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Incorporation of Equity into Infrastructure Decision-Making: Development of an Equity Metric for Infrastructure Retrofitting

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ABSTRACT: *Infrastructure service outages after natural hazards lead to widespread impacts that are often disproportionately clustered among vulnerable groups. These groups also tend to be serviced by more fragile infrastructure that is more prone to outages which circuitously increases the occurrence of impacts. These inequities need to be incorporated into infrastructure decision-making so that service performance can be equitably provided and ultimately increase community resilience. Therefore, an equity metric with multiple decompositions derived from Theil's T is presented to enable the evaluation of retrofit scheme's equity towards groups of differing vulnerabilities and the ultimate incorporation of equity considerations into retrofit prioritization. The metric is derived to promote the equitable spread of electrical distribution network service performance to household structures, yet the metric could be extended to any other infrastructure system. The metric is illustrated in a compact study region with empirically set varying nodal vulnerabilities for an electric distribution network. The results confirm the metric and its various decompositions accurately detect service equity changes from which insights for future decision-making implementation are garnered.*

1. INTRODUCTION

Infrastructure service outages, especially those occuring after natural hazards, are not felt uniformly across a community but are clustered disproportionately among vulnerable groups (e.g., low income, minority, etc.). Compounding, these areas with the greatest vulnerability are often serviced by less adept infrastructure and more prone to outages which circuitously increases the likelihood of these disproportionate impacts (Highfield et al. 2014; Van Zandt 2019). To equitably serve a community, these inequitable impacts should be accounted for during infrastructure decision-making. Community resilience can be better supported when enacted measures strive to reduce these disproportionate impacts and bolster the most vulnerable within a community. As such, equity is becoming an ever-increasing component of interest to support community resilience (Matin et al. 2018; National Academies of Sciences 2022). Yet, limited works incorporate equity considerations in infrastructure retrofit prioritization by which disproportionate impacts could be alleviated and ultimately increase community resilience. For these reasons, an equity-based infrastructure performance metric is needed to enable equity to be considered in tandem with other system performance, cost, and temporal metrics commonly considered in infrastructure decision-making. Thus, this work develops and applies an equity metric to evaluate infrastructure's service performance equity between groups with differing vulnerabilities for use in infrastructure component retrofit prioritization. The equity metric herein targets distribution pole retrofits to ensure a more equitable electric power service performance spread to household structures and their resultant adverse impacts. Yet, the metric could be applied to other infrastructure systems.

The equity metric stems from Theil's Index, specifically Theil's T (Theil 1967), which evaluates the inequality present in a scarce resource's distribution. The scarce resource herein is taken as service performance measured by household infrastructure outage likelihood, yet other scarce resources, such as a household's resilience score, could be defined. Additionally, this metric is decomposable allowing for the evaluation of equity differences between specific vulnerable and less vulnerable populations and transformable to relative measures for ease of understanding. This additional decomposition provides decision-makers with a refined understanding of service inequity to better shape resultant mitigation.

The remaining paper is organized as follows: A brief literature review of existing infrastructure equity metrics is presented. Then the methodology for our equity metric and its decomposition is presented. The metric is then applied for a simple hypothetical electric power network with varying nodal vulnerability scenarios for metric demonstration, followed by the conclusion and future applications.

2. OVERVIEW OF EXISTING EQUITY METRICS IN INFRASTRUCTURE

While equity concerns have become of increasing prominence, equity-based metrics for infrastructure decision-making have been sparsely developed and are in their nascent phase. The equity metrics for infrastructure in the current literature fall under the following: threshold-based, min-max, or the Gini Index.

Threshold-based strives to improve or reduce the percentage of the community that meets a threshold level of service performance, thus prioritizing an equal service level. This is widely applied for infrastructure to meet service levels equally. Yet, this metric can do little to flag the often-large gaps between the least and most vulnerable to go beyond equity defined as blindly weighted equal service provision. *Min-max* strives to equitably reduce the difference in service provision across a community, as seen in France-Mensah et al. (2019) and Kothari and O'Brien (2022). These metrics can be tailored for comparing differences between vulnerable groups, yet these can be skewed by outliers.

The *Gini Index* strives to measure the distribution of a resource across a community and how that distribution differs from a perfect equal distribution. The use of Gini guides the prioritization to reach an equality of service performance as utilized in Dhakal and Zhang (2023) and Seyedashraf et al. (2022). Gini is favored for its bounded and easy to interpret values falling between 0-1. Yet, Gini produces only one aggregate index for a community that does not differentiate between vulnerable groups.

Due to these previous limitations, a Theil's T-based metric is believed to be better suited to evaluate equity due to its additional representation of inequities attribution between vulnerable groups. Through this, vulnerable groups known to be facing inequities can be specifically prioritized. A Theil's T-based metric represents a new equity metric category which strives to address inequity by reducing the inequality in service provision specifically to vulnerable groups. As such, inequality is calculated and referenced in this paper in order to assess equity's current state. More inequality is indicative of more inequity and vice versa.

3. METHODOLOGY

3.1. Theil's T-Derived Infrastructure Equity Metric

Theil's T is an entropy-based measure to quantify the inequality present in the dispersion of a scarce resource. With its origin in the economic sector, the scarce resource was initially intended to be income to measure income inequality. The metric has rarely been implemented in infrastructure applications except for pavement assessments (France-Mensah et al. 2019; Kothari and O'Brien 2022).

Theil's T has a strong benefit of being decomposable into between-zone inequality (BZI) and within-zone inequality (WZI) with zones defined as a group with vulnerable and non-vulnerable characteristics of concern (e.g., minority vs. non-minority, high-income vs. non-high-income). This decomposition enables the understanding of inequality's root causes. BZI indicates the amount of inequality due to resource differences between vulnerable and less vulnerable groups, while WZI indicates the amount of inequality due to the resource differences among that singular group.

Theil's T also presents other beneficial features rendering it advantageous over other metrics. First, Theil's T exhibits scale invariance, which means the resource's units or scale do not alter the distribution's assessment. Secondly, it upholds Dalton's principle of transfers which states that a transfer of a resource from an individual with a greater amount of the resource to one with less of a resource should reduce inequality (Dalton 1920). Lastly, Theil's T is sensitive to transfers at the lower end of the scale, which means prominence will be given toward inequality exhibited by those with less.

As noted previously, Theil's T is decomposable into BZI and WZI and is defined through these components as shown in Eqs. (1), (2), and (3) (Liao 2016; Peacock et al. 1988):

$$T_T = BZI + WZI \tag{1}$$

$$BZI = \sum_{k=1}^{k} y_k \cdot \ln \frac{\overline{x_k}}{\bar{x}} \tag{2}$$

$$WZI = \sum_{k=1}^{k} y_k \cdot \left(\sum_{i=1}^{n_k} y_{ik} \cdot \ln \frac{x_{ik}}{\overline{x_k}} \right) \tag{3}$$

where T_T is the Theil's T, y_k is the kth zone's (i.e., group's) collective scarce resource share of the cumulative amount of resource, $\bar{x}$ is the overall mean amount of resource, $\overline{x_k}$ is mean resource of the kth group, y_{ik} is the ith individual's or entity's resource share of the kth group's cumulative amount of resource, and,x_{ik} ith individual's or entity's resource amount. T_T gives the inequality present in the scarce resource's distribution, and ranges from 0 to a maximum value of $\ln(N)$, with N being the number of individuals whose service performance is being considered. The larger T_T the more inequality present.

These terms can be mathematically redefined relative to their maximum values to provide relative measures (Theil 1967). The relative for Theil's T is defined by Eq. (4):

$$T_T^{rel} = T_T / \ln(N) \tag{4}$$

A value closer to 1 will indicate the greatest amount of inequality, and 0 indicates the corollary. Since BZI and WZI sum to constitute T_T, this additive property can be utilized to render relative metrics. By dividing by the T_T, which is the maximum possible value for either, relative metrics are defined in Eqs. (5) and (6).

$$BZI^{rel} = BZI / T_T \tag{5}$$

$$WZI^{rel} = WZI / T_T \tag{6}$$

A value closer to 1 indicates that the inequality observed can be attributed to distribution between or within vulnerable groups, respectively. A higher BZI is indicative of existing inequities between groups (i.e., unequal service provision between the groups), while a higher WZI is indicative of inequities within the separate groups (i.e., unequal service provision within a group) that can be attributed to structural differences in service performance based on physical placement within the network. All metric values are computed in this study; however, only a few may prove needed in implementation or be of concern to decision-makers. A discussion on which metric values are best suited for implementation follows in the application section.

3.2. Scarce Resource in Infrastructure Applications

The critical element for implementation of Theil's T in infrastructure application is the modification of the scarce resource from income to an infrastructure relevant term. The infrastructure scarce resource should be found at a resolution to enable decomposition into vulnerable and non-vulnerable groups based on affiliated demographic characteristics. Thus, the scarce resource can be taken as any quantity that is indicative of how well or poorly an individual is receiving infrastructure service at their affiliated structure (e.g., household, business, etc.). The service performance is influenced by the collection of components whose functioning dictates the service delivery to an individual at their related structures. As more vulnerable communities are often serviced by less adept infrastructure, the service performance for individuals in this area will subsequently be lower than that of their less vulnerable counterparts serviced by other regions in the network. The scarce resource amount is needed for every individual and should also be selected as a term that is quantifiable at such a resolution. Yet, terms such as the age of an entity's nearest infrastructure component of service should not be taken as the scarce resource due to a neglect of compounding system vulnerability that could drastically alter the realized scarce resource and obscure the present inequity.

As such, the indicative service performance quantity is taken as the household structure's likelihood of outage, P_{outage}. This can be found using system reliability principles where all critical components must be functioning for service provision and defined via Eq. (7).

$$P_{outage} = 1 - P_{service} = 1 - P_s^1 P_s^2 \cdots P_s^m \qquad (7)$$

where $P_{service}$ is the probability of service provision, $p_s^{1,2,\dots,m}$ are the individual distribution pole components' survival probabilities respectively, and m is the total number of components required for service provision to that structure. While this formulation may neglect some network redundancies, it is believed to serve as lower bound estimate of service inequality present. Many redundancies tend to be clustered away from areas of the greatest vulnerability which would only further increase the service performance disparities. With all P_{outage} defined for all individuals, the Theil's T and derived values can be computed.

4. APPLICATION & RESULTS

4.1. Application Area & Trials

The metric is demonstrated for the electric distribution network (EDN) in a hypothetical compact study area shown in Figure 1. This area is based on a small sub-region in Galveston, TX, USA, whose network stems from Darestani and Padgett (2022) and household structure locations from county accessor data (Galveston Central Appraisal District n.d.). The study area is subject to extreme wind events, and the EDN's service performance is assessed relative to this hazard.

In this hypothetical study, the inequity is not assessed between demographic groups due to the area's small size but is assessed relative to only two zones, Zones 1 and 2. These zones are defined based on the network topology with all components in Zone 1 feeding into the bottom most right component that then connects to the main distribution line and the corollary for Zone 2 but on the left. Between these zones the pole quality is varied to create service performance inequities of differing levels. In this application, pole age is taken as the indicative characteristic of pole quality. One age is assigned to all poles in a particular zone. Scenarios where the same age is assigned in both zones are also utilized. The zone age assignment scenarios conducted are the following with Zone 1's age listed first and Zone 2's second: [0-25 0-25], [40 40], [60 60], [80 80], [100 100], [0-25 40], [0-25 60], [0-25 80], [0-25 100], [100 0-25], [80 0-25], [60 0-25], [40 0-25].

The extreme wind distribution pole fragility model by Darestani and Shafieezadeh (2019) is selected to characterize the poles' component performance. This fragility model incorporates an age parameter that enables the scenario age variation. All other input parameters are held constant. For ages from 0 – 25 years, the fragility model produces no variability, as such this age range is noted with this notation.

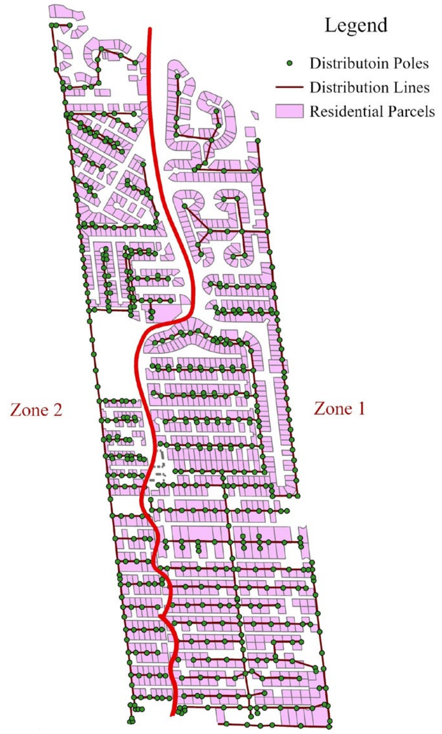

Figure 1: Study area schematic and zone division

4.2. Household Outage Likelihood Calculation

The household structures and the individuals they serve are connected to the infrastructure system via the component closest to the structure, the base node. This is conducted using GIS software for the nearest neighbor. With the base node identified, the shortest path between this node and the terminal node of service, taken as the connection into the main line, is found through graph properties. The components identified on the shortest path from the base node to terminal node are taken to only be distribution poles. The distribution lines that connect the poles could be viewed as critical components. However, there is sparse availability of wire failure fragilities and thus neglected. The individual pole's component survival likelihood is found via Eq. (8).

$$P_s = 1 - \int_{\mathbb{R}} P(outage|v) \cdot f(v)dv \qquad (8)$$

where $P(outage|v)$ is the distribution pole fragility at a given age assignment and assuming outage is synonymous with failure, and $f(v)$ is the PDF of extreme event hazard intensity v. For this region, Yeo et al.'s (2014) extreme wind characterization for Galveston, TX, is utilized. Through the characterization of each component's age and resultant survival likelihood, each individual structures' outage likelihood, the scarce resource, and ultimately the equity metric can be found.

4.3. Equity Metric Results and Discussion

The resultant equity metric values, both original and relative, for this application are reported in Table 1 for the scenarios with the same zone age assignment and Table 2 for the scenarios with varying age assignments. Additionally, Figure 2(a) displays the WZI and BZI decomposition for the varying age scenarios and Figure 2(b) plots T_T and BZI against the zone age differences for the varying scenarios. These results demonstrate the metric accurately reflects the varying amounts of inequity stemming from the scenarios' service performance distributions. The T_T and T_T^{rel} demonstrate less measured inequality in the uniform scenarios relative to those in the varying as expected. As the age assignment increases across the uniform scenarios, the T_T and inequality present slightly decreases which is attributed to severe component aging. When all components perform badly, all individuals receive poor service, which in turn mollifies the network structure-induced service variation and interpreted inequity.

While in the varying scenarios as the zone age difference increases to magnify service inequity, the reported T_T and T_T^{rel} increase as expected. However, when Zone 2 is varying in age and Zone 1's age is held constant, the inequality is markedly larger compared to the corollary. The magnification is attributed to the Theil property of increased sensitivity to transfers at the lower end. Zone 1 is approximately two -thirds of the region while Zone 2 occupies one third. This slightly smaller

Table 1: Uniform age assignment across zones

Age of Zones (yrs)	0-25	40	60	80	100
Theil's T (T_T)	0.1197	0.1178	0.0897	0.0102	0.0003
Relative Theil's T (T_T^{rel})	0.0170	0.0167	0.0127	0.0014	0.0000
BZI	0.0009	0.0009	0.0005	0.0000	0.0000
WZI	0.1188	0.1170	0.0893	0.0102	0.0003
Relative BZI	0.0074	0.0073	0.0053	0.0011	0.0003
Relative WZI	0.9926	0.9927	0.9947	0.9989	0.9997

Table 2: Varying zone age assignment

Age of Zone 1 (yrs)	0-25	0-25	0-25	0-25	100	80	60	40
Age of Zone 2 (yrs)	40	60	80	100	0-25	0-25	0-25	0-25
Zone Age Difference (yrs)	15	35	55	75	75	55	35	15
Theil's T (T_T)	0.2224	0.8586	0.9485	0.9461	0.3917	0.3993	0.4380	0.2184
Relative Theil's T (T_T^{rel})	0.0316	0.1218	0.1346	0.1343	0.0556	0.0567	0.0622	0.0310
BZI	0.1077	0.7702	0.9350	0.9433	0.3907	0.3888	0.3466	0.0990
WZI	0.1147	0.0884	0.0135	0.0029	0.0010	0.0105	0.0913	0.1194
Relative BZI	0.4843	0.8971	0.9858	0.9970	0.9974	0.9737	0.7915	0.4532
Relative WZI	0.5157	0.1029	0.0142	0.0030	0.0026	0.0263	0.2084	0.5468

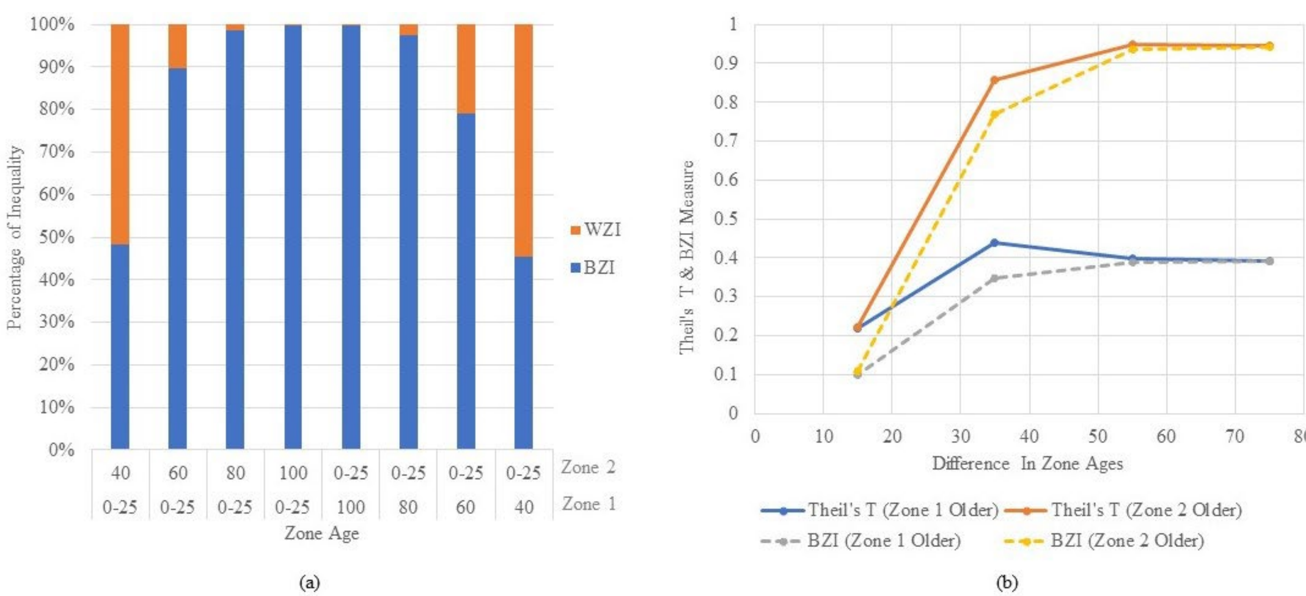

Figure 2: (a) Theil's T inequality decomposition into relative WZI and BZI for varying zone age assignments (b) Theil's T and BZI values across differences in zone age assignments

serving zone receiving markedly lower service performance renders it the lower end of the scale compared to corollary and is why more inequality is measured. This highlighted phenomena makes the metric especially helpful to investigate inequity between a vulnerable minority group and a less vulnerable majority group. In the varying scenarios where Zone 1 varies and Zone 2 is held constant, there is a drop in T_T and T_T^{rel} indicating less inequality. This is also attributed to severe component aging phenomena as seen in the uniform cases.

The decomposition results are also indicative of the expected inequities. In the uniform scenarios, the inequality is primarily WZI, as anticipated without any between-zone age differences introduced. The WZI can be attributed to the network topology rather than any difference stemming from between zones. Households farther away from the mainline naturally have more critical components which increases the likelihood of service loss. In the varying age scenarios, BZI increases as the age difference increases indicating increased service inequity between the two zones. This identification is important to be able to detect how different groups are being provided better or worse service comparatively. Even in the scenarios with Zone 2 varying and Zone 1 constant, while inequality decreases, the BZI increases. BZI therefore serves as a better value to capture continued components of inequity.

4.4. Metric Values for Decision-Problem Implementation

While all the metric values accurately capture the inequality and decomposition to portray the expected equity trends collectively, not all these values would be necessary for an equity-considered retrofit prioritization due to redundancy. By selecting a limited number of these values, the computational burden in complex network decision problems can be decreased. However, if decision-makers are concerned, all values could be incorporated.

In light of the application results, two values best suited for decision problem implementation are the BZI^{rel} and T_T^{rel}. The relative values are selected over the original due to ease of interpretation and invariance of scale. If the number of individuals varies between scenarios (e.g., looking across time points in a changing community), the original values would vary in maximum range leading to incomparable inequality values and decompositions. In these instances, the relative metrics provide more stable comparability and are easier to interpret.

BZI is selected because by measuring and striving to reduce its value during retrofitting, the service inequity apparent between a vulnerable and less vulnerable group could be improved. This targeted reduction appears most promising for communities to reduce their documented inequities. Selecting which vulnerable group for decomposition will be a challenge. This may lead to multiple BZIs relative to differing sets of vulnerabilities, and consideration of multiple BZI should be explored in future work. While in this application BZI performed as expected, the inclusion of T_T^{rel} as a secondary value is important to ensure that no plan may reduce BZI while increasing overall inequity. As the metric is fully incorporated into decision-making, more integrational challenges will surely become apparent and will be explored in future study.

5. CONCLUSIONS & FUTURE WORK

Natural disasters and their induced infrastructure outages have major impacts on communities. Yet, these impacts do not occur uniformly but are disproportionately clustered among vulnerable groups who are also often serviced by infrastructure more likely to fail. To reduce this inequity, an infrastructure equity metric is needed for integration into infrastructure decision problems so that inequity's reduction can be weighed alongside classic decision parameters of cost and system performance.

Thus, this work presents and implements an infrastructure equity metric that stems from Theil's T which measures the inequality present in a scarce resource's dispersion by which inequity can be assessed. In infrastructure applications, the scarce resource can be viewed as the service performance level at a household structure measured by the probability of service outage. The metric also produces decomposed values indicating what amount of inequity is attributed to differences between vulnerable groups. The metric was implemented for an EDN in a compact study area and decomposed between two network zones with varying pole qualities. The results confirm the derived values accurately pick up on inequity.

The metric should be further explored in future study and further integrated into a complete decision problem to evince how equity drives retrofit selections among other economic, network and temporal considerations. Future work should also explore the definition of the scarce resource and how its definition alters the equity's evaluation to drive different retrofit selections. An alternative scarce resource could be taken as the robustness measured through the probability of service survival.

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