

Nonlinear Vibro-Acoustic Behaviour in a Circular Membrane Oscillator

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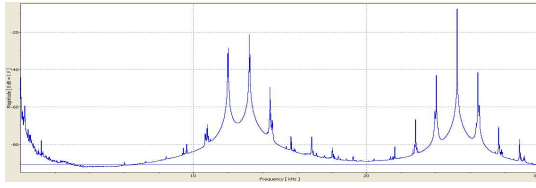
ABSTRACT

Oscillating systems that produce noise at a frequency of excitation may have as an abatement solution the increase of the frequency of excitation to above the audible frequency limit, viz. 20kHz. This approach may not be possible for systems subject to non-linear behavior as sub-harmonics can be generated within the audible range. This paper describes the analysis of a small metal annulus over which a thin membrane is stretched. The membrane is indirectly excited via a piezoelectric transducer attached to the annulus and has an application as an appliance in the electronics sector. The generation of noise is unwanted but found to occur occasionally. The analysis confirms the non-linear behaviour of the system with sub-harmonics resulting from a non-linear interaction with a low natural frequency. An experimental technique using a 2-D Scanning Laser Vibrometer (Polytec PSV-400) allows the vibrational modes of the annulus and membrane to be measured as a function of frequency and presented graphically and demonstrates the membrane to be radiating the noise. The paper discusses with the aid of both an analytical and numerical solution to the Duffing Equation how an increase in damping will suppress the sub-harmonics. A practical damping solution is found for the design which successfully eliminates the noise without negatively effecting its performance.

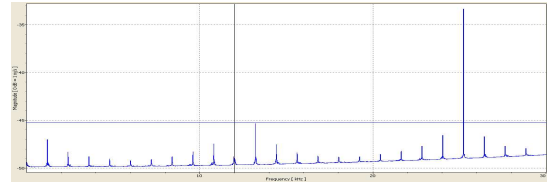
Figure 1a illustrates the problem case. This figure plots the sound radiated from the assembly and demonstrates significant acoustic energy at subharmonics of the excitation frequency. Figure 1b measures the membrane to be vibrating at these frequencies displaying the correlation between the vibration and the sound. The frequencies of figure 1b can be displayed as mode shapes and superimposed upon the membrane as seen in figure 2. Figure 2a shows a strong mode shape at a frequency of high acoustic amplitude demonstrating that it is the membrane that is radiating the sound. Figure 2b confirms the greatest vibration at the excitation frequency to be at the piezoelectric shaker location. The damping in the system was increased by simply wrapping a rubber band around the annulus. The results of this are to be seen in figure 3. All subharmonics are suppressed and hence the audible noise eliminated. A numerical study of the Duffing equation of equation 1 illustrates for a simple single degree of freedom system how subharmonics may be removed by increasing the damping. This simulation was achieved using Matlab and by expressing the Duffing equation in state space form as given in equations 2 and solving using Matlab's ODE45 (Runge Kutta) function. Sample results can be seen in figure 4. This can also be established analytically using an approach similar to that found in Hayashi [1].

$$m\ddot{x} + c\dot{x} + kx + \mu x^3 = F \cos(\omega t) \quad (1)$$

$$\dot{x}(1) = x(2); \quad \dot{x}(2) = \frac{F}{m} \cos(\omega t) - 2\zeta\omega_n x(2) - \omega_n^2 x(1) - \frac{\mu}{m} x(1)^3 \quad (2)$$

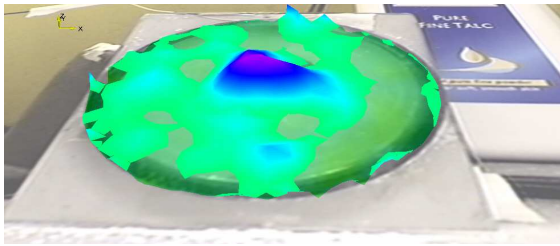


(a) Pressure spectrum from microphone located in near field. Sound is measured at the frequency of excitation but also at sub-harmonics.

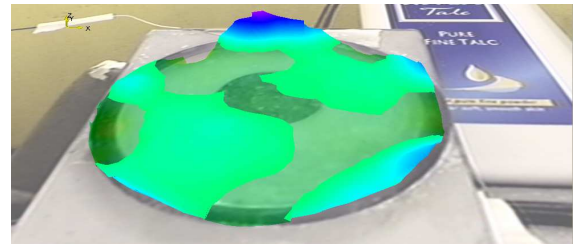


(b) Averaged vibration (velocity) spectrum over membrane. Peak at excitation frequency plus sub-harmonics.

Figure 1: **Low damping set-up.** Excitation frequency of 25.25kHz

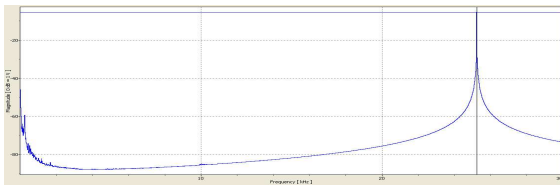


(a) Mode shape of membrane at 13.25kHz - corresponding to a frequency of maximum acoustic output.

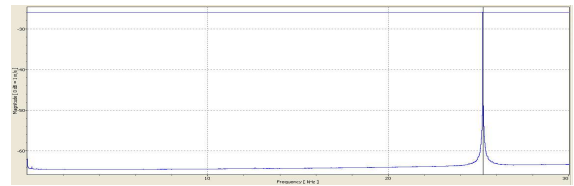


(b) Mode shape of membrane at 25.25kHz - corresponding to the frequency of excitation.

Figure 2: **Low damping set-up.** Velocity mode shape of membrane at two distinct frequencies.

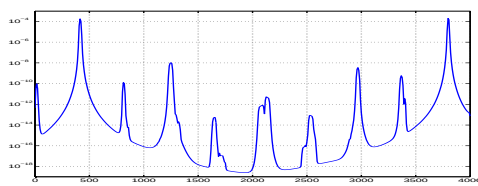


(a) Pressure spectrum from microphone located in near field. Sound at sub-harmonics has been suppressed.

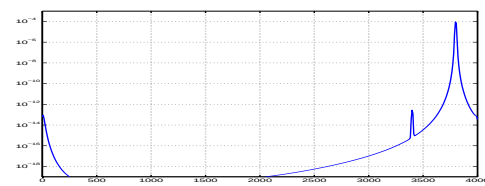


(b) Averaged vibration (velocity) spectrum over membrane. Vibration at sub-harmonics has been damped.

Figure 3: **High damping set-up.** Excitation frequency of 25.25kHz



(a) High excitation force with **low** damping.



(b) High excitation force with **high** damping.

Figure 4: Numerical solution of the Duffing equation using Matlab's ODE45 (Runge Kutta) function. Single degree of freedom system with a natural frequency of 200Hz and excitation frequency of 3.8kHz

References

- [1] C. Hayashi, *Nonlinear Oscillations in Physical Systems*. Princeton University Press, Princeton, New Jersey, 1985.