on log scale for 2  $\mu\text{m}$  square in diamond orientation. Light is incident at  $80^\circ$  from normal in the positive x direction (from the left hand side.) **(b)** Shaded area shows the collection solid angle of the microscope objective over which the Stratton-Chu equations are used.

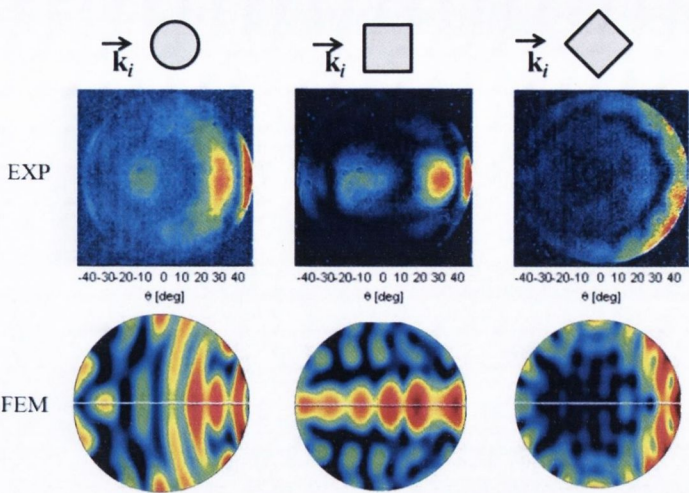


The shaded area in Fig. 7.15(b) is the collection solid angle of the microscope objective. The far field distribution is calculated on this area and then mapped onto a plane using a linear transformation in the same way as the experimental results. The back focal plane results for scattering of s polarised light from the various structures of side  $2\mu\text{m}$  are shown in the top row of Fig. 7.16. The shape and orientation of the scatterer measured are displayed above the experimental image. The numerical results are underneath for comparison. The main features observed experimentally for scattering from each shape are reproduced in the numerical results however we are not able to take into account the reflection of the scattered field from the substrate so some differences are observed. The differences may also be due to interference for light originating for different sides of the disk due to use of a finite diameter pinhole experimentally. Another factor is that some of the low intensity data may not have been recorded by the CCD method. Finally the experimental method has a limited resolution determined by the range of angles collected. This may have caused fine details to blur into each other in the experimental image. Most likely a combination of the above causes the differences in the intensity distributions, however the main features of scattering are evident.



**Figure 7.16** Back focal plane results for various scattering structures of side  $2\mu\text{m}$  compared with the results extrapolated from 3D FEM simulations for the s polarised input case. The experimental results are on the top row, with the corresponding FEM results underneath. The shape, orientation and incident light direction are depicted above the experimental images.

The back focal plane results for a sample of the structures for the p polarised case are shown in Fig.7.17. The numerical results are calculated using the same methods as the s polarised case however now the Fresnel coefficients for the p polarised case are used to calculate the background field.



**Figure 7.17** Sample back focal plane results for various scattering structures of side  $2\mu\text{m}$  compared with the results extrapolated from 3D FEM simulations for the p polarised input case. The experimental results are on the top row, with the corresponding FEM results underneath. The shape, orientation and incident light direction are depicted above the experimental images.

Again the results are qualitatively similar, verifying that the back focal plane method is working in calculating the out of plane far field scattering distribution.



## 7.5 Conclusions

The out of plane scattering from a range of  $\text{Si}_3\text{N}_4$  structures was measured experimentally by imaging the back focal plane of the microscope objective. These results were then compared to the expected out of plane scattering calculated using 3D FEM and the Stratton-Chu equation.

The out of plane scattering was first measured for MF microspheres on a  $\text{Si}_3\text{N}_4$  substrate to test the BFP method with the known analytical solution for a sphere. The comparison of experimental and analytical results are shown in Fig. 7.6. The number of lobes was correctly reproduced however the exact amplitude and position of each lobe did not match. This discrepancy is due to the fact that the analytical solution for a sphere in free space does not take into account the effect of the substrate.

The out of plane scattering was then measured for microdisks and the same four cases outlined in Chapter 6.2 namely, the `square_face`, `square_vertex`, `triangle_face` and `triangle_vertex` cases. The results are shown in Fig. 7.8. Light is scattered predominantly at high angles of  $\phi$  for the microdisk and `square_vertex` cases as we would expect from looking at the image plane results (Fig. 5.7 and Fig. 6.3). The `square_face` and `triangle_vertex` cases scatter light strongly out of plane into lobes which propagate along the  $\phi=0^\circ$ ,  $\phi=180^\circ$  line. The `triangle_face` scatters light out of plane in two main lobes which are at angles of  $\phi$  matching the in-plane lobes from the IPM. (Fig. 6.4)

The number of out of plane lobes is shown in Fig. 7.14 to vary linearly with the size of the structures, in a similar fashion to the in-plane scattering lobes. The 3D FEM method with Stratton-Chu extrapolation outlined in Chapter-2 is used to find the far-field scattering distribution in the cone of the objective. The numerical results correlate well with the experimental images and reproduce the main features of the out of plane scattering highlighted in the previous paragraph. The images do not match exactly as the scattered light reflected from the  $\text{SiO}_2$ -Si interface was not included in the 3D FEM simulation due to memory restrictions on the domain size.

Overall we have measured the out of plane scattering from the structures studied in previous Chapters. This out of plane scattering is due to the finite height of the scatterers and responsible for the patterns observed inside and on the boundaries of the scatterers in the image plane method. The BFP method thus completes our understanding of the image formation in IPM.



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## Chapter 8 Cartesian Ovals

### 8.1 Introduction

In Chapter-5 and Chapter-6 the light scattering distribution from microdisks and low symmetry shapes were investigated. The experimental images obtained in these chapters were explained as a combination of light which is scattered in plane and then from the substrate, and light which is scattered out of plane (Chapter-7). The experimental intensity distribution outside of the scatterer was shown to match well with 3D FEM simulations of the scattered near-field. The next logical step is to use this experimental technique to study structures which are designed to create desired optical effects.

Using the EBL technique outlined in Chapter 4 arbitrary shapes can be created with nanoscale resolution. It is desirable to create small focal regions with a large intensity contrast over the background intensity. Typical commercially available single lenses have spherical surfaces which are easy to manufacture to high precision using grinding and polishing techniques [1, 2]. This however is not the ideal shape to focus light. These lenses suffer from spherical aberration [3]. That is the focal position depends on the radial distance from the principle axis. This limits the distance from the principle axis of the lens which is “usable”, and results in a more elongated and larger focal spot. In general combinations of single lenses or expensive aspherical lenses [4] are used to compensate for spherical aberration.

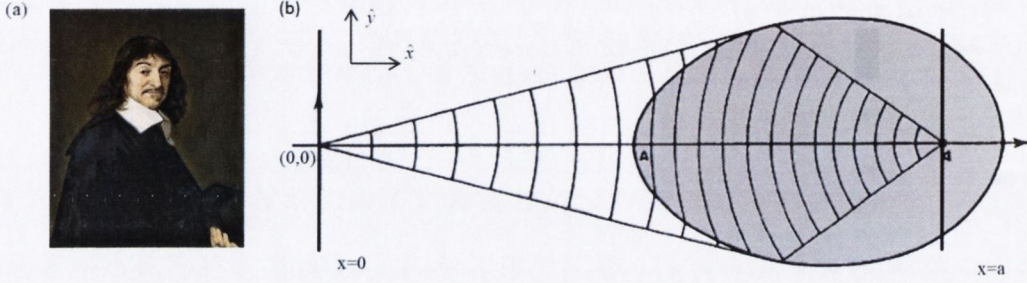
The ideal shape for creating an image without spherical aberration was first discovered by the famous philosopher and mathematician René Descartes in 1637 (Fig. 8.1 (a)), then later studied by Newton in 1664 [5] and Maxwell in 1864 [6]. Specifically the shape can produce two perfectly conjugated planes. That is in the 2D case of Fig. 8.1(b), all the rays coming from point  $p_1$  the line  $x=0$ , are mapped to a single point  $p_2$  on the line  $x=d$  and have the same optical path length. The shape is known as a Cartesian oval and an algebraic expression for the curve using the co-ordinate system in is given in the top left hand corner of Fig. 8.1 is given by [7]

$$n_0 \sqrt{x^2 + y^2} + n_1 \sqrt{y^2 + (x-d)^2} = n_0 A + n_1 (d - A) \quad (8.1)$$

where  $n_1$  and  $n_0$  are the refractive indices of the oval and the surrounding material respectively. The parameter  $d$  is the distance from the object plane to the image plane, and  $A$  is the point of intersection of the oval and the  $x$ -axis. Eliminating the radicals gives an algebraic equation of 4<sup>th</sup>



order, which describes two curves. Only the curve with positive signs for the radicals is used in this case.



**Figure 8.1 (a)** Portrait of Rene Descartes by Frans Hals 1648. **(b)** Cartesian oval: Light from the source at  $x=0$  is refracted at one boundary and forms a converging cylindrical wave with focus at the point  $x=d$  inside the oval..

This shape can also be derived by matching a diverging circular wave-front from the point  $(0,0)$  on the  $x$  axis to a circular wave-front converging to the point  $(d,0)$ .

For the case of an incident plane wave (i.e. object plane at infinity), the solution is found by matching an incoming plane wave-front with a converging circular wave-front at the interface. The algebraic equation for the shape reduces to

$$\frac{\left(x - a \frac{n_1}{n_1 + n_0}\right)^2}{a^2 \left(\frac{n_1}{n_1 + n_0}\right)^2} + \frac{y^2}{a^2 \frac{n_1 - n_0}{n_1 + n_0}} = 1 \quad (8.2)$$

The origin is taken on the centre line of the first boundary of the shape and  $a$  is the distance from the origin to the image plane. Comparison with the standard equation for an ellipse

$$\frac{X^2}{A^2} + \frac{Y^2}{B^2} = 1 \quad (8.3)$$

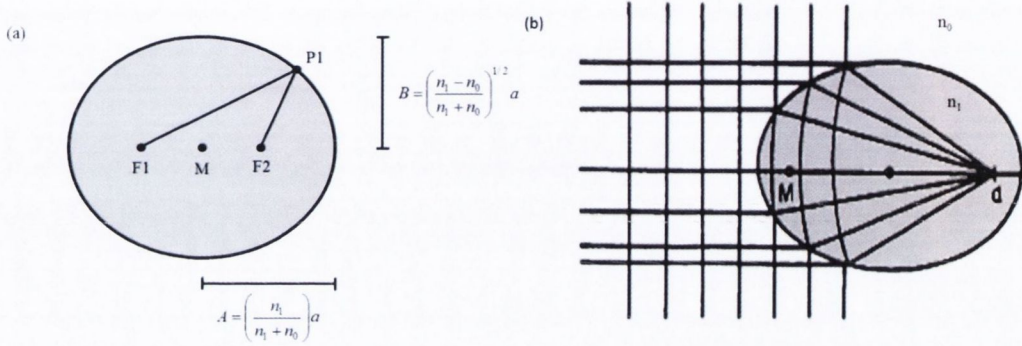
shows that this describes an ellipse with semi-major axis  $A$ , semi-minor axis  $B$  and centre  $M$  given by

$$A = \frac{an_1}{n_1 + n_0} \quad (8.4)$$

$$B = a \left( \frac{n_1 - n_0}{n_1 + n_0} \right)^{1/2} \quad (8.5)$$

$$M = a \frac{n_1}{n_1 + n_0} \quad (8.6)$$

This ellipse has a specific ratio of major to minor axes which depends on the refractive index contrast. The structure thus only depend on one parameter  $a$  as shown in Fig.8.2(a).



**Figure 8.2** (a) Schematic of Cartesian oval for focusing a plane wave. (b) Ray diagram of focusing effect.

When light is incident in the positive x- direction as shown in the ray diagram of Fig. 8.2 (b) it is refracted at the first interface to form a converging cylindrical wave with a diffraction limited focus at point  $a$ . The point  $a$  happens to be second focus of the ellipse F2. Any ellipse contains two focal points F1 and F2 which are at an equal distance from the centre of the ellipse that have the property that the distances between the two foci and any point P1 on the surface of the ellipse are equal to twice the major radius:

$$d(F1, P1) + d(P1, F2) = 2A \quad (8.7)$$

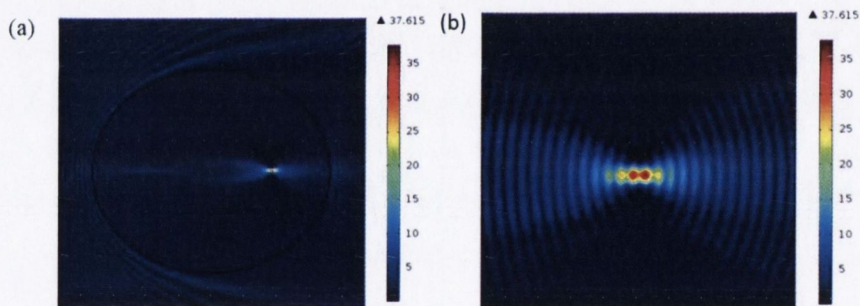
Revolution of Eq 8.1 around the x-axis gives a 3D lens which can create an image free from spherical aberration, however inside the lens. Cartesian ovals and similar curves are commonly used in the design of aspheric optical lenses [8]. Recently the shape has been proposed to create a microresonator with high Q and directional emission, by placing a scatterer at the focus of an auxiliary ellipse with the dimensions of a Cartesian oval. This collimates the light scattered from the defect [9]. Elliptical [10-12] and limaçon shaped resonators [13, 14] have been studied with respect to chaotic whispering gallery modes which they exhibit [15]. However this is the first study of light scattering from 2D elliptical shapes especially with the fixed parameters defined by Eq 8.4 and 8.5. We wish to use the ovals as 2D focusing elements which, either with free space or waveguide illumination, produce diffraction limited focal spots in a planar system.



## 8.2 Cartesian ovals

The derivation in the previous section was based on geometrical optics, so these structures need to be slightly larger than the microdisks, squares and triangles previously studied. Cartesian ovals with parameter  $a$  ranging from 20  $\mu\text{m}$  down to 4  $\mu\text{m}$  were created using the EBL and Ar ChF3 etch outlined in Chapter 4. The refractive index of  $\text{Si}_3\text{N}_4$  was taken at 2. For  $\text{Si}_3\text{N}_4$  in air the optimum major and minor axes given by Eq 8.4 and 8.5 are  $A = (2/3)a$  and  $B = \sqrt{(1/3)}a$  respectively. This means that the largest ovals created had a major diameter of 26.66  $\mu\text{m}$ . These structure scale and can be made any size, but we restrict our study to less than  $a=20 \mu\text{m}$  so that the scattering distribution remains within the field of view of the microscope objective used.

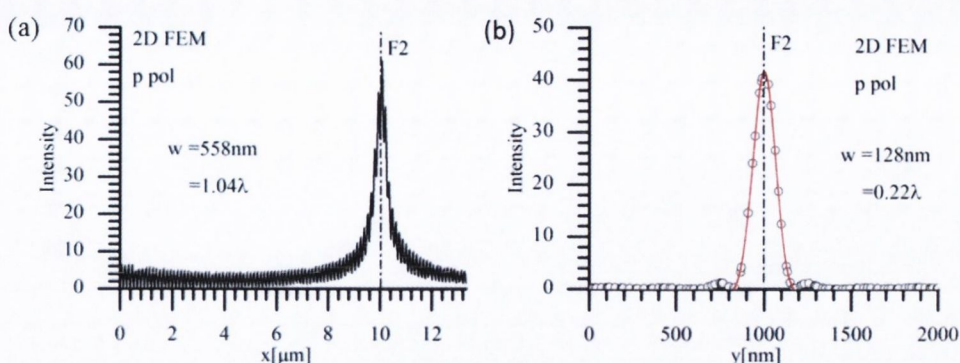
Light focusing by Cartesian ovals can be simulated using the 2D Finite element method (FEM). The structures are too large to simulate the full 3D scattering, however for these larger structures the 2D simulations match well with the experimental results for the intensity in the focal region. The simulations predict a diffraction limited spot at the focal point F2 inside the oval as shown in Fig. 8.3 (a).



**Figure 8.3** (a) 2D FEM of slight focusing by Cartesian oval of parameter  $a=10 \mu\text{m}$ . Light is incident from the left with s polarisation and a wavelength of 532nm.

Light is reflected back from the second interface causing a standing wave interference pattern. The period of this is half the wavelength in of light in the  $\text{Si}_3\text{N}_4$  layer or 133nm ( $\lambda_0 / 4$ ). Cross sections of the total intensity distribution inside the oval are shown in Fig. 8.4. Figure. 8.4 (a) is a cross section along the x axis through the centre of the oval and Fig. 8.4 (b) shows the cross section along the y axis through the focal spot. This is fitted with a Gaussian in order to estimate the  $1/e^2$  waist of the focal spot.

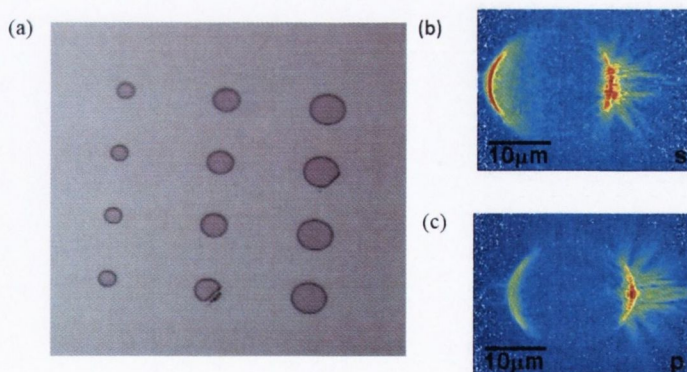




**Figure 8.4** (a) Intensity cross section along the x-axis inside the oval from 2D FEM. (b) Intensity cross section in y direction at  $x=F2$ .

Fitting the profiles we can extract  $w_x$  and  $w_y$  the  $1/e^2$  half waist in along the x and y axis respectively. The focal spot has waist  $w_x = 558\text{nm} = 1.04\lambda$  and  $w_y = 128\text{nm} = 0.22\lambda$ . The small waist in the transverse direction is possible as the focus is formed inside a material of refractive index 2.

A typical optical image of a sample of Ovals with parameter  $a = 20\mu\text{m}$ ,  $15\mu\text{m}$ ,  $10\mu\text{m}$  with 4 repeats of each size is shown in Fig. 8.5 (a). Some ovals in this particular sample were damaged due to measurement of the resist height with a profilometer before the etching step. The experimental images taken using the IPM described in Chapter-5 are shown on a log scale in Fig. 8.5 (b) for s polarised incident light, and Fig. 8.5 (c) for p polarised incident light. The out of plane incident angle  $\theta_i$  was held constant at  $80^\circ$  in all of these measurements.



**Figure 8.5** (a) Optical images of ovals with parameter  $a = 20\mu\text{m}$ ,  $15\mu\text{m}$ , and  $10\mu\text{m}$ , with 4 repeats in each column. (b) Light scattering from oval with parameter  $a = 20\mu\text{m}$  in log scale with s polarised input. (c) Same oval with p polarised input.



The focal spot is not observable as light from F2 does not scatter into the cone of the microscope objective. Only light scattered at the boundaries of the oval and a pattern outside the oval formed from secondary scattering from the substrate are observable. The out of plane scattering in the centre of the structure is negligible. In these structures we cannot gain access to the high intensity focal spot from outside of the oval.

### 8.3 Truncated ovals

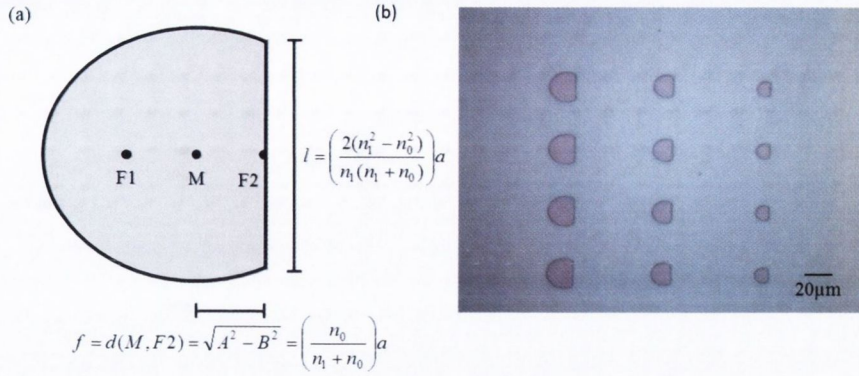
The conversion from plane wave to converging cylindrical wave takes place by refraction at the first interface only. Thus we can truncate the oval at the second focus, with minimal disruption to the fields at this position. This allows us to gain access to the focal spot. A schematic of the geometry is shown in Fig. 8.6 (a). By definition the sum of the distances from any point P on the ellipse to those two foci is constant and equal to the major axis ( $PF_1 + PF_2 = 2A$ ). Using this property and Pythagoras' theorem it is easy to show that the length of the chord  $l$  defining the truncation is given by

$$l = \left( \frac{2(n_1^2 - n_0^2)}{n_1(n_1 + n_0)} \right) a \quad (8.8)$$

whereas the distance between the centre of the ellipse and the new boundary is given by

$$f = d(M, F2) = \sqrt{A^2 - B^2} = \left( \frac{n_0}{n_1 + n_0} \right) a \quad (8.9)$$

For the case of  $\text{Si}_3\text{N}_4$  ( $n_1=2$ ) in air ( $n_0=1$ ), the length of the chord  $l$  is equal to  $a$ , and the distance from the centre of the ellipse to the boundary is  $a/3$ . Truncated ovals with values of  $a$  of 20 $\mu\text{m}$ , 15 $\mu\text{m}$ , 10 $\mu\text{m}$ , 8 $\mu\text{m}$ , 6 $\mu\text{m}$ , 4 $\mu\text{m}$  were fabricated. Figure. 8.6 (b) shows a typical optical image of a fabricated sample. Each column is 4 repeats of a single parameter size  $a$ . The first, second and third columns have  $a=20 \mu\text{m}$ ,  $a=15 \mu\text{m}$ , and  $a=10 \mu\text{m}$  respectfully. The separation between each oval is 50  $\mu\text{m}$ .

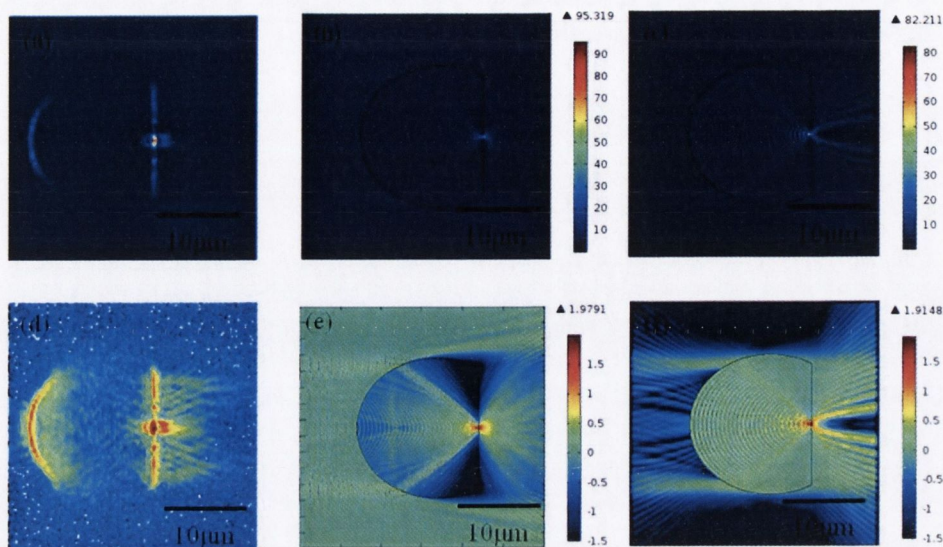


**Figure 8.6** (a) Schematic of truncated oval showing the truncated chord length  $l$  (b) Fabricated sample of truncated ovals with varying parameter  $a$ . Specifically each column is 4 repeats of  $a=20 \mu\text{m}$ ,  $a=15 \mu\text{m}$  and  $a=10 \mu\text{m}$  respectively.

A plane wave-front incident along the positive  $x$  direction will be refracted at the curved interface and transformed into a converging cylindrical wave-front with focus at F2. The samples are measured using the imaging technique outlined in Chapter 5. The incident light is linearly polarised with either  $s$  or  $p$  polarisation. The wavelength used was 532 nm and the input angle was fixed at  $80^\circ$ . Later samples used for optical measurement had a spacing of 100 μm between each oval to avoid inference of light scattered from adjacent ovals.

Figure. 8.7 shows the experimental results for a truncated oval with parameter  $a=15 \mu\text{m}$ , compared with 2D FEM results. The images are taken over a  $30 \mu\text{m} \times 30 \mu\text{m}$  area, with a 46.5 nm image pixel size. Using the FEM method we can plot the norm of total near-field ( $|\vec{E}_i|^2 + |\vec{E}_s|^2$ ) shown in Fig. 8.7 (b) and (e), or just the norm of the scattered field ( $|\vec{E}_s|^2$ ) shown in Fig. 8.7 (c) and (f). Both are displayed below for comparison with the experimental images and normalised with respect to the incident intensity. The first row of Fig. 8.7 shows the results on a linear scale, the second row on a log scale. The experimental results show a small focal spot at position F2 and some lower intensity lobes around the edges of the oval due to scattering from the finite height. These compare well to the 2D FEM results for the total and scattered intensity in Fig. 8.7 (b) and (c) respectively. The FEM predicts an intensity 90 times the background intensity at the focal spot for  $a=15 \mu\text{m}$ .



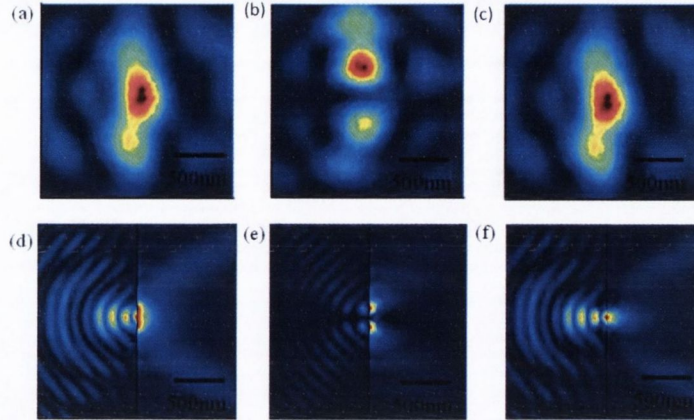


**Figure 8.7** Light scattering from Cartesian oval with parameter  $a=15\ \mu\text{m}$ . Light is incident from the left with a wavelength of 532 nm and s polarisation. **(a)** Experimental measurement linear scale **(b)** 2D FEM total field linear scale **(c)** 2D FEM scattered field on linear scale. **(d)** Experimental measurement on log scale. **(e)** 2D FEM total field log scale **(f)** 2D FEM scattered field log scale.

The experimental image Fig 8.7 (d) shows that the light coming from the edge of the truncated oval consists of a number of spots. The period of the spots on the curved boundary is around 860 nm ( $1.6\ \lambda$ ). The flat boundary shows a line of higher intensity which looks like peaks which are too close to be resolved using the 0.7 NA MO. There are a number of lobes outside the disk which are visible due to scattering of the total field (incident + scattered) from the  $\text{SiO}_2$  surface surrounding the oval. This is qualitatively similar to the 2D FEM results in Fig. 8.7 (e) and (f), however the number of lobes differ from the simulations. This is likely due to the fact that the 2D simulations do not accurately represent the 3D structures and cannot take into account the effect of the substrate layer.

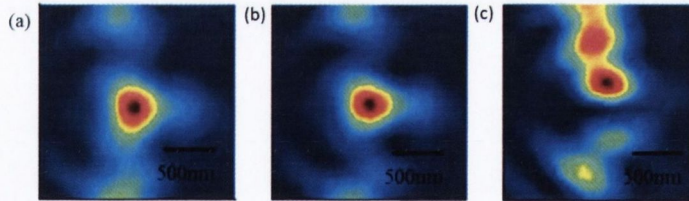
By placing a polariser at position P1 between the objective and the CCD we can map the polarisation of the light coming from the focal region. This matches well with the expected near field intensity distribution from 2D FEM simulations. The top row of Fig. 8.8 shows the experimental images for the s polarised case from a truncated oval with parameter  $a=15\ \mu\text{m}$  and with (a) no analyser (b) x polarised (c) y polarised. The images are cropped over a  $2\ \mu\text{m} \times 2\ \mu\text{m}$  ( $43 \times 43$  pixel) region centred on F2. The second row of Fig. 8.8 shows the 2D FEM results cropped over the same region for (d) the total intensity (e) the intensity polarised along the x-axis  $|E_x|^2$  and (f) the scattered intensity polarised along the y-axis  $|E_y|^2$ .





**Figure 8.8:**  $2\text{ }\mu\text{m} \times 2\text{ }\mu\text{m}$  close-up of region around the focus for a truncated oval with  $a = 15\text{ }\mu\text{m}$ . All images are on a linear scale for s polarised case. The top row are experimental measurements with (a) no analyser (b) x-polarised (c) y polarised. The second row are results of 2D FEM with (d) total field (e)  $|E_x|^2$  and (f)  $|E_y|^2$ .

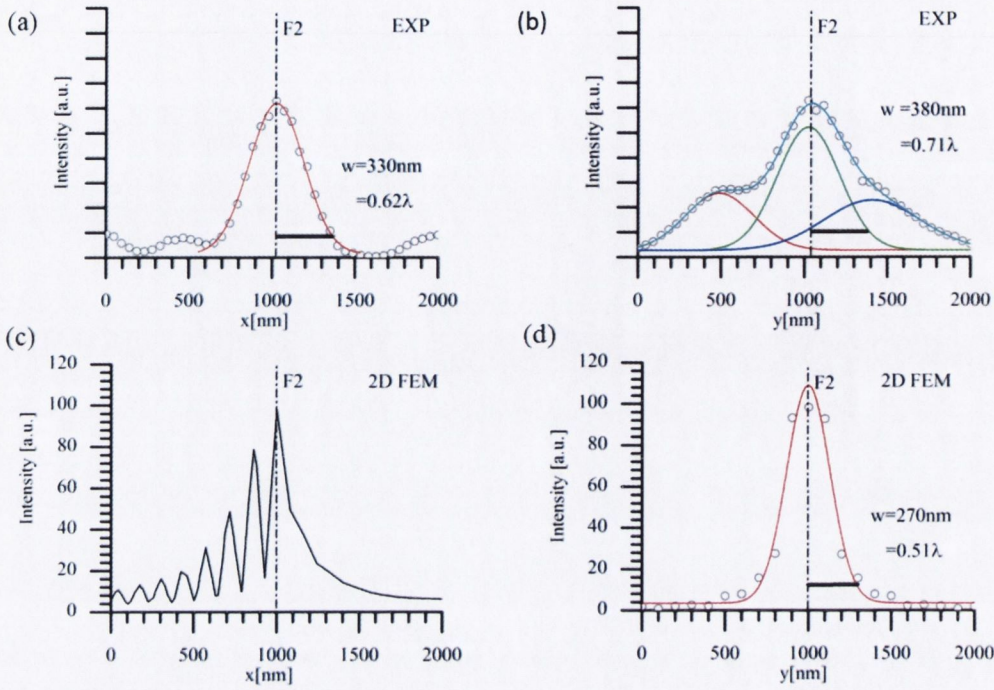
The images show a small focal region which is dominated by the y polarised component which is in the same direction as the incident polarisation. The experimentally measured x- polarised component is 1/100 the intensity of the y-polarised component. The experimental images for the p-polarised case are similar as shown in Fig .8.9, however in this case the field is mostly polarised in the x direction.



**Figure 8.9**  $2\text{ }\mu\text{m} \times 2\text{ }\mu\text{m}$  close-up of region around the focus for a truncated oval with  $a = 15\text{ }\mu\text{m}$ . All images are on a linear scale for p polarised case.

The 2D FEM results in this case predict light to be polarised in the z direction only, so cannot be used to match to the experimental results. The 2D FEM cannot take into account the out of plane angle of the incident light which results in this experimental polarisation dependence. Figure 8.10 shows cross sections of these intensity images. Figure, 8.10(a) is the profile along the x axis through the point F2 and (b) the cross section along the y direction (across the flat surface of the truncated oval).



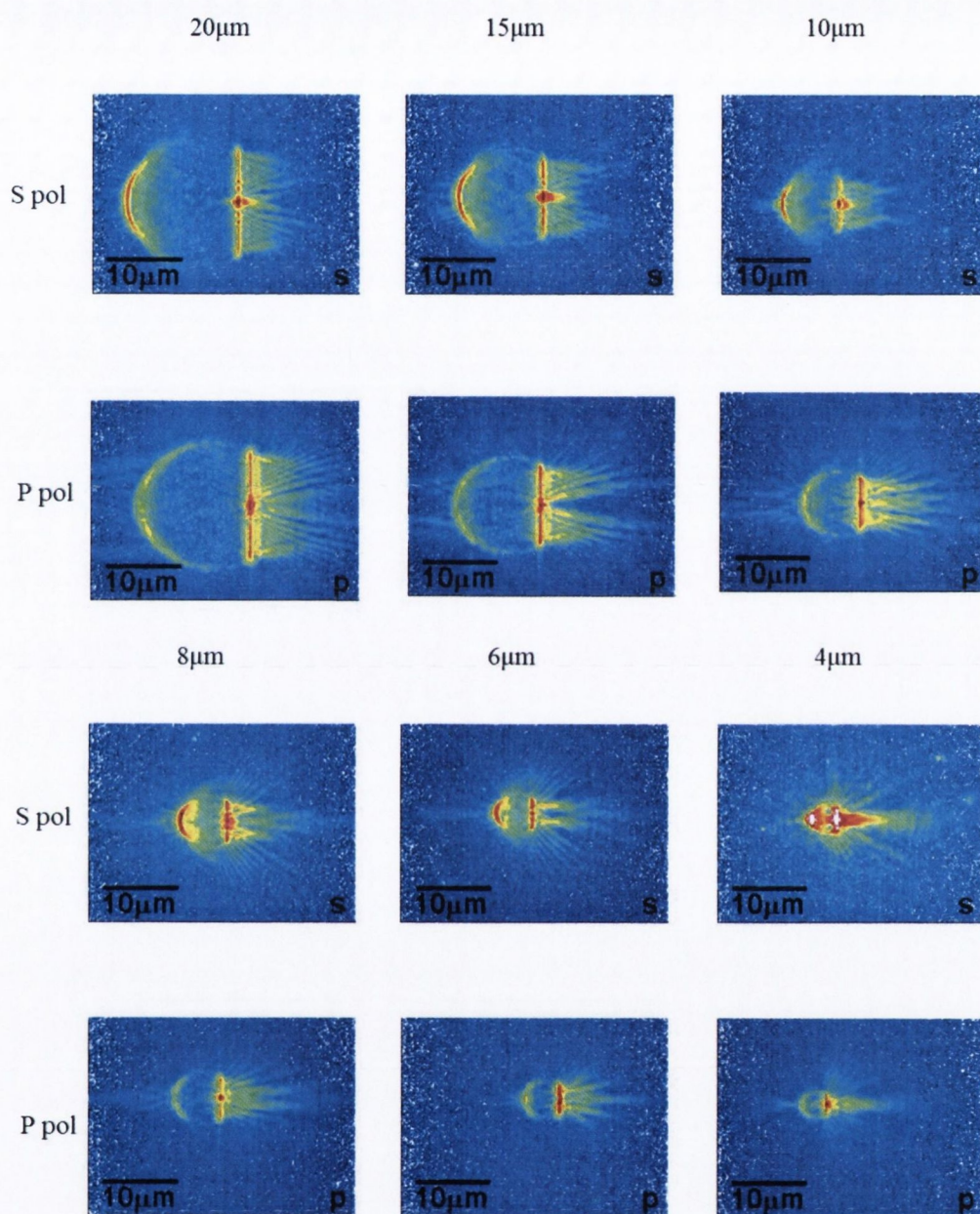


**Figure 8.10** (a) Experimental cross section along  $x$  axis through F2 for a truncated oval with  $a = 15 \mu\text{m}$  and  $s$  polarised incident light. (b) Experimental cross section along  $y$  axis through F2. (c) 2D FEM cross section along  $x$  axis (d) 2D FEM cross section along  $y$  axis. The experimental results are fitted with Gaussians.

The parameter of interest is  $w$  which is the half the  $1/e^2$  width of the Gaussian fit. For a focused Gaussian beam this is limited to  $w \approx \frac{\lambda}{NA\pi}$  which can be as small as  $\lambda/3$  in air. The experimental cross sections in Fig. 8.10 (a) and (b) show a small spot with  $w_x=330\text{nm}$  and  $w_y=380\text{nm}$ . The spot is asymmetric consisting of an overlap of three spots along the  $y$ -axis. Fig. 8.10 (c) and (d) show the near field intensity profiles calculated from 2D FEM along the same lines. The intensity falls off exponentially from the surface in the  $x$  direction and is Gaussian in shape with  $w=270 \text{ nm}$  in the  $y$  direction. The experimental images are blurred by the 0.7 NA of the microscope objective used to collect the light and we would expect to be able to see a minimum spot size of  $w=242 \text{ nm}$  under ideal conditions.

The size dependence of the scattering for truncated ovals with parameter  $a$  ranging from 4-20  $\mu\text{m}$  for both  $s$  and  $p$  polarised incident light is shown in Fig. 8.11. The images are in log scale to emphasis the detail outside of the oval. As the oval gets smaller the effect of light diffracting around the object becomes more prominent and the single focal spot is destroyed.

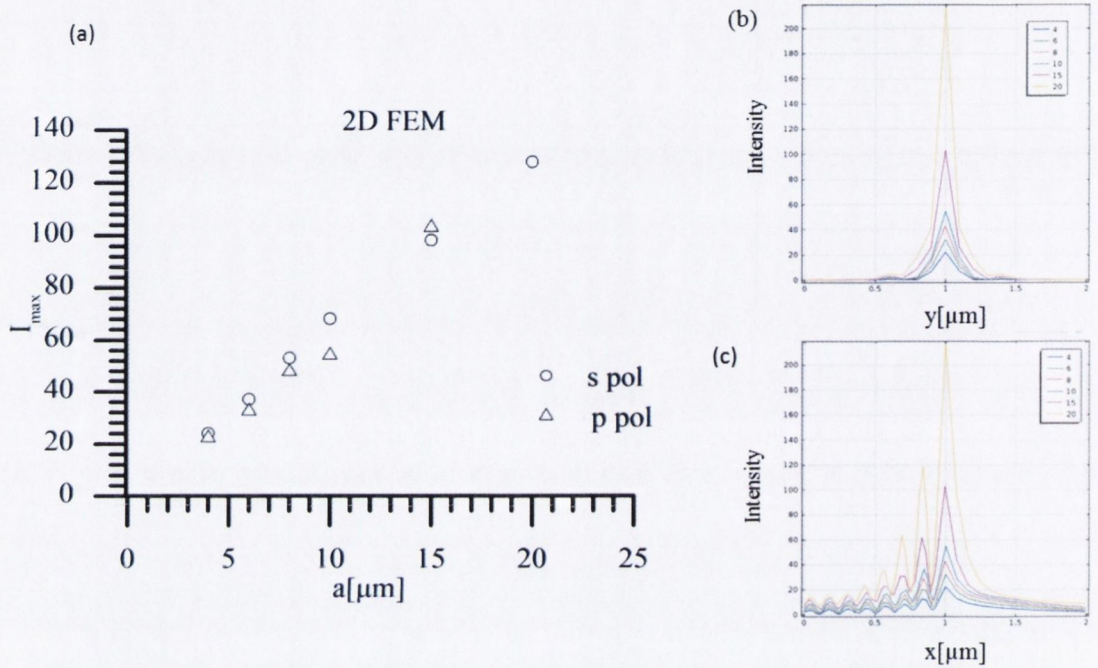




**Figure 8.11** Size dependence of light scattering from truncated Cartesian ovals with parameter  $a$  from  $20\mu\text{m}$  to  $4\mu\text{m}$ . Log scale with incident light s and p polarised as labelled



The intensity of the focal spot increases linearly with the parameter  $a$ . The dependence calculated from the 2D FEM simulations is plotted in Fig. 8.12(a). The cross sections along the  $y$  and  $x$  axis centred on the focal spot as shown for the different values of  $a$  in (b) and (c) respectively.

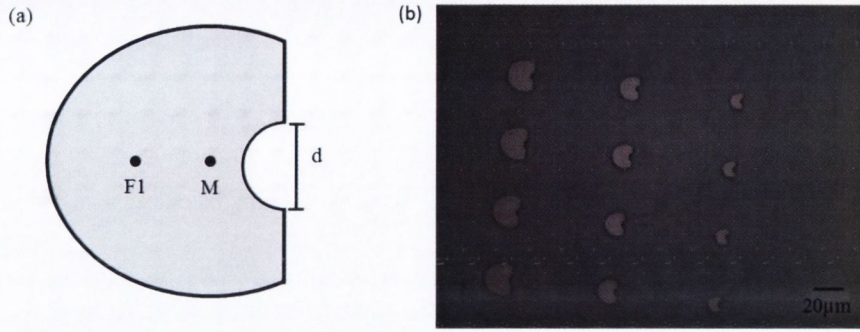


**Figure 8.12** (a) 2D FEM results of maximum in the near field intensity in the focus  $I_{\max}$  relative to the background intensity for s polarised and p polarised cases as a function of  $a$ . (b) Intensity  $y$  axis cross sections for all different sizes of  $a$  examined (c) Intensity profile along the  $x$ -axis for the different sizes.

The  $1/e^2$  half width  $w$  is constant for increasing size at  $0.51 \lambda$ . The field decays exponentially from the surface of the truncated oval with a  $1/e^2$  length of  $450\text{m}$  ( $0.84\lambda$ ).

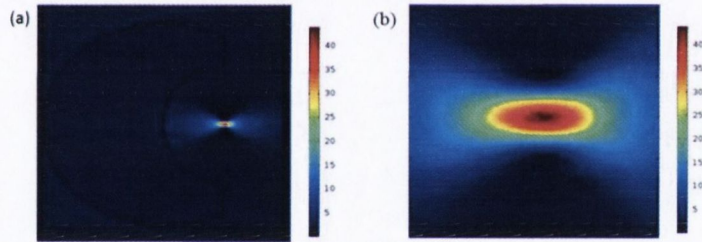
## 8.4 Truncated ovals with cut-out

Since the truncated oval creates a perfectly converging cylindrical wave-front by refraction at the first interface, a circle can be cut out from the shape with centre at the second focus of the ellipse without changing the wave-front across the interface as shown in Fig.8.13 (a). This allows further access to the focal spot. This shape referred to as a truncated\_cut oval is examined for a range of different diameters  $d$  of the cut-out cylinder. It leads to a diffraction limited spot size with an NA which depends on the ratio of the size of the ellipse to the size of the cut-out. An optical image of a typical sample is shown in Fig.8.13 (b).



**Figure 8.13 (a)** Schematic of the truncated cut ovals showing parameter  $d$ . **(b)** Optical image of typical sample. These samples have a constant cut-out diameter of  $5\text{ }\mu\text{m}$ . The columns are 4 repeats of  $a=20\text{ }\mu\text{m}$ ,  $15\text{ }\mu\text{m}$  and  $10\text{ }\mu\text{m}$  respectively.

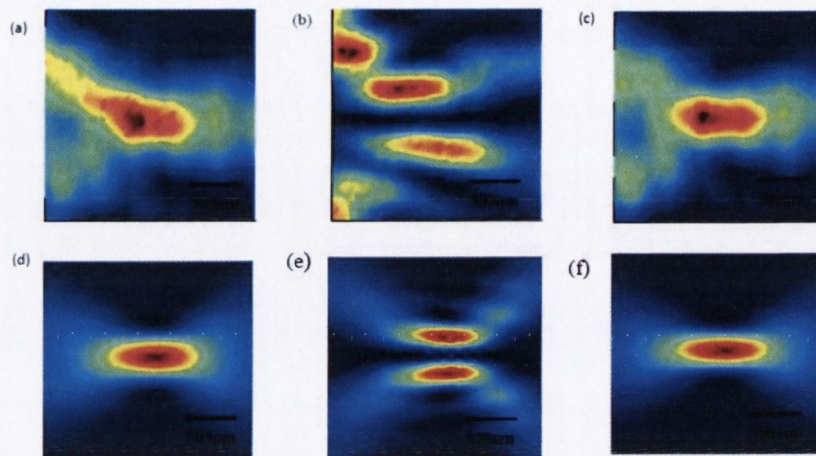
The 2D FEM simulations predict the focal point is still at position P2, however it is now more similar to the focal spot of a focused Gaussian beam. The intensity distribution is shown in Fig. 8.14 for a truncated\_cut oval with parameters  $a=15\text{ }\mu\text{m}$  and  $d=10\text{ }\mu\text{m}$ . A  $2\text{ }\mu\text{m} \times 2\text{ }\mu\text{m}$  region of the intensity distribution around the focus is shown in Fig. 8.14 (b)



**Figure 8.14 (a)** 2D FEM results for light focusing by a truncated cut Cartesian oval with parameters  $a=15\text{ }\mu\text{m}$  and  $d=10\text{ }\mu\text{m}$  **(b)** Close up of the focal region.

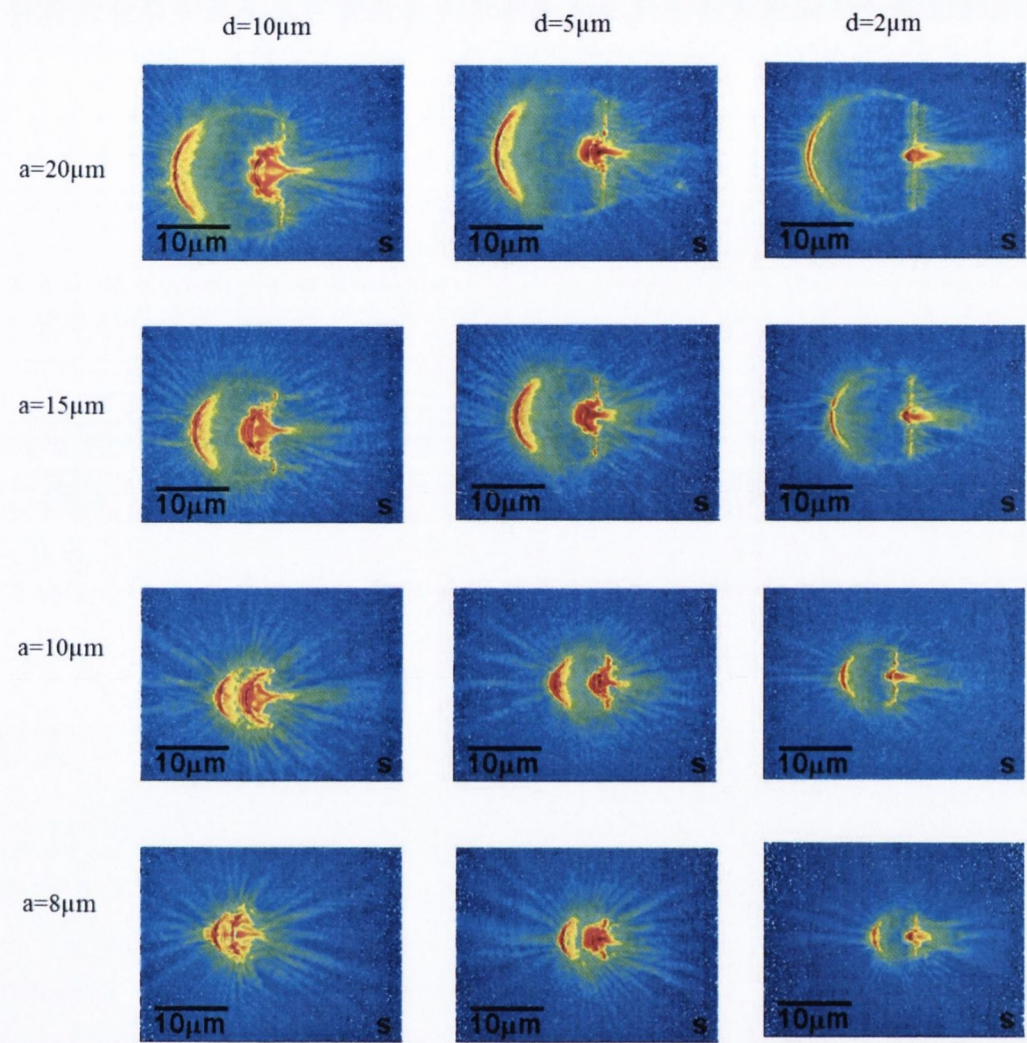
The experiment and 2D FEM results are compared in the focal region of a truncated\_cut oval with parameters  $a=15\text{ }\mu\text{m}$  and  $d=10\text{ }\mu\text{m}$  in Fig.8.15. The first row shows the experimental results for s polarised illumination with (a) no analyser, (b) x-polarised and (c) y-polarised cases. The 2D FEM simulations are on the bottom row with (d) total intensity (e)  $|E_x|^2$  and (f)  $|E_y|^2$ .





**Figure 8.15**  $2\mu\text{m} \times 2\mu\text{m}$  close-up of region around the focus all on linear scale for s polarised case. The top row are experimental measurements with **(a)** no analyser **(b)** x-polarised **(c)** y polarised. The second row are results of 2D FEM with **(d)** total field **(e)**  $|E_x|^2$  **(f)**  $|E_y|^2$ .

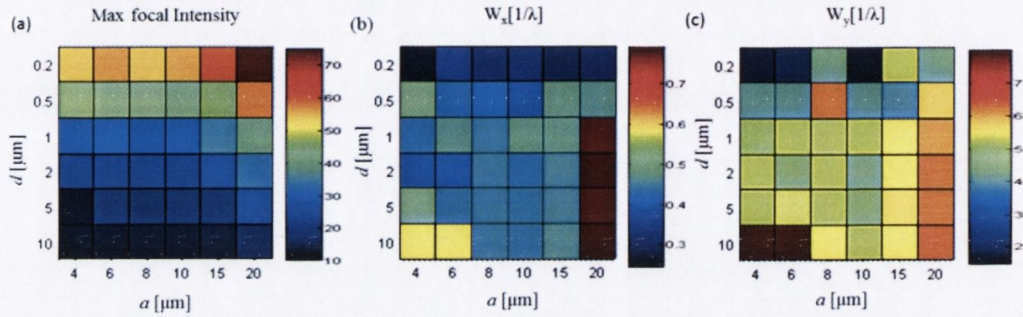
The 2D FEM results compare well with the experimental results. The parameters  $a$  and  $d$  are varied experimentally and a selection of the parameter space is displayed in Fig.8.16. The images are in a log scale.



**Figure 8.16** Parameter dependence of light scattering from truncated cut Cartesian ovals with row showing parameter  $a$  from  $20\mu\text{m}$  to  $8\mu\text{m}$  and columns showing parameter  $d$  from  $10\mu\text{m}$  to  $2\mu\text{m}$ . Images on log scale with incident light  $s$  polarised.

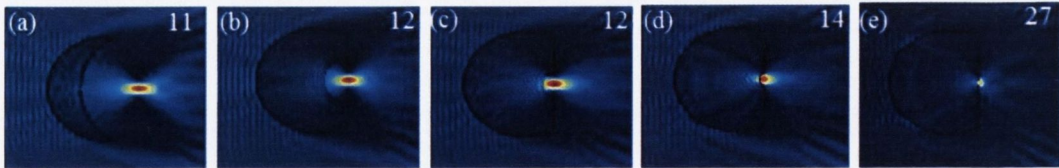


A plot of the intensity and spot size for all values of a parameters  $a$  and  $d$  are shown in Fig.8.17. The spot size is estimated by Gaussian fitting. The parameters  $w_x$  and  $w_y$  are half the  $1/e^2$  width in the x and y directions at the focus respectively. The maximum intensity is the value of the focal intensity compared the incident intensity.



**Figure 8.17** All values calculated from 2D FEM results for the s-polarised input case. (a) The max focal intensity relative to the incident intensity for the different values of parameters  $a$  and  $d$  (b) The  $1/e^2$  half width of the focal spot in the x direction as a function of  $a$  and  $d$  in units of wavelength . (c) The  $1/e^2$  half width of the focal spot in the y direction as a function of  $a$  and  $d$  in units of wavelength .

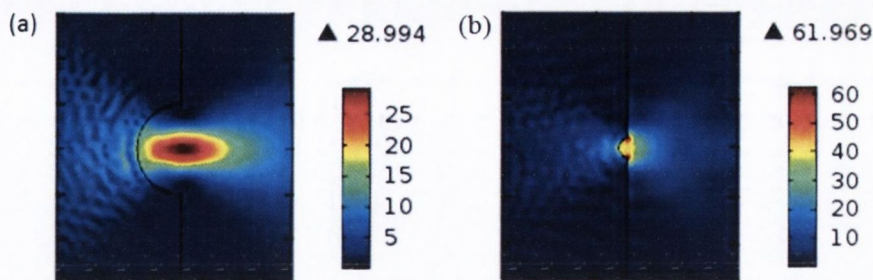
The intensity of the focal spot increases with increasing parameter  $a$ , and decreasing  $d$ . When  $d$  is larger than  $a$  (bottom left hand corner of plots in Fig. 8.17) some of the boundary which refracts the light is cut off resulting in a smaller range of angles being focused and effectively decreasing the NA. This results in a weaker focus as shown in Fig.8.18 (a). Figure 8.18 shows the near-field intensity distribution for a truncated\_cut oval with parameter  $a=4 \mu\text{m}$  and (a) parameter  $d=5 \mu\text{m}$ , (b)  $d=2 \mu\text{m}$ , (c)  $d=1 \mu\text{m}$ , (d)  $d=500 \text{ nm}$  and (e)  $d=200 \text{ nm}$ .



**Figure 8.18** 2D FEM intensity for light scattering from a truncated\_cut oval with parameter  $a$  fixed at  $4 \mu\text{m}$  and (a) parameter  $d=5 \mu\text{m}$ , (b)  $d=2 \mu\text{m}$ , (c)  $d=1 \mu\text{m}$ , (d)  $d=500 \text{ nm}$  and (e)  $d=200 \text{ nm}$ .

There are two interesting size regimes for the parameter  $d$ . When  $d \gg \lambda$  it has little effect on the intensity and spot size. However when  $d \leq \lambda$  we get a confinement effect where the intensity increases and the effective spot size decreases. The spot becomes non-Gaussian. An example of the focal spot in both regimes is shown in Fig. 8.19.

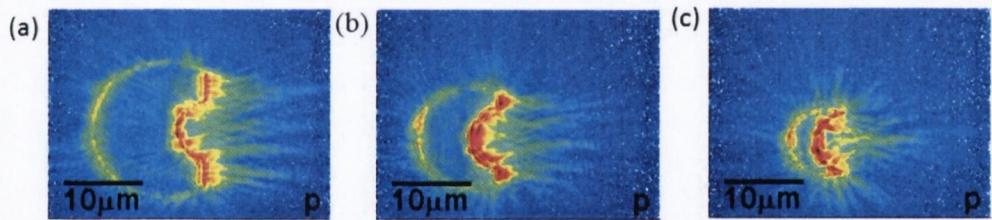




**Figure 8.19** (a) ( $d > \lambda$ ) Focal spot for s polarised input and  $a=10 \mu\text{m}$ ,  $d=1 \mu\text{m}$  (b) ( $d < \lambda$ ) Focal spot for s polarised input and  $a=10 \mu\text{m}$ ,  $d=0.2 \mu\text{m}$ .

The p-polarised case did not produce such a tight focus. This is likely due to the effect of the substrate. P-polarised light will experience a smaller effective index than s-polarised light resulting in the dimension of the Cartesian oval parameters not being optimum. The structures could easily be changed for the p polarised case using Eq 8.4 and 8.5 to define the major and minor semi-axes using the effective index. Some images of the p-polarised cases are shown in Fig.8

20.



**Figure 8.20** Light scattering p polarised case for truncated cut ovals with parameters (a)  $a=20 \mu\text{m}$ ,  $d=10 \mu\text{m}$  (b)  $a=15 \mu\text{m}$ ,  $d=10 \mu\text{m}$  and (c)  $a=10 \mu\text{m}$ ,  $d=10 \mu\text{m}$



## 8.5 Conclusions

The image plane method developed in Chapter 5-7 has been employed to map the near field intensity distribution of a complex shaped scatterer. Ellipses with a specific ratio of major and minor axes were created known as Cartesian ovals. These structures can focus light to a diffraction limited spot without any spherical aberration using a single element.

By truncating the Cartesian ovals at the second focus of the ellipse new structures referred to as truncated ovals were formed which could focus light to a sub wavelength spot on the boundary of the structure. Since the focusing effect is brought about by a single refraction at the first interface this truncation has little effect on the properties of the focal spot.

The size dependence of the structures was explored and a transition from geometrical optics to a diffraction dominated regime was observed in Fig. 8.11. The maximum intensity of the focal spot was shown to increase linearly with the parameter  $a$  over the range explored from  $a=4\text{ }\mu\text{m}$  to  $a=20\text{ }\mu\text{m}$  in Fig. 8.12.

The shape of the curve on the boundary of the Cartesian oval is such that an incident plane wave-front is converted to a cylindrical wave-front converging on the second focus of the ellipse F2. Therefore, removing a cylinder of material with centre at F2 will have a minimal effect on the focusing properties of the shape, as the wave-front will be parallel to the interface. Truncated ovals with a cylindrical cut out were created and the dependence of the focusing properties on the ratio of the size of the oval and the diameter of the cut out were examined.

These structures had three regimes. When the cut out diameter was smaller than the wavelength an intensity enhancement and small evanescent focal spot was created at the cut-out. When the cut-out was a larger than the wavelength, but smaller than the size of the oval parameter  $a$ , the cut-out had little effect on the focal size and intensity. Finally when the cut-out diameter  $d$  was larger than the parameter  $a$  the length of the first interface of the oval used to concentrate the light was reduced, effectively reducing the NA. This resulted in a lower intensity and less confined focal spot.

To conclude, focal spots with dimensions as small as those obtainable with microscope objectives can be achieved using these micron scale focusing elements. The structures behave best when  $a$  is larger than  $8\text{ }\mu\text{m}$  to avoid diffraction around the structures. Their relatively large size and quasi 2D nature make them easy to manufacture using standard processing techniques from the microelectronics industry such as UV lithography explained in Chapter 3. The shapes can be optimised for many different materials, and positioned accurately on the substrate. The truncated ovals allow access to the focal region from outside of the oval. The light is focused onto the boundary and a small spot with a transverse  $1/e^2$  half width  $w=0.71\lambda$  is formed. Two

dimensional FEM simulations suggest that in the near-field region the intensity can be enhanced by up to 120 times the background (Fig. 8.12 (a)), and can be increased by making the structures larger. The spot decays exponentially from the surface creating a very small focal volume.

These structures could be used to create small focal volumes for detection of nanoparticles or fluorescence enhancement or FCS. They are particularly useful for applications such as FCS where it is important to have a small focal volume, but the intensity enhancement is not crucial. When a circle centred on the second focus is removed from the truncated oval the focal spot is completely outside the structure and is similar to a perfect focused Gaussian beam. The NA of the focal spot can be controlled by changing the ratio of the cut-out diameter to the size of the oval. This may be useful for coupling to rectangular and rib waveguides.



## 8.6 References

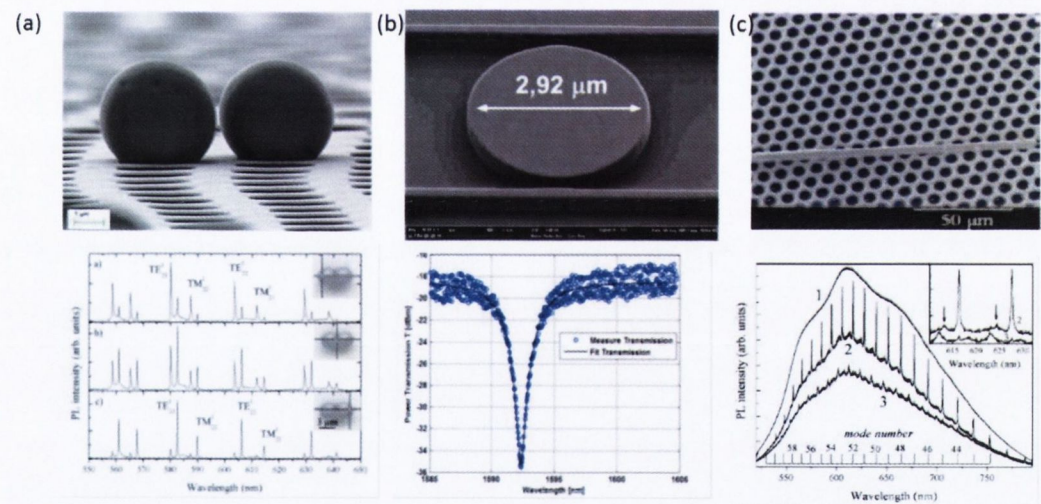
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# Chapter 9      Whispering gallery modes in microdisk resonators

## 9.1 Introduction

High quality factor MDRs exist in microdisks and are commonly referred to as whispering gallery modes (WGMs) due to an analogy with the acoustic effect explained by Lord Rayleigh in the dome of Saint Pauls Cathedral [1].WGMs can exist in any system with spherical or cylindrical symmetry, some examples of which are displayed in Fig. 9.1.These modes are intricately connected to the light scattering distributions as we will see later on in this chapter. In the frequency domain these modes have sharp Lorentzian profiles, with a frequency which depends on the particular mode, and a linewidth which depends on the losses in that mode.

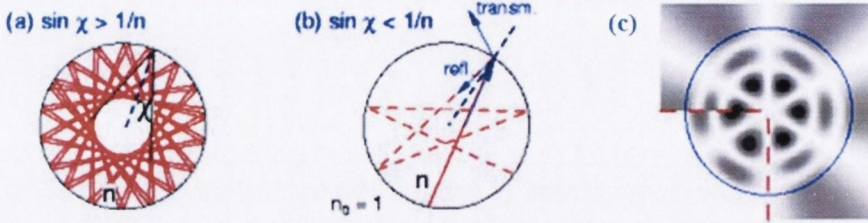


**Figure 9.1** Modal structure of resonators with spherical and cylindrical symmetry. **(a)** *Upper*:Two 2.5 μm diameter Melamine Formaldehyde (MF) spheres coated in Rh6G dye. *Lower*: Coupling is observed between the spheres [2] **(b)** *Upper*: microdisk in silicon on Insulator platform, coupled to by an input and output bus strip waveguides [3]. *Lower*: mode measured as drop in transmission **(c)** *Upper*: microtube made from aluminosilicate, on a periodic substrate. *Lower*: modes measured through PL [4]

From a ray optics picture, these WGMs can be seen as light circulating close to the boundary of the microdisk reflecting multiple times from the discontinuity of refractive index at the interface. From a physical optics point of view these rays interfere to form standing wave



patterns in the intensity distribution. Multiple modes are possible which have discrete frequencies, widths and spatial distributions. An example of one particular mode in both the ray optics and physical optics description is shown in Fig. 9.2.



**Figure 9.2** Comparison of ray optics and physical optics interpretation of WGMs. (a) Mode formed by total internal reflection from the interface. (b) Angle less than critical angle and no mode formed (very lossy). (c) Intensity distribution calculated from physical optics. Adapted from [5].

Before beginning a discussion on WGMs in microdisks, it would be prudent to introduce a few terms which will be referred to regularly

**Size Parameter ( $x_s$ ):** 
$$x_s = \frac{2\pi R}{\lambda} = \frac{2\pi R}{c} \nu$$

The distribution of the fields and the position of the resonances do not depend on the diameter of the sphere and the wavelength of the incident light independently, but rather on the ratio of the diameter ( $2R$ ) to the wavelength ( $\lambda_0$ ). The size parameter  $x_s$  is thus the natural parameter to use in these calculations and is defined as the ratio of the sphere circumference to the free space wavelength.

**Quality factor (Q):** 
$$Q = \frac{\lambda_{res}}{\Delta\lambda_{res}} = \frac{\nu_{res}}{\Delta\nu_{res}} = \omega\tau$$

The modes of these microresonators are lossy, in that a photon has a finite lifetime ( $\tau$ ) in the cavity mode and will eventually escape the resonator through a number of loss mechanisms. Theoretically this results in complex eigenfrequencies for the modes. The quality factor (Q) is a measure of how lossy a mode is and is related to the width of the mode in frequency. The higher the Q, the longer a photon is trapped in the resonator.

**Free spectral range (FSR):**  $FSR = \nu_{m+1} - \nu_m$

The free spectral range (FSR) of a cavity is the distance between adjacent modes. This is an important quantity for lasing and communications, where the goal is to have only one mode in the gain bandwidth and channel, respectively. The FSR is inversely proportional to the diameter of the resonator, so it can be increased simply by decreasing the resonator size. This however results in a decrease in  $Q$  as the radiative loss is increased.

**Mode classification:**  $TE_{n,m,h}$ ,  $TM_{n,m,h}$

Modes in a disk resonator are labelled according to their polarisation (TE or TM), radial mode number  $n$ , azimuthal mode number  $m$  and axial mode number  $h$ . The mode numbers take values of non negative integers. For a mode with integers  $n, m$ , and  $h$ , the intensity distribution will have  $n$  anti-nodes in the radial direction,  $2m$  anti-nodes in the angular direction and  $h$  anti-nodes in the  $z$  direction (along the axis of the disk.). For example the mode displayed in Fig. 9.2 is the  $TE_{2,3,1}$  mode. The origin of this nomenclature will become clear after the physical optics derivation.

**Mode Volume(V):** 
$$V = \frac{\iiint \epsilon(\vec{r}) |E(\vec{r})|^2}{\epsilon(\vec{r}_M) |E(\vec{r}_M)|^2}$$

The mode volume (V) is a measure of the spatial extent of the cavity mode and is an important quantity for the interaction of the mode with an emitter. The vector  $\vec{r}_M$  is the position of the electric field maximum.

An interesting phenomena is observed when a light emitter is placed in or close to a WGM cavity. The rate at which an emitter spontaneously radiates in vacuum is given by Fermi's golden rule [6]. This calculates the transition probability from one energy eigenstate of a quantum system to a continuum of eigenstates. When an emitter is placed inside or close to a cavity the density of photonic states is not a continuum and the expression for Fermi's golden rule changes. The emission is modified, enhanced at the resonance wavelengths of the cavity and suppressed off resonance. This is known as Purcell's effect [6]. On resonance the lifetime of the emitter is shortened resulting in more light emission at that wavelength (more photons per second.) The enhancement factor was calculated by Purcell in 1946 [7] and the dependence on the width and mode volume of the resonance through the following equation



$$F_p = \frac{3}{4\pi^2} \left( \frac{\lambda_c}{n} \right)^3 \left( \frac{Q}{V} \right) \quad (9.1)$$

where  $\lambda_c / n$  is the wavelength in the material, and  $Q$  and  $V$  are the quality factor and mode volume respectively. (Eq. 9.1 is for an emitter ideally placed and orientated in the maximum of the electric field in a cavity). Placing a single photon emitter in a cavity has multiple advantages over free space. Matching the emission spectrally with a cavity mode allows use of the Purcell effect to shorten the decay time below the de-coherence time of the emitter which is crucial for obtaining indistinguishable photons [8]. The direction and polarisation of emission is modified by the cavity and can be used to increase the excitation and collection efficiency [9].

From Eq.9.1 we see that in order to maximise the Purcell effect it is important to choose a cavity with high  $Q$  and low  $V$ . The ultimate  $Q$  factor achievable in a cavity system is limited by a number of loss mechanisms. The total quality factor can be written as [10]:

$$Q^{-1} = Q_{rad}^{-1} + Q_{s.s.}^{-1} + Q_{cont}^{-1} + Q_{mat}^{-1} \quad (9.2)$$

where  $Q_{rad}^{-1}$  denotes radiative losses (curvature losses) due to coupling of light to free space;  $Q_{s.s.}^{-1}$ , scattering losses due to surface roughness;  $Q_{cont}^{-1}$ , losses introduced by surface contaminants (scattering and absorption); and  $Q_{mat}^{-1}$ , intrinsic material losses. Experimentally the highest  $Q$  can be obtained in silica microsphere cavities and is around  $9 \times 10^9$  at a wavelength of 850 nm [11]. These are formed by melting silica with a  $\text{CO}_2$  laser. On cooling surface tension creates microspheres that can have atomically smooth surfaces. Thus the main limitation on  $Q$  factor is material absorption. The fundamental (equatorial) modes have the lowest mode volume and can be singled out from the many degenerate modes by phase matching using for example tapered optical fibres [12-14]. Microdisk resonators fabricated by lithography techniques can be made from higher refractive index materials allowing smaller mode volumes. The  $Q$  factor of these devices is limited by the surface roughness induced from the etching process. Optimisation of etching can obtain  $Q$  factors as high as  $3 \times 10^6$  for Si microdisks on  $\text{SiO}_2$  substrate [15]. The high  $Q$  of surface tension formed microspheres and the low mode volume of microdisk resonators can be combined in the form of microtoroid resonators. These are disks formed by lithographic processes which are then heated above melting temperature using a  $\text{CO}_2$  laser and then allowed to reform using surface tension leading to  $Q$  factors as high as  $1 \times 10^8$  [16]. The high  $Q$  factor is not the only consideration as the Purcell enhancement depends on



simultaneously maximising  $Q$  and minimising  $V$ . This in fact occurs at a diameter of  $7.26 \mu\text{m}$  for silica microspheres [17].

In this project WGMs in microdisks with diameters between  $1 \mu\text{m}$  and  $10 \mu\text{m}$  were examined.  $Q$  factors in the range of  $10^3$  were observed, limited by the resolution of the spectrometer. The mode spectrum is analysed and compared with an effective index analytical model. The cavities are fully characterised and then finally the transition from a microdisk to a microring is examined.

Microdisk cavities have numerous applications such as filters in communication [15], precise material characterisation [18, 19], optical traps, and single photon control. They were first investigated as a cavity for lasing by McCall et al in 1991 [20]. In particular microdisks in Silicon Nitride have been of recent interest due to its low optical absorption in the visible range and easy processing [21-23].

## 9.2 Whispering gallery modes: Analytical work

The EM eigenmodes of a finite height disk do not have an exact analytical solution. The modes can however be modelled approximately by considering the modes of an infinite cylinder with an effective index of refraction which takes into account the finite height. This approach known as the effective index approximation (EI) is used to identify modes in this project. Other methods for calculating mode positions, volumes and  $Q$  factors include S matrix calculations [24] and numerical techniques such as FDTD and FEM [15, 25, 26]. Although numerical techniques provide more accurate results for specific parameters little insight is gained into the mechanism behind the mode formation. In this section we outline the analytical methods used and compare results with numerical methods in the literature. Conformal mapping [27] and a comparison with the Schrödinger equation can also be used to find characteristic equations for the eigenmodes analytically [6].

### 9.2.1 Modes of an infinite cylinder

In order to find the positions and widths of the modes of a microdisk resonator we first consider the eigenmodes of an infinite cylinder. A separation of variables approach is used to solve the wave equation in the cylindrical co-ordinates  $\rho, \phi, z$ . The vector wave equation can be reduced to solving the scalar Helmholtz equation by proper choice of generating function  $\psi$  [28, 29].

$$\nabla^2 \psi - k^2 n_{mat}^2 \psi = 0 \quad (9.3)$$



where  $k = n\omega/c$ , is the free space vector and  $n_{\text{mat}}$  is the refractive index of the material. In cylindrical co-ordinated this read as

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \psi}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 \psi}{\partial \phi^2} + \frac{\partial^2 \psi}{\partial z^2} + k^2 n_{\text{mat}}^2 \psi = 0 \quad (9.4)$$

We seek solutions to this equation which are separable in  $r$ , and  $z$  of the form

$$\psi(\rho, \phi, z) = R(\rho)\Phi(\phi)Z(z) \quad (9.5)$$

Subbing this into Eq.9.4 separates into three ODEs

$$Z'' + (k^2 n_{\text{mat}}^2 - a^2)Z = 0 \quad (9.6)$$

$$\Phi'' - m^2 \Phi = 0 \quad (9.7)$$

$$\rho^2 R'' + \rho R' + (a^2 \rho^2 - n^2)R = 0 \quad (9.8)$$

Equations 9.6 and 9.7 are harmonic oscillator equations. Equation 9.8 is known as Bessel's equation and is common in problems involving cylindrical symmetry. The solutions to these equations are

$$Z_{k,a}^{\pm} = Z_0 e^{\pm i \sqrt{k^2 - a^2} z} \quad (9.9)$$

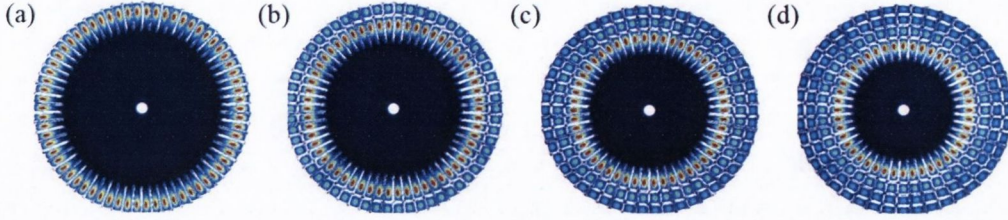
$$\Phi_m^{\pm} = \Phi_0 e^{\pm i m \phi} \quad (9.10)$$

$$R_{a,n} = R_0 J_n(a\rho) \quad (9.11)$$

The constants  $a$ ,  $m$  and  $n$  have been introduced. The constant  $a$  is real valued and  $0 \leq a \leq k$  and represents the propagation of the light along the cylinder axis. The linearly independent solutions of Bessel's equation are known as Bessel functions and are given the notation  $J_n$  and  $Y_n$ . Inside the disk we use  $J_n$  as it is finite at the origin. The constants  $m$  and  $n$  take integer values and are used to label the modes. The full solution for  $\psi_{\text{in}}$  is then:

$$\psi_{\text{in}}^{\pm}(\rho, \phi, z) = A_{nm} J_n(n_{\text{cyl}} k \rho) e^{\pm i m \theta} e^{i \sqrt{k^2 - a^2} z} \quad (9.12)$$

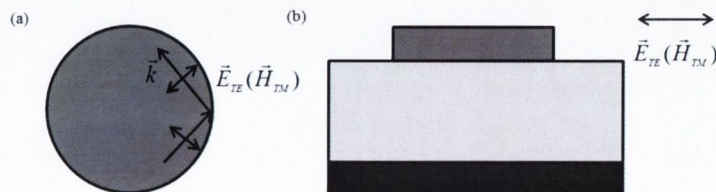
We can now use Eq.9.12 to plot the intensity distribution in the  $z=0$  plane which is proportional to  $|\Psi_{\text{in}} \Psi_{\text{in}}^*|$  for modes with different radial and azimuthal orders  $n$  and  $m$ . (Fig.9.3)



**Figure 9.3** Intensity plot of first four radial order modes of a 4  $\mu\text{m}$  diameter microdisk. The azimuthal order was chosen so that all the modes had a wavelength around 640 nm. **(a)**  $\text{TE}_{1,33}$  @  $\lambda = 646$  nm, **(b)**  $\text{TE}_{2,29}$  @  $\lambda = 635$  nm, **(c)**  $\text{TE}_{3,25}$  @  $\lambda = 638$  nm, and **(d)**  $\text{TE}_{4,22}$  @  $\lambda = 632$  nm.

The modes are labelled according to their polarisation and the radial and azimuthal orders. The intensity distribution has  $n$  anti-nodes in the radial direction and  $2m$  anti-nodes in the angular direction. Modes with higher radial order penetrate more deeply into the disk. These higher order radial modes correspond to modes with a larger angle of incidence when the light hits the boundary in the ray optics picture [30]. Each mode is doubly degenerate with modes which travel in different directions around the circumference of the disk. Outside the cylinder the mode has an exponential like decay. We choose a Hankel function  $H_m^{(1)} = J_m + iY_m$  for the radial dependence outside of the disk to satisfy the condition that the field should be out-going at infinity.

To find the frequencies and widths of these modes we apply the EM boundary conditions at the boundary of the cylinder and match the continuous components of electric field. At this point the vector nature of the electric field becomes important and two separate classes of mode can be identified which are in general labelled transverse electric (TE) and transverse magnetic (TM). The nomenclature here depends on convention and the definitions used in this project are displayed in Fig.9.4. We choose to use the convention of a slab waveguide. For the TE case the electric field is non-zero only in the plane of the cylinder, whereas in the TM case the electric field is out the plane of the cylinder and the magnetic field has only a component in plane.



**Figure 9.4** **(a)** Polarisations of TE and TM modes from above. **(b)** Polarisations from side.



Application of the EM boundary conditions for each case leads to two different characteristic equations given by:

$$J_m(n_{cyl}kR)H_m^{(1)}(kR) = PJ'_m(n_{cyl}kR)H_m^{(1)}(kR) \quad (9.13)$$

with  $P = n_{eff}$  for TM polarisation and  $P = 1/n_{eff}$  for TE polarisation and  $R$  is the radius of the disk. Physically this means TE and TM modes with the same mode numbers are separated in frequency. Re-arranging Eq. 1.13 gives:

$$f(kR) = J_m(m_c kR)H_m^{(1)}(kR) - PJ'_m(m_c kR)H_m^{(1)}(kR) = 0 \quad (9.14)$$

The zeros of this function  $f(kR)$  give complex values of  $k$ , which represent the lossy nature of WGMs. The position of the resonances can be found from the real component of  $k$  and their width can be found from the imaginary component. If we write  $k = k_1 + ik_2$  then the mode position and  $Q$  are given by:

$$k_{res} = \sqrt{k_1^2 + k_2^2} \quad (9.15)$$

$$Q = -\frac{k_1}{2k_2} \quad (9.16)$$

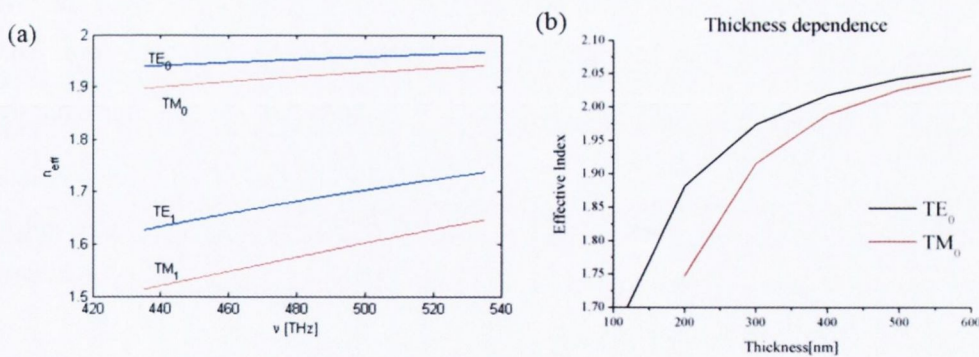
From Eq.1.14 we see that the properties of the modes depend only on the size parameter  $x_s$  and the refractive index contrast  $m_c$ . This concludes a method of characterising modes in an infinite cylinder, but a finite height micro disk is far from infinite. Luckily the insight gained from this model still applies. The finite height of the disk introduces modes in the  $z$  direction which in turn have an effect on the position of the modes in the plane. This can be taken into account by defining an effective index which takes into account the longer optical path length of the rays due to the out of plane angle. This method is the subject of the next section.

### 9.2.2 Modes of a finite cylinder (microdisk)

A finite height microdisk can be treated as an infinite cylinder in the  $z$  direction with an effective index ( $n_{\text{eff}}$ ) given by the ratio of the propagation constant of a mode of a planar waveguide with the same dimensions to the free space wave vector. This effective index describes the 2D propagation of light inside a slab waveguide. It will always be between the index of the core material and the highest index cladding layer for the mode to propagate:

$$1.45 \leq n_{\text{eff}} = \beta / k_0 \leq 2.1 \quad (9.17)$$

Two types of slab waveguide modes exist, transverse electric (TE) and transverse magnetic (TM) as depicted in Fig. 9.4. The modes are labelled according to the number of nodes in the  $z$  - direction and the polarisation. The  $TE_0$  and  $TM_0$  are the fundamental modes and have a maximum in the centre of the core. The  $TE_1$  and  $TM_1$  modes have one node in the  $z$  direction and so on. The effective index calculation is a standard procedure in slab waveguides and is outlined in appendix B. The effective index  $n_{\text{eff}}$  depends on the type of slab waveguide mode (TE or TM) and is dependent on frequency, core height and indices of the core and cladding (Fig.9.5). The refractive index dispersion measured from ellipsometry is quite small in the range we are observing and so the refractive index of the  $\text{Si}_3\text{N}_4$  is held fixed at 2.04 (values range from 2.05-2.03) . The effective index dispersion however is significant and must be included to find the mode positions.



**Figure 9.5** (a) The effective index for The  $TE_0$ ,  $TM_0$ ,  $TE_1$  and  $TM_1$  modes as a function of frequency. (b) Effective index as a function of disk height.

From Fig. 9.5(a) we see that the effective index of the TM mode is always smaller than the TE mode. This means that the TM modes are less confined and results in slightly higher loss (wider) TM modes. The effective index drops off rapidly for higher order modes along the  $z$  axis, thus these modes are of significantly lower quality than the fundamental ones. The



thickness dependence shows that the effective index can be easily controlled by adjusting the thickness of the disks which can be done in the etching step.

### 9.2.3 Numerical implementation of mode finding

In reality it is not trivial to find the complex roots of Eq.9.14 numerically. It is a complex valued function with complex values arguments, and numerical techniques must be used to find the poles in the complex plane. To do this we need to know an approximate place to start looking in the complex plane. A first approximation is to assume that the resonator is closed. i.e. that the modes do not radiate. This imposes the boundary condition

$$\psi_{in}(R, \Theta) = 0 \quad (9.18)$$

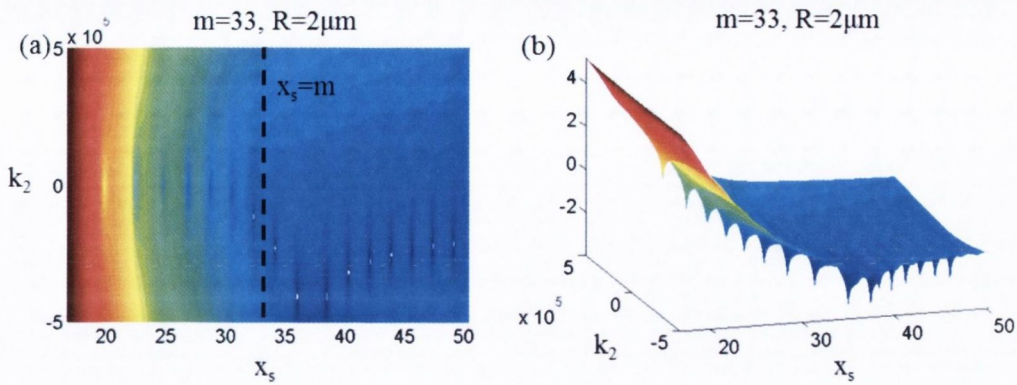
which is true if

$$J_m(n_{eff}kR) = 0 \quad (9.19)$$

The modes in this closed case are not lossy so  $k$  is real. If we let  $X_{nm}$  be the  $n^{\text{th}}$  root of the  $m^{\text{th}}$  order Bessel function, the resonance frequency of the mode is given by:

$$\nu_{nm} \approx \left( \frac{c}{2\pi R n_{eff}} \right) X_{nm} \quad (9.20)$$

Using this method the position of the higher radial order modes can also be approximated. In general the modes of a closed resonator will be lower frequency than that of the open one. We are looking for points where both the real and complex parts of Eq. 9.14 are zero. This corresponds to points where  $|f(kR)|^2$  is zero. Figure 9.6 shows the surface plot over the complex plane of  $|f(kR)|^2$  for the  $m=33$  mode of a microdisk of diameter  $4 \mu\text{m}$ . A log scale is used for the colour axis in (a), (z axis in (b)) to make the poles sharper and easier to see graphically. The x axis is in units of size parameter and the y axis is in units of  $k_2$ .

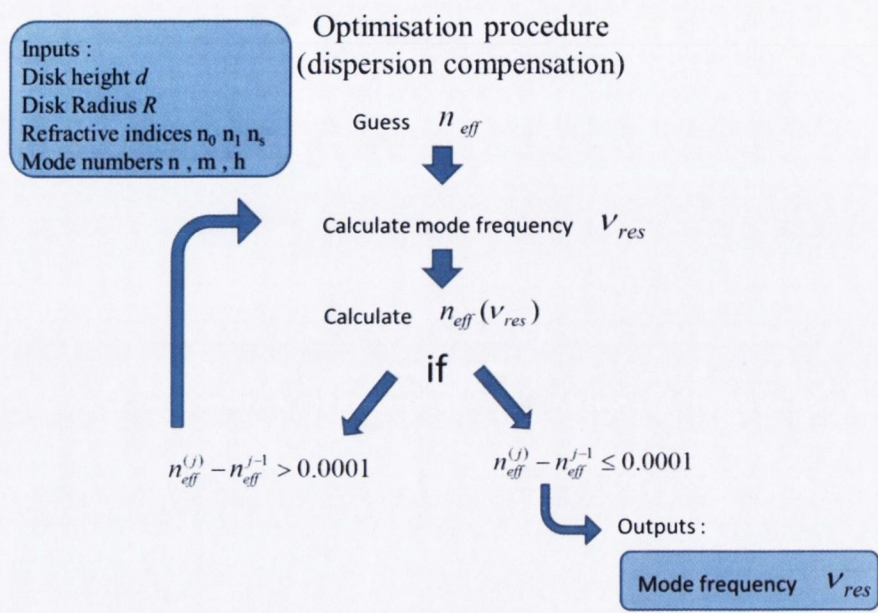


**Figure 9.6 (a)** Surface plot of  $\log_{10}(|f(x_s)|^2)$  with  $f$  taken from Eq.9.14. The x axis is in units of size parameter  $x_s$  and the y axis is the imaginary component of the k vector  $k_2$  **(b)** Same plot in 3D to show the sharpness of the poles.

The lowest frequency (lowest size parameter) pole corresponds to the first radial order mode  $n=1$ . The next poles are the higher order radial modes with azimuthal order  $m=33$ . The quality of the modes is inversely proportional the absolute value of  $k_2$ , so the further from zero they are the lower the Q. The mode we are interested in are the quasi-bound whispering gallery modes with  $x_s < m$ . The Q of the modes decreases rapidly with increasing radial order. Fig.9.6(b) shows a 3D surface plot, to emphasis the sharpness of the poles. The poles would go to zero if the resolution of the plot grid was increased. To find these poles numerically we use a multidimensional unconstrained non-linear approximation technique. The technique is based on the Nelder-Mead method [31]. This method finds the real and imaginary components of  $k$  at the poles ( $[k_1, k_2]$ ). The starting value of  $k_1$  is given by the closed resonator approximation and the starting value of  $k_2$  is set to zero. This succeeds in finding the poles most times, however sometimes it can converge to the wrong pole and common sense must be used in interpretation of the results. A better approximation of a starting point can be made from observing the pole graphically and then running the technique again.

Finally, since the mode frequency depends on the refractive index which is now  $n_{\text{eff}}$ , and  $n_{\text{eff}}$  depends on the mode wavelength an optimisation procedure is necessary to converge on the position of the mode. This is achieved by guessing an initial value of  $n_{\text{eff}}$  and calculating the position of the mode and then using this mode wavelength to calculate the effective index. This is iterated  $n$  times until the difference between the  $n^{\text{th}}$  and  $n^{\text{th}}+1$  value of effective index is below a designated tolerance. The optimisation procedure is outlined in Fig.9.7 below.





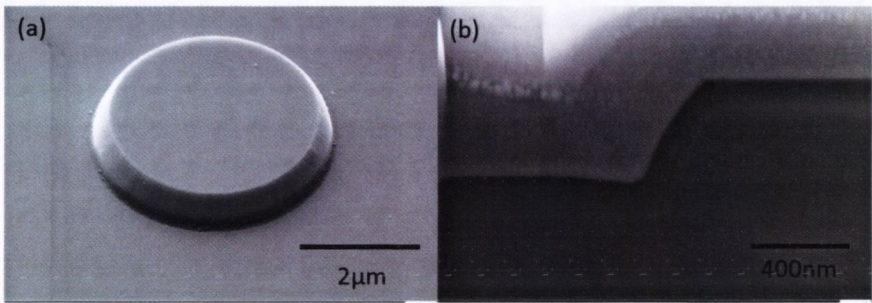
**Figure 9.7** Optimisation procedure for determination of mode frequency.

This leads to the correct solution of the mode positions taking into account effective index dispersion. The EI method is good at predicting the mode positions, however the quality factor is underestimated, as we are actually solving for the case of an infinite cylinder with index  $n_{eff}$ . In the case of a disk the confinement in the  $z$  direction actually increases the  $Q$  factor of the modes. S matrix techniques and FDTD techniques provided better estimation of  $Q$  factor [24]. These methods however are more time consuming and computationally expensive. Some debate exists over the suitability of FDTD for these sort of resonant simulations [32] We are however more interested in finding the positions of the peaks with a quick and reliable algorithm and so opt for the EI method.

9.3 Samples and experimental measurement

Microdisks with diameters ranging from 1-10  $\mu\text{m}$  were fabricated using the EBL technique and the  $\text{SF}_6$   $\text{CHF}_3$  ICP etching process outlined in Chapter-4 (Fig. 9.8 (a)). Cross sectioning using a focused ion beam (FIB) milling method show a significant sidewall angle of  $56^\circ$ . This means that the diameter of the disk changes by 5% from the top of the disk to the bottom. This has interesting effects on the modal structure of the cavity. The sidewall is also convex (Fig. 9.8 (b)). Later samples were fabricated using the Ar  $\text{CHF}_3$  etching recipe which resulted in much straighter sidewalls but larger surface roughness.

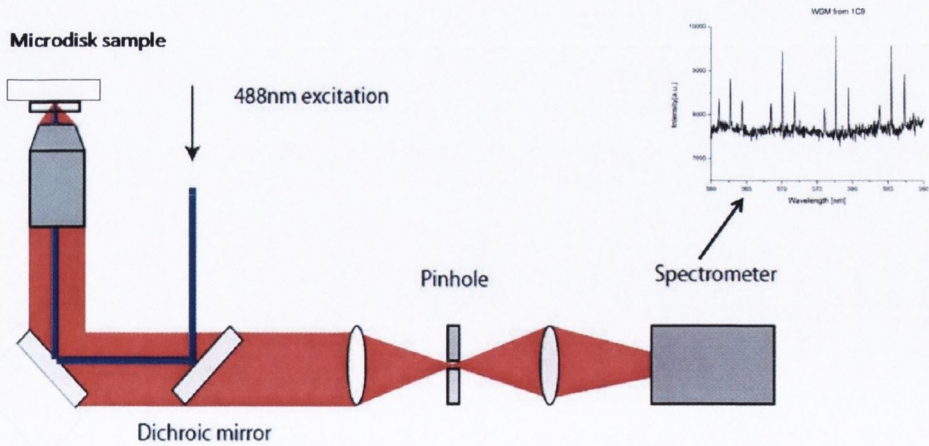




**Figure 9.8** (a) Microdisk with design diameter of 4  $\mu\text{m}$ . (b) The  $\text{SF}_6/\text{CHF}_3$  etching process results in a significant sidewall angle of  $124^\circ$ , (i.e.  $56^\circ$  from substrate). The convex shape of the wall may help in retaining high quality modes.

Due to the LPCVD fabrication of the material system defect states exist in the  $\text{Si}_3\text{N}_4$  layer. These defects produce a broad weak PL emission in the wavelength range 500 nm-900 nm [33]. The PL is then modified by the cavity. The modes of the microdisks are then measured using a simple confocal Photoluminescence (PL) technique depicted in Fig. 9.9.

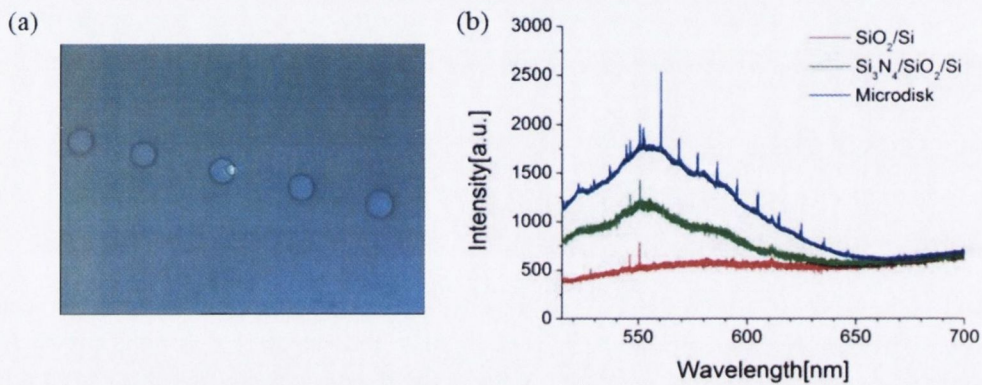
Light from a 488 nm  $\text{Ar}^+$  laser is focused onto the sample using a  $100\times$  0.7 NA objective. (The same long working distance objective as used in the scattering experiments.) The PL from the sample is collected through the same objective. Scattered light from the laser is then filtered using the dichroic mirror. The PL is focused through a pinhole to ensure that the light is collected only from the  $\text{Si}_3\text{N}_4$  layer. The spectrum of the light is examined using a 2400l/mm grating. The PL is observed in the range 500 nm-800 nm. The resolution of the spectrometer in this range limits the observable Q factor to less than  $5\times 10^3$ .



**Figure 9.9** Experimental setup for WGM observation. Light at 488 nm is focused onto the sample using a 100x objective. The scattered light is collected and focused through a pinhole onto the spectrometer. The pinhole ensures that the signal collected comes from the  $\text{Si}_3\text{N}_4$  layer. The inset shows a typical spectrum .



An optical image of a typical measurement is shown in Fig. 9.10 (a). Light from a linearly polarised  $\text{Ar}^+$  laser at 488 nm is focused onto the edge of the microdisk. (Fig. 9.10 (a)). This produces the structured PL shown as the blue line in Fig. 9.10 (b). For comparison the PL spectrum was taken from sample with the no disk (green) and a sample with the  $\text{Si}_3\text{N}_4$  layer removed by etching (red). This confirms that the PL is coming from the  $\text{Si}_3\text{N}_4$  layer and the modes are due to the microdisk cavity. The sharp lines evident in all spectra are Raman scattering peaks are observed from the Si and  $\text{SiO}_2$  layers.



**Figure 9.10** (a) Excitation position on the rim of a microdisk of 10  $\mu\text{m}$  in diameter. (b) PL from  $\text{SiO}_2/\text{Si}$  (red) where the  $\text{Si}_3\text{N}_4$  layer has been stripped. PL from the  $\text{Si}_3\text{N}_4$  layer without any structure (green). PL from a  $\text{Si}_3\text{N}_4$  microdisk (blue).

### 9.3.1 Mechanism of light collection

Light from the 488 nm laser is absorbed by the defect states in the  $\text{Si}_3\text{N}_4$  layer and re-emitted by spontaneous emission in all directions. The wavelength dependence of the SE is modified by the cavity modes. Only a fraction  $\beta$  of the spontaneous emission is trapped in a WGM [34]. This fraction is important for calculating the threshold of lasing in WGM based microdisk lasers [35]. Light from the WGM can escape to free space if the curvature is large enough. The divergence angle of this light however is such that none of it will be directly collected in the MO used for the experiment. The signal observed in the spectrometer is thus the modified PL which was not coupled into the WGM, and light scattered from the surface roughness of the edge of the disk into the collection cone of the microscope objective.

## 9.4 Results

### 9.4.1 Dependence on disk diameter

Initially insight can be gained into the expected dependence of the modes on disk diameter from simple approximations. Since the first order modes circulate quite close to the circumference of the disk with grazing incidence, the simplest approximation is to say that a resonance occurs when the circumference is an integral multiple of the wavelength of light in the planar waveguide mode, which in frequency domain leads to

$$\nu_{1,m} \approx m \left( \frac{c}{2\pi R n_{\text{eff}}} \right) \quad (9.21)$$

This gives an estimate of the positions of the first order modes and allows an estimate for the FSR:

$$FSR = \nu_{m+1} - \nu_m \approx \left( \frac{c}{2\pi R n_{\text{eff}}} \right) \quad (9.22)$$

So this important result shows that if we neglect effective index dispersion the FSR of first radial order modes should be constant .i.e. the modes are equally spaced in frequency. The FSR is shown to be inversely proportional to the radius of the disk  $R$  and the effective index  $n_{\text{eff}}$ . In reality the modes are composed of light traveling in polygonal paths, so the optical path length (OPL) can never be the length of the circumference, and will be shorter. This means that the mode frequency predicted with this method will be lower than the true frequency. Only the first radial order modes can be approximated with this method, as higher order radial modes do not circulate close to the surface, and therefore would not have a path length close to the circumference length.

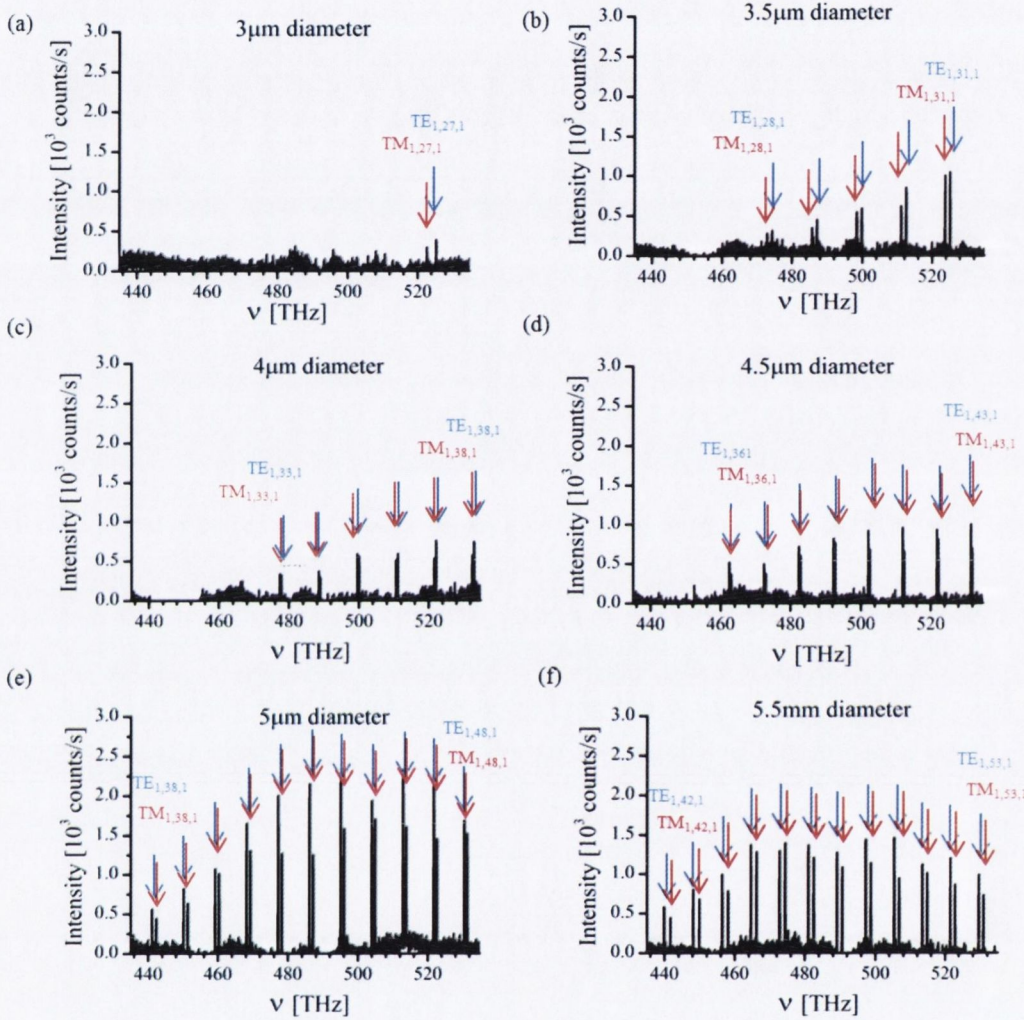
The mode positions are at a fixed size parameter for a specific mode. Thus the dependence of the mode position is linear with wavelength or in the frequency domain:

$$\nu = \left( \frac{c}{2\pi R} \right) x_s \quad (9.23)$$

where  $x_s$  is the size parameter. By making microdisks separated in diameter by 500 nm we are able to probe the entire space of  $x_s$  and observe all the azimuthal resonances. Microdisks with diameters ranging from 1.0-9.5  $\mu\text{m}$  in 550 nm were fabricated with four repeats of each as described in Chapter-4. The spectra were collected and the backgrounds subtracted. The smaller



disks have a less dense mode spectrum which is easier to interpret so we start with them. The results from microdisks with diameters ranging from 3-5.5 $\mu\text{m}$  are shown in Fig. 9.11.

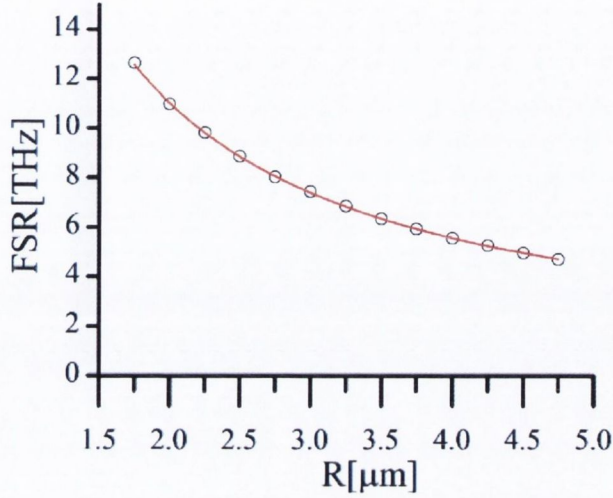


**Figure 9.11** Modes from microdisks in frequency range of 435-535 THz for disk diameter of (a) 3  $\mu\text{m}$ , (b) 3.5  $\mu\text{m}$ , (c) 4  $\mu\text{m}$ , (d) 4.5  $\mu\text{m}$ , (e) 5  $\mu\text{m}$ , and (f) 5.5  $\mu\text{m}$ .

The modes are identified as the TE and TM first order radial modes. Their TE and TM character is later confirmed from the polarisation dependence of the collected light. The TM modes are of lower Q which results in the experimental peak heights being smaller. Since the effective index for these modes is slightly different the TM modes have a slightly larger FSR. This effect results in the crossing of modes observed from the 3.5  $\mu\text{m}$  diameter disk where the TM modes are on the left to the 5.5  $\mu\text{m}$  diameter disk where the TM modes are on the right.

The modes can be labelled using the EI method. Varying diameter allows us to observe modes with azimuthal order from  $m=27$  to  $m=97$ . We observe two sets of equally spaced modes

with FSR of  $\sim 5$ -12 THz (depending on radius.) The FSR as a function of radius taken at 520 THz for the  $TE_{1,m,1}$  modes is given in Fig.9.12



**Figure 9.12** FSR as a function of radius. Fitting gives a  $1/R$  dependence with an effective index of  $1.85 \pm 0.09$

Fitting the FSR shows a  $1/R$  dependence. Comparison with Eq. 9.22 gives a value of effective index of  $1.85 \pm 0.09$ . The actual effective index at 525THz is 1.9638. The discrepancy is due to the approximation in the derivation of Eq.9.22.

The main observation is that there are no modes observed for disks with diameter smaller than 3  $\mu\text{m}$ . This means that there is a lower limit on mode quality factor needed for modes to be observable with this experimental method. As the radius decreases and we go to modes of lower  $m$  we would expect the  $Q$  factor to decrease exponentially as the radiative loss becomes dominant. From tunnelling arguments the radiative  $Q$  factor ( $Q_{\text{rad}}$ ) is given by[20]:

$$Q_{\text{rad}} \approx (1/b) \exp(2mJ) \quad (9.24)$$

where

$$J = \tanh[\sqrt{1 - 1/n_{\text{eff}}^2}] - \sqrt{1 - 1/n_{\text{eff}}^2} \quad (9.25)$$

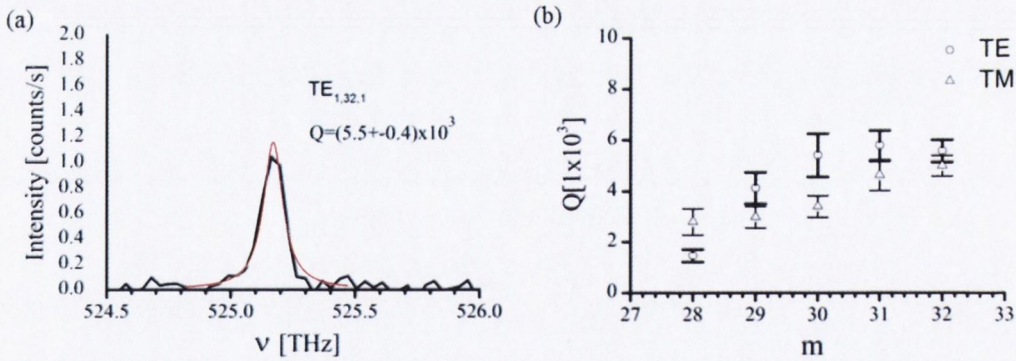
and  $b=6.5$  is calculated by matching with experiments on WGMs in spheres. The modes radiate into a relatively narrow range of angles compared with the diffraction angle associated with the disk thickness. The half angle of this divergence can be estimated from tunnelling arguments to be  $\sim 2/\sqrt{m}$  [rad][20]. This angle is too small to be collected by the microscope objective in the experimental setup. This results in the sharp cut off observed in the experiments. Modes with azimuthal order less than  $m=28$  cannot be observed. The  $Q$  of these modes is limited by material absorption. This material absorption increases with decreasing frequency and is responsible for



the lower peak height of the lower frequency modes in Fig. 9.11. The Q factor due to material losses  $Q_{\text{mat}}$  is approximately given by:

$$Q_{\text{mat}} = \frac{k_0 n_{\text{eff}}}{\alpha} = \frac{n_{\text{eff}}}{k_2} \quad (9.26)$$

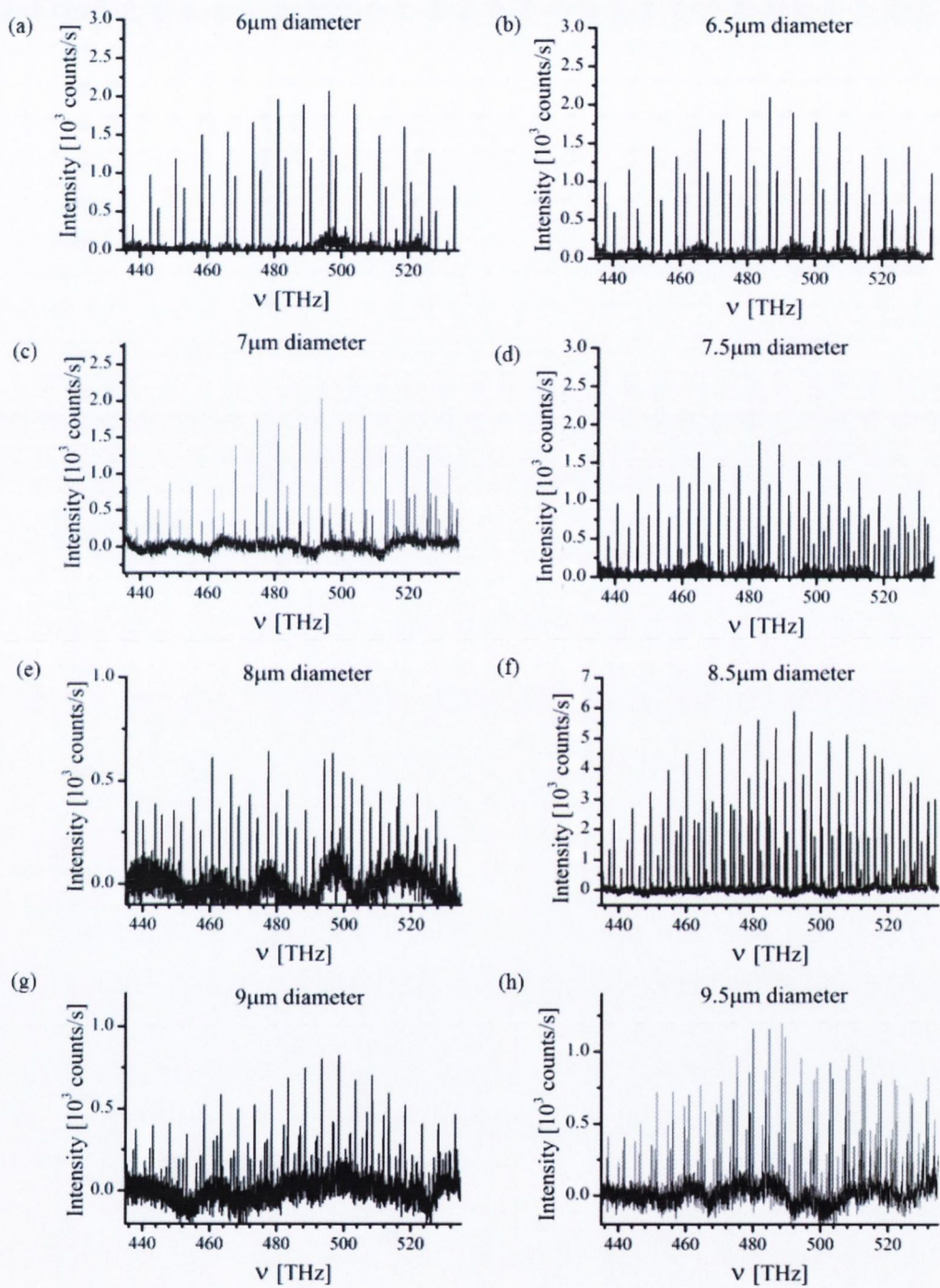
Taking the value of  $k_2$  is measured to be  $\sim 6 \times 10^4$  from ellipsometry, which leads to a material limited Q factor of order  $10^3$ , which is what we observe. The experimentally measurable Q factor is limited from above by the spectral resolution of the grating used in the experiment. The grating used had 2400 l/mm. The spectral resolution is frequency dependent and can be measured by measuring the linewidth of a CW laser, which is well below it. The mode Q factor can be estimated by fitting the modes with a Lorentzian and extracting the FWHM (Fig. 9.13 (a)). The calculated Q factors for modes from  $m=28$  to  $m=32$  (4  $\mu\text{m}$ ) diameter disk are shown in Fig. 9.13 (b). The Q factor increases with increasing mode number until they reach the spectral resolution of the system which is  $\sim 1 \times 10^4$ . This experimental setup is not particularly suited to measuring Q factors as it only covers a small range of azimuthal modes with  $28 < m < 31$ . Modes with higher  $m$  are still visible in the spectra, but they will be broadened by the spectral resolution of the spectrometer.



**Figure 9.13** (a) Example of Lorentzian fitting of a single mode ( $\text{TE}_{1,32,1}$ ) from a 3.5  $\mu\text{m}$  diameter microdisk. (b) Q factor as a function of azimuthal mode order for mode numbers 28-33.

For disks with diameters from 3.0-5.5  $\mu\text{m}$ , only the first order radial modes have high enough Q to be observed using this experimental method. The higher radial orders, although present, are not of high enough Q to be observed.

When the disk diameter is greater than 6  $\mu\text{m}$  more modes start to appear in the spectra. The spectra for disks with diameter ranging from 6  $\mu\text{m}$  to 9.5  $\mu\text{m}$  are displayed in Fig. 9.13.

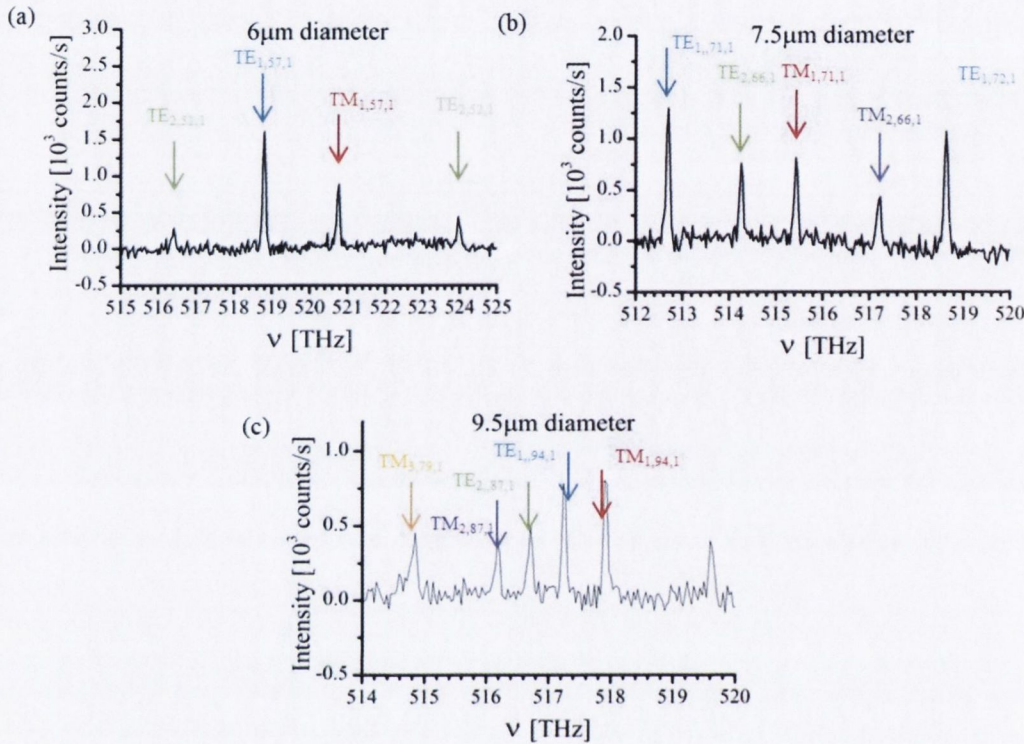


**Figure 9.14** Dense mode spectra in larger diameter microdisks. The disk diameters are (a) 6  $\mu\text{m}$  (b) 6.5  $\mu\text{m}$  (c) 7  $\mu\text{m}$  (d) 7.5  $\mu\text{m}$  (e) 8  $\mu\text{m}$  (f) 8.5  $\mu\text{m}$  (g) 9  $\mu\text{m}$  and (h) 9.5  $\mu\text{m}$

From these graphs we can see the overall effect of the decreasing free spectral range with increasing diameter and higher radial orders mode becoming observable. It is difficult to



decipher the modes in these graphs so we look over a smaller frequency range to identify the modes. The first extra mode appears at 517THz in the 6 $\mu$ m diameter disk with a Q of  $5.6 \times 10^3$ .



**Figure 9.15** (a) Spectra in narrow frequency range for disks with diameter (a) 6  $\mu$ m (b) 7.5  $\mu$ m and (c) 9.5  $\mu$ m.

The modes in the 9.5  $\mu$ m are identified by their position and FSR to the first, second and third radial order modes. The mode identification is an estimate based on the analytical theory which only approximates the microdisks and may be off by  $m \pm 1$  for the 9.5  $\mu$ m diameter disk as the modes become dense enough to make it difficult to label them accurately.

The disks used in this project are thick enough to allow multiple waveguide modes. Calculating the cut-off frequencies for the TE and TM modes shows that the first 2 orders are supported in a waveguide this thick, whereas the 3<sup>rd</sup> order mode is cut off in the range we observe. We examine the spectra in the frequency range 435 THz to 535 THz. The cut-off frequencies are the frequencies below which the modes will not propagate and are determined by the thickness of the waveguide and the refractive indices of the core and cladding given by: (Here only the slab waveguide mode indices are given)

$$TE_0 = 52.48 \text{ THz} \quad TM_0 = 104.27 \text{ THz}$$

$$TE_1 = 313.88 \text{ THz} \quad TM_1 = 365.0 \text{ THz}$$

$$\text{TE}_2=575.21 \text{ THz } \text{TM}_2= 626.94 \text{ THz}$$

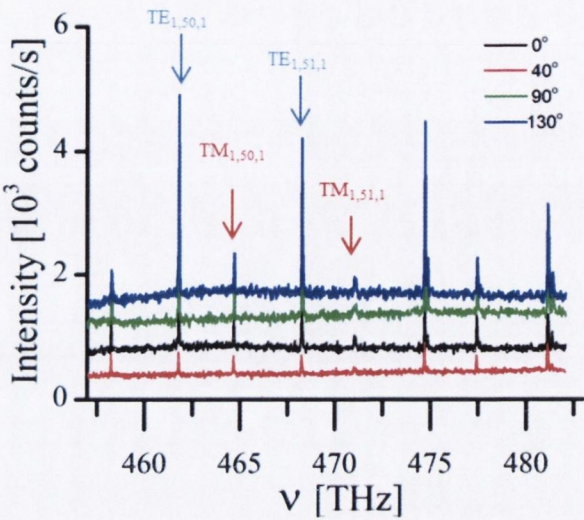
Therefore in the frequency range we observe only fundamental and first order modes are allowed to propagate in the slab waveguide. The first order modes however have significantly lower effective indices as shown in Fig. 9.5 This results in lower Q modes with larger FSR. No modes with FSR large enough to match the first order modes were observed in the microdisk size range observed experimentally. These higher order modes should have a FSR which is 20% longer than the fundamental modes and a significantly reduced Q factor due to material absorption. We conclude that only fundamental waveguide modes are observed in the frequency range of interest, however with larger disks or higher frequencies the first order modes would become observable.

#### 9.4.2 Polarisation dependence

From the analytical theory we know that the modes of a microdisk can be classified into two distinct groups according to their polarisation. This provides an experimental technique to classify the modes before fitting the theory. These two cases undergo different phase shifts on reflection, which results in a frequency difference in the positions of the modes. i.e. TE and TM modes may have the same radial and azimuthal quantum numbers, but they will have different resonant wavelengths. Again we use the TE and TM conventions for the modes of the slab waveguide shown in Fig. 9.4. If we look from the point of view of reflections at the disks edge though we can see that the TE and TM modes of the waveguide undergo p type and s type reflections respectively. The TE modes are discontinuous at the boundary which results in higher confinement and higher Q than the TM modes which are continuous.

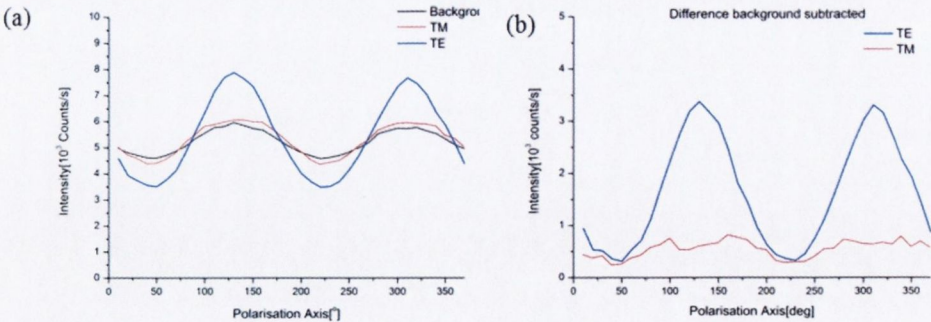
The same experimental setup as in Fig. 9.9 was used with a linear polariser inserted after the dichroic mirror. A quarter wave-plate was inserted in the excitation beam path to change the excitation polarisation from linear to circular. Spectra were taken with the polariser axis at different orientations. The resulting spectrum from a 6  $\mu\text{m}$  diameter





**Figure 9.16** Microdisk Spectrum from 455 THz to 485THz with 4 different positions of the polariser axis. Spectra from a microdisk with 7.5  $\mu\text{m}$  diameter.

There is a clear dependence of the spectrum on polarisation. We take a closer look at the peaks at 461.80 THz and 464.69 THz. These can be classified using the analytical theory as the  $\text{TE}_{1,m,1}$  and  $\text{TM}_{1,m,1}$  modes. The height TE peak varies appreciably with polarisation, whereas the TM remains constant. This is as expected as the Electric field of the TE mode is confined in the plane of the disk is polarised along the radius of the disk. The Electric field of the TM mode is in the plane perpendicular to this and will not depend on polarisation when collected from above. The background emission is also partially polarised. Taking the height of the TE and TM peaks and plotting as a function of the polariser axis we can clear see the polarisation dependence of the modes (Fig. 9.17 (a)).



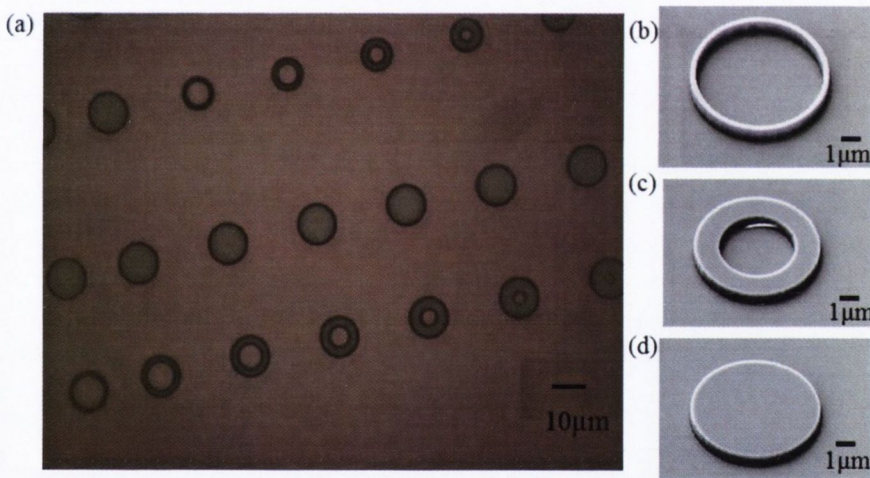
**Figure 9.17:** (a) Intensity as a function of polariser axis on two different modes and on the background. Measurements were taken at  $10^\circ$  intervals, with the same excitation position. (b) The differential increase of each peak is its height above the background signal



Figure 9.18 (b) shows the differential increase, which is the height of the peak above the background. This dependence on polarisation is repeated in every second mode across the visible range for this disk suggesting two sets of different modes. It is the high quality one which shows this dependence. Plotting the differential increase of each mode as a function of polariser axis clearly shows that the TE mode changes while the TM remains constant.

## 9.5 Concentric microdisk resonators

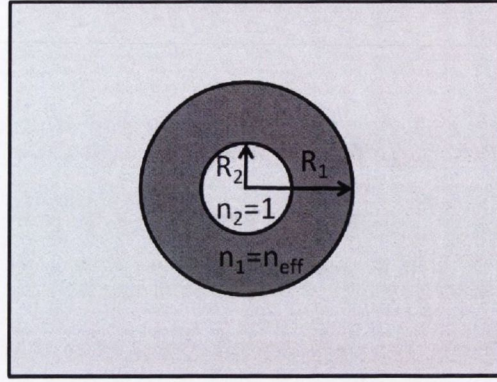
An interesting way to confirm the higher radial order modes are in fact higher order modes experimentally, is to examine a microdisk with a concentric cut-out in the centre. Matched pairs of microdisks and disks with concentric circles cut out were fabricated. These were etched using the Ar CHF<sub>3</sub> method resulting in vertical sidewalls. An optical image of a typical sample is displayed in Fig. 9.18 (a) with SEM images (b)-(d).



**Figure 9.18** (a) Optical image of typical sample. Matched pairs of microdisks are shown, one complete microdisk and one with a hole in the centre. (b) SEM image of microring with outer radius 3 μm and inner radius 2.8 μm (c) SEM of microring with outer radius 3 μm and inner radius 2 μm (d) SEM of complete microdisk of diameter 10 μm .

This effectively shows the transition from a microdisk to a microring resonator and allow us to identify which modes are the higher order radial modes, as these are affected more by the inner hole than the lower radial orders. There is analytical solution for the case of a concentric hole. , off centre causes chaotic modes and more complex behaviour. The analytical solution is found in a similar manner to the characteristic equation of a disk [5]. The parameters of the disk are outlined in Fig. 9.19.





**Figure 9.19** Parameter definition for a microdisk with a concentric disk cut out.

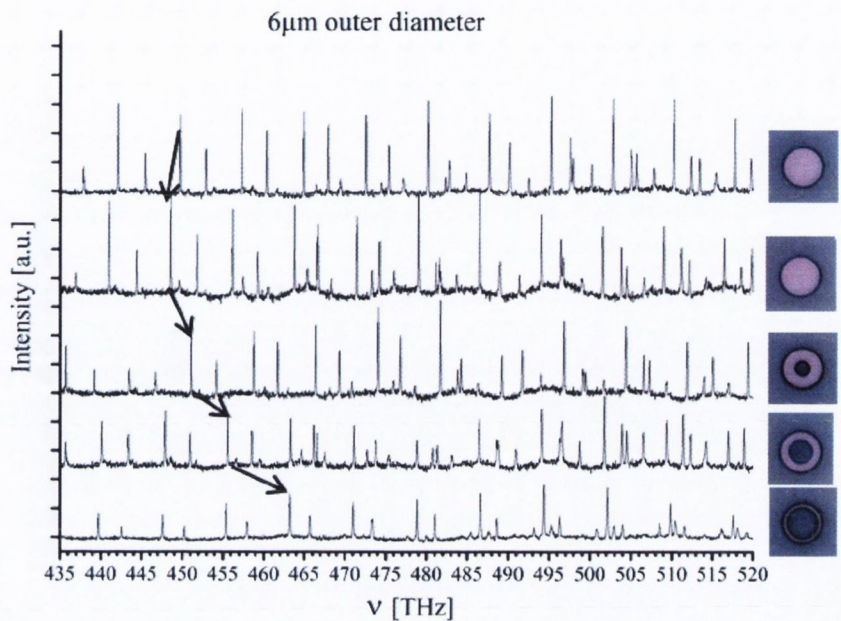
For the TM case, the mode positions and widths are given as the complex solutions of the following characteristic equation[5]:

$$\begin{aligned}
 & n_1 J_m(k_2 R_2) H_m^{(1)'}(k_0 R_1) [H_m^{(2)'}(k_1 R_2) H_m^{(1)}(k_1 R_1) - H_m^{(1)'}(k_1 R_2) H_m^{(2)}(k_1 R_1)] \\
 & - n_1^2 J_m(k_2 R_2) H_m^{(1)}(k_0 R_1) [H_m^{(2)'}(k_1 R_2) H_m^{(1)'}(k_1 R_1) - H_m^{(1)'}(k_1 R_2) H_m^{(2)'}(k_1 R_1)] \\
 & - n_2 J_m(k_2 R_2) H_m^{(1)'}(k_0 R_1) [H_m^{(2)}(k_1 R_2) H_m^{(1)}(k_1 R_1) - H_m^{(1)}(k_1 R_2) H_m^{(2)}(k_1 R_1)] \\
 & + n_1 n_2 J_m(k_2 R_2) H_m^{(1)}(k_0 R_1) [H_m^{(2)}(k_1 R_2) H_m^{(1)'}(k_1 R_1) - H_m^{(1)}(k_1 R_2) H_m^{(2)'}(k_1 R_1)] = 0
 \end{aligned} \tag{9.27}$$

Note that Eq.9.27 reduces to Eq.9.14 for the case of  $n_2=n_1$  or  $R_2=0$ . The case we observe is  $n_2 < n_1$ . This causes a shift in the resonances to higher frequency which can be understood by defining an average index:

$$n_{av} \stackrel{def}{=} \left( 1 - \frac{R_2^2}{R_1^2} \right) n_1 + \frac{R_2^2}{R_1^2} n_2 \tag{9.28}$$

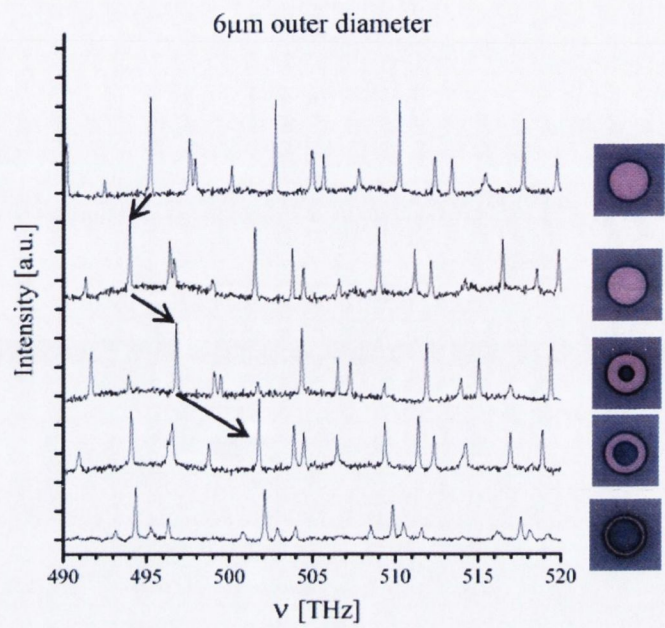
The results for a 6  $\mu\text{m}$  outer diameter disk ( $R_1=3 \mu\text{m}$ ) in the frequency range from 435-520THz are shown in Fig. 9.21. The values of  $R_2$  are varied from 0 to 2.5  $\mu\text{m}$  in steps of 500 nm. The largest value of  $R_2$  corresponds to a ring resonator with waveguide width of 500 nm. Spectra are shown overlaid with the largest value of  $R_2$  at the bottom and corresponding optical images of the resonators to the right. The proximity effect causes the centre to fill in after a value of  $R_2=1.5 \mu\text{m}$ .



**Figure 9.20** Spectra for microdisks with concentric holes in the range 435-520THz. The outer radius  $R_1$  is fixed at  $3\mu\text{m}$ . From the bottom up  $R_2=2.5\mu\text{m}$ ,  $R_2=2\mu\text{m}$ ,  $R_2=1.5\mu\text{m}$ ,  $R_2=0\mu\text{m}$  and  $R_2=0\mu\text{m}$  again. Spectra from two different microdisks are shown to display the size variation due to process variation in the diameter of the disk.

Care must be taken in interpreting the positions of the resonances. The top two spectra show that there is small variation in the resonance position due to process variation in the diameter of the disk alone. As  $R_2$  is increased the resonances shift significantly to higher frequencies. For the largest value of  $R_2$  the ring resonator is thin enough to cut off the higher radial order modes at low frequencies, but then at higher frequencies these resonances reappear. Their positions with respect to the first order resonances are shifted with respect to the case of the microdisk. Looking at a smaller frequency range shows the effect with more clarity (Fig.9.22).





**Figure 9.21** Spectra for microdisks with concentric holes in the range 490-520 THz. The outer radius  $R_1$  is fixed at  $3\text{ }\mu\text{m}$ . From the bottom up  $R_2=2.5\text{ }\mu\text{m}$ ,  $R_2=2\text{ }\mu\text{m}$ ,  $R_2=1.5\text{ }\mu\text{m}$ ,  $R_2=0\text{ }\mu\text{m}$  and  $R_2=0\text{ }\mu\text{m}$  again.

## 9.6 Conclusions

In this chapter the microdisks fabricated for scattering experiments were re-examined as resonant cavities. The idea of whispering gallery modes was introduced. The analytical solution of the eigenmodes of an infinite cylinder showed that the WGMs occur at frequencies where the denominator of the expansion co-efficient of the scattered field derived in Chapter 2 goes to zero. This shows that WGMs are responsible for the complex frequency dependence of light scattering from microdisks. The effective index approximation was introduced as a method to label the modes while taking into account the finite height of the disks and the resulting effective index dispersion.

A simple  $\mu$ PL method using defect emission from the  $\text{Si}_3\text{N}_4$  layer allowed the modes in the cavities to be experimentally measured. The mode spectrum for a range of size parameters from  $x_s=6$  to  $x_s=60$  was observed by changing the microdisk diameter from  $1\mu\text{m}$  to  $9.5\mu\text{m}$  in  $500\text{ nm}$  steps. Using this method first order radial modes with azimuth mode orders  $m$  from  $m=27$  to  $m=96$  were observed. The method was shown to require a critical  $Q$  factor before the modes could be observed. This meant that modes with azimuthal order below  $m=27$  could not be examined. Material absorption measured by ellipsometry was shown to be the limit the  $Q$  factor of the modes to  $\sim 10^3$ - $10^4$ . The upper limit on the measurement of  $Q$  factor was set by the spectrometer resolution at  $1 \times 10^4$ . The  $\mu$ PL method was determined to be unsuitable for measuring the  $Q$  factors of the modes as the  $Q$  factor was only within the lower bound set by material absorption and the upper bound set by the spectrometer for azimuthal mode orders  $m=28$  to  $m=32$ . The FSR range of the modes was shown to follow a  $1/R$  dependence as a function of microdisk radius  $R$  as expected from simple analytical arguments. The spectra for small diameter disks ( $D < 6\mu\text{m}$ ) showed only first order radial modes. These appeared in pairs with one high  $Q$  and one lower  $Q$  mode. By experimentally measuring the polarisation dependence the high  $Q$  mode was identified as the TE mode. This mode is expected to have higher  $Q$  than the TM mode from theory as it is slightly more confined due a discontinuity at the boundary of the microdisk.

For larger microdisk diameters ( $D > 6\mu\text{m}$ ) the mode spectrum quickly became denser as  $2^{\text{nd}}$  and  $3^{\text{rd}}$  order radial modes become of sufficient Quality to be observed experimentally. These modes could be identified using the EI approximation. Although the first order waveguide modes ( $\text{TE}_{x,y,1}$ ) were not cut off in the frequency range observed, their effective index was significantly lower than the fundamental modes and they were not observed in the size parameter range investigated.



Finally an experiment was performed to verify the labelling of the higher order radial modes, and to investigate the transition from a microdisk to a microring. Matched pairs of microdisks and disks with concentric circles of increasing radius cut out were fabricated. This effectively created microrings with varying waveguide thickness. The modes were shown to shift to higher frequency as the ring width decreased in accordance with theory. The higher radial order modes were first to feel the effect of decreasing the width of the waveguide and were shifted relative to the first order modes. Due to process variation more experiments are needed to explore this transition from microdisk to microring in more detail.

These cavities are not of particularly high Q factor however they may be useful for as an experimental tool to observe other phenomenon. The well-spaced, easily identifiable modes in the microdisks with diameters less than 6  $\mu\text{m}$  could be used in mode coupling experiments. Coupled cavities are receiving attention as a means to produce low threshold lasers with directional emission [36-38]. The position of the modes in these cavities can be measured quickly and easily. The mode frequency is sensitive to the refractive index of the surrounding medium in close proximity of the disk. These structures could be used as gas, nanoparticle or bio-sensors. By displacing the cut-out from the centre of the disk chaotic modes could be observed and studied in the system [5].

In conclusion we have fabricated microdisk resonators from  $\text{Si}_3\text{N}_4$  using a simple process with few steps. These cavities exhibit modes with moderate Q factor of  $10^3$  limited by material absorption. The case of a microdisk with a concentric shown to be a useful system for studying and labelling WGMs. This system and the case of an off centre cut-out is of interest for future work.

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## Chapter 10    General conclusion and future outlook

In this work we have shown that light scattering from particles in the intermediate size regime, between Rayleigh scattering and Geometrical optics, can produce unique intensity distributions in the near-field. A simple imaging technique was shown to provide information about the near-field intensity distribution using a non-invasive far-field method. The out of plane scattering in the far-field was characterised by imaging the back focal plane of a collection objective onto a CCD. The experimental results were in agreement with a full 3D numerical FEM model of the near-field distribution.

Using a fabrication method adapted from the microelectronics industry such as UV lithography or EBL, scattering structures could be made with arbitrary shape. Controlling the shape of the particle allowed precise control of the near-field intensity distribution. The size dependence and the polarisation dependence of the light scattering were fully explored. Linear relationships were found between the number of lobes in the scattering distribution and the size of the particle. The number of lobes matched well with the 3D FEM simulations when the substrate layer was included. These structures showed twice as many lobes as expected from an infinite cylinder. This showed that the finite height of the disk and the substrate layer had an influence on the in-plane scattering distribution. Different shapes exhibited very different intensity distributions in the near-field which could be amenable to a range of applications. We proposed that the squares could be used as tools for FCS, trapping and nanoparticle detection experiments, where added functionality could be achieved using the polarisation dependence of the scatters. The triangles could be used as micron scale beam splitters with a splitting ratio that depended on the incident angle and polarisation. After gaining a full understanding of the mechanism of image collection using the image plane method it was applied to more complex structures. These Cartesian ovals were designed specifically to create high intensity focal spots free from spherical aberration. By modifying these structures the effective numerical aperture could be controlled. This could be useful for coupling to rectangular and rib waveguides.

The scattering structures examined could produce focal spots of the same size as expensive microscope objectives. For comparison a high NA objective can cost from €3000-€5000, whereas a single scattering structure has a material cost of  $2 \times 10^{-6}$  cent. These scattering structures cannot reproduce the full functionality of a high NA objective, however for applications such as FCS where the size of the point spread function (PSF) is more important than the intensity contrast, a microdisk combined with a cheap focusing lens could be used



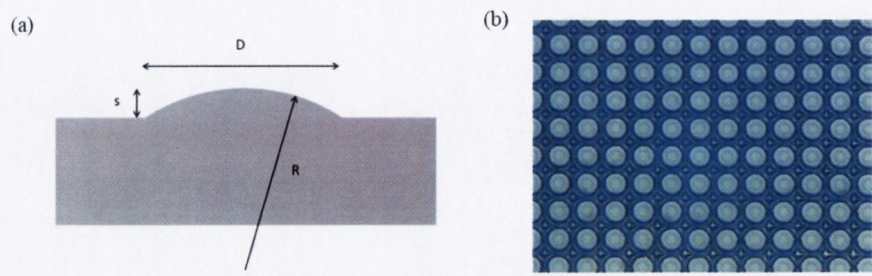
instead of a more expensive high NA microscope objective . These scattering structures show promise for large scale production of small focal spots, or large arrays of focal spots. The diffraction based focusing elements can produce focal spots which are more intense and smaller than microlenses of similar dimensions [1]. They can be integrated with planar waveguides to produce novel planar dielectric focusing devices.

Photonic nanojets were observed in water when the refractive index contrast was 1.5. These PNJ provide a small interaction volume outside of the disk and could be used for bio-sensing and nanoparticle detection in liquids With perpendicular focused illumination a sub-wavelength waist beam can be created with a divergence angle over 10 times smaller than expected from Gaussian focusing. More work is needed on characterisation of the scattered field in aqueous environments and for different input amplitude and phase distributions.

A general conclusion of this thesis is that micron scale structures defined using planar fabrication can be used as optical elements to control the propagation of light in-plane, close to the surface of a substrate. These structures would introduce loss due to out of plane scattering. However if a certain amount of loss is acceptable in the system, these structures could provide a host of new functionalities.

10.1 Applications

Refractive microlenses (MLs) are well known micrometre scale devices used to focus light [1]. They are in general spherical sections on a planar substrate and can be fabricated using machining [2], grey-scale lithography[3] and etching techniques[4] or more recently inkjet printing [5] . The total lensing effect is a combination of refraction at the spherical surface and propagation through the finite thickness of the substrate. A typical ML is depicted in Fig. 10.1 (a).



**Figure 10.1(a)** Schematic cross section of single microlens. Aperture  $D$ , sag height  $s$  and radius of curvature  $R$  displayed. **(b)** Typical optical image of a microlens array (reproduced from [4])



For a spherical ML the relationship between the radius of curvature  $R$ , the aperture diameter  $D$ , and the lens thickness (sag height)  $s$  are given by the basic geometry:

$$R = \frac{(D/2)^2 + s^2}{2s} \quad (1.1)$$

This puts a limit on the achievable numerical apertures from  $0.15 \leq NA \leq 0.45$ . Typical aperture diameters are from  $8 \mu\text{m}$  to  $1200 \mu\text{m}$ . Due to the fabrication techniques arrays of MLs can easily be

mass produced (Fig. 10.1 (b)). Microlens arrays (MLAs) are commonly used to enhance the performance of CCD cameras, and in adaptive optics as wavefront sensors [6] whereas single MLs are used to couple light to fibres. Due to the fabrication process it is not easy to create a ML that focuses light in the plane of the substrate. Some designs based on free standing out of plane MLs have been proposed and demonstrated but these remain relatively exotic [7].

The quasi-2D scatterers explored in Chapters-5-8 could be used to create in-plane micro lenses. Thus all the common applications of MLs and MLAs could be transferred into 2D systems. Since the direction of the PNJ from a disk depends on the incident wave-front direction and shape these devices could be used as 2D wave-front sensors to measure spatial coherence in planar waveguides.

Coupling from large multimode rib waveguides to single mode waveguides can be achieved using long adiabatic tapers over lengths of  $100 \mu\text{m}$  or longer [8]. For applications requiring low device footprint these tapers are not suitable. Using a 2D scattering element may serve to match the NA of the single mode rib waveguide over a much shorter distance. Light will be lost to scattering at the interfaces of these devices and out of plane diffraction, however it may be an acceptable loss in order to reduce device footprint in certain applications.

Due to the high spatial frequency components in PNJs [9] they may have applications in coupling to plasmonic structures such as Metal-Insulator-Metal (MIM) waveguides [10]. A 2D planar solid immersion mirror using reflection to focus light was proposed by Challenger et al [11] to couple to a plasmonic near field transducer (NFT) for heat assisted magnetic recording (HAMR).

PNJs from spheres were proposed for high resolution optical recording [12]. The planar structures proposed in this thesis may be easier to fabricate and integrate with an optical recording head. Other applications can be found by transferring previously demonstrated applications of PNJs in microspheres to a planar system. For example PNJ from microdisks in water could be used for nanoparticle detection [13], optical trapping [14], PL enhancement and single molecule detection [15]. By interfering light from two spatial coherent scattering structures smaller spots could be created in analogy to 4 pi microscopy [16, 17]. Since these



scatterers can be embedded into a material of higher index integration with microfluidic channels [18, 19] would be possible in order to create interesting devices such as polarisation controlled on-chip optical sorting.

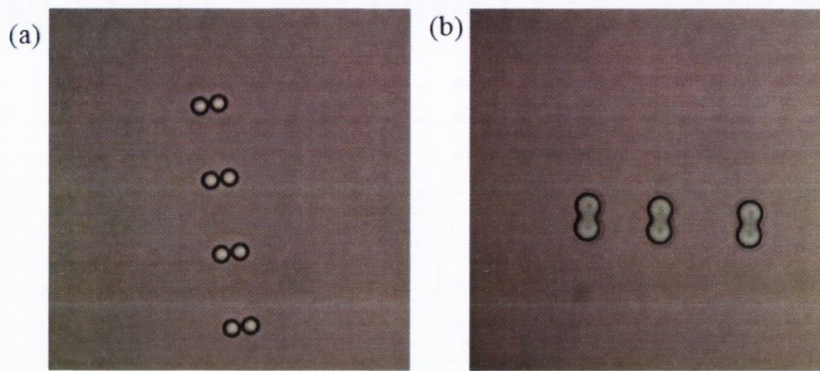
## 10.2 Future outlook

### 10.2.1 Light scattering and PNJs

In this project we have shown a way to easily fabricate and characterise these microscale scattering based structures. This opens the way for much more applications based research on these structures, such as the examples mentioned in the previous section. A starting point would be to examine the behaviour of the scatterers when butt-coupled to a planar waveguide. The scattering intensity distribution would be expected to be close to that of the plane wave case for excitation from a fundamental mode with cross section larger than the scatterer, however this would change dramatically for higher order waveguide modes. More characterisation of the near field could be carried out with other techniques such as scattering based SNOM.

### 10.2.2 Whispery gallery modes

Due to their cylindrical symmetry microdisk cavities generally emit light equally in all directions. This is a disadvantage over conventional Fabret-Perot and DBR resonators, as the emitted power in any one direction is greatly reduced. In order to overcome this problem symmetry breaking methods must be introduced. This can be achieved by numerous methods including positioning dipole emitters and/or defects at specific locations in the cavity [20], deforming/patterning the cavity [21, 22], and creating regions with higher curvature [23, 24], and thus more radiative loss. Multiple cavities can be coupled through their evanescent fields to introduce this symmetry breaking. The goal of these symmetry breaking techniques is to maintain the high quality factors modes associated with these resonators, focusing the output power into particular directions [25]. The first experimental observation of mode splitting due to coupling between individual photonic resonators was observed in 1998 [26]. They observed mode splitting from two square GaAs cavities coupled by a narrow bridge. The GaAs layer was surrounded by highly reflecting GaAs/AlAs Bragg mirrors and a 7 nm wide  $\text{In}_{0.14}\text{Ga}_{0.86}\text{As}$  quantum well was used as the optically active medium.



**Figure 10.2** Resist profiles for production of 5  $\mu\text{m}$  diameter coupled microdisks (a) Separated disks with sub 1  $\mu\text{m}$  gap. (b) Microdisks overlapped due to proximity effect.

During this project bi-disks of diameter 5  $\mu\text{m}$  and separation varying from 100 nm up to 2  $\mu\text{m}$  as shown in Fig. 10.2, however no modes were observed from disks with separation under 500nm. We attribute this to a proximity effect in the fabrication process causing an asymmetry in the disks and destroying the high Q modes (Fig. 10.2 (b)). With further optimisation of the EBL, these  $\text{Si}_3\text{N}_4$  microdisks combined with the  $\mu\text{PL}$  measurement could be a simple system in which to observe mode splitting in the visible range.

By varying the etching time we have microdisks with different thicknesses can be produced. These microdisks would have a rib waveguide cross section, as opposed to rectangular. We would expect these devices to have a different FSR, as they will have a smaller effective index. It has been suggested that specific heights would forbid out of plane radiate loss resulting in higher Q modes [27]. This claim could be explored experimentally using the microdisk fabrication and measurement technique outlined in this thesis.



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## Appendix A :Analytical scattering programs.

These programs were written for Matlab R2011b, to calculate near field intensity distributions and far field scattering plots for the a sphere and an infinite cylinder illuminated by a plane wave. Near-fied.m calculates the near field intensity for an infinite cylinder illuminated by a linearly polarised plane wave in the plane perpendicular to the cylinder axis. This code was used in Chapter-2 to study the properties of PNJs. Far-field.m calculates the far field scattering distribution for the same case. This code was used to verify the 2D FEM and Stratton-Chu equation numerical technique. Sphere\_far\_field.m calculates the scattered far field from a sphere in the scattering plane and in the plane perpendicular to this. This code was used to verify the 3D FEM and Stratton-Chu numerical technique.

### (A.1) Infinite cylinder

#### Near\_field.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Program to calculate the scattered and internal fields of an      %
% infinite cylinder illuminated by a linearly polarised plane      %
% wave. The fields are calculated over a 15*12um field of view with%
% a 46.5nm pixel size to match with experimental results.        %
%                                                                    %
%Inputs:                                                            %
%       pol (input polarisation TM=> along cylinder axis)         %
%       D (cylinder diameter)                                     %
%       lambda (incident wavelength)                             %
%       m (complex refractive index)                             %
%                                                                    %
%Outputs                                                            %
%TE/TM case                                                         %
%       Is (Scattered intensity outside cylinder)                 %
%       Itot (Total internal, scattered and incident intensity    %
%       Iext (Total outside of cylinder excluding internal        %
%       field.)                                                    %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all;

%Input parameters

pol=input('Incident polarisation(TE/TM): ','s');
D=input('Cylinder Diameter [um]: ');
lambda=input('Incident wavelength [nm]: ');
m=input('Complex refractive index contrast(n+j*k): ');

%conversion
```



```
D=D*1E-6;
lambda=lambda*1E-9;
xs=pi*D/lambda;
k=2*pi/lambda;
rho=m*xs;

%defining grid to calculate field on and conversion to cylindrical
%co-ordinates.

xrange=[-(6E-6):0.0465E-6:9E-6];
yrange=[-(6E-6):0.0465E-6:6E-6];
[X,Y]=meshgrid(xrange,yrange);
[theta,ro]=cart2pol(X,Y);
s=size(X);

%Wiscombe condition for truncating the infinite sums

nc=round(xs+4.05*xs.^(1/3)+2);

%TM case
if strcmp(pol,'TM')==1

%Initialising variables.

Es_z=zeros(s(1),s(2));
Es_z_sum=zeros(s(1),s(2));
Eint_z_sum=zeros(s(1),s(2));
Eint_z=zeros(s(1),s(2));
%Calculation of expansion coefficients.
b0=(-
m*besselj(1,rho).*besselj(0,xs)+besselj(0,rho).*besselj(1,xs))./(-
m*besselj(1,rho).*besselh(0,xs)+besselj(0,rho).*besselh(1,xs));
d0=(1/besselj(0,rho))*(besselj(0,xs)-b0*besselh(0,xs));

for n=1:nc
dj=besselj(n-1,xs)-(n./xs).*besselj(n,xs);
djrho=besselj(n-1,rho)-(n./rho).*besselj(n,rho);
dh=besselh(n-1,xs)-(n./xs).*besselh(n,xs);
b(n)=(m.*djrho.*besselj(n,xs)-
besselj(n,rho).*dj)./(m.*djrho.*besselh(n,xs)-besselj(n,rho).*dh);
d(n)=(1/besselj(n,rho)).*(besselj(n,xs)-b(n).*besselh(n,xs));

Es_z_sum(:, :)=Es_z_sum(:, :)+( (sqrt(-
1))^n.*b(n).*besselh(n,k*ro(:, :)).*cos(n.*theta(:, :)));
Eint_z_sum(:, :)=Eint_z_sum(:, :)+(sqrt(-
1))^n.*d(n).*besselj(n,m*k*ro(:, :)).*cos(n.*theta(:, :)));
end

%Calculation of electric fields
Ei_z(:, :)=exp((sqrt(-1))*k.*ro.*cos(theta));
Eint_z(:, :)=d0.*besselj(0,m*k*ro(:, :))+2*Eint_z_sum;
Es_z(:, :)=(-b0.*besselh(0,k*ro(:, :))-2.*Es_z_sum(:, :));

for i=1:s(1)
    for j=1:s(2)
        if ro(i,j)<= D/2
```

```
        Es_z(i,j)=0;
    else
        Eint_z(i,j)=0;
    end
end
end

Is=(Es_z).*conj(Es_z);
Itot=(Es_z+Ei_z).*conj(Es_z+Ei_z)+Eint_z.*conj(Eint_z);

%TE case
elseif strcmp(pol,'TE')==1

%Initialising variables.
Es_r=zeros(s(1),s(2));
Es_r_sum=zeros(s(1),s(2));
Es_phi=zeros(s(1),s(2));
Es_phi_sum=zeros(s(1),s(2));
Eint_r=zeros(s(1),s(2));
Eint_r_sum=zeros(s(1),s(2));
Eint_phi=zeros(s(1),s(2));
Eint_phi_sum=zeros(s(1),s(2));

%Calculation of expansion coefficients.
a0=(-
besselj(1,rho).*besselj(0,xs)+m*besselj(0,rho).*besselj(1,xs))./(-
besselj(1,rho).*besselh(0,xs)+m*besselj(0,rho).*besselh(1,xs));
c0=(1/besselj(0,rho))*(besselj(0,xs)-a0*besselh(0,xs));

for n=1:nc
dj=besselj(n-1,xs)-(n./xs).*besselj(n,xs);
djrho=besselj(n-1,rho)-(n./rho).*besselj(n,rho);
dh=besselh(n-1,xs)-(n./xs).*besselh(n,xs);
a(n)=(djrho.*besselj(n,xs)-
m.*besselj(n,rho).*dj)./(djrho.*besselh(n,xs)-m.*besselj(n,rho).*dh);
%(Determines TE mode positions)
c(n)=(1/besselj(n,rho))*(besselj(n,xs)-a(n)*besselh(n,xs));

Es_r_sum(:,:)=Es_r_sum(:,:)+((sqrt(-
1))^n).*a(n).*n.*besselh(n,k.*ro(:,:)).*sin(n.*theta(:,:)));
Es_phi_sum(:,:)=Es_phi_sum(:,:)+( (sqrt(-1))^n).*a(n).*(besselh(n-
1,k.*ro(:,:))-
(n./(k.*ro(:,:))).*besselh(n,k.*ro(:,:))).*cos(n.*theta(:,:)));

Eint_r_sum(:,:)=Eint_r_sum(:,:)+((sqrt(-
1))^n).*c(n).*n.*besselj(n,m*k.*ro(:,:)).*sin(n.*theta(:,:)));
Eint_phi_sum(:,:)=Eint_phi_sum(:,:)+((sqrt(-1))^n).*c(n).*(besselj(n-
1,m*k.*ro(:,:))-
(n./(m*k.*ro(:,:))).*besselj(n,m.*k.*ro(:,:))).*cos(n.*theta(:,:)));

end

%Calculation of electric fields
Ei_r(:,:)=sin(theta(:,:)).*exp((sqrt(-1))*k.*ro.*cos(theta));
Ei_phi(:,:)=cos(theta(:,:)).*exp((sqrt(-1))*k.*ro.*cos(theta));
```



```
Eint_r(:, :)=-(2*sqrt(-1)./(m^2).*k.*ro(:, :)).*Eint_r_sum;
Eint_phi(:, :)= -(sqrt(-1)./m).*(c0.*(-
besselj(1,m.*k.*ro(:, :))+2.*Eint_phi_sum(:, :));
Es_r(:, :)=(2.*sqrt(-1))./(k.*ro(:, :)).*Es_r_sum;
Es_phi(:, :)=sqrt(-1).*(a0.*(-besselh(1,k.*ro(:, :)))+2.*Es_phi_sum);

for i=1:s(1)
    for j=1:s(2)
        if ro(i,j)<= D/2
            Es_r(i,j)=0;
            Es_phi(i,j)=0;
        else
            Eint_r(i,j)=0;
            Eint_phi(i,j)=0;
        end
    end
end

Is=(Es_r+Ei_r).*conj(Es_r+Ei_r)+(Es_phi+Ei_phi).*conj(Es_phi+Ei_phi);
Iint=(Eint_r).*conj(Eint_r)+(Eint_phi).*conj(Eint_phi);
Itot=Is+Iint;
end
```

### Far\_field.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Program to calculate the far field scattering distribution along %
%  $\phi=0$  for  $\theta=0:180^\circ$  for an infinite cylinder illuminated by a plane %
% wave incident in the plane perpendicular to the cylinder axis %
%
%Inputs:
%       pol (input polarisation TM=> along cylinder axis)
%       D (cylinder diameter)
%       lambda (incident wavelength)
%       m (complex refractive index)
%
%Outputs
%TE/TM case
%       ITE Far-field intensity along  $\phi=0$ ,  $\theta=0:180$  for TE case
%       ITM Far-field intensity along  $\phi=0$ ,  $\theta=0:180$  for TE case
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all;

%Input parameters

pol=input('Incident polarisation(TE/TM): ','s');
D=input('Cylinder Diameter [um]: ');
lambda=input('Incident wavelength [nm]: ');
m=input('Complex refractive index contrast(n+j*k):');

phi=[0:0.005:2*pi];
s=size(phi);
D=D*1E-6;
lambda=lambda*1E-9;
```

```
xs=pi*D/lambda;
k=2*pi/lambda;

%Wiscombe condition for truncating the infinite sums
nc=round(xs+4.05*xs.^(1/3)+2);

if strcmp(pol,'TM')==1
%Initialising variables.
ITM_sum=zeros(1,s(2));
ITM=zeros(1,s(2));

%Calculation of expansion coefficients.

b0=(-
m*besselj(1,rho).*besselj(0,xs)+besselj(0,rho).*besselj(1,xs))./(-
m*besselj(1,rho).*besselh(0,xs)+besselj(0,rho).*besselh(1,xs));

for n=1:nc
dj=besselj(n-1,xs)-(n./xs).*besselj(n,xs);
djrho=besselj(n-1,rho)-(n./rho).*besselj(n,rho);
dh=besselh(n-1,xs)-(n./xs).*besselh(n,xs);
b(n)=(m.*djrho.*besselj(n,xs)-
besselj(n,rho).*dj)./(m.*djrho.*besselh(n,xs)-besselj(n,rho).*dh);

ITM_sum=ITM_sum+b(n).*cos(n.*phi);
end

ETM=real(b0+2.*ITM_sum);
ITM=(b0+2.*ITM_sum).*conj(b0+2.*ITM_sum);

elseif strcmp(pol,'TE')==1
%Initialising variables.
ITE_sum=zeros(1,s(2));
ITE=zeros(1,s(2));

%Calculation of expansion coefficients.
a0=(-
besselj(1,rho).*besselj(0,xs)+m*besselj(0,rho).*besselj(1,xs))./(-
besselj(1,rho).*besselh(0,xs)+m*besselj(0,rho).*besselh(1,xs));

for n=1:nc
dj=besselj(n-1,xs)-(n./xs).*besselj(n,xs);
djrho=besselj(n-1,rho)-(n./rho).*besselj(n,rho);
dh=besselh(n-1,xs)-(n./xs).*besselh(n,xs);
a(n)=(djrho.*besselj(n,xs)-
m.*besselj(n,rho).*dj)./(djrho.*besselh(n,xs)-m.*besselj(n,rho).*dh);

ITE_sum=ITE_sum+a(n).*cos(n.*phi);
end

ETE=real(a0+2.*ITE_sum);
ITE=(a0+2.*ITE_sum).*conj(a0+2.*ITE_sum);
End
```



## (A.2) Sphere

### Sphere\_far\_field.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Program to calculate the far field scattering distribution along %
%  $\phi=0$  for  $\theta=0:180^\circ$  for an sphere illuminated by a plane wave %
% incident in the plane perpendicular to the cylinder axis %
% %
%Inputs: %
%      D (cylinder diameter) %
%      lambda (incident wavelength) %
%      m (complex refractive index) %
%
%Outputs %
%      i1 Far-field scattering distribution perpendicular to %
%          the scattering plane %
%
%      i2 Far-field scattering distribution perpendicular to %
%          scattering plane %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

D=2E-6; %sphere diameter
m=2; % refractive index contrast
lambda=532E-9; % incident wavelength.
pi=3.14;
xs=pi*D/lambda; % size parameter
nc=round(xs+4.05*(xs^(1/3))+2); % cutoff Wiscombe condition
z=m*xs;

% angular range of interest:
t1=0; %Start angle in degrees
t2=180; %end angle in degrees
c=pi/180;
theta =[t1*c:0.2*c:t2*c];

%Create the angle dependence functions from the recurrssion relations
in B&H, this works very fast for nc up to 30%

s1=size(theta);

pp=zeros(nc,s1(2));
tao=zeros(nc,s1(2));

pp0=zeros(1,s1(2));
pp(1,:)=ones(1,s1(2));
pp(2,:)=((2*2-1)/(2-1)).*cos(theta).*pp(1,)-(2/(2-1)).*pp0;

tao0=0;
tao(1,:)=cos(theta);
tao(2,:)=2*cos(theta).*pp(2,)-(2+1).*pp(1,);

%theta dependences

for n=3:1:nc+1,
```

```
pp(n,:)=((2*n)-1)/(n-1)*cos(theta).*pp(n-1,:)-(n)/(n-1)*pp(n-2,:);

tao(n,:)=(n-1)*cos(theta).*pp(n,:)-(n)*pp(n-1,:);
end

% Get the expansion co-efficients

j0=sqrt(pi/(2*xs))*besselj((0)+1/2,xs);
y0=sqrt(pi/(2*xs))*bessely((0)+1/2,xs);
h0=j0+sqrt(-1)*y0;

jz0=sqrt(pi/(2*z))*besselj((0)+1/2,z);
yz0=sqrt(pi/(2*z))*bessely((0)+1/2,z);
hz0=jz0+sqrt(-1)*yz0;

for n=1:nc+1 %j(1) corresponds to j0 in B&H notation
    j(n)=sqrt(pi/(2*xs))*besselj((n)+1/2,xs);
    y(n)=sqrt(pi/(2*xs))*bessely((n)+1/2,xs);
    h(n)=j(n)+sqrt(-1)*y(n);

    jz(n)=sqrt(pi/(2*z))*besselj((n)+1/2,z);
    yz(n)=sqrt(pi/(2*z))*bessely((n)+1/2,z);
    hz(n)=jz(n)+sqrt(-1)*yz(n);
end

drj(1)=xs*j0-(1)*j(1);
drh(1)=xs*h0-(1)*h(1);

drjz(1)=z*jz0-(1)*jz(1);
drhz(1)=z*hz0-(1)*hz(1);

for n=2:nc+1
    drj(n)=xs*j(n-1)-(n)*j(n);
    drh(n)=xs*h(n-1)-(n)*h(n);

    drjz(n)=z*jz(n-1)-(n)*jz(n);
    drhz(n)=z*hz(n-1)-(n)*hz(n);
end

for n=1:nc
    a(n)=(m^2)*jz(n)*drj(n)-j(n)*drjz(n)/(m^2*jz(n)*drh(n)-h(n)*drjz(n));
    b(n)=(jz(n)*drj(n)-j(n)*drjz(n))/(jz(n)*drh(n)-h(n)*drjz(n));
end

E1=zeros(nc,s1(2));
E2=zeros(nc,s1(2));

for n=1:nc
    const(n)=(2*n+1)/(n*(n+1));
    E1(n,:)=const(n).*(a(n).*pp(n,:)+b(n).*tao(n,:));
    E2(n,:)=const(n).*(a(n).*tao(n,:)+b(n).*pp(n,:));
end
```



```
end
  E1t=sum(E1);
  E2t=sum(E2);

i1=E1t.*conj(E1t);
i2=E2t.*conj(E2t);
```

## Appendix B: Effective index calculation

A brief summary of the approach of the standard derivation of effective index dispersion adapted from [1]. We wish to solve Maxwell's Equations in the domain of a slab waveguide illustrated in Fig.1

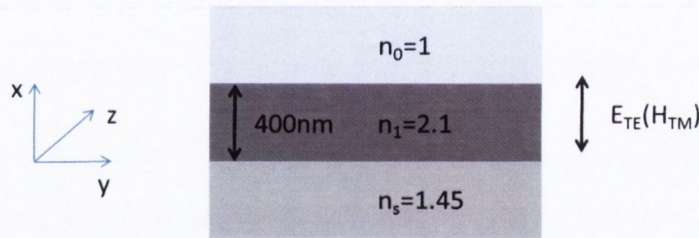
$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad (1)$$

$$\nabla \times \vec{H} = -\epsilon_0 n^2 \frac{\partial \vec{E}}{\partial t} \quad (2)$$

We are interested in plane wave propagation of the form

$$\vec{E} = \vec{E}_0(x, y) e^{i(\omega t - \beta z)} \quad (3)$$

$$\vec{H} = \vec{H}_0(x, y) e^{i(\omega t - \beta z)} \quad (4)$$



**Figure 1** Layer structure and co-ordinate system used in the planar waveguide in order to calculate the TE and TM effective indices.

Subbing Eq.4 and Eq.5 into Eq.1 and 2 we get the following set of equations for the electromagnetic field components:

$$\begin{cases} \frac{\partial H_z}{\partial y} + i\beta H_y = i\omega\epsilon_0 n^2 E_x \\ -\frac{\partial H_z}{\partial x} - i\beta H_x = -i\omega\epsilon_0 n^2 E_y \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = -i\omega\epsilon_0 n^2 E_z \end{cases} \quad (5)$$

$$\begin{cases} \frac{\partial E_z}{\partial y} + i\beta E_y = -i\omega\mu_0 H_x \\ -\frac{\partial E_z}{\partial x} - i\beta E_x = -i\omega\mu_0 H_y \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -i\omega\mu_0 H_z \end{cases} \quad (6)$$

In the geometry of a slab waveguide, the electromagnetic fields do not vary in the y direction. Therefore we can set  $\partial \vec{E} / \partial y = 0$  and  $\partial \vec{H} / \partial y = 0$ , in Eq.5 and Eq.6.



The result is two independent electromagnetic modes known as the transverse electric (TE) mode and the transverse magnetic (TM) mode. The TE mode satisfies:

$$\frac{d^2 E_y}{dx^2} + (k^2 n^2 - \beta^2) E_y = 0 \quad (7)$$

where

$$H_x = -\frac{\beta}{\omega \mu_0} E_y \quad (8)$$

$$H_z = -\frac{i}{\omega \mu_0} \frac{dE_y}{dx} \quad (9)$$

and

$$E_x = E_z = H_y = 0 \quad (10)$$

In this case the electric field lies in the plane transverse to the z direction as  $E_z=0$ , hence the name TE.

For the TM case

$$\frac{d}{dx} \left( \frac{1}{n^2} \frac{dH_y}{dx} \right) + \left( k^2 - \frac{\beta^2}{n^2} \right) H_y = 0 \quad (11)$$

where

$$E_x = -\frac{\beta}{\omega \epsilon_0 n^2} H_y \quad (12)$$

$$E_z = -\frac{i}{\omega \epsilon_0 n^2} \frac{dH_y}{dx} \quad (13)$$

and

$$E_y = H_x = H_z = 0 \quad (14)$$

In this case the magnetic field lies in the plane transverse to the z direction as  $H_z=0$ , hence the name TM. For the TE case we can calculate the dispersion equation by solving Eq.7 .

The electromagnetic fields are confined to the core region and decay exponentially in the cladding regions so we look for a solution of the form

$$E_y = \begin{cases} A \cos(\kappa x - \phi) e^{-\sigma(x-a)} & (x < a) \\ A \cos(\kappa x - \phi) & (-a \leq x \leq a) \\ A \cos(\kappa x + \phi) e^{\xi(x+a)} & (x < -a) \end{cases} \quad (15)$$

where  $\kappa$ ,  $\sigma$ , and  $\xi$  are wavenumbers along the x-axis in the core and cladding regions.

Applying the boundary conditions that  $E_y$  and  $H_z$  are continuous at the boundaries we get the eigenvalue equations

$$\tan(u + \phi) = \frac{w}{u} \quad (16)$$

$$\tan(u - \phi) = \frac{w'}{u} \quad (17)$$

where we have introduced the normalised propagation constants

$$\begin{cases} u = \kappa a \\ w = \xi a \\ w' = \sigma a \end{cases} \quad (18)$$

Expanding Eq.16 and Eq.17 ,

$$\begin{aligned} u &= \frac{m\pi}{2} + \frac{1}{2} \tan^{-1}\left(\frac{w}{u}\right) + \frac{1}{2} \tan^{-1}\left(\frac{w'}{u}\right) \\ \phi &= \frac{m\pi}{2} + \frac{1}{2} \tan^{-1}\left(\frac{w}{u}\right) - \frac{1}{2} \tan^{-1}\left(\frac{w'}{u}\right) \end{aligned} \quad (19)$$

(m=0,1,2,...)

The transverse wavenumber in the core  $\kappa$  should be real for the main part of the field to be confined to the core. This is true if:

$$n_s \leq \frac{\beta}{k} \leq n_1 \quad (20)$$

The effective index of the mode is defined as

$$n_{eff} = \frac{\beta}{k} \quad (21)$$



A mode will exist only if its effective index satisfies Eq.20. By introducing some new parameters we can simplify Eq. 19 to

$$2\nu\sqrt{1-b} = m\pi + \tan^{-1}\sqrt{\frac{b}{1-b}} + \tan^{-1}\sqrt{\frac{b+\gamma}{1-b}} \quad (22)$$

where:

$$\nu = \sqrt{k^2 a^2 (n_1^2 - n_s^2)} \quad \text{Normalised frequency} \quad (23)$$

$$b = \frac{n_{eff}^2 - n_s^2}{n_1^2 - n_s^2} \quad \text{guided modes} \Rightarrow 0 \leq b \leq 1 \quad (24)$$

$$\gamma = \frac{n_s^2 - n_0^2}{n_1^2 - n_s^2} \quad \text{Asymmetry parameter (0 for symmetric waveguide)} \quad (25)$$

A similar approach for TM modes gives the equation:

$$2\nu\sqrt{1-b} = m\pi + \tan^{-1}\left(\frac{n_1^2}{n_s^2}\sqrt{\frac{b}{1-b}}\right) + \tan^{-1}\left(\frac{n_1^2}{n_0^2}\sqrt{\frac{b+\gamma}{1-b}}\right) \quad (26)$$

The cut-off frequencies for the TE and TM modes are given by:

$$\nu_{c,TE} = \frac{m\pi}{2} + \frac{1}{2}\tan^{-1}\sqrt{\gamma} \quad (27)$$

$$\nu_{c,TM} = \frac{m\pi}{2} + \frac{1}{2}\tan^{-1}\left(\frac{n_1^2}{n_0^2}\sqrt{\gamma}\right) \quad (28)$$

For a symmetric waveguide  $\gamma=0$  and there is no cut-off frequency for the fundamental TE mode, this is not true for the asymmetric case.

## References

1. k. Okamoto, *Fundamentals of optical waveguides* (Academic Press, 2005).

## Appendix C :Programs to find WGM of a microdisk using the EI approximation

The function WGM\_open.m is used to calculate the frequencies and Q factors of modes close to a given input frequency. The effective index of a mode is calculated using eff.m and the program WGM\_closed.m gives an approximate position of the modes using the closed resonator approximation. WGM\_open.m calls both eff.m and WGM\_closed.m so all files need to be in the same folder. Finally complex\_plot.m is used to plot the characteristic equation in the complex plane and find an approximate mode position graphically if the Nelder-Mead method does not work the first time.

### eff.m

```
function [neffTE,neffTM]=eff(fr)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Function to find the effective index of a slab waveguide at a      %
% given frequency. The refractive indices and waveguide dimensions %
% are fixed to those used in the project.                          %
%                                                                    %
% Inputs:                                                            %
%          fr Frequency[Hz]                                          %
% Outputs  [nTEeff,nTMeff] Effective indices                    %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Parameter definition

m=0;
d=400E-9;
a=d/2;                % disk thickness
n0=1;
n1=2.04;
ns=1.45;
g=(ns^2-n0^2)/(n1^2-n0^2); %asymmetry parameter
del=(n1^2-n0^2)/(2*n1^2);  %relative refractive index difference

k0=(2*pi*fr)/3E8;
v=k0*a*sqrt(n1^2-n0^2);

%Define characteristic equation

int=0;
```



```
for b=0:0.0001:1
int=int+1;

u(int)=v*sqrt(1-b);
w(int)=v*sqrt(b);
wp(int)=v*sqrt(b+g);

fte(int)=2*v*sqrt(1-b)-(m*pi)-atan(sqrt(b/(1-b)))-atan(sqrt((b+g)/(1-b)));

ftm(int)=2*v*sqrt(1-b)-(m*pi)-atan(((n1^2)/(ns^2))*sqrt(b/(1-b)))-
atan(((n1^2)/(n0^2))*sqrt((b+g)/(1-b)));
end

% Find zeros of characteristic equation
bv =[0:0.0001:1];

s=size(bv);
for k=1:s(2)-1

    if sign(fte(k))>sign(fte(k+1))
        z1=(bv(k)+bv(k+1))/2;
        break
    end
end

for k=1:s(2)-1

    if sign(ftm(k))>sign(ftm(k+1))
        z2=(bv(k)+bv(k+1))/2;
        break
    end
end

%output results

neffTE=sqrt(z1*(n1^2-ns^2)+ns^2);
neffTM=sqrt(z2*(n1^2-ns^2)+ns^2);
```

**WGM\_closed.m**

```
function [xs]=WGM_closed(n,m,s)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Rough approximation to find resonance frequencies using the      %
% closed resonator approximation                                %
% Requires: eff.m                                              %
%                                                              %
% Inputs:                                                       %
%         n radial mode number                                %
%         m azimuthal mode number                             %
%         s 1 for TE or 2 for TM mode                          %
% Outputs:                                                       %
%         xs size parameter of resonance                       %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

x=0.01:0.01:2*m;
sx=size(x);

[neffTE, neffTM]= eff(525E12);

if s==1
    nnn=neffTE;
elseif s==2
    nnn=neffTM;
end

b=besselj(m,x);
bp=0.01+besselj(m,x); % work around for issues of finding zeros
                      % to early

%Find zeros of bp

i=0;

for k=1:sx(2)-1
    if sign(bp(k))~=sign(bp(k+1)) && i<1
        i=i+1;
        k1=k;
        zz(i)=(x(k)+x(k+1))/2;
    end
end

% use zero of bp as a starting point to find zero of b

i2=0;
for k=k1:-1:2
    if sign(b(k))~=sign(b(k-1)) && i2<1
        i2=i2+1;
        zz2(i2)=(x(k)+x(k-1))/2;
    end
end
```



```
end

% for k=k1:1:sx(2)-1
%   if sign(beTE(k))~=sign(beTE(k+1)) && iter2<4
%       iter2=iter2+1;
%       zz2(iter2)=(x(k)+x(k-1))/2;
%   end
% end

xs=zz2/nnn;
```

### WGM\_open.m

```
function [vr,Qr]=WGM_open(n,v,R,s)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Code to find the resonance positions of a microdisk using EI      %
% approximation method. The disk height and the refractive indices % %
of the layers are fixed for this specific project.                %
%                                                                    %
% Requires: eff.m                                                  %
%           WGM_closed.m                                           %
%                                                                    %
% Inputs:                                                          %
%           n radial mode number                                   %
%           v frequency of experimental resonance                  %
%           s TE or TM mode                                        %
%           R disk radius [m]                                      %
%                                                                    %
% Outputs                                                         %
%           vr frequency of the open resonator resonance[Hz]      %
%           vc frequency of the closed resonator resonance[Hz]    %
%           Qr Q factor of the modes                             %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Parameter definition

n1=1.0;
[n2TE,n2TM]=eff(v);
ns=1.45;

if s=='TE'
    n2=n2TE;
    yita=n1/n2;
    a=1;
elseif s=='TM'
    n2=n2TM;
    yita=n2/n1;
    a=2;
end

mg=round((2*3.14*R*n2*v)/c);

i=0;
mvec=[mg-5:1:mg+5];
for m=mg-5:1:mg+5
    i=i+1;
    vS=WGM(m,a);
```

```
krealS=vS/R;
kimagE=0;

kVec=fminsearch('equK_fminsearch',[krealS,kimagE],optimset('disp','off
','tolX',1.0e-16,'tolFun',1.0e-16),m,n1,n2,R,yita);
vopen(i)=(3E8/(2*pi))*kVec(1)
vclosed(i)=vS
Q(i)=-kVec(1)/2/kVec(2)
end

vr=vopen;
Qr=Q;
```

### Complex plot.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Script to plot the characteristic equation over the complex plane%
% to aid with mode identification.                                     %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
clear all;

pol=input('Incident polarisation(TE/TM): ','s');
R=input('Input microdisk radius [um]: ');
v=input('input frequency [nm]: ');
m=input('Input approximated mode number m');

n1=1;
[neffTE,neffTM]=eff(v);

if s=='TE'
    n2=neffTE;
    yita=n1/n2;
elseif s=='TM'
    n2=neffTM;
    yita=n2/n1;
end

v1=(v-100E12):1E12:1200E12;
k1=(2*3.14/(3E8))*v1;
k2=[-5E5:1E3:5E5];
[K1,K2]=meshgrid(k1,k2);
k=K1+i.*K2;
t= -yita.*(besselj(m-1,k.*n2.*R)-
besselj(m+1,k.*n2.*R)).*besselh(m,1,k.*n1.*R)+...
(besselh(m-1,1,k.*n1.*R)-
besselh(m+1,1,k.*n1.*R)).*besselj(m,k.*n2.*R);
yy=abs(t);

[K3,K4]=meshgrid(v1,k2);

figure, surface(K1*R,K2,log10(yy)); shading interp;
```



