

The $\bar{B} \rightarrow D^* \ell \bar{\nu}$ form factor at zero recoil and the determination of $|V_{cb}|$

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We summarize our lattice QCD study of the form factor at zero recoil in the decay $\bar{B} \rightarrow D^* \ell \bar{\nu}$. After careful consideration of all sources of systematic uncertainty, we find, $h_{A_1}(1) = 0.913^{+24+17}_{-17-30}$, where the first uncertainty is from statistics and fitting while the second combined uncertainty is from all other systematic effects.

1. INTRODUCTION

A precise value for CKM matrix element $|V_{cb}|$ is an important ingredient in the study of CP violation and the determination of Wolfenstein parameters $(\bar{\rho}, \bar{\eta})$ via the unitarity triangle.

Experimental studies of the $\bar{B} \rightarrow D^* \ell \bar{\nu}$ decay rate determine the combination $|V_{cb}| h_{A_1}(1)$ [1–3]. The hadronic form factor $h_{A_1}(1)$ at zero recoil must be computed by theoretical means in order to extract $|V_{cb}|$.

Heavy quark symmetry imposes powerful constraints on $h_{A_1}(1)$: it requires $h_{A_1}(1) \rightarrow 1$ in the infinite mass limit [4] and it determines the structure of $1/m_Q^p$ corrections [5]. These power corrections correspond to long distance matrix elements in Heavy Quark Effective Theory (HQET). Previous determinations of $h_{A_1}(1)$ have employed sum rules [6] or have appealed to quark models [7].

We present a determination of $h_{A_1}(1)$ from lattice QCD using a new double ratio method designed so that the bulk of correlated statistical and systematic errors cancel [8]. In fact, with this method nearly all errors – including quenching – scale with $1 - h_{A_1}$ rather than with h_{A_1} .

Our result is computed in the quenched approximation. The method lends itself to a completely model independent determination of $h_{A_1}(1)$ once unquenched gauge configurations become available. The full details of our analysis are found in Reference [8].

2. THE METHOD

In the heavy quark expansion [8],

$$h_{A_1}(1) = \eta_A \left[1 - \frac{\ell_V}{(2m_c)^2} - \frac{\ell_V^{(3)}}{(2m_c)^3} - \frac{\ell_P}{(2m_b)^2} - \frac{\ell_P^{(3)}}{(2m_b)^3} + \frac{2\ell_A}{2m_c 2m_b} + \frac{\ell_A^{(3)}}{2m_c 2m_b} \left(\frac{1}{2m_c} + \frac{1}{2m_b} \right) + \frac{\ell_-^{(3)}}{2m_c 2m_b} \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) + \dots \right]. \quad (1)$$

The short distance quantity η_A relates HQET to QCD and is known to two loop order [9]. The ℓ 's are the matrix elements in HQET which we determine from lattice QCD.

We construct double ratios [8] from the lattice matrix elements $\langle D | V_4 | B \rangle$, $\langle D^* | A_1 | B \rangle$ and $\langle D^* | V_4 | B^* \rangle$. The matching between lattice and HQET currents is known to one loop order [10]. By varying the “charm” and “bottom” quark masses in the lattice simulations, we are able to extract all of the required ℓ 's in Eq. 1, except $\ell_-^{(3)}$. The truncation uncertainty in our $h_{A_1}(1)$ determination is the size of this unknown term plus higher order terms in the heavy quark expansion.

3. RESULT FOR $h_{A_1}(1)$

We find

$$h_{A_1}(1) = 0.913^{+24+17}_{-17-30}. \quad (1)$$

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Table 1

Budget of statistical and systematic uncertainties for $h_{A_1}(1)$ and $1 - h_{A_1}(1)$. The row labeled “total systematic” does not include uncertainty from fitting, which is lumped with the statistical error. The statistical error is that after chiral extrapolation.

uncertainty	$h_{A_1}(1)$	$1 - h_{A_1}(1)$ %
stats and fits	+0.0238 – 0.0173	+27 – 20
adj. m_c & m_b	+0.0066 – 0.0068	+ 8 – 8
α_s^2	± 0.0082	± 9
$\alpha_s(\bar{\Lambda}/2m_Q)^2$	± 0.0114	± 13
$(\bar{\Lambda})^3/(2m_Q)^3$	± 0.0017	± 2
a dependence	+0.0032 – 0.0141	+ 4 – 16
chiral extrap.	+0.0000 – 0.0163	+ 0 – 19
quenching	+0.0061 – 0.0143	+ 7 – 16
total syst.	+0.0171 – 0.0302	+20 – 35
stat $\oplus$ syst	+0.0293 – 0.0349	+34 – 40

The first uncertainty is the statistical error added in quadrature to the uncertainty from fitting procedures. The second uncertainty is the combined error, in quadrature, from the other sources listed in Tab. 1.

3.1. Systematic Uncertainties

• **Statistics and fitting procedures.** The statistical error in our result is after the “chiral” extrapolation of the spectator quark. The fitting procedure error includes the effect of excited state contamination and also bounds variations in $h_{A_1}(1)$ from alternate plausible time ranges in fitting three-point ratio plateaus. Minimum Chi-square fits were obtained for all fits. Fits include the data correlation matrix and produce bootstrap error determinations. Small poorly determined eigenvalues of correlation matrices were rejected in a SVD decomposition.

• **Adjustment of m_c and m_b .** The charm and bottom masses are determined by adjusting the bare quark mass until the kinetic mass of a lattice meson matches the physical D_s and B_s masses respectively. These kinetic masses tended to be quite noisy. Our uncertainties are taken to encompass our m_c and m_b determinations from quarkonia. The agreement between heavy-light

and -onia quark mass determinations is better for charm than bottom. Hence, our charm quark mass has the smaller uncertainty.

• **Matching, α_s^2 .** Although the matching between HQET and QCD is known to two loop order[9], the matching between lattice and QCD is only known to one loop order[10]. Hence, we match among HQET, lattice and QCD schemes consistently to one loop order, choosing scales for the QCD coupling according to the BLM prescription[11]. The uncertainty in Tab. 1 reflects our estimate of non-BLM terms at and beyond two loop order in the perturbative expansions.

• **Undetermined $(\bar{\Lambda}/(2m_Q))^3$ terms.** We did not determine $\ell_-^{(3)}$ in Eq. 1. Taking nominal values for m_c and m_b and $\ell_-^{(3)} \approx \bar{\Lambda} \approx 0.5 \text{ GeV}$ we estimate the size of the unknown term to be ~ 0.0017 . This estimate is consistent with the size of all other $1/m_Q^3$ terms that *are* included in our result.

• **Action and currents $\alpha_s(\bar{\Lambda}/(2m_Q))^2$.** The action and currents are tuned to tadpole-improved tree level. A careful analysis[8] of the form factors shows that remaining corrections are of order $\alpha_s(\bar{\Lambda}/(2m_Q))^2$. Using nominal values, we estimate $\alpha_s(\bar{\Lambda}/(2m_c))^2 \approx 0.008$. We also estimate this uncertainty by repeating our analysis using tree level quark masses instead of the “quasi one-loop” masses used in our standard analysis. We estimate the uncertainty in $h_{A_1}(1)$ to be ± 0.0114 by this method.

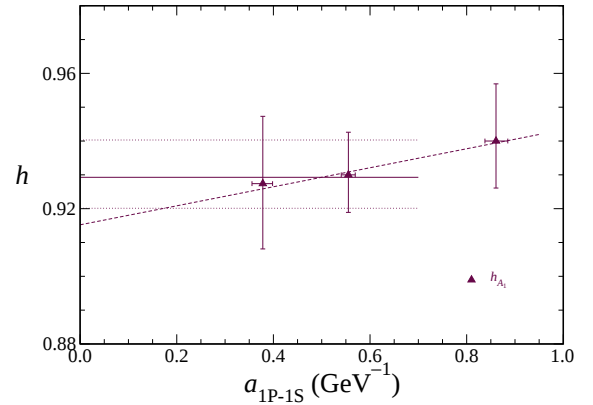


Figure 1. Lattice spacing dependence for $h_{A_1}(1)$.

• **Lattice spacing dependence.** We computed $h_{A_1}(1)$ for a strange-mass spectator quark at three lattice spacings corresponding to $\beta = 6.1$, 5.9 and 5.7. The lattice spacing dependence is shown in Figure 3.1. We take the weighted average (horizontal line and error envelope) of values at our two finest lattice spacings as our best determination of $h_{A_1}(1)$. The value from our coarsest lattice is not used in this average since it suffers larger heavy quark discretization errors. We do include the coarsest lattice in our bound on discretization errors. This bound is shown by the linear fit to all three points.

• **Chiral extrapolation.** We have studied the spectator mass dependence for $h_{A_1}(1)$ at our two finer lattice spacings. Figure 3.1 shows the linear extrapolation to the physical down quark mass for $\beta = 5.9$. We observe a similar slope, in dimensionful units, for $\beta = 6.1$. Our best value of $h_{A_1}(1)$ with a strange-mass spectator quark is shifted downwards by the amount shown in the figure to yield our final value of $h_{A_1}(1)$ with a bottom spectator quark. Statistical errors are increased as indicated in the figure.

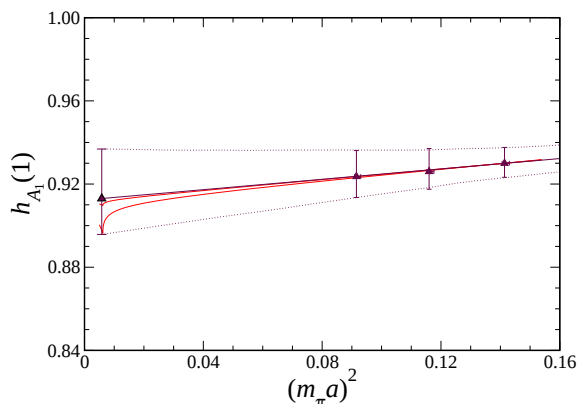


Figure 2. Spectator quark mass dependence at $\beta = 5.9$.

Randall and Wise[12] have computed pion loop effects upon $h_{A_1}(1)$ in Chiral Perturbation Theory. The curve with the cusp shown in Fig. 3.1 is this prediction for the spectator mass depen-

dence[8]. The departure from linear behaviour near the down quark mass adds an additional uncertainty to the chiral extrapolation.

• **Quenched QCD.** Our result is obtained in quenched QCD. The quenched coupling runs incorrectly: the short-distance quantity η_A changes by $+0.0050$ when quenched. Long distance form factors, such as h_{A_1}/η_A , are also affected by quenching. With our method, however, quenching errors only affect the deviations from unity. Guided by studies of decay constants f_B and f_D , we expect this error to be less than 10%. Our quenching uncertainty reflects estimates of both long and short distance effects.

4. $|V_{cb}|$

Using our result, we find:

$$10^3 |V_{cb}| = \begin{cases} 45.9 \pm 2.4^{+1.8}_{-1.4} & \text{CLEO [1]} \\ 38.7 \pm 1.8^{+1.5}_{-1.2} & \text{LEP [2]} \\ 39.3 \pm 2.5^{+1.6}_{-1.2} & \text{Belle [3]} \end{cases}, \quad (2)$$

where the second error results from adding all our uncertainties in quadrature. This result includes a $+0.007$ QED correction to $h_{A_1}(1)$.

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