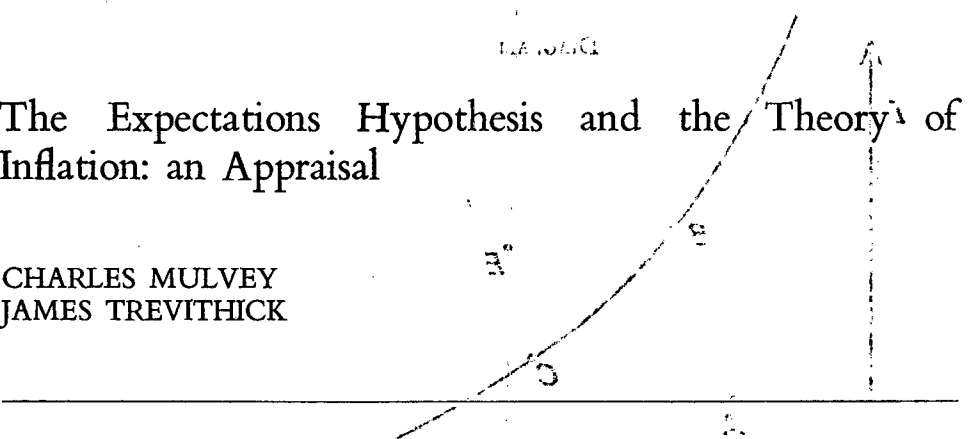


The Expectations Hypothesis and the Theory of Inflation: an Appraisal

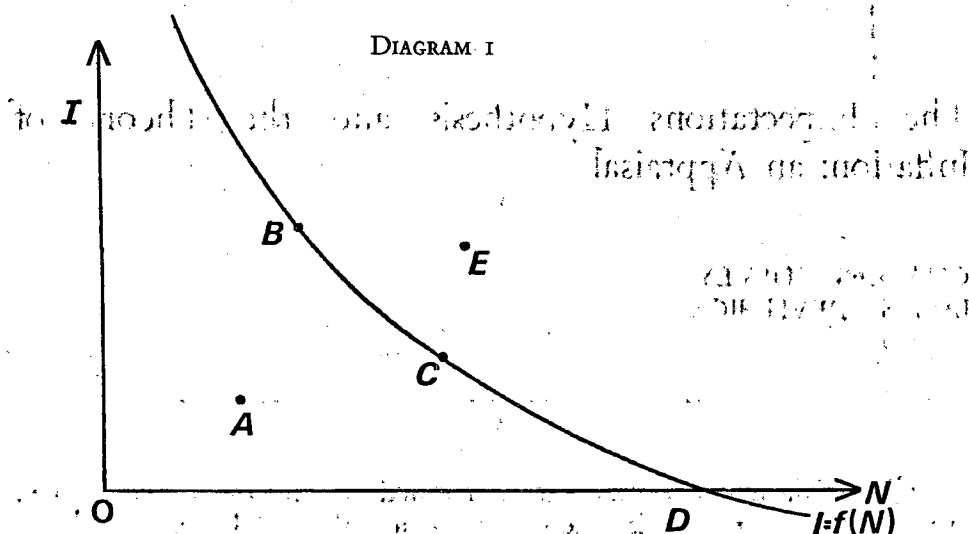
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ONE of the most interesting and significant developments to have emerged in recent years from the discussion of the nature of the "Phillips Curve" has centred around the role of expectations in the inflationary process. Quite apart from the importance of this issue for the theory of inflation the debate on the role of expectations has great significance for economic policy at the moment since it calls into question the long-run stability of the inflation/unemployment trade-off relation. The purpose of this paper is to review the most important contributions to the expectations debate, to present them in a coherent and accessible form paying particular attention to the adaptive expectations hypothesis which has been extensively used in the literature on the subject.

The acceptance of the existence of a stable Phillips curve relationship as a matter of empirical fact has obvious, and wide-ranging, policy implications. The relationship describes the effective constraint which any government faces when its policy objectives consist, *inter alia*, of full-employment and the elimination of inflation. It presents in the clearest possible light the stark inconsistency of a policy which aims at driving unemployment down to its absolute floor while, at the same time, attempting to eradicate inflation. "Low" unemployment is bought at the price of a positive rate of inflation; the rate of inflation can be reduced to zero only by a policy designed to raise the level of unemployment. The Government must face up to the set of policy alternatives indicated by the trade-off relationship and any attempt to obtain an unemployment-inflation mix such as the one indicated by point A in diagram 1 can only be interpreted as unrealistic, wishful thinking. The only set of points which represent a feasible choice between unemployment (N) and inflation (I) are points such as B, C and D.

Even if the trade-off relation is accepted as an accurate description of the constraint which a government faces in planning for "low" inflation and "low" unemployment, several questions remain unanswered. Given the many combinations of unemployment and inflation actually feasible, which combination will



the government in fact select? An answer to this question will be attempted in section 1 of this paper.

Of more fundamental importance is the question: Does the trade-off relation, in fact, exist as a stable relationship over time or will it shift under certain circumstances? Many writers on the subject of inflation believe that the trade-off relation is only a short-run set of policy alternatives open to the government; that if a particular combination of unemployment and inflation is selected and is such that a certain amount of unanticipated inflation occurs, then considerations of economic rationality would set forces in motion which would cause the trade-off to shift over time. This is what we shall call the "expectations hypothesis" advanced by Friedman [1], Phelps [2, 2a] and others; it is clearly of vital importance for government policy towards unemployment and inflation. The theoretical implications of introducing expectations into a model of inflation will be examined in sections 2 and 3 of this paper.

The question of the long run stability of the trade-off relation can only be answered by the application of empirical methods to the available data. In section 4 of this paper, recent econometric work on the expectations hypothesis is cited and evaluated. The main conclusion of this section is that, although expectations play an important role in the process of inflation, the case for the extreme expectations hypothesis (*viz.* that no long run trade-off exists) cannot be maintained.

Section 1: The Optimal Combination of Unemployment and Inflation

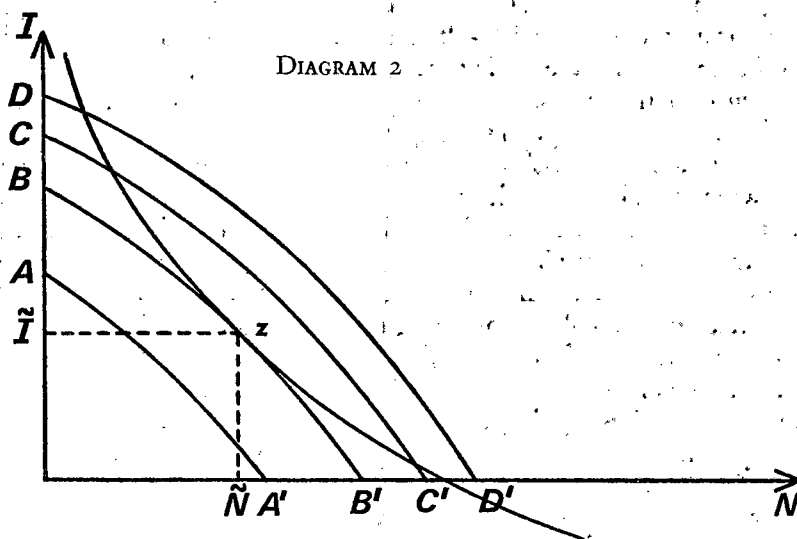
The underlying assumption of this section of the paper is that there exists a stable long run trade-off relation between inflation and unemployment, the latter being a proxy variable for the pressure of aggregate demand. That is to say, we assume that if the government were to choose any point on the trade-off relation,

combining a given rate of inflation and a given percentage rate of unemployment, then this policy combination will remain feasible over a considerable period of time; the trade-off relation will show no tendency to shift bodily with the passage of time. If this assumption can be shown to be correct, we will see below that standard static utility maximisation techniques may be applied to the problem of finding the optimal inflation-unemployment mix.

Consider a social utility function indicating the way in which social utility (evaluated by political decision-makers) is affected by differing combinations of unemployment and inflation. The dependence of social utility (U) upon the rate of inflation and the unemployment rate may be expressed symbolically as follows:

$$U = \phi(I, N) \quad (1)$$

The problem of finding the optimum unemployment-inflation mix may be solved by classical maximisation techniques. The government must select a certain combination of inflation and unemployment, (I, N) , which will maximise $\phi(I, N)$ subject to the single constraint $I = (fN)$. A diagrammatic interpretation, analogous to the indifference curve analysis of elementary consumer theory, is given below (re Lipsey (3)).



The curves AA' , BB' and DD' are the concave indifference curves, generated by the social utility function $U = \phi(I, N)$. It is evident that those indifference curves situated near the origin indicate the enjoyment of a higher level of social utility than those situated at a greater distance from the origin. It is assumed that the economic decision-makers will aim at arriving at the lowest possible indifference

curve, i.e., that indifference curve situated nearest to the origin (furthest to the "south-west").

The first order condition for a welfare optimum requires that the combination of unemployment and inflation actually selected by economic decision-makers should be such that the marginal rate of substitution between unemployment and inflation should be equal to the slope of the trade-off constraint. As can be seen from diagram 2, this condition is fulfilled at point Z ; at point Z , the constraint is tangential to the lowest attainable social indifference curve BB' . Further study of the same diagram makes it clear that social welfare is maximised when unemployment is $\bar{N}$ and inflation is I ; Lipsey terms $\bar{N}$ and I the "acceptable unemployment rate" and the "acceptable rate of inflation" respectively. Bearing in mind that the unemployment rate is amenable to considerable manipulation by the monetary and fiscal policies of the economic authorities, a rational government will aim at adjusting the overall pressure of demand in the economy to such a level as is consistent with an unemployment rate $\bar{N}$; a rate of inflation I^* will be a necessary consequence of such an unemployment rate if we assume a "demand-pull" interpretation of the trade-off relation.

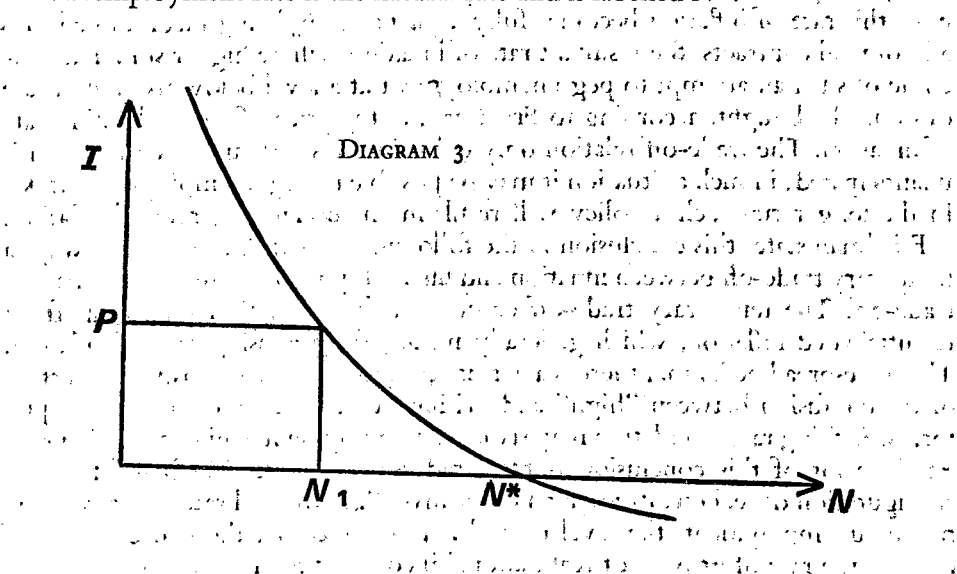
Section 2: The Stability of the Trade-off Relation

Following upon the initial enthusiasm which greeted the largely empirical work of Phillips [3], Lipsey [4] *et al.*, there has arisen a growing scepticism concerning the long run properties of the trade-off relation. While conceding that a government may be able to use the policy instruments at its disposal to arrive at an unemployment-inflation mix such as the one indicated in diagram 2, many writers (Friedman [1], Phelps [2], Cagan [5], etc.) contend that such a policy combination can only be maintained in the short run. In the medium and long run, forces will be at work which will have the ultimate result of making point Z a non-optimal (and infeasible) policy choice. Crucial to this objection to a stable trade-off relation is the role that expectations play in determining the current rate of inflation.

The concept of the "natural" rate of unemployment plays a large part in Friedman's attack on the stability of a long run unemployment-inflation relation. "At any moment of time", writes Friedman, "there is some level of unemployment which has the property that it is consistent with equilibrium in the structure of real wage rates." This equilibrium level is termed by Friedman the "natural rate of unemployment". If this rate of unemployment were to be maintained over a period of time, real wage rates could be expected to rise in line with labour productivity. If, as a result of government economic policy, the actual unemployment rate was to fall below the natural unemployment rate, there would be "... an excess demand for labour which will produce upward pressure on real wage rates". Similarly there would be a downward pressure on real wage rates if the actual rate of unemployment were to exceed the natural rate. Friedman goes to great pains to point out the dissimilarity between his analysis and that of Phillips. Whereas Phillips' original contribution to the study of wage inflation was couched entirely in terms of money wage rates, Friedman believes that the

central subject of study should be the behaviour of *real* wage rates at various levels of unemployment. On the other hand, an examination of how money wage rates respond to varying levels of aggregate demand may not be too misleading when the "average" rate of change of prices has remained fairly constant over a substantial period of time; in such a situation there is no tendency for the anticipated rate of price increase to change appreciably. In contrast, in situations where the anticipated rate of price increase varies considerably, the trade-off relation will be ill-defined.

Much of Friedman's analysis refers to the stability of the Phillips curve (which, it will be remembered, attempts to demonstrate an association between the rate of change of *money wage* rates and the unemployment rate). Exposition will be greatly simplified without changing the crux of the argument if we switch to the trade-off relation $I = f(N)$ in which, once again, the variable on the left-hand side of the equation is the rate of change of *prices*. The trade-off relation which we have used up to this point helps greatly in following Friedman's chain of reasoning. Fundamental to Friedman's objection to the existence of a stable trade-off relation is the notion that, for each anticipated rate of inflation, there corresponds a particular trade-off relation; the higher the anticipated rate of inflation, the further to the north-east will be the trade-off relation. In the example we use below, we will assume that prices have been stable over a long period of time. We assume the unemployment rate is at a level which will perpetuate this stability in prices. This unemployment rate is the natural rate and is labelled N^* .



Since price stability is presumed to have existed for a considerable period in the past, it would be natural to expect that the anticipated rate of price increase would be zero. More will be said later in this paper about the exact formation of expectations in such models of inflation; suffice it to say at this point that Friedman assigns

an all-important role to past experience of price inflation in determining the anticipated or expected rate of price inflation. For example, if the rate of inflation has been 5 per cent per annum over a long period in the past, then such models usually assume that the anticipated rate of inflation will be somewhere in the region of 5 per cent per annum.

Returning to diagram 3, it is now assumed that the government decides that its target rate of unemployment is to be N_1 , $N_1 < N^*$, and that it takes the appropriate monetary and fiscal action to achieve this desired unemployment rate. The "market" unemployment rate of N_1 will then be below the "natural" rate. Initially prices will rise and there will be a *decline* in the real wage rate despite the fact that the higher level of labour demand indicated by an unemployment rate of N_1 would lead workers to expect a *rise* in real wage rates. But this situation will not last for long. "Employees will start to reckon on rising prices of the things they buy and to demand higher nominal wages for the future". Real wages will start to rise to their initial level, or perhaps higher. If a rate of inflation of p per cent were to persist as a result of the unemployment-inflation choice of the government (see section 1), such a rate of inflation would be taken into account in all contracts fixing nominal factor prices. Once this positive rate of inflation is fully anticipated by all the parties involved in the fixing of nominal contracts, the rate of inflation will rise; and once the rate of inflation has risen, anticipations of inflation will be revised in an upward direction. Hence an unemployment rate of N_1 will give rise to an initial unanticipated rate of inflation of p per cent; but once this rate of inflation becomes fully anticipated by being incorporated into all nominal contracts, the resultant rate of inflation will be higher still. The outcome of such an attempt to peg unemployment at a level below its natural rate can only be bought, according to Friedman, at the price of an accelerating rate of inflation. The trade-off relation only exists in the short run when inflation is unanticipated; in such a situation it may be possible to peg unemployment at N_1 . In the longer run such a policy will result in an accelerating rate of inflation.

Friedman states this conclusion in the following terms: "... there is always a temporary trade-off between inflation and unemployment; there is no permanent trade-off. The temporary trade-off comes not from inflation *per se*, but from unanticipated inflation, which generally means, from a rising rate of inflation. The widespread belief that there is a permanent trade-off is a sophisticated version of the confusion between "high" and "rising" that we all recognise in simpler forms. A rising rate of inflation may reduce unemployment, a high rate will not." At the root of this conclusion is that traditional neo-classical belief that the configuration of real quantities in an economy—"... the real rate of interest, the rate of unemployment, the level of real national income, the real quantity of money, the rate of growth of real national income or the rate of growth of the real quantity of money"—is determined independently of the rate of price inflation once this rate of inflation has come to be fully anticipated.

In order to provide a framework for a discussion on the econometric work on this "expectations hypothesis", it is necessary to translate Friedman's analysis into

a form more amenable to empirical scrutiny. The existence of a permanent trade-off between unemployment and inflation implies that: $I = g(Z)$ where Z is a vector of real variables which could conceivably have an effect on I ; for example, Z could include such variables as the rate of increase of import prices, the rate of increase of labour productivity etc. For expositional simplicity it will be assumed that these other factors exert a negligible influence on the rate of inflation and that the two dimensional trade-off relation represented by $I = f(N)$ describes the fundamental process of inflation. This is the extreme position of those who believe in a permanent trade-off.

On the other hand, the extreme expectations hypothesis implies the following relationship:

$$I = f(N) + x \quad (3)$$

where x is the expected or anticipated rate of inflation. Rearranging equation (3) will make the expectations hypothesis clearer:

$$I - x = f(N) \quad (4)$$

The current rate of inflation can be divided into two parts: that part which is fully anticipated: x ; and that part which is unanticipated: $I - x$, i.e., the left hand side of equation (4). The expectations hypothesis states that only unanticipated inflation will vary with the unemployment rate. A further implication of this hypothesis is that the only situation in which inflation is fully anticipated is when unemployment is at its natural rate. The natural rate of unemployment may be found very easily (in theory at least); when inflation is fully anticipated, $I = x$ so that the natural rate of unemployment is found by evaluating the root of $f(N) = 0$.

But the idea of an *accelerating* rate of inflation cannot be deduced from the foregoing analysis. It depends upon an assumption concerning the formation of expectations which is implicit in Friedman's account. The current expected rate of inflation, x , is supposed to depend systematically upon the actual rates of inflation experienced in the past. Symbolically this may be written:

$$x_t = h\{I_{t-1}, I_{t-2}, I_{t-3}, \dots, I_{t-n}, \dots\} \quad (5)$$

The exact manner in which the expected rate of inflation depends upon past rates of inflation needs to be spelt out in greater detail. Since expectations are unobservable and unquantifiable in their own right, *a priori* restrictions must be imposed upon the formation of such a variable as x . One convenient and, therefore, common way of describing the emergence of expectations is that of "adaptive expectations". The adaptive expectations model states that the expected rate of inflation in period t is a weighted average of all past rates of inflation. The

weights are selected in such a way that greater importance is attached to more recently experienced rates of inflation; more specifically, I_{t-1} , I_{t-2} , I_{t-3} , etc., are assigned the respective weights $(1-\psi)$, $\psi(1-\psi)$, $\psi^2(1-\psi)$, etc., where ψ is greater than or equal to zero but less than one; a weighting system which follows this pattern is normally described as being one of geometrically decreasing weights since the coefficients of the distributed lag equation (6) below decline in geometric progression. The adaptive expectations model therefore postulates that expectations are formed according to the following scheme:

$$x_t = (1-\psi)I_{t-1} + (1-\psi)\psi I_{t-2} + (1-\psi)\psi^2 I_{t-3} + (1-\psi)\psi^3 I_{t-4} + \dots$$

or

$$x_t = (1-\psi) \sum_{j=1}^{\infty} \psi^{j-1} I_{t-j} \quad (6)$$

The assumption that ψ satisfied that property that $0 \leq \psi < 1$ is important on two accounts: firstly it ensures that expression (6) has a finite limit, i.e., that the geometric progression converges; secondly it produces the economically meaningful result that the more distant is the rate of inflation, the less is the importance attached to it.

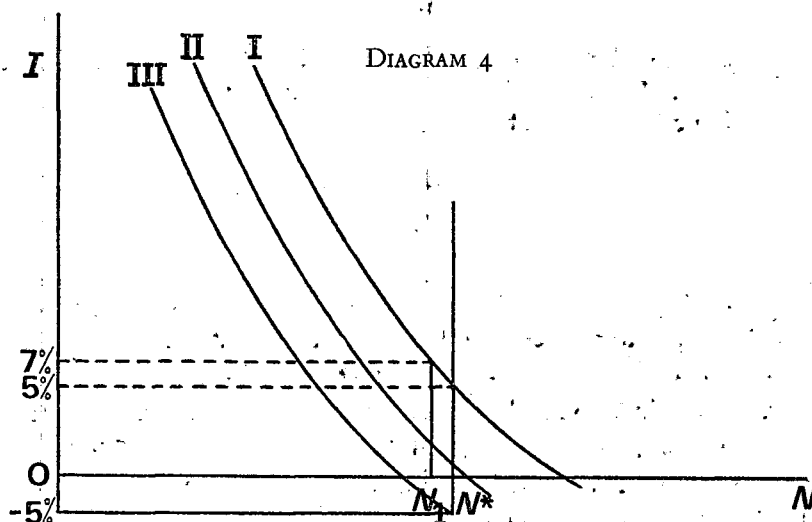
It can be easily verified that the adaptive expectations model described in equation (6) is equivalent to the following statement:

$$x_t - x_{t-1} = (1-\psi)(I_{t-1} - x_{t-1}) \quad (7)$$

This description of how expectations evolve is extensively used in the theory of inflation (re. Cagan [5] and Solow [7]) and will be discussed further in Section 4 of this paper.

If it can be assumed, as the adaptive expectations model postulates, that the current expected rate of inflation depends systematically upon past rates of inflation, and reacts most sensitively to the most recently experienced rate of inflation, the notion of an accelerating rate of inflation becomes clearer. When the actual rate of inflation exceeds the expected rate of inflation in a given period, a higher expected rate of inflation will be generated in the subsequent period, leading to a higher *actual* rate of inflation in this period. Since there is no tendency for the unanticipated part of inflation to vanish over time if the government pursues a policy of maintaining unemployment at a level below its natural rate, there will always be a tendency for expectations to be revised upwards in a vain attempt to catch up with the actual rate of inflation. It is this continuous upward revision in the expected rate of inflation which produces the acceleration in the actual rate of inflation. The only situation in which such revision will not occur is when unemployment is at its natural rate.

From the foregoing discussion it is clear that the natural rate of unemployment is compatible with any rate of anticipated price inflation.



In diagram 4, three temporary trade-off relations are drawn. Curve I refers to a situation in which the expected rate of inflation is 5 per cent; curve II to one in which the expected rate of inflation is 0 per cent and curve III refers to a situation in which the expected rate of price deflation is 5 per cent.

For the sake of example consider curve I. The actual rate of inflation will only equal the expected rate when unemployment is at its natural rate N^* . In this case there will be no tendency for the rate of inflation to accelerate since there is no necessity for expectations to be revised upwards. On the other hand if the government aimed at keeping unemployment down to a level N_1 , the rate of inflation would be 7 per cent per annum, 2 per cent higher than anticipated. Expectations will be revised upwards so that the whole trade-off relation will drift upwards. The upward displacement of the trade-off relation will continue so long as the unemployment rate falls short of N^* . The same applies, *mutatis mutandis*, to a situation in which the expected rate of inflation is initially zero (as along curve II) or initially negative (as along curve III).

The fundamental relation underlying the extreme expectations hypothesis is equation (3) which states, once again, that

$$I = f(N) + x. \quad (3)$$

It is essential that x should enter equation (3) with a coefficient of unity. If it could be demonstrated that the coefficient of x is significantly different from unity, then a permanent trade-off relation could be said to exist. Suppose the coefficient of x were θ and assume that the rate of inflation is fully anticipated. The revised version of equation (3) would then be:

$$I = f(N) + \theta x \quad (8)$$

Since we have assumed that the current rate of inflation is fully anticipated, it follows that $I = x$. In such a situation, a permanent trade-off relation would emerge, as is indicated by equation (9).

$$I = \frac{f(N)}{1-\theta}, \quad 0 \leq \theta < 1 \quad (9)$$

Equation (9) has important implications for the existence of a stable trade-off relation. For an arbitrarily selected level of unemployment, any divergence between the actual and expected rates of inflation will disappear over time and the eventual rate of inflation will be $\frac{f(N)}{1-\theta}$. Hence it is not only the natural rate of unemployment that is capable of sustaining a constant rate of inflation; any positive rate of unemployment will eventually produce a constant rate of inflation (which may, of course, be positive or negative inflation).

Two important questions of empirical fact remain to be answered. The first is the speed of reaction of the expectations variable x to recently experienced rates of inflation. If the rate of inflation were to take a dramatic turn in an upward direction, how quickly would the complete upward revision in the expectations variable be accomplished? An answer to this question will depend upon the exact value of ψ in equation (6). If ψ is near zero, only the most recently experienced rates of inflation will count and complete revision will be rapid. In contrast, if ψ is nearer unity, temporally remote rates of inflation will be assigned greater importance and the process of complete revision will be very slow indeed. This question has wide-ranging implications for economic policy. If a government is certain that expectations take a long time to be revised either upwards or downwards, it may be able to behave as if a permanent trade-off between unemployment and inflation existed. We return to this question in Section 4 of this chapter.

The second question to be answered relates to the precise value of θ . A value of θ which turns out to be not significantly different from unity will constitute strong evidence in favour of the extreme expectations hypothesis. A value of θ which is not significantly different from zero is evidence in favour of the extreme version of the permanent trade-off hypothesis.

Such a value (i.e. one which is not significantly different from zero) will imply the existence of widespread "money illusion" and many writers on the subject subscribe to the belief that whereas such a phenomenon may be possible in the short run, it cannot prevail in the long run. "I must admit to the view", writes Cagan [6], "that in the long run there is no money illusion and that the coefficient of price expectations variables should add up to unity." This *a priori* conviction seems to be quite pervasive in many of Friedman's writings. Professor Tobin [8] has subjected this conviction to extensive criticism. "I must confess to a strong

sense of *déjà vu*," he writes, "The current debate strikes me as the Keynes-classics debate of the 1930's removed thirty years and one time derivative." According to Tobin, "The Phillips curve idea is a reincarnation in dynamic guise of the original Keynesian idea of irrational 'money illusion' in the supply of labour." The extent of such money illusion will be indicated by the magnitude of the parameter.

Section 3: The Implications of the Expectations Hypothesis for Optimal Employment over Time

In Section 1 we examined the procedure by which a government would select the optimum combination of unemployment and inflation on the assumption that the trade-off relation was stable in the long run. This assumption has been questioned by the advocates of the expectations hypothesis and we examined their views in Section 2. It will have become apparent by now, however, that a theory which postulates that the trade-off relation is shifting continually (unless unemployment is kept at its "natural" level) will adopt a fundamentally different approach to the problem of an optimal unemployment-inflation mix from that presented in Section 1. In that section, the standard techniques of static utility maximisation can be validly applied to arrive at an optimal combination of unemployment and inflation which, on the assumption of an unchanging utility function, *can be maintained forever*. Such techniques cannot be applied to situations in which the relation is continually shifting due to the selection of particular rates of inflation. The problem of intertemporal optimisation assumes an all-important role and techniques appropriate to the solution of this problem must be called upon.

The first attempt at framing the problem in terms of intertemporal optimisation was contained in a celebrated article by Edmund Phelps [2] and the analysis presented below is a simplification of his pioneering contribution. Parts 1 to 4 will set out the basic ingredients of Phelps's analysis; part 5 will state the nature of the problem of intertemporal optimisation in terms of these ingredients; and in part 6 an attempt will be made at solving this problem and at deriving an optimal rule concerning the utility maximising level of demand.

1. The Quasi-Phillips Curve

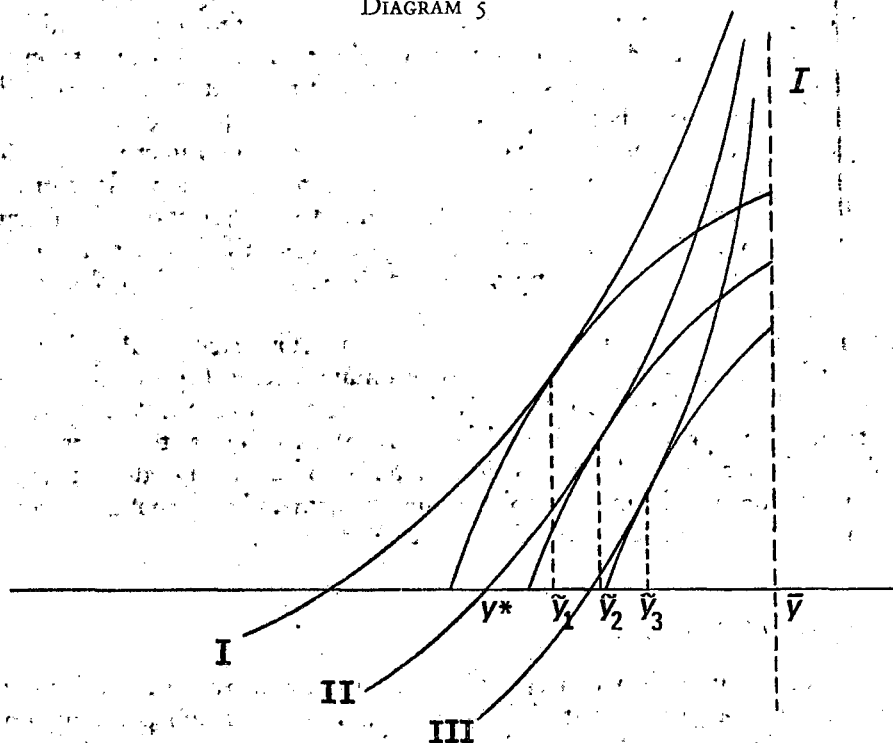
Phelps starts off with a concept with which we are by now familiar—the Phillips curve—but presents it in a new light. Instead of concentrating upon the relationship between the rate of change of money wage rates and the unemployment rate, as Phillips and others have done, he focuses attention on the relationship between the rate of change of prices and the level of capacity utilisation, y , the latter being inversely correlated with the unemployment rate. In this guise, the strict expectations hypothesis becomes:

$$I = f(\gamma) + x \quad (10)$$

Certain restrictions are imposed upon the variation in $f(\gamma)$: (i) $f'(\gamma) > 0$ which means that the higher the level of capacity utilisation, the greater is the unanticipated rate of inflation for a given x . (ii) $f''(\gamma) > 0$ which is the counterpart of a convex Phillips curve where $g''(N) > 0$. (iii) the upper bound of γ is $\bar{\gamma}$, which is determined by considerations of population and labour supply; moreover, $\lim_{\gamma \rightarrow \bar{\gamma}} f(\gamma) = \infty$. (iv) μ is some minimal level of γ "... such that output will only

be large enough to permit production of the programmed investment, leaving no employed resources for the production of consumption goods." The $f(\gamma)$ relationship has another interesting feature; when $I = x$, $f(\gamma) = 0$ and $\gamma = \gamma^*$, i.e. $f(\gamma^*) = 0$; γ^* will be recognised as that level of capacity utilisation which will entail a level of unemployment equal to the natural rate of unemployment. These restrictions and properties are illustrated in diagram 5 below.

DIAGRAM 5



Three hypothetical quasi-Phillips curves are shown in diagram 5; each corresponding to a different expected rate of inflation. Curve I corresponds to a positive expected rate of inflation ($x_1 > 0$); curve II corresponds to a zero expected rate of

inflation ($x_2 = 0$); and curve III corresponds to a negative expected rate of inflation ($x_3 < 0$).

The statical utility maximising utilisation ratio is $\bar{\gamma}_1$ when $x = x_1$, $\bar{\gamma}_2$ when $x = x_2$ and $\bar{\gamma}_3$ when $x = x_3$. In general the statical optimal utilisation ratio will be assumed to be a decreasing function of the expected rate of inflation. We shall assume that the initial anticipated rate of inflation, x_0 , is a datum so that we may refer to a single statical optimal utilisation ratio $\bar{\gamma}$.

2. The Adaptive Expectations Mechanism

We have already encountered this notion in Section 2 so that little further explanation is necessary, except to rewrite equation (7) in continuous form:

$$\begin{aligned}\dot{x} &= \frac{dx}{dt} = a(I - x) \\ &= a(\{f(\gamma) + x\} - x) \quad \text{by equation (10)} \\ &= af(\gamma) \\ &= G(\gamma).\end{aligned}\tag{11}$$

Hence, the rate of change of the expected rate of inflation, $\dot{x}$, is an increasing function of the utilisation ratio ($G'(\gamma) > 0$) and the rate of acceleration increases as γ increases ($G''(\gamma) > 0$). Moreover, when $\gamma = \gamma^*$, $G(\gamma) = 0$, i.e. when the expected and actual rates of inflation are equal, there is no tendency for individuals to revise their expectations in either direction.

3. The Utility Function

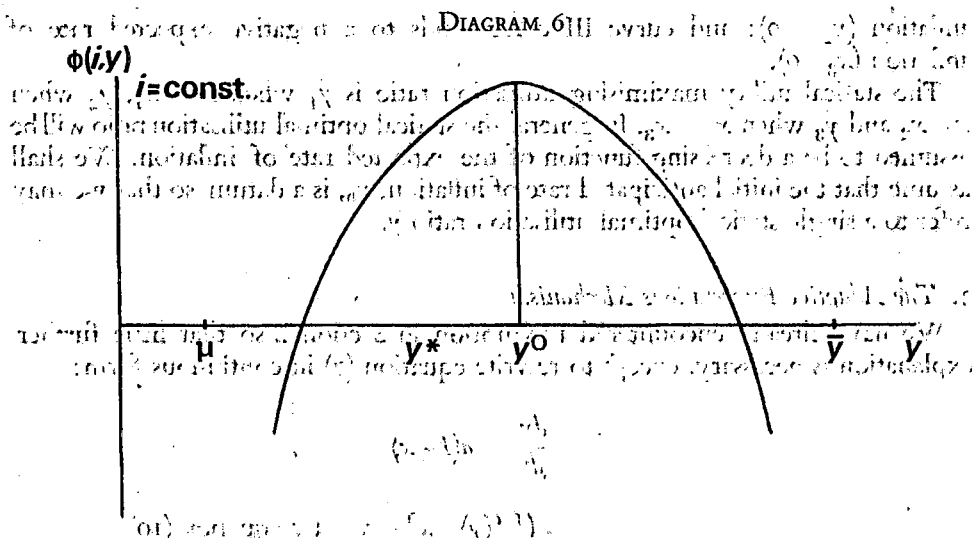
Phelps starts out with a utility function which at first sight may appear to have little relevance to the problem discussed in Section 1. At any point of time, utility is assumed to depend upon two variables: (i) the level of capacity utilisation, γ , and (ii) the money rate of interest, i . In symbols, this dependence may be written:

$$U = \phi(i, \gamma)\tag{12}$$

The exact nature of the dependence of U on γ and i needs to be spelt out in greater detail before further analysis of the optimisation can take place.

(i) The Relationship between U and γ .

The fundamental dependence of U on γ is best illustrated by assuming that the money rate of interest does not vary and by constructing the following diagram.



It is assumed that there are two basic desiderata entering any utility function—consumption and leisure. As y rises, consumption rises and undue incursions into the community's leisure time are not made so that the utility is rising as y rises in this range (in other words the marginal utility of utilisation is positive in this range). As y increases still further, however, greater demands will be made upon leisure time. Beyond a certain point, y^0 in the above diagram, the disutility associated with incursions made upon leisure time by a higher y will outweigh increased utility from higher consumption. To the right of y^0 , higher consumption is not worth the sacrifice of leisure required to obtain it.¹ As full capacity is approached, incursions on leisure become intolerable so that as y approaches $\bar{y}$, the $\phi(i, y)_{i=\text{const.}}$ curve tends to minus infinity. Similarly as y falls towards μ , the reductions in consumption become more and more alarming as y approaches μ , the $\phi(i, y)_{i=\text{const.}}$ curve tends to minus infinity. These assumptions account for the shape of the curve in diagram 6.2.

(ii) *The Relationship between U and i .*

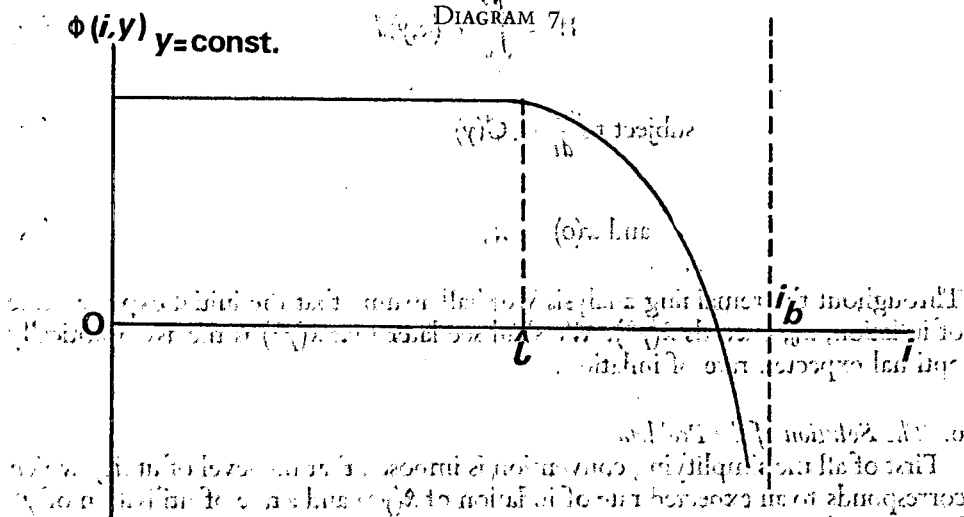
We can examine the dependence of U on i in a similar manner. Assume that the utilisation ratio is constant and examine the variation in U consequent upon variation in i .

If i varies within the range $0 \leq i \leq \bar{i}$, no diminution in utility will occur as a result of either an increase or a decrease in i . Once i rises beyond a certain interest rate $\bar{i}$, however, further increases in i will bring about a reduction in the level of utility since it will pay to economise on the holding of cash balances by more frequent visits to the bank, etc. Such attempts to economise on the holding of transactions

1. The existence of involuntary unemployment implies that the equilibrium utilisation level y^* will not coincide with y^0 . In the diagram it is assumed that $y^* < y^0$.

$\phi(i, y)$ $y = \text{const.}$

DIAGRAM 7



balances will produce disutility for the holder of such balances by reducing his leisure time. As i approaches $\bar{i}$, which Phelps calls the *barter point*, the monetary system breaks down since the holding of any transactions balances becomes prohibitively expensive. If $i \leq \bar{i}$, full liquidity is said to exist.

4. The Real and Money Rates of Interest

The money rate of interest, i , is equal to the real rate of interest, r , plus the expected rate of inflation, x , i.e. $i = r + x$. The real rate of interest is assumed to vary directly with the level of capacity utilisation so that $r = r(y)$. Clearly $r(y) > 0$ and $r'(y) > 0$; a simplifying but not a crucial assumption is that $r''(y) \geq 0$. Hence if $i = r(y) + x$, $U_i = \phi(r(y) + x, y) = U(x, y)$ (13).

The expected rate of inflation which is just sufficient to permit full liquidity at $i = \bar{i}$, given the level of capacity utilisation, is denoted by $\hat{x}$. $\hat{x}$ satisfies the relation $\bar{i} = r(y) + \hat{x}$ for a given y . The greater is y , the smaller the expected rate of inflation which is consistent with full liquidity. We may therefore write $\hat{x} = \hat{x}(y)$ where $\hat{x}'(y) < 0$. Of particular interest in subsequent analysis is that value of x at which there is full liquidity ($i = \bar{i}$) and equilibrium ($y = y^*$); this value of x is denoted by $\hat{x}(y^*)$.

5. The Problem to be Solved

The dynamic problem to be solved consists of the maximisation of social welfare over time. It involves the discovery of an optimal time path for $y(t)$ (or $x(t)$): Social welfare, W , is the sum of future utilities from $t = 0$ onwards. This must be maximised subject to equation (11) and the initial condition that $x(0) = x_0$. Stated more formally the problem is one of maximising

$$W = \int_0^{\infty} U(x, \gamma) dt \quad (14)$$

$$\text{subject to } \frac{dx}{dt} = G(\gamma) \quad (11)$$

$$\text{and } x(0) = x_0 \quad (15)$$

Throughout the remaining analysis we shall assume that the initial expected rate of inflation, x_0 , exceeds $\hat{x}(\gamma^*)$. We shall see later that $\hat{x}(\gamma^*)$ is the asymptotically optimal expected rate of inflation.

6. The Solution of the Problem

First of all the simplifying convention is imposed that the level of utility which corresponds to an expected rate of inflation of $\hat{x}(\gamma^*)$ and a rate of utilisation of γ^* be set equal to zero, i.e. $U(\hat{x}(\gamma^*), \gamma^*) = 0$. This means that appropriate units of utility measurement are chosen so as to make $U(\hat{x}(\gamma^*), \gamma^*)$ equal to zero.

Since $\dot{x} = G(\gamma)$ and G is a monotonically increasing function of γ , we may write $\gamma = G^{-1}(\dot{x})$. Substituting into equation (13) the following functional remains to be maximised:

$$W = \int_0^{\infty} V(x, \dot{x}) dt = \int_0^{\infty} V(x, G) dt \quad (16)$$

$$\text{subject to } x(0) = x_0 \quad (15)$$

This is one of the simplest problems encountered in the calculus of variations and the solution has been well known to economists since the Ramsey-Keynes problem of optimal savings was posed. The necessary condition for a maximum is that

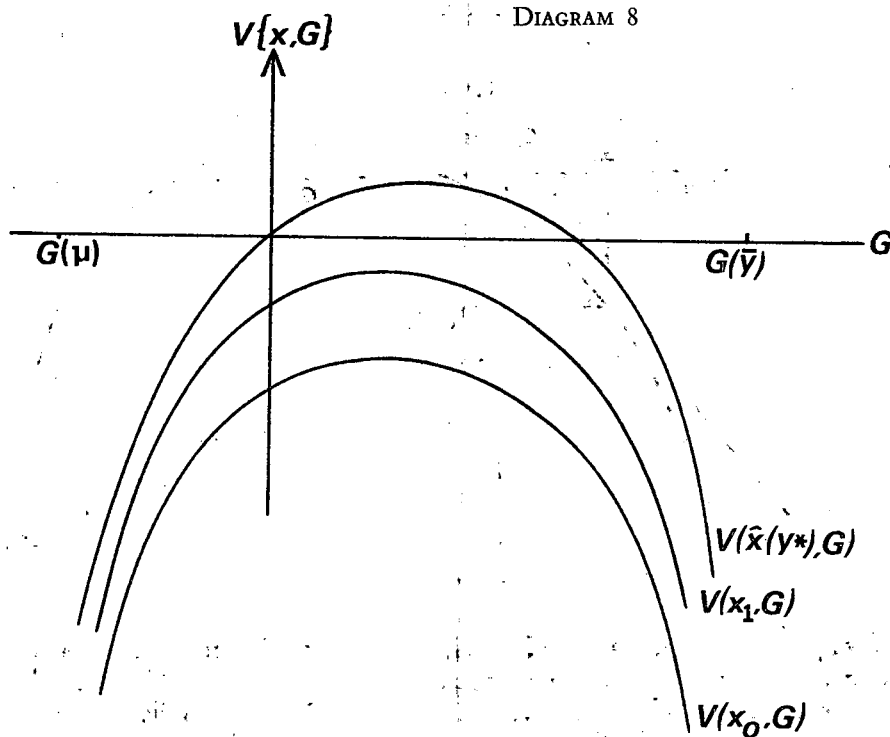
$$V(x, G) - G V_G(x, G) = 0 \quad (17)$$

$$\text{or } \frac{V(x, G)}{G} = V_G(x, G) \text{ where } V_G(x, G) = \frac{\partial V}{\partial G}$$

This optimal condition must be met at all points in time from $t = 0$ onwards. The value of G which satisfies equation (17) at a point in time is the optimal rate of change of the expected rate of inflation and is labelled G_0 .

It will be useful to illustrate graphically the $V(x, G)$ relation. Since we assume that the expected rate of inflation is initially constant at x_0 , we may invoke the restrictions placed on the utility function $\phi(i, \gamma)$ and deduce that the shape of the $V(x_0, G)$ function is as follows.

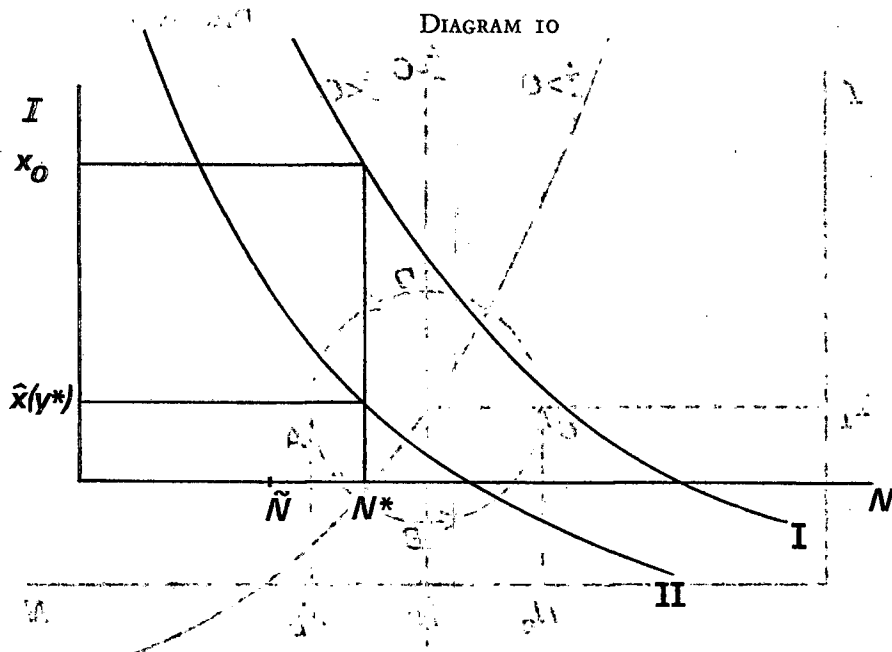
DIAGRAM 8



Suppose now that the expected rate of inflation fell to x_1 : the level of utility is approached for all G , i.e. the curve will have shifted upwards. This upward drift will continue until the curve reaches its limiting position which coincides with the curve $V(x(y^*), G)$ which, due to our felicitous choice of units, passes through the origin. The curve will cease to drift once $x = \hat{x}(y^*)$ since full liquidity will have been reached and no increase in utility will be achieved by a further fall in the expected rate of inflation. We are now in a position to interpret the optimality rule set out in (17) in terms of diagram 9 below.

At a point in time, the optimal value of G (and hence of γ) is found by drawing a line AO through the origin in such a way that it is tangential to the $V(x_0, G)$ curve. Denote the point of tangency by the symbol ψ . At ψ , the slope of the $V(x_0, G)$ function viz. $V_G(x_0, G)$, is equal to the slope of the tangent, $\frac{V(x_0, G)}{G}$.

This optimality condition is thus satisfied at point ψ . It follows that the optimal rate of decrease of the expected rate of inflation is G_0 , which is found by drawing a vertical line from ψ to the $G(\gamma)$ axis. Noting that the optimal rule at time $t = 0$ requires a negative $G(\gamma)$, we deduce that initially a government must follow a policy of *underutilisation* so as to drive down the expected rate of inflation.



employment can be steadily expanded so that in the limit, the unemployment rate will approach N^* and the expected rate of inflation will fall to $\hat{x}(y^*)$.

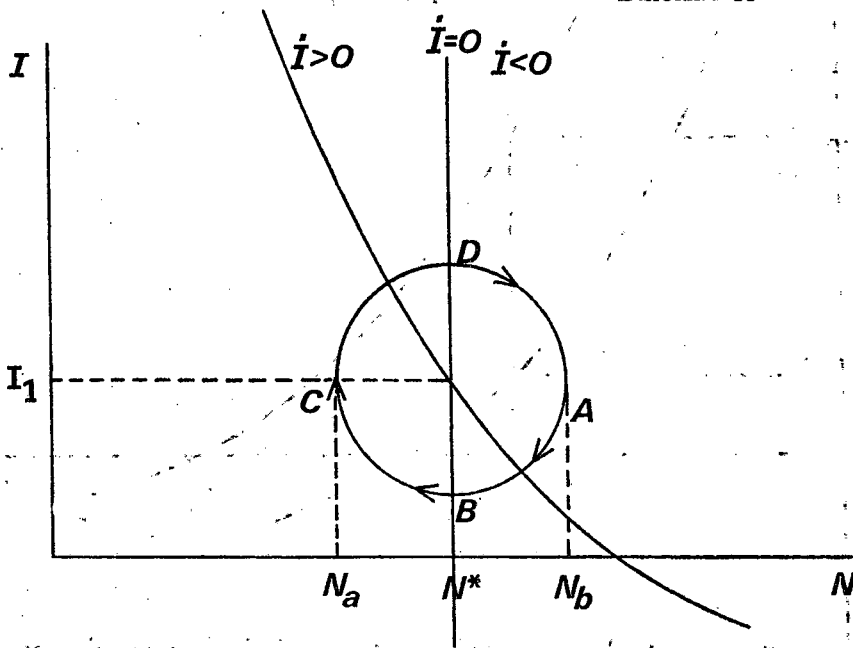
It will also be noted that, starting with an excessively high expected rate of inflation, a *unique* expected rate of inflation is optimal in the limit viz. $\hat{x}(y^*)$. This conclusion does not square with the static approach set out in diagram 4 in which *any* expected rate of inflation can yield an optimum.

Section 4: Empirical Evidence on the Expectations Hypothesis

The first attempt at testing empirically the Friedman-Phelps expectation hypothesis was undertaken by Brechling [9]. He makes use of the particular prediction of this hypothesis which states that when unemployment is below its natural rate, inflation will be accelerating; that when unemployment is at its natural rate, there will be a steady, fully anticipated rate of inflation; and that when unemployment is above its natural rate, inflation will be decelerating. This prediction implies the existence of some sort of loop around the short run trade-off relation which is shown in diagram II.

To start with it is assumed that there is a positive anticipated rate of inflation I_1 . In accordance with the expectations hypothesis, we deduce that at all points to the left of N^* , the natural unemployment rate, $\dot{y} > 0$; that at all points to the right of N^* , $\dot{y} < 0$; and that when the unemployment rate equals N^* , $\dot{y} = 0$ and $I = I_1$. Moreover, it is assumed that the initial unemployment rate is N_0 , so that the observed inflation-unemployment combination corresponds to point A in the diagram. Brechling now imposes arbitrary limits upon the variation of the

DIAGRAM II



unemployment rate. He makes the simplifying but expendable assumption that the unemployment rate varies smoothly between a lower limit N_a and an upper limit N_b (which for the sake of convenience coincides with the initial unemployment rate). If N^* falls within the interval (N_a, N_b) , then the expectations hypothesis will predict that clockwise loops will occur around the $f(N)$ relation. To illustrate how this phenomenon arises, we have assumed that the unemployment rate has just risen to its upper limit N_b and that from now on the government will undertake over a period of time such policies as will bring the unemployment rate down to its lower limit N_a . Since $N_b > N^*$, there will be a tendency for the rate of inflation to fall as N falls from N_b to N^* ; this tendency will be arrested when N reaches N^* ; and it will be reversed (i.e. inflation will accelerate) as N rises beyond N^* . The nearer the unemployment rate is pushed towards N_a , the faster the rate of inflation accelerates. If we make the additional assumption that once N_a is reached the government will undertake the appropriate measures needed to drive N steadily upwards towards N_b , the opposite process will occur: the rate of inflation will tend to decelerate. When point D is reached, the rate of inflation will have reached its maximum and will start falling as the unemployment rate is raised towards N_b . It is clearly an implication of the expectations hypothesis that if such smooth variations in unemployment were actually to occur, then clockwise loops of the sort described would develop even in the short run, provided, of course, that N^* is less than N_b and greater than N_a (an alternative test being appropriate when this proviso cannot be met).

As a test for this hypothesis using American data, Brechling assumes that $N_a = 3$ per cent, that $N_b = 9$ per cent and that there is a smooth cycle between these two limits. Using his own trade-off equation to predict the variation in the rate of inflation which would be produced by the smooth cycle of unemployment between its two limits, he discovered that the loops so generated rotated in an *anticlockwise* direction, which is contrary to the prediction of the expectation hypothesis. As further support for his contention, Brechling points to numerous econometric studies which discovered that when the *rate of change* of unemployment was included as an additional explanatory variable in regression equations, it in general entered with a negative coefficient. That is, it was generally found that when unemployment is falling, the rate of inflation is higher than when it is rising. This phenomenon produces anticlockwise loops and lends support to Brechling's tentative refutation of the expectations hypothesis.

Following upon the work of Brechling, a more searching econometric study was undertaken by Professor Solow [7]. Although his data referred to both British and American experience, the author himself placed greater trust, for a variety of reasons, in the results for the American economy and these will be subjected to closer scrutiny in this section.

The fundamental relationship which he attempts to test directly is an equation similar to equation (8) of Section 2; in that equation all other real variables apart from the unemployment rate were assumed, for expositional purposes, to exert an insignificant influence upon the rate of inflation. Such an approach is highly suspect, however, when one attempts to construct a realistic econometric model of the process of price inflation; many more real variables will have to be included before the model can be considered as even a plausible approximation to reality. This may be overcome by rewriting equation (8) in the following way:

$$I = f(Z) + x \quad (10)$$

where Z is a vector of real variables: The components of the Z vector are somewhat different in Solow's study from those included in other studies of the trade-off relation.

For a measure of aggregate demand, Solow prefers to use a measure of capacity utilisation rather than the overall unemployment rate; moreover, his measure of capacity utilisation is highly non-linear and is a transformation of the capacity utilisation index published by the Wharton School at the University of Pennsylvania.

Solow also attempts to separate out the distinct effects of variations in the money wage rate from variations in labour productivity: since the rate of change of unit labour cost, ulc , is equal to the rate of change of money wage rates, w , plus the reciprocal of the rate of change of output per man, r , a theory which states that an increase in unit labour cost arising from an increase in money wage rates will have the same effect on prices as an increase in unit labour costs arising from a fall in output per man will not require any breakdown of the unit labour

cost variable—the rate of change of unit labour cost—as a single independent variable will account satisfactorily for both effects. Since $ulc = w + r$, this approach is tantamount to assuming that if w and r were to be included as separate explanatory variables in a trade-off equation they would have identical coefficients. Solow resists such an assumption and allows the coefficients of w and r to be freely determined in the estimation process.

Two dummy variables are also included. The first one, K , is designed to allow for the effect of the Korean War upon the process of price formation. The second, G , is designed to catch the effects of the informal incomes policy which had been in operation since 1962.

Equation (10) may now be rewritten in explicit linear form:

$$I_t = a_0 + a_1 w + a_2 r + a_3 F + a_4 NCU + a_5 K + a_6 G + a_7 x \quad (11)$$

F is a modified index showing the effect of farm prices on the rate of change of private domestic nonfarm business prices (I), NCU is the non-linear index of capacity utilisation and a_7 is the coefficient θ , which we mentioned previously.

The only problem left to be tackled is the selection of a suitable price expectations variable. In his first attempt at constructing such a variable Solow made use of the adaptive expectations model mentioned in Section 2 of this chapter whereby

$$x_t = (1 - \psi) \sum_{j=1}^{\infty} \psi^{j-1} I_{t-j} \quad (6)$$

We know nothing about ψ except that it is greater than or equal to zero and less than or equal to unity so that, depending upon the value of ψ selected, there are an infinite number of expectations series compatible with (6). One way around this problem is to pick a finite number of specimen values of ψ and construct a price expectations series for each specific value of ψ . This is the approach adopted by Solow. He allows ψ to assume eleven particular values 0.0, 0.1, 0.2, . . . 0.9, 1.0 and examines the performance of each series so generated. In general, the higher the value of ψ the better the statistical fit; the best fit is obtained for a value of unity for ψ . There are, however, reasons to believe that a value of ψ around 0.4 or 0.5 is nearer the mark. Due to the construction of the I_t index as a four quarter change in the price index, a value of $\psi = 1$ implies that the x variable is simply equal to I_{t-1} which has three quarters in common with I_t . On the other hand most of the improvement in fit has already been achieved by the time $\psi = 0.4$. Moreover, at this value of ψ , a_1 and a_2 seem to make more sense for small or intermediate values of ψ . (in Solow's notation.) Solow plumps for a value of ψ of 0.4 which implies an equation of the following type:

$$I_t = 0.0061 + 0.2492w + 0.2495r + 0.7262F + 0.0107NCU + 0.0147K \\ - 0.0028G + 0.4029x$$

(1.622) (3.342) (4.180) (4.498) (2.183) (3.755) (1.588) (7.838)

t -values are in parentheses; $R^2 = 0.9057$.

It is clear that the coefficient θ (α_7 in equation (11)) is significantly different from unity. Moreover θ remains significantly different from unity for all values of ψ selected for the x series. If this is so then it constitutes strong evidence in favour of the notion that a permanent trade-off surface does in fact exist even though expectations of inflation do play an important role in determining the actual rate of inflation. Furthermore, if any confidence can be placed in a value of ψ of around 0.4 it implies that the revision of expectations takes a long time to occur. In conclusion Solow states: "Whatever may be true of Latin-American-size inflations or even smaller perfectly steady inflations, under conditions that really matter—irregular price increases with an order of magnitude of a few percent a year—there is a trade-off between the speed of price increase and the real state of the economy. It is less favorable in the long run than it is at first. It may not be 'permanent'; but it is long enough for me."

The empirical research of Brechling and Solow outlined above would appear to point to the conclusion that a permanent trade-off between inflation and unemployment (or some other measure of aggregate demand) does in fact exist. The work of Brechling draws out certain qualitative implications of the expectations hypothesis which, on the basis of the evidence cited, fail to be met in reality. Solow's approach is more direct in that he attempts to estimate the values of the parameters θ and ψ . He deduces that ψ is probably around 0.4, which implies that complete adjustment to a new higher rate of inflation will probably be quite prolonged. More important still, he estimates that the value of θ is significantly different from unity which constitutes a refutation of the strict expectations hypothesis. Further research clearly needs to be carried out in this field and the last word has by no means been said so far in the debate. Anticipating many of the future studies on the subject, Professor Tobin [8] predicts that "... sooner or later an econometric study will surely enough produce the magic number one for the price coefficient ...". Not a very hopeful note to end a paper with.

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