A SAMPLING BASED APPROACH TO LINE SCRATCH REMOVAL FROM MOTION PICTURE FRAMES

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ABSTRACT

We address the problem of detecting, and subsequently removing, 'line scratch' distortion in motion picture frames (see figure 4). A model for the lines' interaction with the image data is constructed. A sampling based algorithm based on the Reversible Jump Markov chain Monte Carlo framework is developed which enables automatic determination of both the unknown *number* of lines present, together with the lines' parameters. Previous work has not attempted to automatically determine the number of lines present [1]. Our approach is widely applicable in many object recognition problems, where the number of objects is unknown.

1. REVERSIBLE JUMP MCMC

Recently, Markov chain Monte Carlo (MCMC) methods (*e.g.* the Gibbs Sampler, the Metropolis-Hastings algorithm [2], [3]) have come into more widespread use for inference tasks, especially in image analysis [4]. They provide a method of exploring a complex probability distribution, usually $p(\theta|\mathbf{i})$, the posterior distribution of the parameters of interest. The Metropolis-Hastings algorithm is constructed by proposing changes to one or more of the components of θ at each iteration. If $\pi(d\mathbf{x})$ is the distribution of interest, and $q(\mathbf{x}, d\mathbf{x}')$ is the proposal distribution for the changes, then accepting the changes with probability

$$A(\mathbf{x},\mathbf{x}') = \min\left(1, \frac{\pi(d\mathbf{x}')q(\mathbf{x}',d\mathbf{x})}{\pi(d\mathbf{x})q(\mathbf{x},d\mathbf{x}')}\right)$$

can be shown to construct a transition kernel, $T(\mathbf{x}, d\mathbf{x}')$ which satisfies the *detailed balance* condition,

$$\int_A \int_B \pi(d\mathbf{x})T(\mathbf{x},d\mathbf{x}') = \int_B \int_A \pi(d\mathbf{x}')T(\mathbf{x}',d\mathbf{x})$$

a sufficient condition to ensure that the sampler does indeed converge to $\pi(d\mathbf{x})$ [2].

The standard forms of MCMC algorithms are limited, however, in that the dimensionality of $\mathbf{x}$ must be fixed. For our application, detection of an unknown number of lines, the dimensionality of the parameter vector is initially unknown. The Reversible Jump MCMC framework [5] enables an algorithm to be constructed which samples from $p(k, \theta^{(k)}|\mathbf{i})$, where k indexes the models of differing dimensionality, $\theta^{(k)}$ are the parameters of the kth model and $\mathbf{i}$ is the data (in this case the degraded image). We need to construct the transition kernel $T(\mathbf{x}, d\mathbf{x}')$ to maintain detailed balance when moving between the different subspaces.

When the current state is $\mathbf{x}$, choose to make a move of type m which will take the current state to $d\mathbf{x}'$ with probability $q_m(\mathbf{x}, d\mathbf{x}')$ (this includes the probability of choosing this move type). In [5], subject to a number of technical conditions, the acceptance probability is derived as

$$A_m(\mathbf{x},\mathbf{x}') = \min\left(1, \frac{\pi(d\mathbf{x}')q_m(\mathbf{x}',d\mathbf{x})}{\pi(d\mathbf{x})q_m(\mathbf{x},d\mathbf{x}')}\right)$$

The main condition when using this in practice is that the proposal density must be constructed such that $\pi(d\mathbf{x})q_m(\mathbf{x}, d\mathbf{x}')$ has finite density with respect to a symmetric measure. This can be done by constructing the proposals such that $\pi(d\mathbf{x}')q_m(\mathbf{x}', d\mathbf{x})$ and $\pi(d\mathbf{x})q_m(\mathbf{x}, d\mathbf{x}')$ are of the same dimensionality (even though $\mathbf{x}$ and $\mathbf{x}'$ are of differing dimensionality), by making the q_m's functions of a number of new random variables, which are introduced to match the dimensions. If $\mathbf{x}'$ is of higher dimension than $\mathbf{x}$, and is made to be some function of $\mathbf{x}$ and u, the new random variables, then the acceptance probability is

$$\min\left(1, \frac{p(k+1,\theta^{(k+1)}|\mathbf{i})}{p(k,\theta^{(k)}|\mathbf{i})q_k(u^{(k)})}\left|\frac{\partial(\theta^{(k+1)})}{\partial(\theta^{(k)},u^{(k)})}\right| \times \frac{p(\text{choosing to propose the reverse jump})}{p(\text{choosing to propose the forwards jump})}\right)$$

where $q_k(\cdot)$ is the probability density function of the new random variables, and $\left|\frac{\partial(\theta^{(k+1)})}{\partial(\theta^{(k)},u^{(k)})}\right|$ is the Jacobian of the transformation from $\{\theta^{(k)}, u^{(k)}\}$ to $\{\theta^{(k+1)}\}$.

2. LINE SCRATCH REMOVAL

A common defect in old motion picture material is 'line scratches'. They are visible as narrow, bright or dark, vertical (or near-vertical) lines, which persist for many frames.

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The scratches are formed by the film material running against an object in the camera or projection equipment. If the abrasion causes the removal of a proportion of the thickness of the emulsion, and the silver particles are uniformly distributed within the emulsion, then the effect is to reduce the density by a multiplicative factor. If the line scratch is formed on a positive print then reducing the thickness of the emulsion will result in a bright line; if the line scratch affects a negative print then in the subsequent positive copy a dark line will be visible. Dark lines are more common, and so the factor by which the original grey level at the line scratch location has been multiplied is parameterised by calling it the 'depth', d, where we take $0 \leq d < d_{max}$ where d_{max} is the largest value which results in a visible line scratch. Lines with values of d even very close to one are extremely visible due to their structure. In the experiments described later $d_{max} = 0.955$ was used.

Each scratch can be parameterised spatially by its starting position, y, and its width w. (Here we assume that the line extends the full height of the frame and is vertical; relaxing these constraints is straightforward.)

2.1. Constructing the Reversible Jump Sampler

To perform inference with regard to the number of line scratches and the scratches' parameters we must sample from $p(k, \{y, w, d\}^{(k)}|\mathbf{i})$ where $\{y, w, d\}^{(k)}$ denotes the k tripples of parameter values for the k lines. This can be written using Bayes' Theorem as

$$p(k, \{y, w, d\}^{(k)}|\mathbf{i}) \propto p(\mathbf{i}|k, \{y, w, d\}^{(k)})p(\{y, w, d\}^{(k)}|k)p(k)$$

So, up to a normalising constant, the target distribution is specified by the likelihood, the prior on the line scratches' parameters conditioned on the number of scratches, and the prior on the number of scratches.

The likelihood The effect of the line scratch is to multiply the original grey scale value by the factor d. Thus the likelihood is the probability of observing the image, where the affected areas have their values multiplied by d. For computational reasons, we used the simplest image model consistent with accurate detection. This is the model that assumes that the image is a sample from a space-varying Gaussian process, where the means and variances are estimated from the image data in a block-based manner, using overlapping blocks. The size of the blocks was chosen to be larger than the width of the line scratches. Then, when a pixel lies within a line, it is taken to be drawn from $N(d \times \mu, d \times \sigma)$ rather than from $N(\mu, \sigma)$. It is acknowledged that the presence of line scratches on the image will tend to reduce the estimated values of μ, and raise the estimated values of σ. This will weaken the detection, but in the experiments in section 3 it was found to be a sufficiently robust assumption.

The priors The prior on k gives information about the expected number of line scratches present. This was taken to be a Poisson distribution, conditioned on the maximum possible number of line scratches being known. The parameter λ gives the expected number of scratches.

$$p(k) \propto \exp(-\lambda)\frac{\lambda^k}{k!}, \qquad k = 0, 1, 2, \ldots, k_{max}$$

Note that this *includes* the case $k = 0$ to deal with the situation of no line scratches being present in the same framework.

The remaining priors, on the line scratches' parameters, are all conditioned on k.

The prior on $\{y\}$ tells us where the line scratches are expected to appear in the image. The physical mechanism of the lines production means that the lines are independent. Hence a uniform prior is used on the position variables. For simplicity the prior on the widths of the line scratches was taken to be a uniform distribution, with each w being drawn independently from $[1, w_{max}]$. The 'depth' parameters were also taken to be from a uniform distribution on $[0, d_{max}]$, where $d_{max} = 0.955$ as discussed above.

The proposal and acceptance probabilities To finally construct the reversible jump sampler two things must be achieved – the proposal distributions for each move type, conditioned on k must be specified, and the corresponding acceptance probabilities determined.

The available move types are

1. to change the position of an existing line
2. to change the width of an existing line
3. to change the depth of an existing line
4. to introduce a new line – a *birth*
5. to remove an existing line – a *death*

The proposal probabilities are respectively p_k, w_k, d_k, B_k, D_k where $p_k + w_k + d_k + B_k + D_k = 1$. In [5] Green proposed choosing B_k and D_k to be proportional to the stationary distribution based just on the prior for the number of scratches, *i.e.*

$$B_k = c \min\left(1, \frac{p(k+1)}{p(k)}\right) \quad D_k = c \min\left(1, \frac{p(k)}{p(k+1)}\right)$$

where c was chosen to be the largest value that gave $B_k + D_k \leq 0.9$ for all k. The remaining probability was distributed equally between the remaining moves, conditioned on $B_{k_{max}} = 0$ and $p_0 = w_0 = d_0 = D_0 = 0$. For the line scratch detection problem this resulted in an inefficient sampler; better results were obtained by decreasing the chances of proposing a death. The reasons for doing this will be made clear in the discussion of the action of the algorithm in section 3.

At each iteration, once the type of move has been selected by sampling from the distribution over the available move types, the acceptance probability must be determined. For proposed changes to the position, width and depth of an existing line scratch, changes which do not involve changing the dimensionality of the parameter space, the acceptance probabilities are as for the standard Metropolis Hastings algorithm.

Position, width and depth New values for these variables were proposed by perturbing the current values, drawing a new value uniformly in some interval around the current value. Because of the use of uniform priors and symmetric proposal distributions, the acceptance probabilities were all min (1, (likelihood ratio)).

Birth The birth step increases the number of parameters by three – a value must be assigned to the $\{y, w, d\}$

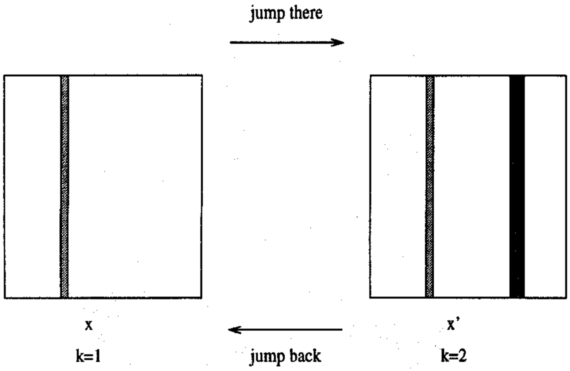

Figure 1: Illustration of the jumps involved in the birth step

triple associated with the new line scratch. Because each scratch occurs independently of any other, the existing parameter values cannot be used to guide the generation of the new parameters, and so $\{y', w', d'\}$ are drawn from their respective priors, conditioned on the new line not overlapping any existing one.

The acceptance probability for a change in state from $\mathbf{x}$ to $\mathbf{x}'$ involving a change in dimensionality was derived as

$$A(\mathbf{x},\mathbf{x}') =$$

$$\min\left(1, \frac{p(k+1,\theta^{(k+1)}|\mathbf{i})}{p(k,\theta^{(k)}|\mathbf{i})q_k(u^{(k)})}\left|\frac{\partial(\theta^{(k+1)})}{\partial(\theta^{(k)}u^{(k)})}\right|\frac{D_{k+1}}{k+1}\frac{1}{B_k}\right)$$

where the final two terms, $D_{k+1}/(k+1)$ and $1/B_k$ are the probabilities of proposing the *specific* changes in dimensionality. (The choice of *which* of the $k+1$ lines to remove is made randomly.) This is essentially

(the probability of jumping *back*) divided by
(the probability of jumping *there*).

Figure 1 illustrates this proposed jump.

In terms of the line scratch detection problem these are

$$p(k+1,\theta^{(k+1)}|\mathbf{i}) = p(\mathbf{i}|k+1,\{y,w,d\}^{(k+1)})\times$$
$$p(\{y,w,d\}^{(k+1)}|k+1)p(k+1)$$

$$p(k,\theta^{(k)}|\mathbf{i}) = p(\mathbf{i}|k,\{y,w,d\}^{(k)})p(\{y,w,d\}^{(k)}|k)p(k)$$

and

$$q_k(u^{(k)}) = p(y',w',d') = \frac{1}{L}\times\frac{1}{W}\times\frac{1}{d_{max}}$$

from the uniform priors on y, w and d which these proposals are drawn from. The Jacobian term, $\left|\frac{\partial(\theta^{(k+1)})}{\partial(\theta^{(k)}u^{(k)})}\right| = 1$ as each of the variables in the $k+1$ dimensional case is equivalent to exactly one of either the existing variables or the newly introduced variables.

Substituting into equation 2.1 results in the acceptance probability for the birth move being

$$A(\mathbf{x},\mathbf{x}') = \min(1, R)$$

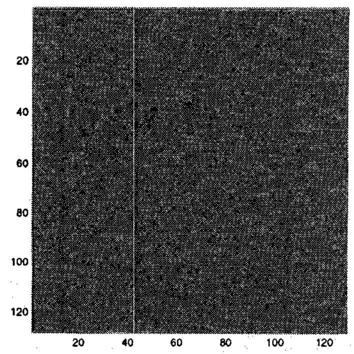

Figure 2: Artificial image

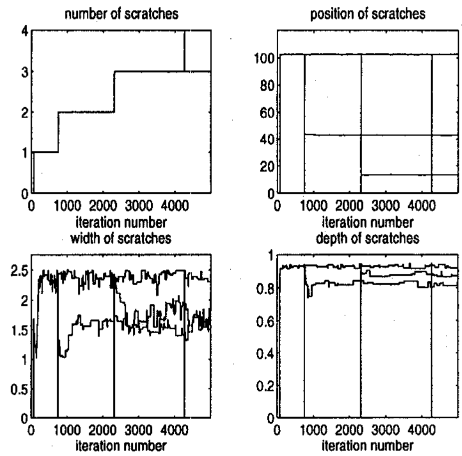

Figure 3: Results

where

$$R = \frac{(\text{likelihood})^{(k+1)}}{(\text{likelihood})^{(k)}}\times\frac{p(k+1)}{p(k)}\times\frac{D_{k+1}}{(k+1)B_k}$$

Death The death move must be constructed to undo exactly the birth move. In this case this involves choosing a line to remove, at random. A similar derivation to that above gives the acceptance probability as $A(\mathbf{x},\mathbf{x}') = \min(1, R^{-1})$

3. RESULTS

Here we present results of line scratch detection algorithm applied to two examples. The first was a 'best case' artificial image, where the image statistics exactly match the likelihood model described above, and line scratches were added to match the assumed line scratch model. The other was a frame digitised from real motion picture material.

Best case frame Figure 2 shows an artificial image, where each pixel in the background is drawn independently from a normal distribution. Three line scratches have been added. Two of these are clearly visible, the third (to the right of the frame) much less so. Table 1 lists the parameters of the line scratches. Figure 3 shows the results for

Figure 4: Frame from the 'knight' sequence

Figure 5: Restored frame

Scratch Number	1	2	3
true y	13.3	42.8	102.6
true w	1.85	1.47	2.41
true d	0.88	0.84	0.93
estimated y	13.3	42.7	102.5
estimated w	1.75	1.55	2.38
estimated d	0.88	0.82	0.93

Table 1: Actual and estimated line scratch parameters for the artificial example

running the reversible jump MCMC detector on this image starting from $k = 0$.

The correct number of line scratches has been identified, and their parameters estimated accurately - the parameter values estimated from forming a histogram of the samples are also listed in table 1.

Figure 3 requires more comment to explain the action of the algorithm. Because of the nature of the line scratches, that they are isolated, independent features, the algorithm functions much as a random search – lines are proposed in the birth step and are not accepted until one happens to be proposed at the location of an actual scratch. Once the line is accepted, subsequent proposals to alter its parameters cause their values to explore the posterior distribution – the estimates in table 1 are maximum a posteriori values estimated by smoothing the histograms of the sample values. Proposals to remove a true line were always rejected. Figure 3 also illustrates that it is possible for lines to be accepted which are not true lines, but these are rapidly removed.

Knight Figure 4 shows a frame from the 'knight' sequence, which is degraded by the presence of a bright line. Applying the line scratch detection algorithm to a 'negative' version of this image correctly located the single line at position $\hat{y} = 166.6$, with width $\hat{w} = 1.2$. Figure 5 shows the results of restoring this frame using a version of the interpolators described in [6]. Further experiments on this and other naturally degraded image sequences have demonstrated the robustness of the algorithm.

4. SUMMARY

In this paper we have discussed some of the theory of reversible jump MCMC samplers, which enable distributions defined over an unknown number of parameters to be sampled. This theory was used as the computational engine behind the development of an algorithm for the detection of 'line scratch' defects on motion picture material. The workings of this algorithm on artificially and naturally degraded images was explored. The detection algorithm proved accurate in determining the number and locations of the line scratches, and enabled visually pleasing restorations to be achieved.

5. REFERENCES

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