

Vulnerabilities in Water Distribution Systems Using $N - k$ Contingency Analysis

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ABSTRACT: Adequate performance of water distribution systems is at the core of community resilience. Water distribution systems are vulnerable to a variety of hazards, as they are susceptible to direct physical damage (e.g., earthquakes, winter storms, floods, and deterioration), and to service interruptions from failures of critical infrastructure they depend on (e.g., power and telecommunication systems). In this study, we adapt the $N - k$ contingency analysis, which is widely used to study power system security, to assess the vulnerability of water distribution systems. Multi-stage $N - k$ contingency analyses support resilience analysis: first by sampling possible system failure scenarios conditional to k component failures, independent of the hazard; second by evaluating system functionality loss (e.g., water service availability loss) under failure scenarios through physics-based models; and third by identifying critical failure scenarios that cause significant functionality loss and ranking the criticality of corresponding components for future intervention. We demonstrate our approach on two cases, one synthetic with Net3 in the Water Network Tool for Resilience (WNTR), and one practical with the water distribution system of the City of Lumberton, NC, which is a community testbed on the Interdependent Networked Community Resilience Modeling Environment (IN-CORE). The $N - k$ contingency analysis of these cases reveals different vulnerability patterns mediated by network layout and operation settings. In particular, the Net3's most vulnerable locations are where its backbone distribution lines lay because it is not feasible to shift their flow loads to adjacent assets due to capacity limitations. The most vulnerable area of the Lumberton network is where large, concentrated demands co-exist with sparse distribution lines that limit alternative flow paths. Overall, vulnerable areas relate to a lack of global and local connectivity redundancy, and local flexibility in system flow routing. Our $N - k$ approach can reveal intrinsic system vulnerabilities relevant to any hazard events. It complements existing hazard-dependent resilience analyses to support decisions and help community stakeholders allocate limited resources to improve the future resilience of their water distribution systems.

1. INTRODUCTION

Water distribution systems (WDSs) are lifeline infrastructure, whose adequate performance is at the core of community resilience (ATC 2016, NIST 2020). WDSs are vulnerable to direct physical damage from hazards (e.g., earthquakes, winter storms, floods, and aging) as well as service interruptions from failures of critical infrastructure they depend on (e.g., power and

telecommunication systems) (Davis 2014, Zhang et al., 2016; González et al., 2016). Resilience analysis of WDSs is critical to inform communities about mitigation and restoration policies, particularly by building upon system vulnerability assessments.

In the past decade, research advanced the quantification of resilience of water infrastructure against extreme, high-impact events (Klise et al., 2017; Davis 2018; Chu-Ketterer et al., 2023). Although a variety of resilience metrics are defined for resilience quantification, it is widely

acknowledged that resilience is concerned with the performance loss due to contingencies and the time to recover normal operation. One common aspect of existing studies on resilience quantification is the use of hazard-dependent analysis, where the system resilience is quantified conditional to given hazard scenarios. Hazard-dependent resilience quantification analysis usually consists of component fragility modeling, system damage simulation, and recovery analysis. The inputs mainly include hazard scenarios, component characteristics, component fragility curves, and recovery resource allocation rules (He & Cha, 2018). The output is usually the system functionality variation over time, which is used to calculate the resilience measure. Such hazard-dependent resilience quantification can inform a community about what to expect for their WDS after specific hazards, what mitigation and restoration measures can be adopted to improve its resilience.

Another critical but understudied aspect of resilience analysis of lifeline infrastructure systems, which are complex networks consisting of interconnected components over large spatial areas, is to characterize their inherent system vulnerability that results from their topological layout and commodity flow distribution. This hazard-independent aspect that highlights inherent system characteristics can complement the existing hazard-dependent resilience analysis in two ways. First, there is shift from a bottom-up approach, where the individual components are emphasized in the analysis, to a top-down approach, where the networked system as a whole takes center stage. This shift can improve our understanding about what contributes to an infrastructure system's general resilience. Second, hazard-independence manages uncertainty differently, as hazard and damage scenario modeling with their uncertainties are side-stepped. Instead, we unravel intrinsic system vulnerabilities to various possible contingencies.

In this paper, we introduce and extend the $N - k$ contingency analysis concept, which is widely used for power system security analyses to

k component failures (Sundar et al., 2019; Birchfield et al., 2021), to perform hazard-independent vulnerability analysis for WDSs. The $N - k$ contingency analysis identifies critical component failure scenarios that trigger significant performance loss of an infrastructure system, as well as the corresponding critical system re-configuration intervention measures. The goal is to understand how the intrinsic complexity and interdependency of an infrastructure system and its components determine its capability to withstand disruptions and recover operation.

The rest of this paper is structured as follows: Section 2 explains our methodology for vulnerability assessment of WDSs using $N - k$ contingency analysis. In Section 3, we demonstrate case applications to two WDSs, one synthetic with Net3 in WNTR, and one practical with the WDS of the City of Lumberton, NC, which is a community testbed on IN-CORE. Finally, Section 4 provides concluding remarks, including insights gained from our analysis and potential directions for future research.

2. METHODOLOGY

This section demonstrates how we adapt the traditional $N - k$ contingency analysis to support hazard-independent resilience analysis of WDSs. Our primary focus is on the vulnerability assessment, which relates to the intrinsic system characteristics.

2.1. Functionality analysis

The quality of service of an infrastructure system after disruption is the focus of infrastructure resilience analysis. For quantification purposes, the quality of service is usually described by an appropriate performance or functionality measure. In this study, we define functionality as a quantity between 0 and 1, where 0 denotes system-wide service failure, and 1 represents the desired system service status (satisfied demand with quality).

A WDS's functionality relates to three correlated aspects: quantity, pressure, and quality. In this study, we focus on the functionality with

respect to water quantity because it is usually the most basic requirement for the stability of a community. We define the system-wide *WSA* of a WDS as:

$$WSA(t) = \frac{\sum_{i \in N^*} D_i^d(t)}{\sum_{i \in N^*} D_i^e(t)} \quad (1)$$

where N^* is the set that contains all the nodes that have water demands; $D_i^d(t)$ and $D_i^e(t)$ are the delivered and expected demands at node i at time t , respectively.

We use WNTR developed by the EPA and Sandia National Laboratories (Klise et al., 2017) to perform hydraulic simulations and to estimate the functionality metric *WSA*.

2.2. $N-k$ Contingency modeling

The traditional $N-k$ contingency analysis for power systems aims to identify the set of k components whose simultaneous or near-simultaneous failure maximizes the damage to the system, measured in terms of the amount of load shedding in the system immediately following the disruption (Sundar et al., 2019). In the context of vulnerability analysis of WDSs, we also look for contingency scenarios that can have significant impacts on community resilience, not just the worst-case impact. Thus, the goal of $N-k$ contingency analysis for a WDS is to identify a set of possible severe contingency scenarios that can cause significant functionality loss.

Our $N-k$ contingency modeling consists of contingency identification and evaluation. At a specific contingency level k , the number of contingency scenarios for selection will be $C(N, k)$, where N is the total number of components of interest. The number of contingency scenarios can be combinatorically very large for $k \geq 2$ and values of N on the order of hundreds to thousands.

The contingency evaluation for a WDS is performed through physics-based functionality analysis as discussed in Section 2.1. The functionality loss variation during an extended post-disruption time period is used as the criterion to evaluate a contingency scenario.

In this study, the components of interest are pipes, which account for the vast majority of components in a WDS. We model the failure of a pipe by completely closing it indicating that water cannot flow through it, which is a common approach adopted in practice (Diao 2016; Chu-Ketterer et al., 2023). The pipe closure setting assumes that valves are accessible for each of the pipes to isolate them once they are break or burst.

2.3. System vulnerability identification

With the critical contingency scenarios obtained from the $N-k$ contingency modeling, we are able to identify the system's vulnerable parts, failures of which cause significant system-wide functionality loss. Note that the system's vulnerable parts are independent of hazard disruptions and are only determined by the inherent system topology and operational settings.

A straightforward way to describe a WDS's vulnerability is to rank the components (pipes) according to their contribution to system resilience. Here, the contribution of a pipe p_j is characterized by the functionality difference between WDS W^* , where p_j is functional, and WDS $W^* - p_j$, where p_j is nonfunctional.

To make the system vulnerability characterization tractable and still informative, we focus on $N-1$ and $N-2$ contingency modeling. With $N-1$, we rank the first tier of pipes according to the functionality loss due to their failures. The larger the functionality loss is, the higher the pipe ranking r_j^1 is. A subset of pipes, failure of which can cause system functionality loss, will have a distinctive ranking r_j^1 , and they are considered as the first tier of critical pipes for system vulnerability.

With $N-2$, we have two-round of rankings. The first round is to validate the ranking from $N-1$. It ranks pairs of pipes in the same way as that of $N-1$ and assigns the ranking number $r_{j,l}^2$ to each pipe p_j in pair l , where $l = 1, 2, \dots, N-1$, and $r_{j,l}^2 \in \{1, 2, \dots, \frac{N(N-1)}{2}\}$. Then, it ranks the pipes according to the accumulated rankings in the first round,

$$a_j^2 = \sum_{l=1}^{N-1} r_{j,l}^2 \quad (2)$$

where a pipe with a smaller a_j^2 will have a higher ranking. The basic idea is that a pipe is critical if its joint failure with any other pipe tend to cause large functionality loss. Each pipe will have a ranking number as it will either be in the first-tier pipe set or have joint failure with a pipe in the first-tier pipe set. To distinguish the criticality of pipes in the second round, we remove all the contingency scenarios that include any pipe in the first tier and perform the ranking again using the same method. In this way, we subtract impacts of pipes in the first tier set to unravel distinctive pipe rankings for the rest of the set.

3. CASE STUDY

3.1. Case 1

The first case we use to demonstrate our methods is a synthetic network featured by EPANET and WNTR, dubbed Net3. Net3 consists of 2 water sources, 2 tanks, 2 pumps, and 117 pipes, providing around 2.1 million gallons of water supply to 59 service nodes per day.

The contingency scenario we model is pipe closing for 24 hours beginning at hour 7 (peak-demand hour) in the simulation. All scenarios are simulated for a total duration of 31 hours. There are 117 and 6786 contingency scenarios for $N - 1$ and $N - 2$ contingency modeling, respectively.

Figure 1 shows the hourly functionality loss variation for important $N - 1$ contingency scenarios, which are defined as those that have average functionality loss larger than 0.05. In the early period of the contingency, from hour 8 to hour 16, there are few contingency scenarios that can cause considerable functionality loss. Starting in hour 17, the number of important contingency scenarios increases, and there are several scenarios that lead to 1.0 functionality loss, including may overlapping ones in Figure 1. Note that the 1.0 functionality loss is nominal, as we set the functionality loss to be 1.0 when the hydraulic simulation fails to converge. Hydraulic simulation divergence is observed when the solver cannot

find solutions that meet mass balance and energy conservation requirements under severe damage scenarios—it violates physics or there is severe functionality reduction.

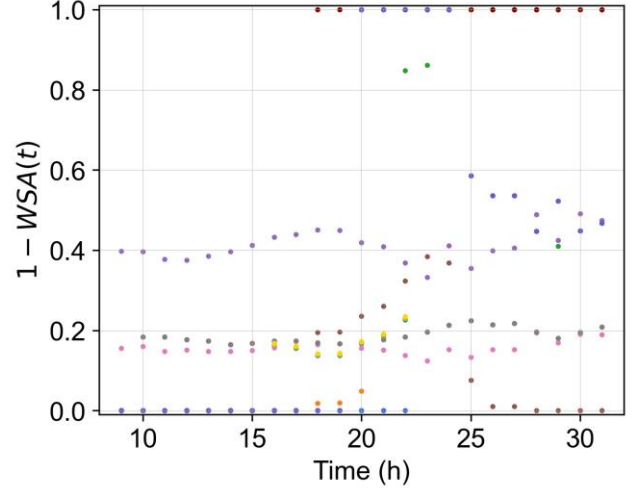


Figure 1: Net3's functionality loss ($1 - WSA(t)$) variation during the contingency. Each node of a unique color represents one $N - 1$ contingency scenario.

There are 14 unique critical $N - 1$ contingency scenarios in total over the entire contingency period for Net3. We rank the criticality of their corresponding pipes according to their average hourly functionality loss, as shown in Figure 2. The criticality ranking goes from 1, the most important, to 14, the least important, for the first tier of 14 pipes.

The key highlight from the critical first-tier pipe ranking is that the criticality of a pipe for system resilience is determined by two joint factors, the pipe's functional capacity, and the system's capacity to shift the pipe's load when it fails. In other words, a pipe is critical for a WDS if it delivers large water flows, and its function cannot be taken over by its neighbor alternatives.

Note that the most critical pipes of Net3 are located in two zones, Zone 1 and Zone 2, as shown in Figure 2. Pipes in Zone 1 are backbone distribution lines with large diameters that connect the upstream water sources with the downstream demand nodes. Pipes that lay nearly parallel with them on their left have small diameters and do not have much capacity to undertake the load shifted due to their closures.

Pipes in Zone 2 transport water flow from source S2, which operates fulltime and provides around 83% of total water supply. Hence, these pipes are critical because only part of the lost supply can be compensated by supply from source S1 and storage in the three tanks as source S1 operates only part of a daily cycle. On the other hand, pipes connected with S1 are not considered as critical because they have alternatives in Zone 2 that can take over their function.

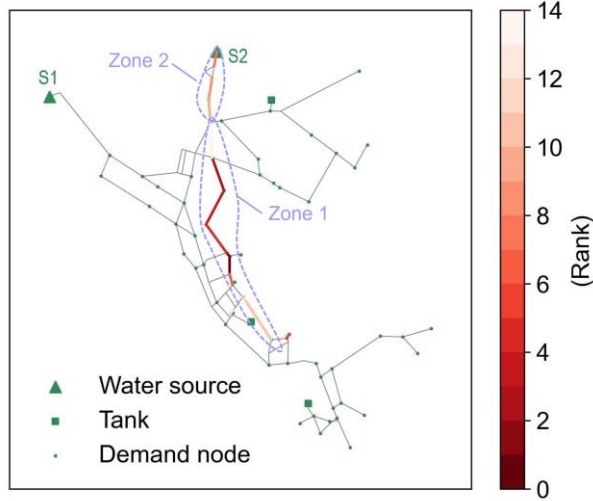


Figure 2: The first-tier critical pipe ranking for Net3.

We use the ranking method in Section 2.3 and the 0.05 functionality loss threshold to perform two rounds of rankings for $N - 2$ contingency scenarios. In the first round, the top 14 pipe ranking is consistent with that of the $N - 1$ contingency analysis. The second round unravels more distinctive criticality rankings for the rest of the pipes, as shown in Figure 3. There are another 34 pipes identified as critical, and their locations are not as concentrated as those of the first 14 pipes. Critical pipes in Zones 1 and 4 are the ones that connect tanks and large-demand end users to the network. Zone 2 is a bottle neck area that links the lower right part of the WDS with the main network. Critical pipes in Zone 3 include the ones that serve as backbone distribution lines, including pipes that connect with the tank. Overall, unlike pipes in the tier one that serve as global flow routing backbone, the pipes in tier two tend to link local supply by tanks and local demand clusters.

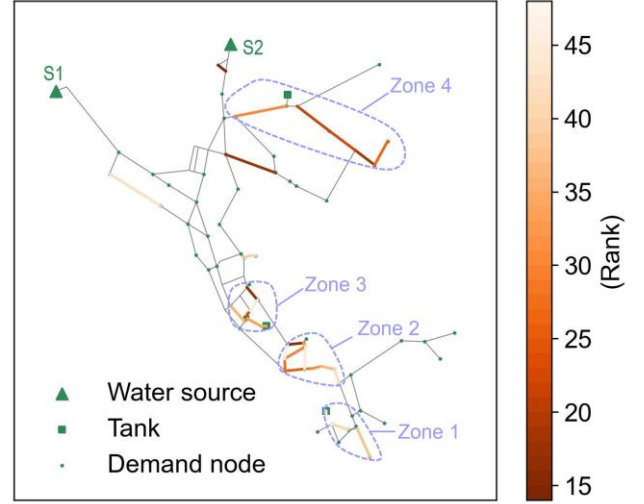


Figure 3: The second-tier critical pipe ranking for Net3.

The $N - 1$ and $N - 2$ contingency analysis on Net3 identifies different groups of vulnerable locations in this system. The first group includes pipes that serve as the flow routing backbone for the whole system, and it highlights the importance of global connectivity redundancy. The second group consists of pipes that connect local storage to local demand clusters, and it suggests the need for distributed local supply redundancy. The criticality ranking of pipes informs where and how resilience mitigation resources could be prioritized. One effective intervention strategy is to introduce alternative capacity to vulnerable locations by upgrading existing pipes or adding extra pipes. The alternative capacity actually provides redundancy and flexibility to a WDS improving its inherent resilience against different hazards. Its idea is different from the classic component strengthening strategy that tries to prevent component failures.

Here, we give one simple example of mitigation strategy for Net3, replacing the pipes that are parallel to those critical pipes in Zone 1 in Figure 3 with large-diameter pipes. Figure 4 highlights the intervened pipes marked in cadet blue, and all of them are of the same diameter size, 30 inches, which is the same size as critical pipes in Zone 1.

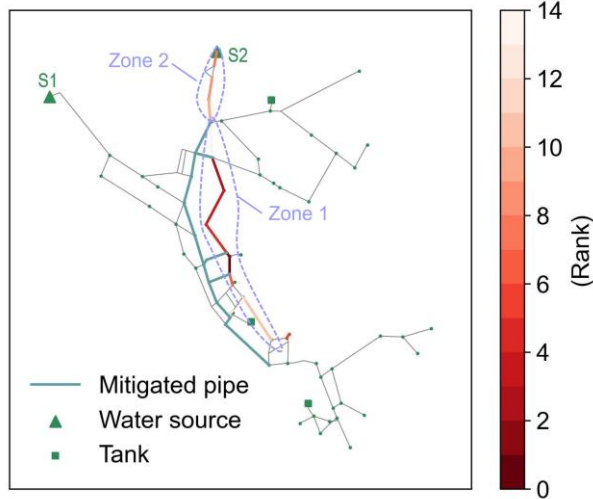


Figure 4: One example of intervention strategy for Net3. The mitigated pipes are ones that are replaced with pipes of diameter 30 inches, the same pipe size as critical pipes in Zone 1.

The $N - 1$ functionality loss variation for the intervened Net3 is presented in Figure 5. Five important $N - 1$ contingency scenarios are identified, which are a subset of the 14 critical scenarios for the original Net3. None of the 5 critical pipes is located in Zone 1, while 3 of them are located in Zone 2. The other 2 pipes are end-branches that connect nodes with large demands to the network, whose corresponding scenarios are surrounded by the dashed rectangle in Figure 5. The intervention strategy turns out to be very effective in mitigating the vulnerable locations in the system.

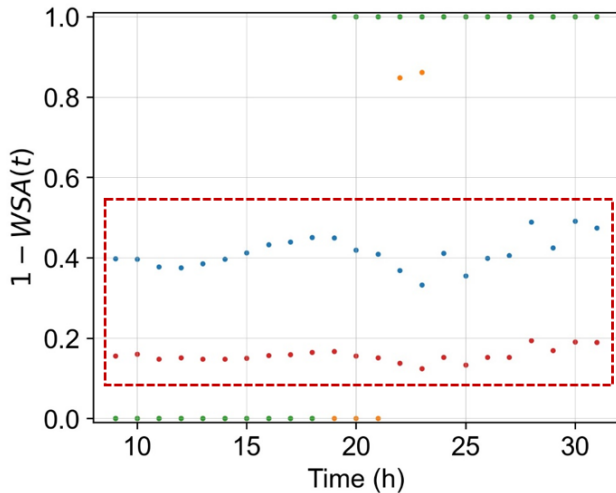


Figure 5: $N-1$ functionality loss ($1 - WSA(t)$) variation for the intervened Net3.

3.2. Case 2

The second case is the water distribution network of the City of Lumberton, NC (called NetL in the following), which is built from realistic data. Lumberton is a community testbed on IN-CORE, which is a community resilience modeling platform. NetL includes 1 water treatment plant, 3 elevated tanks, each having a 1-million-gallon capacity, 672 pipes, and 604 junctions, 394 of which require water supply. Its average daily supply is around 4.4 million gallons. The contingency starts at hour 8 (peak-demand hour) and ends at hour 32. There are 672 and 22546 scenarios for $N - 1$ and $N - 2$ contingency modeling, respectively.

The hourly functionality loss variation for important $N - 1$ contingency scenarios is shown in Figure 6. There are 12 unique important $N - 1$ contingency scenarios that can be categorized into two groups. The first group includes only 1 contingency scenario, which refers to failure of the pipe in Zone 1 in Figure 7 that connects the water treatment plant with the rest of NetL. It has a stepwise functionality loss variation as shown in Figure 6. From hour 8 to hour 16, the functionality loss is 0, which indicates that the system is supplied by its 3 tanks. The functionality loss is 1.0 on hour 17 onwards because the tanks run out of water. The other group consists of 11 pipes connected in series in Zone 2, as shown in Figure 7. The corresponding functionality loss is due to locally concentrated service disconnection, and the variations share the same pattern. These 11 pipes are the only water distribution path for the lower-left part of the system, where there is a node with large demand of 1 million gallons per day, and accounts for nearly 23% of total system supply. The variation over time comes from the varying hourly node demands in the system. Unlike Net3 that has vulnerable locations along the backbone distribution paths, NetL's vulnerability lies in single connections with one water resource as well as the assets conveying the large demand

located far away from the main network.

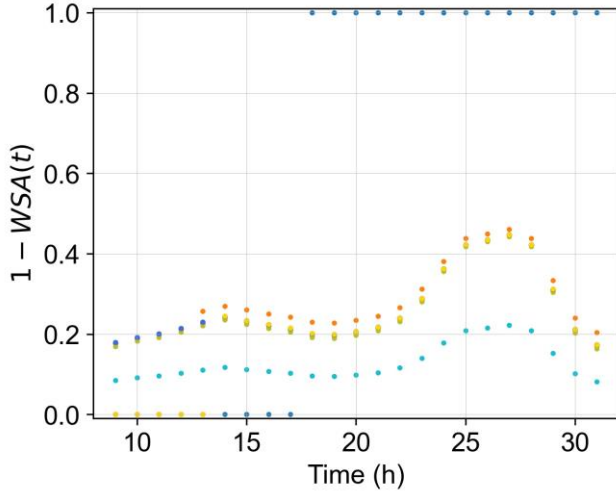


Figure 6: *NetL's functionality loss ($1 - WSA(t)$) variation during the contingency. Each node of a unique color represents one $N - 1$ contingency scenario.*

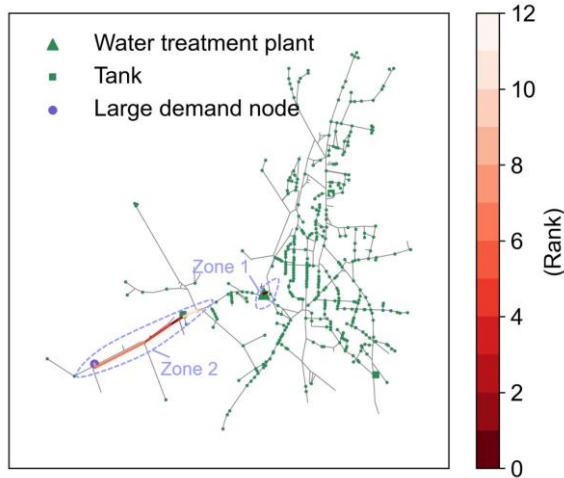


Figure 7: *The first-tier critical pipe ranking for NetL. The large demand node is the one with demand of 1 million gallons per day.*

For the sake of computational feasibility, we select 12396 of 22546 $N - 2$ contingency scenarios according to a preliminary connectivity-based functionality loss estimation that approximates the WSA based on the source-terminal connectivity. The top 12 pipe ranking in the first round is consistent with that of the $N - 1$ contingency analysis. Then, another 18 pipes are identified as second-tier critical pipes (located in the same area, as shown in Figure 8). It turns out that the lower-left part of the system, which has

sparse tree-style connection, is the most vulnerable region.

The vulnerable part supplies water to the industry consumers of the community. One effective vulnerability mitigation strategy is adding extra pipes in this area to have supply loops. Another alternative strategy is adding more onsite tanks in this area to add local storage for large demand industry consumers.

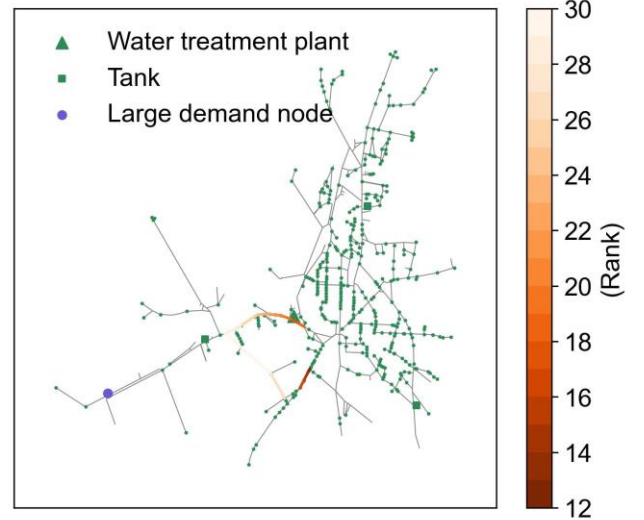


Figure 8: *The second-tier critical pipe ranking for NetL.*

4. CONCLUSIONS AND FUTURE WORK

This study introduces the $N - k$ contingency analysis as a scheme for hazard-independent vulnerability analysis and demonstrates its application to two case WDSs. This approach identifies and ranks critical components, whose failures can cause significant functionality loss, revealing vulnerable locations inherent to a networked system. The $N - 1$ and $N - 2$ contingency analysis of these two cases highlight different vulnerability patterns that result from different network layout and operation settings. The Net3's most vulnerable locations are where its backbone distribution lines lay because it is not feasible to shift flow loads on these pipes to their alternative pathways due to capacity limitations. The NetL's most vulnerable area is the part that has large, concentrated demands and sparse distribution lines, which indicate lack of

alternative flow paths or storage. Different mitigation strategies are identified based on the insights from the results. Overall, the highlight is that the presence of vulnerable areas comes from lack of global and local connectivity redundancy, and flexibility in a system, which can be unraveled by contingency analysis and aided in the future by the combinatorial properties of networks in their own right.

The $N - k$ contingency analysis is a useful tool for WDS vulnerability analysis. This paper primarily focusses on the technical functionality of WDSs, where socio-economic impact analyses could be chained to the current contingency analysis in IN-CORE to understand how the WDS vulnerability impacts community resilience. In addition to identifying the vulnerable part in a system, hazard-independent contingency analyses can be incorporated with optimization models to develop optimal intervention strategies that go beyond common hardening strategies.

5. ACKNOWLEDGEMENT

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