

Efficiency in the Forward Exchange Market: An Application of Cointegration

BRIAN M. LUCEY*

Central Bank of Ireland, Dublin

Abstract: This paper defines a cointegrated system and discusses the implications of cointegration for efficiency in the forward exchange market using Irish daily data. Tests are discussed and carried out and the results are compared and contrasted with those of an earlier paper by Leddin (1988) who examined efficiency in the Irish forward market using different techniques. In contrast with the results which were obtained by Hakkio and Rush (1987), who used the cointegration approach to study efficiency in the US and German forward exchange markets, the study finds substantial evidence of market inefficiency.

I INTRODUCTION

Economic theory in many cases postulates that the time series representations of economic series should trend together. An example would be consumption theory, which would tend to suggest that consumption and income would move in broadly similar directions. More formally, the quantity theory of money, which states that $MV = PY$, would say that the "wedge" between nominal income and money (whatever that measurement is) would be constant if velocity were constant, i.e., if the monetarist model holds. In that case, we should expect to see money and nominal income trending very closely together. Finally, if the market for foreign exchange is an efficient one, in the sense of Fama (1970), then the forward rate and the spot rate should trend together, with the forward rate being an efficient and unbiased predictor of the future spot rate.

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Structural economic models using standard econometric techniques attempt to model the relationships between series and to estimate the parameters which underly the relationship. From these parameter estimates various inferences about the underlying theory can be made, and the process of falsification can proceed. Another way in which trending variables might arise is from the assumption of equilibrium relationships. If it is assumed that equilibrium holds, then it can be seen that the difference between the equilibrium and the actual should not be "large". More generally, however, it is the case that econometric techniques and/or data limitations may not allow for adequate inference to be drawn. Recent advances in the theory of what are called cointegrating vectors allow for the possibility of more general and easier inference to be drawn concerning economic theory. This paper defines cointegration in the context of a simple 2-variable system. Then, the relationship between cointegration and market efficiency is explored. Tests are then carried out on Irish forward and spot exchange rates to ascertain if there is evidence that they form a cointegrated system. Finally, the implications of these tests for market efficiency are discussed.

II DEFINING COINTEGRATION

The Wold decomposition theorem states that a stationary time series process with no deterministic component has an infinite moving average representation. It is easy to show (Box and Jenkins, 1970) that this can be represented approximately by a finite autoregressive moving average (ARMA) process. From this comes the definition of integration.

Definition: A series with no deterministic process which has a stationary invertible ARMA process after differencing d times is said to be integrated of order d . [Represented as $I(d)$]

For purposes where $d=0$ then the level will be stationary, and where $d=1$ then the change is stationary. The main differences between processes which are integrated with $d=0$ and $d=1$, are well known, and are discussed in Engle and Granger (1987). For cases where $d=1$, as is common in economic series, i.e., where the first differences of a series are stationary, then as $t \rightarrow \infty$, $v(x_t) \rightarrow \infty$ where t is time, x_t is the variable integrated of order 1 ($x_t \sim I(1)$) and $v(x_t)$ is the variance of x_t . Put simply, it may appear (and if we wait long enough it will appear) that the levels representation of the series will "explode". A series which is $I(0)$ will be characterised by wide swings, and short run effects will predominate. The effect of an innovation or "news" will be transitory. For $I(1)$ series, the long run will dominate, with permanent effects coming from "news". The close link between error correction mechanism (ECM's) and cointegration can be introduced by observing that if x_t and y_t are both

$I(d)$, then in general, for $a \neq 0$, the linear combination $z_t = x_t - ay_t$ will be also $I(d)$. Engle and Granger (1987), adapting from Granger (1981) and Granger and Weiss (1983) define cointegration as below.

Definition: The components of a vector x_t are cointegrated of order (d,b) , represented by $x_t \sim CI(d,b)$ if:

- (I) x_t is $I(d)$, i.e. all elements of the vector x_t are each $I(d)$ [(II) a non-zero vector exists such that $z_t = \alpha^1 X_t \sim I(d-b), b > 0$ where α is called the cointegrating vector.

Put more simply, if $x_t \sim CI(1,1)$, then it means that x_t achieves stationarity after differencing once, and at least one linear combination of its elements is stationary without differencing, i.e., a reduction of 1 in the number of degrees of differencing required to achieve stationarity. Engle and Granger (1987) state the following, for the $CI(1,1)$ case, it can be said that "... equilibrium will occasionally occur, at least to a close approximation".

Cointegration of order $(1,1)$ therefore implies that two series do not wander too far apart; the difference between them will be stationary, and therefore, we may expect to see close "tracking" of one series by the other; if they were not cointegrated, the value of the difference between them would diverge arbitrarily with probability 1, being itself a $I(1)$ series. Note that as yet, nothing has been said about causal directionality: a least one causal direction is implied, but cointegration does not imply any explicit causal relationship.

The close link between Cointegration and Error Correction has been established by Granger (1981, 1983). If two variables X and Y are cointegrated with a cointegrating factor or cointegrating vector α , then they can be written as the ECM:

$$x_t - x_{t-1} = a[x_{t-1} - \alpha Y_{t-1}] + b[Y_t - Y_{t-1}] + e_t.$$

This standard ECM is similar to that of Davidson, Hendry, Srba and Yeo (1978). It relates the change in the variables to lagged changes and to a lagged linear combination, which is stationary in the levels and may be seen as the equilibrium error. As pointed out by a referee, we may view this ECM as a description of how the economy eliminates the equilibrium error. This therefore implies that theory describes the long run, and that random shocks in the short run can only be slowly adjusted to. While this view of equilibrium sees changes in the variables as arising from the equilibrium error, another view sees the reverse: Campbell and Shiller (1988) see the equilibrium error

1. However, if α has to be estimated from the cointegrating regression of $\Delta \alpha^1 X_t$ on a constant and $\alpha^1 X_{t-1}$, then the D-F significance levels given in Dickey-Fuller (1981) are no longer valid.

as resulting from the incorrect forecasts by agents of the variables. They show how an ECM can be written as a VAR (Vector Autoregressive) system which can then be tested for restrictions. Engle and Granger (1987) infer from the ability of an ECM to be given a VAR representation, that, given that standard VAR models force cointegration via system restrictions, VAR's estimated with cointegrated data tend to be mis-specified when differences are used, and omit valid restrictions (the ECM) if levels are used. Given the increasing popularity of highly aggregated VAR systems for the forecasting of major macroeconomic elements, and given that the efficiency of the VAR lies in the ability to use OLS efficiently, this causes potentially major problems for atheoretic forecasting, which will use biased estimates.

A more recent development, by Hylleberg *et al.* (1988) is the concept of seasonal cointegration. Put simply, this states that although two series may appear not to be cointegrated at, say, the annual frequency, they may be so at other frequencies. If that is the case, then the standard tests discussed below are rendered invalid. The existence of seasonal unit roots has similar consequences for shocks as for the case where the root is present at the annual frequency.

III TESTING FOR COINTEGRATION: ENGLE-GRANGER TESTS

Testing for cointegration is closely tied to the problem of testing for unit roots. Intuitively, one can see a simple test of cointegration as being a test of the stationarity of the difference between two variables: if there is a unit root present in the series $(X_t - Y_t)$ then the variance will be bounded by infinity (explosive variance) and the variable will be non-stationary. Campbell and Shiller (1987) look at the problem this way. Let X_t be the vector of variables to be tested for cointegration. The null hypothesis is that $\alpha^1 X_t$ is stationary. If α^1 , the proposed cointegrating vector, is known (or predicted by economic theory) then modified Dickey-Fuller (1981) tests can be used, with t and F statistics corrected for residual autocorrelation.¹ Engle and Granger provide a number (7) of tests for testing. However, it must be stated that the Engle-Granger tests are somewhat limited in applicability, as

- (a) They assume CI(1,1) to be the true case.
- (b) They are derived for two variables.
- (c) They were derived from Monte Carlo simulations of 100 observations.

Recently, Engle and Yoo (1987) have produced expanded critical values for two of the Engle and Granger's test statistics. These are calculated for 50, 100 and 200 observations and for systems of up to five variables.

Engle and Granger (1987) propose seven tests; they then evaluate their

performance and select one preferred test. Monte Carlo simulations with 10,000 replications were used to obtain the critical values.

(a) The Cointegrating Regression Durbin-Watson Tests (CRDW)

The cointegrating regression (sometimes called the Equilibrium Regression (Hakkio and Rush (1987)) of X_{it} on X_{jt} , $i \neq j$ is estimated. The residuals are then tested for stationarity. If the residuals are non-stationary, then the Durbin-Watson statistic will tend to zero. Thus, if the DW is "too large", the variables can be seen as cointegrated, i.e., a stationary difference is present between them.

(b) Dickey-Fuller Tests (DF)

The residuals from the cointegrating regression are tested by means of an auxiliary regression. This regresses the first difference of the residuals on a lagged residual and the coefficient of the lagged value is then tested for significance via a t test. This assumes that any serial correlation present is first order.

(c) Augmented Dickey-Fuller Test (ADF)

This is the same as that for the DF test, except that additional lags (of the residual) are included in the auxiliary regression. As a result, it is over parameterised if any autocorrelation present is first order, but is correct in higher order cases.

(d)-(g) Vector Autoregression Tests (VAR)

These tests estimate various VAR representations of the implied ECM.

Commenting on the validity, power and usefulness of these tests, Engle and Granger recommend test (c), the ADF. However, they feel that the CRDW test is a very useful first test, due to its simplicity. It is, however, somewhat limited. In empirical tests in the paper, Engle and Granger mainly use the CRDW, DF and ADF tests to examine time series data for the presence of cointegration.

IV COINTEGRATION AND MARKET EFFICIENCY

Market efficiency in the sense of Fama (1970) implies, among other things, that prices have no memory. Prices reflect all available information. For a more detailed discussion, and a valuable review of the literature relating to market efficiency see Levich (1985). Granger (1986) shows that this implies that the prices of assets from two efficient markets cannot be cointegrated. This can be intuitively seen if we consider an ECM. If a market is efficient, then the changes will be unpredictable. Although serially correlated levels are compatible with market efficiency, if there is a time varying equilibrium

about which the rate fluctuates, the first differences should always be serially uncorrelated. If two asset prices are cointegrated, then an ECM can be found to express them. But, this would mean that part of the change in price could be predicted from the ECM. Let X_t and Y_t be two asset prices. Cointegration implies:

$$(X_t - X_{t-1}) = a[X_{t-1} - dY_{t-1}] + b[Y_t - Y_{t-1}] + e$$

where X_t and Y_t are cointegrated with a cointegrating factor of d . However, both X_{t-1} and Y_{t-1} are available in the $t-1$ information set. So, two asset prices from efficient markets cannot be cointegrated.

In the market for foreign exchange, there are further implications for efficiency. If the forward rate is to be an efficient and unbiased estimate of the future spot rate, the forward forecast error, $F_t - S_{t-1}$, must be a serially uncorrelated random variable. Efficiency tests which rely on regressing one on the other would work only in the case where both were stationary. Non-stationarity would lead to explosive standard errors. Cointegration has several implications for efficiency. First, F_t and S_{t-1} must be cointegrated. Were they not to be, then the forward forecast error would be non-stationary and would lead to F_t and S_{t-1} diverging arbitrarily far apart. Secondly, it is necessary for the cointegrating factor to equal 1. This is demonstrated somewhat more formally in Hakkio and Rush (1987). Finally, the ECM must have a certain shape (Hakkio and Rush).

The non-stationarity of the series means that it is difficult to formally test for a cointegration factor = 1, but this factor itself can be observed. Cointegration itself is relatively easily tested. Cointegration also means in the forward/spot market that the risk premium, if present, is stationary, necessarily. If there is a "risk" premium, then

$$S_t = RP + F_{t-1} + e_t$$

where RP is risk (or time varying) premium, e_t is white noise. If spot and forward rates are cointegrated, the forward forecast error is stationary. In the above case, the forward forecast error is $RP + e_t$. But, e_t is white noise, so RP must be stationary. Similarly, the risk premium, in a non-cointegrated system must definitionally be non-stationary. Leddin (1988) studying the sterling/Irish Pound market found that the forward rate was an unbiased predictor of the future spot rate, using quarterly data. Thus, he implicitly found a stationary risk premium, implying cointegration at quarterly frequencies.

V TESTING EXCHANGE MARKET EFFICIENCY IN IRELAND

Daily data on Irish Pound spot and forward rates were analysed, starting in December 1987; 100 observations were used. Forward exchange rates were the one month forward rates, which were matched to the appropriate spot exchange rate. This was done by allocating them to the nearest business day to 30 days ahead. All data was in nominal terms. Thus, if a forward rate was to be matched to a Saturday, it would actually be matched to Monday and so on. It is evident, from looking at the relationship between the forward and spot rates, for both the dollar and the pound sterling that there is a consistent failure to track by the forward rate. This would immediately raise doubts as to the efficiency of the forward rate in predicting the future spot rate.

As cointegration theory does not imply anything about causality it may be felt that it is necessary to test for cointegration of X and Y by looking at X on Y and Y on X. However, if X is cointegrated with Y, definitionally Y is cointegrated with X. They are two elements of a cointegrated system.

The following relationships were tested:

- (a) The spot exchange rates against the dollar and against sterling are part of a cointegrated system;
- (b) The forward rates are themselves part of a cointegrated system;
- (c) The spot and forward rates, for each currency, are part of a cointegrated system.

As stated, in all cases 100 observations were used, to provide strict comparability of the estimated test statistics to those given in Engle and Granger (1987).

Table 1 provides details of the tests carried out on the relationships hypothesised above. Details of the equilibrium and auxiliary regressions may be found in Appendix 1. £ = IR£/£ spot rate, £30 = IR£/£ 30 day rate. Analogously for the dollar.

Table 1: *Testing for Cointegration*

Test	Critical Value		Values of Test Statistic for System Being Analysed			
	1%	5%	£ - \$	£ - £30	\$ - \$30	\$30 - £30
CRDW	.511	.386	.146	.153	.089	.2679
DF	4.07	3.37	0.3741	0.5603	-1.50	-2.64
ADF	3.77	3.17	-0.2979	-0.4961	-1.34	-2.13

Ho: No cointegration. Reject for large values.

Critical values as taken from Table II, Engle and Granger (1987).

VI INTERPRETING THE EVIDENCE

Spot Exchange Rates

The evidence against cointegration is strong. A preliminary conclusion in favour of market efficiency, as evidenced by the seeming absence of cointegration, can be registered. None of the tests reject the null hypothesis, of a non-cointegrated system, all being well below the critical values.

Forward Exchange Rates

Again, there is no evidence on the basis of the test carried out here that the forward dollar and pound rates form a cointegration system. This would again be evidence for market efficiency.

Spot and Forward Rates

As stated, it is necessary, but not sufficient, for market efficiency that the spot and forward rates for a currency be cointegrated.

There is no evidence on the basis of the tests carried out that the spot and forward rates are cointegrated.

It can be seen (Appendix 1) that in no case is the coefficient on the forward rate at all close (in terms of standard errors) to 1. Accordingly, the testing that $-a = b = 1$ in an estimated ECM representation, is not done, as the assumption $d=1$ would not seem to be justified, and one of the necessary conditions is violated. So, I conclude against market efficiency in this case.

VII CONCLUSION

We have defined a cointegrated system as one in which there is a stationary levels linear combination of two non- (levels) stationary variables. The presence of cointegration is closely tied to the concept of equilibrium being some long-run phenomena towards which the market is adjusting. Tests were discussed and the implications for market efficiency also discussed. It was seen that two asset prices or exchange rates from an efficient market could not be cointegrated. Also, in the forward exchange market, for the forward rate to be an efficient and unbiased prediction of the future spot rate, the two had to be cointegrated, with cointegrating factor equal to one. Tests showed that there was no evidence of cointegration in the individual asset prices, and neither for the individual spot-forward systems, thus providing mixed evidence on market efficiency. This goes to some extent contrary to the work of Leddin (1988) which found strong evidence on the efficiency of the (Sterling) market. However, the finding that the implied risk premium was stationary is not inconsistent with the work of Leddin.

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APPENDIX 1

Equilibrium and Auxiliary Regression
(t statistics in brackets)

1. *Equilibrium Regression*

$$\begin{aligned} \text{£} &= 1.115608 + .004369 \$ \\ &\quad (36.018) \quad (.225) \end{aligned}$$

$$DW = .146$$

$$R^2 = .0005$$

$$\begin{aligned} \text{£} &= 1.126961 - .003779 \text{£}30 \\ &\quad (192.390) \quad (-.749) \end{aligned}$$

$$DW = .153$$

$$R^2 = .0057$$

$$\begin{aligned} \$ &= 1.628826 - .017457 \$30 \\ &\quad (24.252) \quad (-.446) \end{aligned}$$

$$DW = .089$$

$$R^2 = .002$$

$$\begin{aligned} \$30 &= 1.766255 - .044798 \text{£}30 \\ &\quad (22.415) \quad (-.657) \end{aligned}$$

$$DW = .267$$

$$R^2 = .004$$

2. *First Auxiliary Regression: Dickey-Fuller Regressions*

Resij = Residuals of equilibrium regression of currency i on currency j.

Difresij = First difference of Resij.

$$\begin{aligned} \text{Difrespd} &= 0.001076 \text{Respd} & DW &= 1.61 \\ &\quad (.73) & R^2 &= -.004 \end{aligned}$$

$$\begin{aligned} \text{Difrespp} &= 0.024702 \text{Respp} & DW &= 1.62 \\ &\quad (.56) & R^2 &= -.005 \end{aligned}$$

$$\begin{aligned} \text{Difresdp} &= -0.133857 \text{Resdp} & DW &= 2.17 \\ &\quad (-2.64) & R^2 &= .067 \end{aligned}$$

$$\begin{aligned} \text{Difresdd} &= -0.044659 \text{Resdd} & DW &= 1.59 \\ &\quad (-1.50) & R^2 &= .022 \end{aligned}$$

3. *Second Auxiliary Regressions: Augmented Dickey-Fuller Regression*

Dependent Variable: Difresdp

<i>Independent Variable</i>	<i>Lag</i>	<i>Coefficient</i>	<i>t Statistic</i>
Resdp	1	-.1260	-2.13
Difresdp	1	-.1209	-1.12
Difresdp	2	-.0215	-0.19
Difresdp	3	.0751	0.69
Difresdp	4	.0984	0.92
Difresdp	5	-.1259	-1.19

 $R^2 = .124$

DW = 2.00

Dependent Variable: Difresdd

<i>Independent Variable</i>	<i>Lag</i>	<i>Coefficient</i>	<i>t Statistic</i>
Resdd	1	-.0428	-1.34
Difresdd	1	.2282	2.18
Difresdd	2	-.1017	-0.95
Difresdd	3	.1476	1.38
Difresdd	4	.0841	-0.77
Difresdd	5	-.1453	-1.36

 $R^2 = .122$

DW = 2.07

Dependent Variable: Difrespd

<i>Independent Variable</i>	<i>Lag</i>	<i>Coefficient</i>	<i>t Statistic</i>
Respd	1	-0.259	.340
Difrespd	1	-.2516	1.94
Difrespd	2	-.0082	.06
Difrespd	3	.0147	0.11
Difrespd	4	.0254	0.15
Difrespd	5	-.0949	-0.55

 $R^2 = .04$

DW = 1.98

Dependent Variable: Difrespp

<i>Independent Variable</i>	<i>Lag</i>	<i>Coefficient</i>	<i>t Statistic</i>
Respp	1	-.0386	-.49
Difrespp	1	.2612	2.00
Difrespp	2	-0.0193	-.15
Difrespp	3	.0741	.56
Difrespp	4	-0.0315	-.19
Difrespp	5	-0.0708	-.42

Naming Conventions

Respp/Resdd are the residuals of the equilibrium regressions of the spot and forward rates for the pound and the dollar. Diffrespp is the first difference of the above.

Data Source: Financial Times.