Possibilistic Reliability Assessment of Geotechnical Foundations using Databases: Application to Wind Turbine Foundations

Alessandro Tombari
Senior Lecturer, School of Architecture Technology and Engineering, University of Brighton, Brighton, UK

Marcus Dobbs
Head of Engineering Geology, British Geological Survey, Keyworth, Nottingham, UK

Luciano Stefanini
Emeritus Professor, Department of Economics Society Politics, University of Urbino, Urbino, Italy

ABSTRACT: Reliability analysis based on probabilistic methods is widely applied in geotechnical engineering. Several analytical and numerical methods have been developed to assess reliability based on the concept of occurrence of failure. Nevertheless, the critical aspect involves the input data used for performing these analyses. Common geotechnical engineering real-world problems deal with scarce and sparse information due to the lack of experimental data, which are not available to a sufficient extent and quality to be used to infer a reliable probability distribution. Therefore, non-probability and hybrid methods based on fuzzy logic and possibility theory as well as imprecise probabilities, have been developed. In this paper, a methodology based on the possibility approach has been proposed to account for an external local database to support the reliability assessment of wind turbine foundations. A Target Reliability Index is proposed to assess the fuzzy reliability. This cardinality is calibrated to be consistent with the probabilistic concept of failure probability, if data were of an ideal random nature. Moreover, fuzzy clusterization is carried out to determine fuzzy partitions to be used for the computation of the fuzzy limit state. The number of subclasses is related to a specific degree of understanding; therefore, the proposed method integrates the practitioner's local experience for finalizing the decision-making process and assessing the reliability of the geotechnical problem. The reliability assessment of the bearing capacity for a wind turbine foundation is provided to highlight the potential of the proposed methodology.

1. Introduction

A safe design of a foundation is achieved when a specific target reliability level is properly met. When dealing with geotechnical engineering problems, various sources of uncertainty affect the reliable assessment of the safety level, such as inherent variability (e.g., Lumb 1966), measurement scatter due to limitations of the experimental techniques as in Phoon and Kulhawy (1999) and Uzielli et al. (2006), and sampling or statistical error as in Ching et al. (2016) because of the limited number of soil samples used in the investigation.

Therefore, a reliability framework should allow the quantification of the various uncertainties affecting the parameters of the engineering problem. According to the nature of the uncertainty, e.g. aleatoric or epistemic, different approaches have been developed. Under the probabilistic framework, well-established approaches can be used: Zorzi et al. (2020) developed a reliability framework with emphasis on verifying the serviceability limit state criterion in terms of maximum allowable rotation during an extreme cyclic loading event assuming the soil parameters are normally distributed; Carswell et al. (2015) used a first-order method (β method) and Monte Carlo simulation (MCS) to estimate the reliability of an offshore wind turbine monopile foundation when the beta distribution has been adopted for modeling the variability of the soil properties.

Nevertheless, practical geotechnical problems are associated with sampling uncertainty resulting from sparse and scarce information. Ching and Phoon (2019) and Ching and Phoon (2020) proposed a Bayesian machine learning method to handle Multivariate, Uncertain and unique, Sparse, and InComplete (*MUSIC*) data. Non-probabilistic approaches have been also applied for geotechnical reliability analyses; nominal soil properties have been treated as fuzzy numbers in Nawari and Liang (2000), or as fuzzy sets in Pramanik et al. (2021). Possibility theory (Dubois and Prade 2004) based on Zadeh's theory of fuzzy sets (Zadeh 1965) provides a simple quantitative framework for statistical reasoning with imprecise probabilities (Beer et al. 2013; Dubois and Prade 2015).

In this paper, a novel definition of reliability based on the possibility approach is proposed. This provides a simple and robust tool for the uncertainty propagation of *MUSIC* data and subjective information. By adopting the probability-possibility consistency (Hose and Hanss 2019), the novel definition of possibilistic reliability equips practitioners with an alternative cardinality of reliability that extends the classical definition.

Data, represented by possibility distributions, are estimated from samples of datasets (e.g., Hose and Hanss 2020) through fuzzy clustering (Oliveira and Pedrycz 2007). Therefore, the database becomes a fundamental support for practitioners for assessing reliability. Moreover, through clustering, a robust approach to account for local experience and engineering judgment is integrated into the proposed reliability assessment of geotechnical systems.

2. PROPOSED POSSIBILITY RELIABILITY APPROACH WITH DATABASE CLUSTERING

2.1. *Possibilistic Reliability*

The structural safety at the Ultimate Limit State (ULS) of a geotechnical system can be described through a limit state function or performance function, G, taking as arguments a vector of d real parameters $(X_1,...,X_d)$ with elements in a nonempty subset $\mathbb{X} \subseteq \mathbb{R}^d$. The safety criterion can be expressed as follows:

$$\begin{aligned} G(X_1,X_2,...,X_d) > 0 &\Rightarrow \text{Safe} \\ G(X_1,X_2,...,X_d) \leq 0 &\Rightarrow \text{Failure.} \end{aligned} \tag{1}$$

The real parameters $(X_1,...,X_d)$ represent the geotechnical properties governing the resistance of a given foundation as well as the external loads applied to it. Because of the uncertainty in the characterization of the soil medium, the Possibility Theory of Dubois (2006) offers a framework to deal with multivariate, uncertain, unique, sparse and incomplete (or *MUSIC*, Phoon 2018) data. Therefore, a subset of the previously defined real-valued quantities $X_j, j = 1,2,...,d$ are now expressed in terms of d fuzzy intervals (or fuzzy numbers) represented by membership functions $u_{X_j} : \mathbb{R} \to [0,1]$ such that, given two closed real intervals $[a_j,b_j]$ and $[c_j,d_j]$ with $a_j \leq c_j \leq d_j \leq b_j$, it is, for $z \in \mathbb{R}$,

$$u_{X_j}(z) = \begin{cases} 0 & \text{if} \quad z < a_j \\ u^L_{X_j}(z) & \text{if} \quad a_j \leq z < c_j \\ 1 & \text{if} \quad c_j \leq z \leq d_j \\ u^R_{X_j}(z) & \text{if} \quad d_j < z \leq b_j \\ 0 & \text{if} \quad z > b_j \end{cases} \tag{2}$$

where $u^L_{X_j} : [a_j,c_j] \longrightarrow [0,1[$ is a non-decreasing right-continuous function, $u^L_{X_j}(z) > 0$ for $\in]a_j,c_j]$, called the *left side* of the fuzzy interval and $u^R_{X_j} : [d_j,b_j] \longrightarrow [0,1]$ is a non-increasing left-continuous function, $u^R_{X_j}(z) > 0$ for $z \in [d_j,b_j[$, called the *right side* of the fuzzy interval. By convention, if $d = 1$ the index j will be omitted in (2).

Zadeh's extension principle (Zadeh 1975) is hence applied to transform the limit state function G in Eq. (1) to a fuzzy function which takes the membership functions of the inputs, u_{X_j}, $j = 1...,d$

and returns the membership function u_G of the output. In analogy with the representation of Eq. (2), u_G takes the following form

$$u_G(g) = \begin{cases} 0 & \text{if} \quad g < a_G \\ u_G^L(g) & \text{if} \quad a_G \leq g < c_G \\ 1 & \text{if} \quad c_G \leq g \leq d_G \\ u_G^R(g) & \text{if} \quad d_G < g \leq b_G \\ 0 & \text{if} \quad g > b_G \end{cases} \tag{3}$$

where the support is given by the interval $[a_G, b_G]$ and the core by $[c_G, d_G]$; $g \in \mathbb{R}$ is a dummy variable that denotes any possible value of function G on its range.

This allows exploitation of important features for the definition of a safety criterion: since a probabilistic cumulative distribution function (CDF) can be transformed to a possibility distribution function (Hose and Hanss 2019), and vice-versa, a possibilistic cardinality based on Eq. (3) can be calibrated to be consistent with the target probability of failure prescribed by the structural code (e.g. EN 1990 British Standards Institution. et al. (2021)).

The conventional safety criterion is hence converted as follows:

$$\begin{aligned} u_G(0) < \alpha^* &\Rightarrow \text{Safe} \\ u_G(0) \geq \alpha^* &\Rightarrow \text{Failure} \end{aligned} \tag{4}$$

where, for a fixed (possibilistic) Target Reliability Value, α^*, the value $G \leq 0$ is possible only at a *small* maximum acceptable possibility level to guarantee structural safety.

In this paper, the probability-to-possibility transformation is represented by the Average Cumulative Function (ACF for short) of u, (see Stefanini and Guerra 2017), stated in terms of the function F_u satisfying the equality

$$u(x) = 2 \cdot min\{F_u(x), 1 - F_u(x)\}. \tag{5}$$

There exists a bijective correspondence between the family of all AC functions F_u and the family of the probabilistic CDFs F_X of a real random variable X. For a fixed F, the function

$$\begin{aligned} &F^{-1}(\alpha) = \inf\{x | F(x) \geq \alpha\} \text{ for all } \alpha \in]0,1] \\ &F^{-1}(0) = a \end{aligned} \tag{6}$$

is called the quantile function of F. Then, given F as above, there exists a unique membership function u with AC function $F_u = F$ such that, for all $x \in \mathbf{R}$, equation (5) is satisfied and the α-cuts of the fuzzy number u coincide with the α-quantiles of the inverse function F_u^{-1} obtained as in Eq. (6) for F_u, for all $\alpha \in]0,1[$ (Stefanini and Guerra 2017).

For example, through the transformation of the Gaussian cumulative distribution function into the average cumulative function, as explained below, it is possible to define a value of membership function consistent with the prescribed failure probability of 7.2348×10^{-5} in EN 1990 British Standards Institution. et al. (2021) for RC2 class with a 50-year reference period; the consistent Target Reliability index is computed as $\alpha^* = 1.447 \times 10^{-4}$ for the same conditions.

Since the membership functions of the $X_j, j = 1,2,...,d$ parameters can be obtained from the empirical cumulative distribution functions, we promote the use of an external database to support the reliability assessment of geotechnical foundations.

2.2. *Integrating Databases and Local Experience*

Given a database of d-dimensional tuples, $\mathbf{x}_i = (x_{i,1}, ..., x_{i,d})$, $i = 1,2,...,m$, with m records for each of the given d attributes, the estimation of the membership function for each of the d parameters taken independently is achieved through the three-step procedure described below. In the rest of this section, the procedure is developed for a single given input parameter X, hence $d = 1$ and the j index is omitted.

2.2.1. *Step 1: Fuzzy clustering and partitioning*

Clustering is essentially an optimization approach that consists of determining a family of n classes $\mathscr{C}_1, ..., \mathscr{C}_n$ and the clustering $(m \times n)$-matrix, $U = [u_{i,k}]$ with $u_{i,k} \in \{0,1\}$ for k-means deterministic clustering or $u_{i,k} \in [0,1]$ for $c-$means fuzzy clustering, in order to minimize an appropriate objective function. This can be based on some distance measure, $d(x_i, \mathscr{C}_k)$, between the i-th record and the k-th class (cluster).

In fuzzy clustering, the component $u_{i,k}$ of the partitioning matrix measures the degree of membership of x_i to class $\mathscr{C}_k$ and $\beta \geq 1$ is a fixed fuzzification coefficient. For $\beta = 1$, the approach will provide the same results as the deterministic *k-means* clustering. Usually, each cluster $\mathscr{C}_k$ is identified in terms of a computed (representative) prototype value $c_k \in [a,b]$, $k = 1,2,\ldots,n$ and the distance $d_{i,k}$ is the euclidean one.

After assigning a degree of membership, $u_{i,k}$, to the i-th data for each of the $k = 1,\ldots,n_c$ classes, a (finite) *fuzzy partition* of the interval $[a,b]$ in terms of a family of n_c fuzzy numbers $\left\{A_k^{n_c} | k = 1,2,\ldots,n\right\}$ is sought. In this paper, the so-called *Ruspini condition* (see, e.g., Holčapek et al. 2015; Novák et al. 2016) is satisfied, namely, $A_k^{n_c}(x) + A_{k+1}^{n_c}(x) = 1$ for all $k = 1,2,\ldots,n_c - 1$ and all $x \in [a,b]$. The usefulness of a fuzzy partition is that each observed datum x_i has either one membership value $A_k^{n_c}(x_i) = 1$ (and it belongs to the core of the fuzzy number $A_k^{n_c}$ representing the k-th cluster), or exactly two positive values $A_k^{n_c}(x_i)$ and $A_{k+1}^{n_c}(x_i) = 1 - A_k^{n_c}(x_i)$, the others being zero.

2.2.2. *Step 2: Determine Empirical Membership Functions*

The *second step* consists of obtaining, for the n_c classes of the Ruspini partition, the empirical membership functions extracted from the observed values $\{x_i | i = 1,2,\ldots,m\}$ in the clustered database. Each k-th Ruspini partition, $A_k^{n_c}$, is converted to an empirical membership function u_k^{ACF}, $k = 1,\ldots,n_c$, through the ACF transformation of the subset of data of each cluster belonging to the support of $A_k^{n_c}$, $k = 1,2,\ldots,n_c$, as described by Eq. (5). The transformation will generate a possibility number consistent with the empirical cumulative distribution of the data.

2.2.3. *Step 3: Derivation of the Design Fuzzy Membership function*

The final step of our procedure is to obtain the fuzzyfied version of the input parameter $X = \hat{x}$, i.e., its membership function $u_{\hat{x}}$ on the range $[a,b]$, for any possible value $\hat{x}$. The core of the fuzzy value $\hat{x}$ is represented by the characteristic or nominal value embodying the aleatory randomness of the considered parameter. The remaining features,

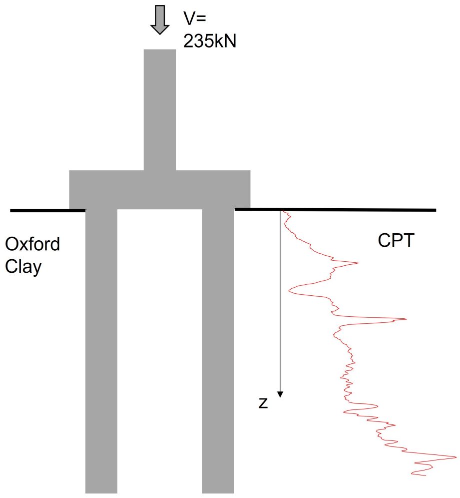

Figure 1: Investigated Wind Turbine Foundation problem

i.e. the support and shape of the membership function, are derived from the information extracted from the database. In particular, two consecutive fuzzy numbers $A_{\hat{k}}^{n_c}$ and $A_{\hat{k}+1}^{n_c}$ of the Ruspini partition $\left\{A_k^{n_c}|, k = 1,2,\ldots,n_c\right\}$ obtained in Step 2 are identified in relation the the position of $\hat{x}$; therefore, a convex combination is used for merging the two adjacent empirical membership functions as follows:

$$u_{\hat{x}} = w\hat{u}_{\hat{k}} + (1-w)\hat{u}_{\hat{k}+1} \tag{7}$$

where the weights w and $1-w$ are given by $w = A_{\hat{k}}^{n_c}$ (or, equivalently, $1 - w = A_{\hat{k}+1}^{n_c}$) . The obtained fuzzy number, $u_{\hat{x}}$, defines the Design Membership Function to be used as an argument of the fuzzy performance function in Eq. (1) and to assess the possibilistic reliability in Eq. (4).

3. Possibility Reliability Assessment of Wind Turbine Pile Foundation

3.1. *Problem Description*

A 65-KW onshore wind turbine founded on a 4×4 pile group is considered. Data of the wind turbine are taken from Austin and Jerath (2017). The tower is 23 m high and its mass is 10700 kg. In this investigated example, the piles have been designed to have a diameter, D, equal to 0.25m and pile length, L, of 4.5m. An equivalent vertical static load of about 235 kN is applied to the pile group founda-

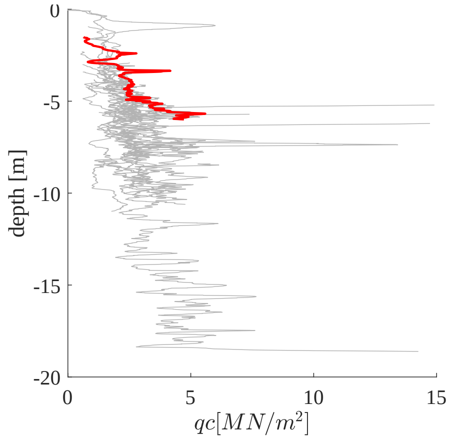

Figure 2: CPT soundings obtained from the BGS database (Self et al. 2012) in grey colour and reference CPT test (red colour curve)

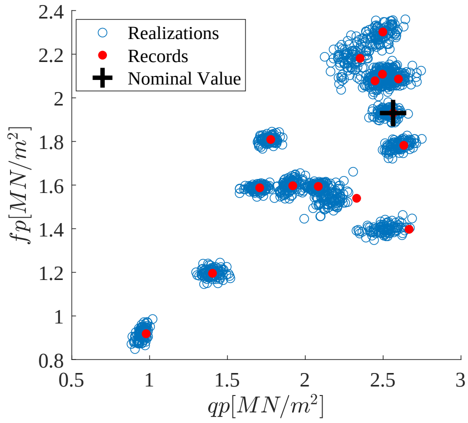

Figure 3: Distribution of the Cone and Frictional Resistances

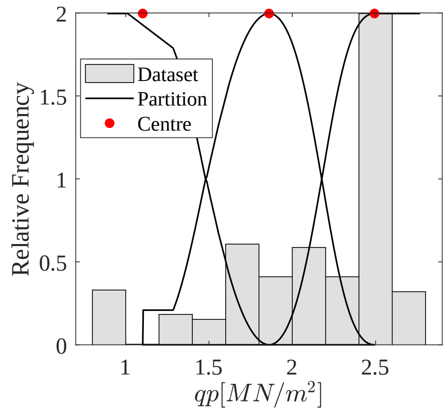

Figure 4: Fuzzy Clustering of the cone resistance for low *DoU* ($n_c = 3$)

tion to consider gravity and wind load. The performance function G to define the structural safety at the Ultimate Limit State (ULS) is defined as the difference between the resistance and the external load V as follows:

$$G = R - V \tag{8}$$

The pile bearing capacity, R, is evaluated directly from the results of a Cone Penetration Test (CPT) sounding carried out in the Oxford Clay Formation (see Figure 1). Therefore, the resistance parameters are evaluated through the Bustamante and Gianeselli approach where the cone resistance, qc, is used to derive the pile unit end bearing, qp, and the pile unit side friction, fp.

3.2. BGS Database Clusterization

To assess the reliability of the designed pile group, the result of the experimental CPT is compared with published data and local experience, as suggested by EN 1990 2021. In this paper, the dataset of CPT values recorded in the BGS National Geotechnical Properties Database (Self et al. 2012) is considered. The considered records are the cone resistances of CPT testing conducted on 30 boreholes in the Oxford Clay formation from 3 different members (i.e., Stewartby, Peterborough and Weymouth). Figure 2 shows with a red-color curve the cone resistance, qc, derived from the experimental CPT sounding used for the pile design whilst the grey curves represents all the records collected in the BGS database. To consider the measurement error, for each record, 100 samples are generated by using the original record as a mean trend and summing up a random, normally distributed, fluctuation (Ching et al. 2018) with coefficient of variation (COV) equal to 0.15, selected as the lower bound of the COV values investigated in Phoon and Kulhawy (1999) and Salgado et al. (2019) for CPT testing. The generated values are represented by a blue circle in Figure 3

Figure 4 and Figure 5 show the distribution of the cone and frictional resistance and their Ruspini partitions by black-curves for 2 different number of subdivisions, i.e., $n_c = 3$ and $n_c = 7$; it is worth emphasizing that, in our application, the number n_c

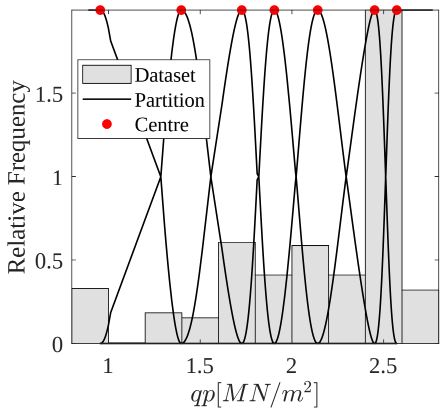

Figure 5: Fuzzy Clustering of the cone resistance for high *DoU* ($n_c = 7$)

of membership functions in a Ruspini partition can be associated with a semantic interpretation, for instance, for $n_c = 3$ classes the DoU is considered *Low*, for $n_c = 5$ classes the DoU is *Typical*, and DoU is *High* for $n_c = 7$ classes.

This property is exploited in the proposed methodology to embed the local experience and engineering judgment in the assessment of the reliability: if the level of confidence in the experimental data, hereinafter called Degree of Understanding (DoU), based on local experience, number of samples and comparison with published data, is high, a higher number of subclasses can be used to reduce the epistemic uncertainty whilst for low DoU (no local experience and few samples), the number of subclasses is reduced by increasing the epistemic uncertainty of the design parameter.

3.2.1. Assessment of the Possibilistic Reliability

The Ruspini partitions are then transformed through the ACF-transformation and converted in empirical membership functions as depicted with black curves in Figure 4 and 5. The membership function of the cone resistance, u_{qp}, and frictional resistance, u_{fp}, for the reliability assessment are obtained by positioning the nominal values used for the pile design (in this case, the cone and frictional resistance obtained by the CPT sounding) and then merged to derive the red-colour membership functions in Figures. 6-7 for different DoU.

It is worth noting that the support of the design membership function is smaller when a higher DoU

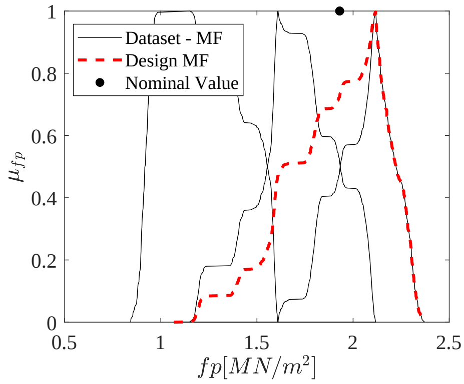

Figure 6: Empirical and Design membership function (MF) for low *DoU*

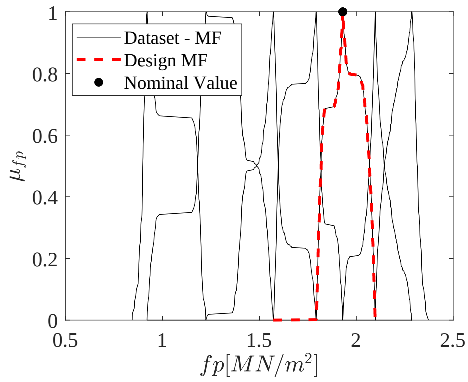

Figure 7: Empirical and Design membership function (MF) for high *DoU*

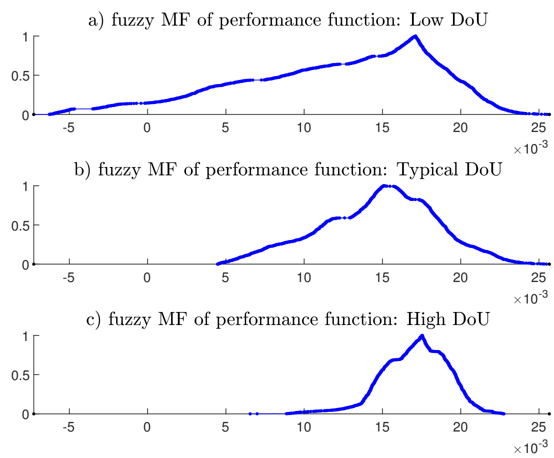

Figure 8: Membership function u_G of the fuzzy performance function G for three DoU levels: a) Low; b) Typical; c) High.

Table 1: Possibilistic Reliability Assessment

DoU	Low	Typical	High
Test Result	FAIL	SAFE	SAFE

(*High* understanding) is considered, demonstrating that by increasing the local experience and engineering judgment, the method is able to capture the decrease of the uncertainty of the parameter. Finally, the possibilistic reliability assessment is conducted for each DoU by verifying Eq. (4).

Because of the monotonicity of the performance function, G, the computation of the membership function, u_G, is simply obtained by calculating the deterministic function, G, for each value of the sides of the design membership function; each value is associated with a certain level of membership in $[0,1]$. If the value of the membership function at $G = 0$ is greater than the Target Reliability Value ($\alpha^* = 1.447 \times 10^{-4}$), the criterion is satisfied (*Safe*), otherwise not and the structure is unsafe (*Fail*). Moreover, as evidenced in Figure 8, an increase in DoU level is associated to a reduction in the support $[a_G, b_G]$ of membership function u_G in Eq. (3) and, consequently, a lower impact of the uncertainties on the result.

Table 1 summarizes the outcome of the proposed possibilistic reliability assessment; for *Low* DoU, the pile design is considered unsafe due to the large variation and weight of the database; for *Typical* and *High* DoU, if local experience and number of samples are sufficient, the design can be considered safe.

4. Conclusions

In this paper, a novel reliability assessment of geotechnical systems is proposed. To deal with *MUSIC* data and subjective information, the Possibility Theory is used to define a fuzzy performance function based on the probability-possibility consistency theorem. This allows the definition of a possibilistic cardinality congruent with the probabilistic concept of failure probability. Moreover, the membership functions of the soil parameters can be estimated from the fuzzy clusterization of an external database, that can be modulated to accommodate different Degrees of Understanding to support the decision-making process based on subjective information such as local experience and engineering judgment.

5. Acknowledgments

Dr Tombari gratefully acknowledges the financial support of the Engineering and Physical Sciences Research Council (EPSRC) through the New Investigator Award EP/W001071/1 "Structural Life-Cycle Enhancement of Next-Generation Onshore and Offshore Wind Farms".

6. References

Austin, S. and Jerath, S. (2017). "Effect of soil-foundation-structure interaction on the seismic response of wind turbines." *Ain Shams Engineering Journal*, 8(3), 323–331.

Beer, M., Ferson, S., and Kreinovich, V. (2013). "Imprecise probabilities in engineering analyses." *Mechanical Systems and Signal Processing*, 37(1-2), 4–29.

British Standards Institution., European Committee for Standardization., British Standards Institution., and Standards Policy and Strategy Committee. (2021). *Eurocode 0, Basis of structural and geotechnical design*. British Standards Institution, London.

Carswell, W., Arwade, S. R., DeGroot, D. J., and Lackner, M. A. (2015). "Soil-structure reliability of offshore wind turbine monopile foundations: Soil-structure reliability of offshore wind turbine monopiles." *Wind Energy*, 18(3), 483–498.

Ching, J. and Phoon, K.-K. (2019). "Constructing Site-Specific Multivariate Probability Distribution Model Using Bayesian Machine Learning." *Journal of Engineering Mechanics*, 145(1), 04018126.

Ching, J. and Phoon, K.-K. (2020). "Constructing a Site-Specific Multivariate Probability Distribution Using Sparse, Incomplete, and Spatially Variable (MUSIC-X) Data." *Journal of Engineering Mechanics*, 146(7), 04020061.

Ching, J., Phoon, K. K., and Wu, S. H. (2016). "Impact of Statistical Uncertainty on Geotechnical Reliability Estimation." *Journal of Engineering Mechanics*, 142(6), 04016027.

Ching, J., Wu, T.-J., Stuedlein, A. W., and Bong, T. (2018). "Estimating horizontal scale of fluctuation with limited CPT soundings." *Geoscience Frontiers*, 9(6), 1597–1608.

Dubois, D. (2006). "Possibility theory and statistical reasoning." *Computational Statistics & Data Analysis*, 51(1), 47–69.

Dubois, D. and Prade, H. (2004). "Possibilistic logic: a retrospective and prospective view." *Fuzzy Sets and Systems*, 144(1), 3–23.

Dubois, D. and Prade, H. (2015). "Possibility Theory and Its Applications: Where Do We Stand?." *Springer Handbook of Computational Intelligence*, J. Kacprzyk and W. Pedrycz, eds., Springer Berlin Heidelberg, Berlin, Heidelberg, 31–60.

Holčapek, M., Perfilieva, I., Novák, V., and Kreinovich, V. (2015). "Necessary and sufficient conditions for generalized uniform fuzzy partitions." *Fuzzy Sets and Systems*, 277, 97–121.

Hose, D. and Hanss, M. (2019). "Possibilistic calculus as a conservative counterpart to probabilistic calculus." *Mechanical Systems and Signal Processing*, 133, 106290.

Hose, D. and Hanss, M. (2020). "On data-based estimation of possibility distributions." *Fuzzy Sets and Systems*, 399, 77–94.

Lumb, P. (1966). "The Variability of Natural Soils." *Canadian Geotechnical Journal*, 3(2), 74–97.

Nawari, N. O. and Liang, R. (2000). "Fuzzy-based approach for determination of characteristic values of measured geotechnical parameters." *Canadian Geotechnical Journal*, 37(5), 1131–1140.

Novák, V., Perfilieva, I., and Dvorák, A. (2016). *Insight into fuzzy modeling*. John Wiley & Sons, Inc, Hoboken, New Jersey.

J. V. d. Oliveira and W. Pedrycz, eds. (2007). *Advances in fuzzy clustering and its applications*. Wiley, Chichester OCLC: ocm75713388.

Phoon, K.-K. (2018). "Probabilistic Site Characterization." *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering*, 4(4), 02018002.

Phoon, K.-K. and Kulhawy, F. H. (1999). "Characterization of geotechnical variability." *Canadian Geotechnical Journal*, 36(4), 612–624.

Pramanik, R., Baidya, D. K., and Dhang, N. (2021). "Reliability assessment of three-dimensional bearing capacity of shallow foundation using fuzzy set theory." *Frontiers of Structural and Civil Engineering*, 15(2), 478–489.

Salgado, R., Ganju, E., and Prezzi, M. (2019). "Site variability analysis using cone penetration test data." *Computers and Geotechnics*, 105, 37–50.

Self, S., Entwisle, D., and Northmore, K. (2012). "The structure and operation of the BGS National Geotechnical Properties Database Version 2." *Internal Report IR/12/056*, British Geological Survey.

Stefanini, L. and Guerra, M. L. (2017). "On possibilistic representations of fuzzy intervals." *Information Sciences*, 405, 33–54.

Uzielli, M., Nadim, F., Lacasse, S., and Phoon, K. (2006). "Soil variability analysis for geotechnical practice." *Characterisation and Engineering Properties of Natural Soils*, K. Phoon, D. Hight, S. Leroueil, and T. Tan, eds., Taylor & Francis, 1–103.

Zadeh, L. (1965). "Fuzzy sets." *Information and Control*, 8(3), 338–353.

Zadeh, L. (1975). "The concept of a linguistic variable and its application to approximate reasoning-III." *Information Sciences*, 9(1), 43–80.

Zorzi, G., Mankar, A., Velarde, J., Sørensen, J. D., Arnold, P., and Kirsch, F. (2020). "Reliability analysis of offshore wind turbine foundations under lateral cyclic loading." *Wind Energy Science*, 5(4), 1521–1535.