

Experimental and computational evaluation of modal identification techniques for structural damping estimation

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Abstract The damping ratio is a key indicator of a structures susceptibility to human discomfort due to dynamic loads. Computational models, wind tunnel studies and empirical data can provide estimates of the damping ratio of a structure. However, the only true way to investigate the damping ratio of a structure is through modal identification using data from field tests carried out on the full-scale finished structure. This paper investigates the efficacy of three modal identification methods for estimating the damping ratio of the first two modes of a structure from ambient data. The three methods considered are the Bayesian Fast Fourier Transform (BFFT), the Random Decrement Technique (RDT), and a hybrid of the RDT which first decomposes the ambient data into sub signals using Analytical Mode Decomposition (AMD) and is referred to as the AMD-RDT. Each method is applied to two case studies in order to investigate the accuracy of their damping estimates. The first case study is experimental and involves the excitation of a scaled model structure using a shake table; the second case study considers a computational model of a tall building under simulated wind loads. It was found that the AMD-RDT was the superior method for damping estimation, particularly for the estimation of damping ratio in the second mode, even when the modes were closely spaced. The length of time series data used and the noise within the data was seen to affect the accuracy of the three methods studied.

1 Introduction

Multiple physical phenomena can contribute to the level of damping displayed in the vibration response of different structures. [1]. Therefore, calculating the damping ratio of a structure is a difficult process which does not always prove accurate [1, 2]. Moreoften, damping estimation of a structure relies on empirical data from previous full scale monitoring campaigns of buildings, with many design codes offering values based on the type of construction and building materials [2]. For this reason, despite advances in technology allowing for better modelling and scaled wind tunnel tests, the only truly accurate method for obtaining the damping ratio of a structure is from full scale monitoring of the completed structure [3, 4].

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Full scale monitoring of a structure can take the form of forced vibration experiments or ambient data collection. Forced vibration experiments involve applying a known input loading to the structure and recording the output. Often this involves an impractically large loading and is deemed uneconomical. Ambient vibration tests capture the unknown but statistically random natural excitation of structures by wind loading and other dynamic loads during operation. This has resulted in them becoming increasingly popular in both structural health monitoring and modal identification technique development [3, 5]. The tests typically involve the use of accelerometers and a data acquisition system to record and save the ambient response data. The ambient data collected can be processed using modal identification techniques to identify the key parameters of the structure such as natural frequencies, damping ratios and mode shapes. There are many different modal identification techniques, some of which operate in the time domain, whilst others operate in the frequency domain. This paper considers three modal identification methods: the Bayesian Fast Fourier Transform (BFFT) which operates in the frequency domain, the Random Decrement Technique (RDT) which operates in the time domain and a hybrid of the RDT which involves first decomposing the response signal into subsignals using Analytical Mode Decomposition (AMD). The third method is here on referred to as the AMD-RDT.

Modal identification using the three methods described in this paper is performed on the acceleration response data from two case studies; one experimental and one computational. The first case study is a three storey scaled model structure excited using white noise applied through a shake table. The second is a computational model of a 105 meter, 35 storey structure excited using simulated wind loads. The modal identification will focus on estimating the damping ratios in the first two modes for each case study. The damping ratio is the key parameter for modal identification chosen in this study as it is the modal parameter which is hardest to predict accurately without full scale testing and is significantly influential in the dynamic response of a structure and occupant comfort.

2 Bayesian Fast Fourier Transform

The BFFT is a method commonly used in modal identification to predict the modal properties of a structure from ambient data and has been investigated extensively [5-7]. The principle of the BFFT is that both the real and imaginary parts of the FFT of the acceleration response of a structure experiencing broad-band excitation will have a Gaussian distribution that can be described analytically by a set of modal parameters, θ . The modal parameters contained in θ are the natural frequency f , damping ratio ζ , mode shape Φ , entries of the spectral density matrix $\{S_{ij}\}$ and the spectral density of the prediction error σ^2 , for any given mode.

This paper employs the BFFT method as described by [5]. In short, the FFT data obtained from ambient vibrations is used to maximise the posterior probability density function (PDF) of the modal parameters and hence, find the most probable value (MPV) of each of the modal properties.

The posterior PDF can be approximated by a Gaussian PDF for a sufficiently large data set [5]. To do this, the second order approximation of the log-likelihood function, $L(\theta)$, is taken by letting θ be the most probable value (MPV) that minimises L . It is then treated as a second order Taylor series about θ with the first-order term vanishing to optimality of θ . The posterior PDF then becomes a Gaussian PDF as shown in Equation 1

$$p(\theta|\{Z_k\}) \propto \exp \left[-\left(\frac{1}{2}\right) (\theta - \hat{\theta})^T \hat{C}^{-1} (\theta - \hat{\theta}) \right] \quad (1)$$

Where Z_k is the joint PDF of the augmented FFT vectors of the ambient data and is considered a zero-mean Gaussian vector; $\hat{C}$ is the posterior covariance matrix of Z_k defined as

$$\hat{\mathbf{C}} = \mathbf{H}_L(\theta)^{-1} \quad (2)$$

where the Hessian of L at the MPV is $\mathbf{H}_L(\theta)$.

The calculation of both the MPV and covariance matrix are essential for the computation of the Gaussian PDF. Therefore, the estimation of these two variables becomes the main computational effort in Bayesian identification problems.

3 Analytical Mode Decomposition

The Random Decrement Technique (RDT) is commonly combined with a signal decomposition method as it has been proven to have poor estimation of damping ratios when the structural modes are closely spaced [8, 9]. Empirical mode decomposition (EMD) is one method which has been shown to yield good results when combined with the RDT [3, 9]. However EMD has many shortfalls, including that it does not always correctly decompose signals where modes are closely spaced or beating occurs [8, 10, 11]. Other investigations involve the use of Analytical Mode Decomposition (AMD) for signal decomposition and found it effective for signals with highly coupled modes [10].

In this paper, the AMD is combined with the RDT for modal identification from ambient data. The AMD decomposes a subsignal into two components, each of which with Fourier spectra that are non-vanishing over mutually exclusive frequency ranges separated by a bisecting frequency ω . Each subsignal can then be analysed using the RDT outlined in Section 4 to extract the free decay response of the structure and determine its damping ratio. The AMD is applied in this paper as described by [8]. A brief description of the method is given here.

Let $\mathbf{x}(t)$ denote the measured acceleration data containing a number of frequency components $(\omega_1, \omega_2, \dots, \omega_n)$ where n is the number of subsignals in which the data is to be decomposed. The subsignals, $\mathbf{x}_n(t)$ have Fourier Spectra $\hat{\mathbf{X}}(\omega)$ covering n mutually exclusive frequency ranges such that $(|\omega| < \omega_{b1}), (\omega_{b1} < |\omega| < \omega_{b2}), \dots, (\omega_{(bn-2)} < |\omega| < \omega_{(bn-1)})$ and $(\omega_{(bn-1)} < |\omega|)$ where $\omega_{bi} \in (\omega_i, \omega_{i+1}) (i = 1, 2, \dots, n-1)$ are the bisecting frequencies. Therefore,

$$\mathbf{x}(t) = \sum_{i=1}^n \mathbf{x}_i(t) \quad (3)$$

Each of the modal responses has a narrow bandwidth in the frequency domain and can be determined by

$$\mathbf{x}_i(t) = s_i(t) - s_{i-t}, \dots, \mathbf{x}_n(t) = \mathbf{x}(t) - s_{n-1}(t) \quad (4)$$

$$s_i(t) = \sin(\omega_{bi}t\mathbf{H}[x(t)\cos(\omega_{bi}t)] - \cos(\omega_{bi}t\mathbf{H}[x(t)\sin(\omega_{bi}t)]) \quad (5)$$

where $\mathbf{H}[\cdot]$ represents the Hilbert Transform. After application of the AMD method to achieve subsignals of the response signal, the RDT is applied to obtain the free vibrational response of the structural system and identify its damping ratio.

4 Random Decrement Technique

The RDT is based on the theory that an ambient, white noise excitation of a structure will result in an acceleration response at the n th DOF in the i th mode, $\mathbf{x}_{ni}(t)$, which consists of the response due to initial displacement $\mathbf{x}_{x_0ni}$, the response due to the initial velocity $\mathbf{x}_{\dot{x}_0ni}$ and the external input force due to random excitations $\mathbf{x}_{Fni}$ such that

$$\mathbf{x}_{ni}(t) = \mathbf{x}_{x_0ni} + \mathbf{x}_{\dot{x}_0ni} + \mathbf{x}_{Fni} \quad (6)$$

The objective of the RDT is to estimate the damping experienced by a linear structural system by using the resulting signature from combining averaged segments of its response [3, 12]. The response segments are those from the time history of the acceleration response which satisfy a threshold condition, X_{pni} [2, 4, 13, 14]. In theory, by averaging a large number of random decrement response segments with identical triggering conditions, the initial velocity and forced vibration responses reduce to zero, leaving only the response due to the initial displacement. Essentially the random component of the response is removed leaving a signal comprising only the free decay response of the structure. The RDT was applied in this paper as described by [15] to obtain the random decrement signatures defined as

$$\delta_{ni}(t) = \frac{1}{N} \sum_{k=1}^N X_{pni}(t_k + \tau) \quad (7)$$

where N = number of subsamples and $\tau = t - t_i$.

Once the free vibrational response for each mode has been determined, the Hilbert transform is applied to each random decrement signature $\delta_{ni}(t)$ to approximate the free decay response and determine the damping ratio ζ_i [9, 14].

5 Case studies

5.1 Shake table tests

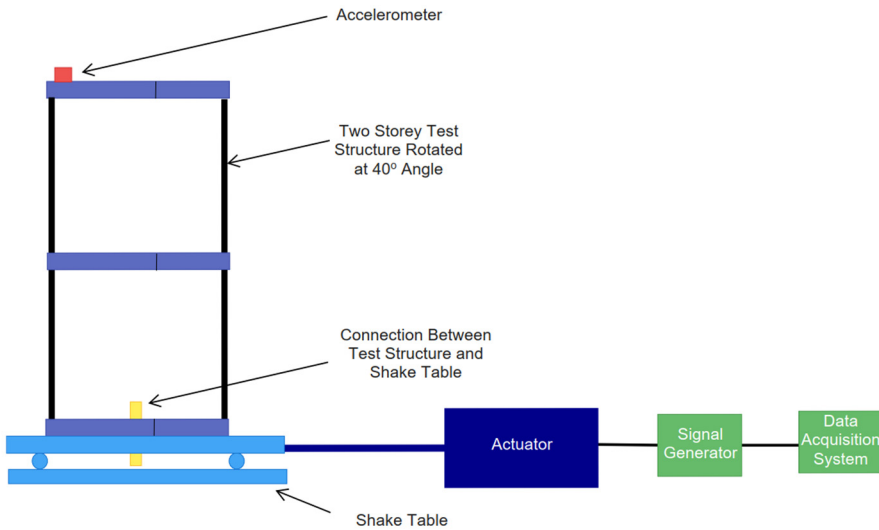


Fig. 1. Schematic of experimental setup.

The first case study is experimental in nature and involves a 2 storey model structure excited on a shake table. The model structure is rotated at a 40° angle in plan in order to excite both the first and second modes which are closely spaced due to the symmetry of the structure.

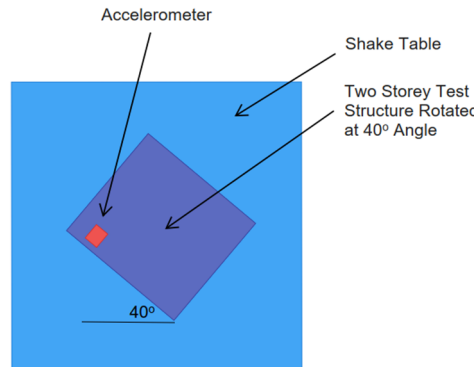


Fig. 2. Plan view of model structure showing rotation of 40° on shake table.

In order to study the reliability of the 3 modal identification methods considered in this paper, the actual modal properties of the test structure in the first two modes were measured. The natural frequency of each mode was obtained from the FFT of the isolated acceleration response for each mode. The damping ratio was obtained through what is considered a forced vibrations test in which a known load was applied to the test structure and then stopped suddenly, resulting in the free vibrational response of the structure. Using the free vibrational responses in the relative plane, the damping ratios of each mode were obtained using the logarithmic decrement technique:

$$\delta = \frac{1}{n} \ln \left(\frac{x(t_0)}{x(t_n)} \right) \quad (8)$$

where δ is the logarithmic decrement; n is the peak number being considered; x is the amplitude of the peak; t_0 is the time at which the initial peak occurred and t_n is the time at which the peak being considered occurred. The damping ratio ξ can then be calculated using Equation 9.

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \quad (9)$$

These equations were applied to the recorded free vibrational response and the damping ratio was taken as an average of the estimates of ξ for each peak.

Table 1. Model Structure Properties.

Mode	f_i (Hz)	ζ_i (%)
1	4.5	0.05
2	5	0.08

To study the reliability of the 3 modal identification methods considered in this paper, a white noise excitation was applied to the structure using the shake table. An accelerometer attached to the top of the model structure was used to capture the structural response at a 50 Hz sampling rate. The structural response captured is statistically random and stationary and is the ambient data used to perform modal identification of the model structure using the BFFT, RDT and AMD-RDT methods.

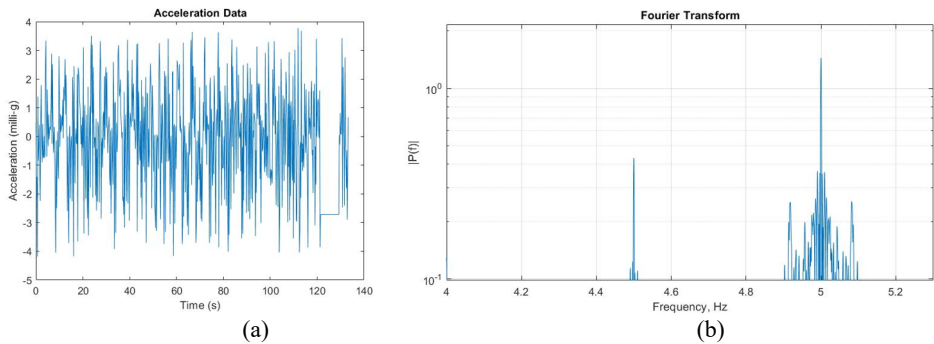


Fig. 3. Exert from (a) the acceleration response and (b) the FFT from shake table data.

Table 2 contains the results which were obtained from applying the three modal identification methods. The error of each estimation is shown in Table 3.

Table 2. Model structure modal identification results.

Mode	f_i (Hz)	Actual ζ_i (%)	BFFT ζ_i (%)	RDT ζ_i (%)	AMD-RDT ζ_i (%)
1	4.5	0.0500	0.0450	0.0470	0.0480
2	5	0.0800	0.0685	0.0970	0.0785

Table 3. Model structure modal identification error.

Mode	f_i (Hz)	BFFT Error (%)	RDT Error (%)	AMD-RDT Error (%)
1	4.5	-10	-6	-4
2	5	-14	+21	-2

It is evident that the AMD-RDT method proved to be the most accurate modal identification method applied in this case study. However, it is worth noting that for both the first and second modes the damping ratio is considerably small and whilst the BFFT and RDT methods did have a larger error, they did result in reliable damping estimates lying close to the actual value.

The purpose of applying the AMD-RDT method was to prove it’s superiority in estimating the damping ratios of a structure with closely spaced modes [8]. The AMD-RDT did identify the damping ratios of the first two closely spaced modes with minimal error (<5%) and prove more accurate than the BFFT and RDT methods alone. It can be seen that for both the BFFT and RDT methods the resulting error from estimating the damping ratio of the second mode was considerably more than the relative errors of the estimates for the first mode. This is not true for the AMD-RDT method and further adds claim to the effectiveness of the AMD-RDT method in situations where modes are closely spaced.

5.2 Simulated response

Ambient data was obtained by simulating wind loading on a computational model of a 105 meter, 35 storey structure. The simulated acceleration response of the structure acts as the ambient data processed for the modal identification using the BFFT, RDT and AMD-RDT methods.

Table 4. Computational Model Properties.

Mode	f_i (Hz)	ζ_i (%)
1	0.3171	1.2700
2	1.4530	1.5000

A simulated wind loading was generated using the Eurocode methods described in [16] assuming a wind climate typical in London, the United Kingdom. This was applied to the computational model in three degrees of freedom at each floor. The acceleration response of the computational model was then captured by applying a time stepping approach. Note, the modes are not closely spaced in this case study as seen in Figure 4 (b).

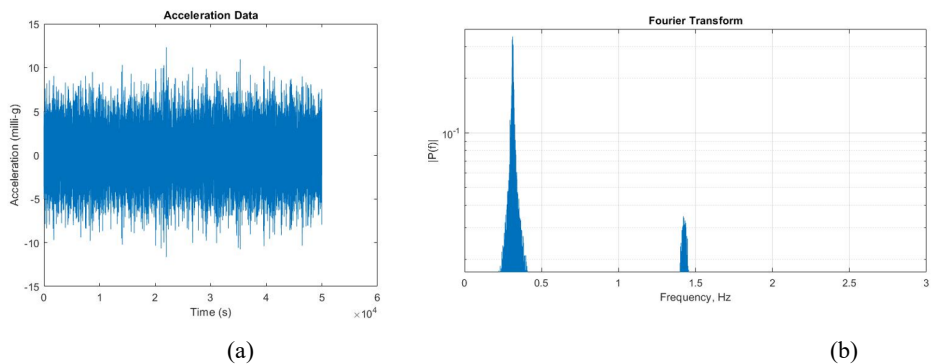


Fig. 4. Exert from (a) the acceleration response and (b) the FFT from computer simulations.

The results from the modal identification using the ambient data from the computer simulations are shown in Table 5, with relative errors shown in Table 6.

Table 5. Computational simulation modal identification results.

Mode	f_i (Hz)	Actual ζ_i (%)	BFFT ζ_i (%)	RDT ζ_i (%)	AMD-RDT ζ_i (%)
1	0.3717	1.2700	1.5000	1.3152	1.2968
2	1.4530	1.5000	1.8746	1.6532	1.4850

Table 6. Computational simulation modal identification error.

Mode	f_i (Hz)	BFFT Error (%)	RDT Error (%)	AMD-RDT Error (%)
1	0.3171	+18.1	+3.6	+2.1
2	1.4530	+25.0	+10.2	-1.0

It can be seen that the BFFT produced estimates with the most significant error for both the first and second modal damping ratios. The RDT also provided results with significant error. However, the error was less than that for the shake table case study in Section 5.1. Again, for both the BFFT and RDT, identification of the damping ratio in the second mode was less accurate than for the first mode. Neither the BFFT or RDT methods decompose the signal into its different components and this is the likely cause of the larger error in the estimate of the damping ratio in the second mode seen here, and in the results presented in Section 5.1. The AMD-RDT decomposes the signals into subsignals for each modal response and this is reflected in the results; the estimates of damping in both the first and second modes for both case studies have similarly small errors.

The AMD-RDT and RDT performed better for the second case study than the first. This can be accredited to the cleaner signal obtained from computer simulations as opposed to laboratory tests. However, the same is not true for the BFFT which predicted with more

accuracy in the first case study. This is possibly due to the longer time segments processed in the first case study. The BFFT relies on analysing the entire ambient data set whereas the RDT and AMD only consider time segments of data over a defined triggering value.

The AMD-RDT gave estimates which were considerably accurate ($< 3\%$) when compared to the other two methods which both had errors upwards of 10% for the estimation of damping in the second mode. Whilst the modes here are not closely spaced, the AMD-RDT still proves the superior method for modal identification of the damping ratio.

6 Conclusion

Three modal identification techniques, The BFFT, RDT and AMD-RDT have been applied to two case studies, one experimental and one computational, to investigate their accuracy in predicting the damping ratio of the first two modes of a structure from ambient data. In the first case study the first two modes were closely spaced, whereas for the second case study the modes were well separated.

It was found that each of the modal identification methods predicted the damping ratio for both modes within 25% of the actual value for each case study. The AMD-RDT method was found to be the most accurate predictor of damping ratio for both case studies, handling the closely spaced modes of the first case study remarkably well.

It is clear that of the modal identification techniques considered in this study the AMD-RDT has proven to be the most efficacious for damping estimation with errors $< 5\%$. It is evidently the preferred method for damping estimation when modes are closely spaced or where more than one mode contributes to the response. However, the AMD-RDT is not without its limitations. Unlike the BFFT, the AMD-RDT does not provide any confidence interval of its damping estimates. More work is needed to develop techniques which allow for statistical descriptions of the predictions from the AMD-RDT to allow for its further application in field tests of structures.

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