



Natural convection from a triangular array of isothermal horizontal cylinders

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ARTICLE INFO

Keywords:

Tubular heat exchangers
Natural convection
Particle image velocimetry
Thermal plume interaction
Flow oscillations

ABSTRACT

Natural convection heat transfer from three isothermally heated horizontal cylinders submerged in water in a triangular arrangement is investigated. Two configurations, named here as one over two and two over one, are characterised for three cylinder spacings ($S = 2, 3$ and 4 cylinder diameters) and Rayleigh number conditions ($Ra = 2 \times 10^6, 4 \times 10^6$ and 6×10^6). Combined local heat flux measurements and flow velocity measurements using particle image velocimetry are employed to examine the governing heat transfer mechanisms. For many cases of the one over two configuration, substantial differences from the single cylinder case are noted. The buoyant plumes generated from the two lower cylinders interact with the flow field and heat transfer around the upper cylinder, altering the local fluid velocities and temperature fields. This effect is dependent on both the inter-cylinder spacing and the Rayleigh number condition. For the two over one setup, both upper cylinders for all cases, except $S = 4D$ and $Ra = 6 \times 10^6$, behave in the same manner as a single cylinder. This is due to the competing effects of the buoyant plume rising from the lower cylinder and the restriction of flow through the gap between the two upper cylinders.

1. Introduction

Convective heat transfer from cylinder arrays occurs in many industrial and domestic applications. While time-averaged external forced convective heat transfer from these systems is well known, natural convection heat transfer from horizontal cylinder arrays is still an underdeveloped topic. This is particularly the case for closely packed cylinder arrays where strong interactions occur between adjacent thermal plumes. Previous research in this field has largely focused on time and spatially averaged heat transfer characteristics either for a single cylinder [1–6] or cylinder arrays [7]. Cylinder confinement has been explored in the vertical [8–10] and horizontal directions [11–14]. Only a limited number of researchers have investigated the thermal interaction between neighbouring cylinders in an array. In many situations, the cylinders do not behave like a collection of individual horizontal cylinders, and so the correlations and predictions made for a single cylinder cannot be applied. These changes in behaviour are due to the interactions between the buoyant plumes that are generated by each cylinder, which may impinge on downstream cylinders. When this happens, the Nusselt number is no longer only a function of the Rayleigh number (Ra) and the Prandtl number (Pr) but is also affected by the presence of the upstream cylinder(s).

A detailed review of the field has been previously conducted by some of the present authors [15–17]. As such only pertinent research

will be discussed here. Numerical [18,19] and experimental [17] studies have been carried out to investigate the local heat transfer distribution around an isothermal cylinder. Reymond et al. [17] observed a peak in local heat transfer at the bottom of the cylinder due to the thinnest boundary layer and consequent greatest temperature gradient in this region. Pera and Gebhart [20] investigated the wake formation over cylindrical surfaces in natural convection flows. Kitamura et al. [21] explored the heat transfer and fluid flow for natural convection around large horizontal cylinders to determine the turbulent transition and its influence on heat transfer. Fiscoletti et al. [22] demonstrated a critical Rayleigh number above which the generated plume from a single cylinder begins to oscillate. Eckert and Soehnngen [23] investigated the interaction of the developing thermal plumes and thermal boundary layers of two vertically aligned cylinders. They showed that the plume generated from the lower cylinder could either enhance or diminish the heat transfer performance of the upper cylinder depending on the applied spacing (S). Sparrow and Niethammer [24] studied natural convection of a vertical cylinder pair in air. They found that the lower cylinder was unaffected by the upper cylinder for all cases. This result was corroborated by Reymond et al. [17]. Cianfrini et al. [25] found a minimum critical spacing below which the upper cylinder decreases the heat transfer performance of the lower cylinder. Persoons et al. [16] investigated natural convection

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Nomenclature

D	Cylinder diameter [m]
g	Gravitational acceleration [m s^{-2}]
h	Heat transfer coefficient [$\text{W m}^{-2} \text{K}^{-1}$]
k	Thermal conductivity [$\text{W m}^{-1} \text{K}^{-1}$]
q	Heat transfer rate [W]
L	Length [m]
q''	Heat flux [W m^{-2}]
S	Centre-to-centre cylinder spacing [m]
T	Temperature [K]
t	Time [s]
U	Velocity [m s^{-1}]

Dimensionless Numbers

Nu	Nusselt number, $Nu = hD/k$ [–]
Pr	Prandtl number, $Pr = \nu/\alpha$ [–]
Ra	Rayleigh number, $Ra = g\beta(T_s - T_\infty)D^3/(\nu\alpha)$ [–]

Greek Symbols

α	Thermal diffusivity [$\text{m}^2 \text{s}^{-1}$]
β	Thermal expansion coefficient [K^{-1}]
ν	Kinematic viscosity [$\text{m}^2 \text{s}^{-1}$]
θ	Heat flux sensor angular position [$^\circ$]

Subscripts

∞	Bulk or infinity
θ	Local
f	Film
m	Circumferential average
s	Cylinder surface
wf	Working fluid
x, y, z	Coordinate direction

Acronyms

PIV	Particle image velocimetry
PMMA	Polymethyl methacrylate
PU	Percentage uncertainty
RTD	Resistance temperature detector
SU	Standard uncertainty

heat transfer and fluid dynamics for a pair of horizontal cylinders on the same vertical axis. Generally, a heat transfer enhancement of the upper cylinder was observed in comparison to a single cylinder. Their simultaneous particle image velocimetry and heat transfer data demonstrated two separate alternating flow regimes around the upper cylinder. In their follow-up work, Pelletier et al. [15] developed a numerical model to study the interaction between the two buoyant plumes and the relationship between unsteady fluid flow and heat transfer enhancement. They determined that the fluctuations of the Nusselt number at various points on the upper cylinder circumference were caused by two factors. The first was a large time scale oscillation of the lower generated plume (0.01 Hz), the second, a faster oscillation associated with smaller-scale vortices that periodically form alongside the plume at inflection points (0.067 Hz). The slower swaying plume was found to have a dominating impact on the overall heat transfer. Other researchers [26–34] have investigated natural convection heat transfer from two vertically aligned cylinders over a range of working fluids, Rayleigh numbers, confinement conditions, and spacings.

Corcione et al. [35] studied a pair of isothermal cylinders on the same horizontal plane. For large spacings, the average Nusselt number of each cylinder approached that of a single cylinder in an infinite medium. An optimum spacing for heat transfer was noted as cylinder spacing was decreased due to the chimney effect between the adjacent cylinders. As the Rayleigh number was increased, the optimum spacing for peak average heat transfer decreased. Below the optimum spacing, a full merging of the two boundary layers was observed, leading to a decrease in heat transfer for both cylinders. Corcione et al. [35] also investigated transverse misalignment of a pair of cylinders. They observed that as the tilt angle varied from 0° (vertically aligned) to 90° (horizontally aligned), the contribution of the chimney effect increased while the influence of the plume effect decreased. Sparrow and Boessneck [36] noted for their study of a pair of misaligned cylinders that transverse offsetting caused an increase in the upper cylinder Nusselt number (up to 27%) compared to the inline case. However, at greater vertical separation distances ($S = 5D$), the average Nusselt number decreased (by up to 20%).

A number of authors have examined the heat transfer from cylinder columns both experimentally [7,14,30,37] and numerically [5,38–40]. In general, it was observed that above a minimum spacing, the bottom cylinder behaves almost as if the other cylinders were not present. At large spacings, the heat transfer capability is enhanced up to a maximum spacing, while at closer spacings a decrease in performance compared to the single cylinder case is noted.

Mun et al. [41,42] numerically investigated a confined diamond array. The cylinders were located on the vertical and horizontal centrelines of the enclosure, and they were moved along the centrelines to vary the distance between them. They explored both equal [41] and unequal [42] horizontal and vertical spacing between cylinders. The confining wall-length scaled with the cylinder diameter ($\times 5$). The thermal and flow fields changed from steady state to unsteady state depending on the Rayleigh number and spacing. Lieberman and Gebhart [43] studied the temperature distributions around a flat array of long horizontal wires. They noted a peak average Nusselt number for the different angle-spacings at a tilt angle of 60° . This is in partial agreement with the results of Eckert and Soehngen [23], which suggested that a staggered array would achieve the maximum heat transfer.

Previous research, as reported above, has shown that the heat transfer characteristics of a cylinder array cannot be predicted by simple superposition of single cylinder behaviour. Strong interactions occur between the developing thermal plumes and thermal boundary layers of adjacent cylinders, directly impacting the heat transfer characteristics. This research sets out to investigate the buoyancy-driven heat transfer from three isothermally heated horizontal cylinders, arranged in equilateral triangle formations. Testing is conducted for two primary cylinder configurations: one cylinder over two cylinders (1:2) and two cylinders over one cylinder (2:1), as illustrated in Fig. 1(a) and (b) respectively. Both configurations are explored for three nominal Rayleigh number conditions (2×10^6 , 4×10^6 and 6×10^6) and centre to centre spacings ($2D$, $3D$ and $4D$). Local heat transfer characteristics of the isothermal cylinders in the array are detailed using an embedded heat flux sensor. Particle image velocimetry (PIV) is used to investigate the local fluid flow field around the isothermal cylinders to gain a deeper understanding of their heat transfer mechanisms.

2. Experimental apparatus and data reduction**2.1. Natural convection test facility**

The experimental facility, illustrated in Fig. 2, consists of two primary components, a polymethyl methacrylate (PMMA) vessel and horizontal isothermal copper cylinders. The tank is designed to approximate an infinite fluid medium for the submerged isothermal cylinders, although some three-dimensional effects may be present. The vessel is an open-top box of internal dimensions of $900 \times 300 \times 900 \text{ mm}^3$ ($L \times W \times$

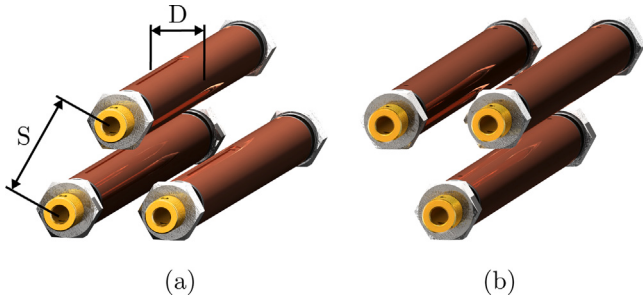


Fig. 1. Cylinder testing configurations. (a) 1:2 and (b) 2:1.

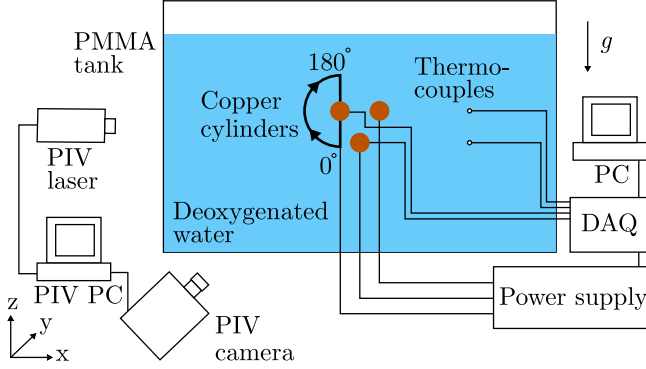


Fig. 2. Experimental facility schematic.

H). Interchangeable PMMA sections (see Fig. 3(g)) are used to vary the copper cylinder configuration and spacing. The designed vessel holds approximately 200 L during testing. The tank is supported on $40 \times 40 \text{ mm}^2$ aluminium profile frame.

2.2. Local heat transfer measurement

The isothermal copper cylinders are $D = 30 \text{ mm}$ in diameter and 330 mm long (see Fig. 4). This sizing mitigates edge effects [8,15,16]. They are mounted horizontally in the centre of the water tank and submerged in deoxygenated water. The effect of horizontal and vertical confinement of the heated cylinders is prevented by ensuring a greater than $3D$ separation from the vessel sides [14] and top of the working fluid [8], as proposed by Atmane [8] for their study on the effects of vertical confinement around a horizontal heated cylinder subjected to natural convection heat transfer with a similar range of Ra . The minimum distance between the edge of any tested cylinder and the vessel boundary or free surface is $10D$. Two 500 W cartridge heaters are embedded along the cylinder axis in the centre of each cylinder for isothermal heating during testing.

The heaters are held in position with thermal adhesive, which also serves to minimise thermal contact resistance. The temperatures of the cylinders are controlled by setting the power supplied to the cartridge heaters using a variable controlled power supply. 20 mm threaded brass cylinders were attached to the ends of each cylinder (Fig. 4(a)). The cylinders were secured to the PMMA section using an o-ring seal (Fig. 4(b)) and an outside nut fastened to the brass section.

Deoxygenated water is employed as the working fluid to provide a stable temperature medium, whose bulk temperature would not be strongly influenced by varying ambient conditions. Water also enabled the establishment of the desired experimental conditions at much lower cylinder wall temperatures in comparison to comparable Rayleigh number conditions in air. The properties of water are evaluated at the film temperature, $T_f = (T_s + T_\infty)/2$, where T_s and T_∞ are the cylinder surface temperature and bulk working fluid temperature respectively.

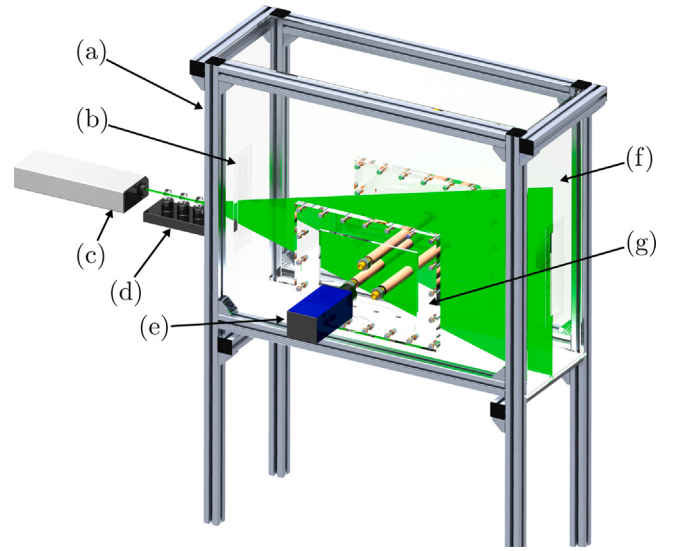


Fig. 3. Experimental facility. (a) Aluminium profile frame, (b) Glass window, (c) New Wave Research Solo II Nd:YAG twin cavity laser, (d) Laser optics, (e) LaVision FlowMaster 3S CCD camera, (f) PMMA vessel and (g) Interchangeable PMMA faceplate.

During testing, a temperature range of $29.6 \text{ }^\circ\text{C} \leq T_s \leq 44.5 \text{ }^\circ\text{C}$ and $26.1 \text{ }^\circ\text{C} \leq T_\infty \leq 38.7 \text{ }^\circ\text{C}$ was observed.

Local heat transfer measurement is facilitated using a 0.127 mm thick, $3.8 \times 2 \text{ mm}^2$ heat flux sensor (RdF Micro-Foil, P/N: 27036-2-RdF) that is embedded flush with the surface of the copper cylinders. The sensor also incorporates an inbuilt T-type thermocouple. All cylinders can be rotated about their axis with an accuracy of $\pm 0.5^\circ$. The emissivity of the Kapton coating on the heat flux sensor is 0.7 . Due to symmetrical considerations, only two of the three cylinders are instrumented with heat flux sensors. Local heat flux (q'') data are recorded using a National Instrument (NI) 9215 data acquisition module (DAQ) in tandem with a Fylde 351UA amplifier. Data acquisition and synchronisation are implemented by a custom-built virtual instrument (VI) in LabVIEW. The heat flux sensors were calibrated in-situ, following the procedures outlined by Reymond et al. [17] and Atmane et al. [8], in which the measured heat transfer was referenced against the correlation for natural convection from a single horizontal cylinder developed by Churchill and Chu [2].

The bulk water temperature is measured using two T-type thermocouples (RS Pro T Type, P/N: 621-2209). These thermocouples are positioned at the same elevation as the centre of their corresponding horizontal isothermal cylinders. Temperature measurements are sampled using a NI 9219 DAQ. The thermocouples were calibrated in situ against a factory calibrated resistance temperature detector (RTD) probe with an Omega CL 26 digital meter (expanded uncertainty $\pm 0.3\%$ at $50 \text{ }^\circ\text{C}$). The desired Rayleigh number is established by setting T_s using a PID controller and comparing it with T_∞ , which is measured at the same elevation as the test cylinder. During testing, the temperature difference between bulk upper and lower thermocouples was typically $< 0.1 \text{ K}$ and the bulk temperature standard deviation was $\pm 0.5 \text{ K}$ and $\pm 0.6 \text{ K}$ for the upper and lower bulk temperature, respectively, across all test points. The Rayleigh number was defined by [44]:

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu\alpha} \quad (1)$$

where g is gravitational acceleration, β is the thermal volumetric expansion coefficient, ν is the kinematic viscosity, and α is the thermal diffusivity of the water. The average Pr ranged between 4.2 – 5.7 across all tests. During experimentation, cylinders were rotated at 10° increments to achieve a complete 360° profile of the local heat flux and surface temperature. At each test point, the facility is allowed to

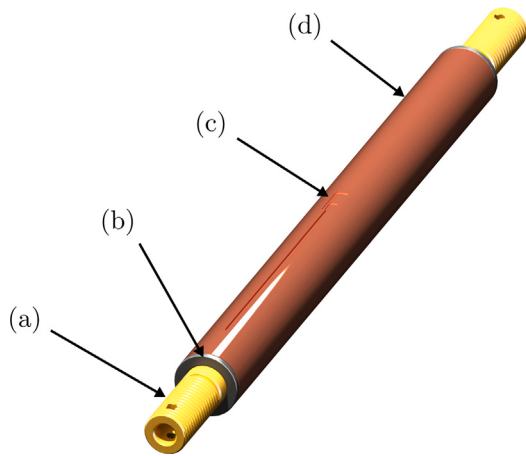


Fig. 4. Isothermal copper cylinder. (a) Brass insert, (b) O-ring seal, (c) Heat flux sensor and (d) copper cylinder.

reach quasi-steady-state conditions before data acquisition. The local heat flux, cylinder wall temperature and bulk water temperature measurements are recorded. Data is acquired at each 10° increment over a 15 min sampling period. Each measurement condition is repeated five times to ensure repeatability. Processing of the acquired data is achieved using custom-built code in MATLAB. From the acquired local heat flux data, the local (Nu_θ) and average (Nu_m) Nusselt number around the isothermal cylinder can be calculated by:

$$Nu = \frac{hD}{k} \quad (2)$$

where h is heat transfer coefficient ($h = q/(T_s - T_\infty)$) and k is the thermal conductivity of the working fluid at the film temperature.

During testing, the circumferential temperature as measured by the embedded sensor, was observed to vary on average by ± 0.7 K for one standard deviation for both the upper and lower cylinders. Considering the experimental uncertainty associated with the cylinder surface temperature outlined in Table 1 and the time required to complete one testing revolution (> 9 h), the isothermal cylinder approximation is suitable for the present study.

2.2.1. Fluid flow measurements

Flow velocity measurements are performed using particle image velocimetry (PIV). The current study employs a New Wave Research Solo II Nd:YAG twin cavity laser (15 Hz, maximum energy: 30 mJ, pulse width: 3 to 5 ns, beam divergence: <3 mrad). Customised optics are used to generate a 1 mm thick light sheet of 532 nm wavelength, aligned as shown in Fig. 3. A LaVision FlowMaster 3S thermoelectrically cooled camera (sensor size: 1280×1024 pixels, pixel size: $6.7 \times 6.7 \mu\text{m}^2$, maximum frame rate: 8 Hz, dynamic range: 12 bit) is used to record the fluid motion with a 105 mm focal length f/2.8 Sigma lens. With the light sheet entering from the left (see Fig. 3), the cylinder casts a shadow where no PIV measurements are possible (e.g. see Fig. 7). The camera is aligned nearly perpendicular to the light sheet, to minimise light reflections from the cylinder surface and to maximise the visible region near the surface of the top cylinder. Because of the slightly off-perpendicular position (viewing angle of a few degrees), a calibration target was positioned in the water tank before the measurements to perform an optical distortion correction using LaVision's Davis 7.2.2 software. Details of the calibration process are given by Yates [45].

Hollow glass spheres with diameters between 5 to 25 μm and a density of 1.10 g cm^{-3} (Spherical 110P8, P/N: 11W111) are used as tracer particles for PIV measurements. For these experiments, a frame rate of 2 Hz is sufficient to capture the transient fluid flow behaviour,

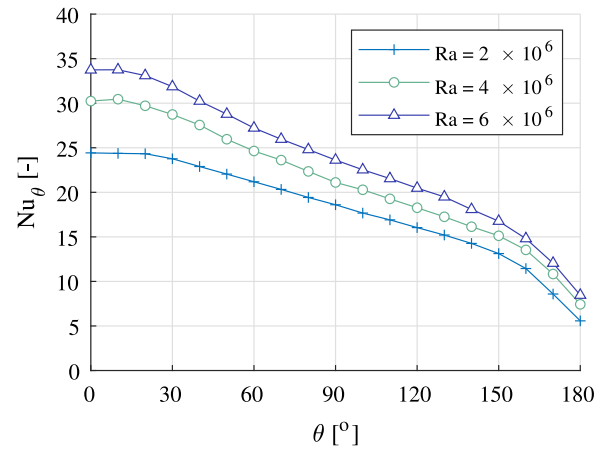


Fig. 5. Effect of Rayleigh number on the time-averaged local Nusselt number along the circumference of a single cylinder (bottom = 0° , top = 180°).

Table 1
Experimental uncertainty.

Parameter		PU	Unit
$T_{s,1}$	$\pm$	0.5	K
$T_{s,2}$	$\pm$	0.6	K
$T_{bulk,1}$	$\pm$	0.6	K
$T_{bulk,2}$	$\pm$	0.6	K
q''	$\pm$	15.0	%
Ra	$\pm$	1.8	%
Nu	$\pm$	15.1	%
U_{wf}	$\pm$	0.6	%

over a capture period of 160 s. The time between the two laser pulses is set at 20 ms. The acquired PIV data are processed with LaVision Davis 7.2.2 software. A multi-grid cross-correlation with continuous window shifting and deformation, and an initial interrogation window size of 32×32 pixels, and a 75% overlap approach is applied by the software on the first pass. The second pass uses a more refined window of 16×16 pixels and a 50% overlap. The resultant vector maps are then processed using MATLAB. In order to effectively plot these streamlines, a moving average of a short series (<20) of vector maps are used to obtain the final results. Average streamline plots over longer time periods are also included in the analysis of the results to show the plume position over time. The maximum Reynolds number ($Re = UD/\nu$) for this study based on flow measurements is estimated to be ~ 1000 .

2.3. Uncertainty analysis

The experimental uncertainty for all parameters is determined using the methodology outlined by Kirkup and Frenkel [46]. A list of the relevant parameters and their associated percentage uncertainty (PU) is outlined in Table 1. All listed values are to a 95% confidence level. First, the standard uncertainty (SU) of the acquired test data is determined. A combined uncertainty approach is then applied to Eqs. (1) and (2). The uncertainty associated with each temperature measurement is determined from their calibration fit. The uncertainty in the local heat flux measurement stems from data correlated by O'Gorman [47] and Cledes et al. [6]. The accuracy of the PIV flow measurements is determined using the methodology outlined by Melling [48]. Using a nominal maximum velocity of 25 mm s^{-1} and Stokes flow approximation for the drag force results in an average fluid velocity (U_{wf}) of 7.89 mm s^{-1} and an average particle velocity of 7.84 mm s^{-1} . This

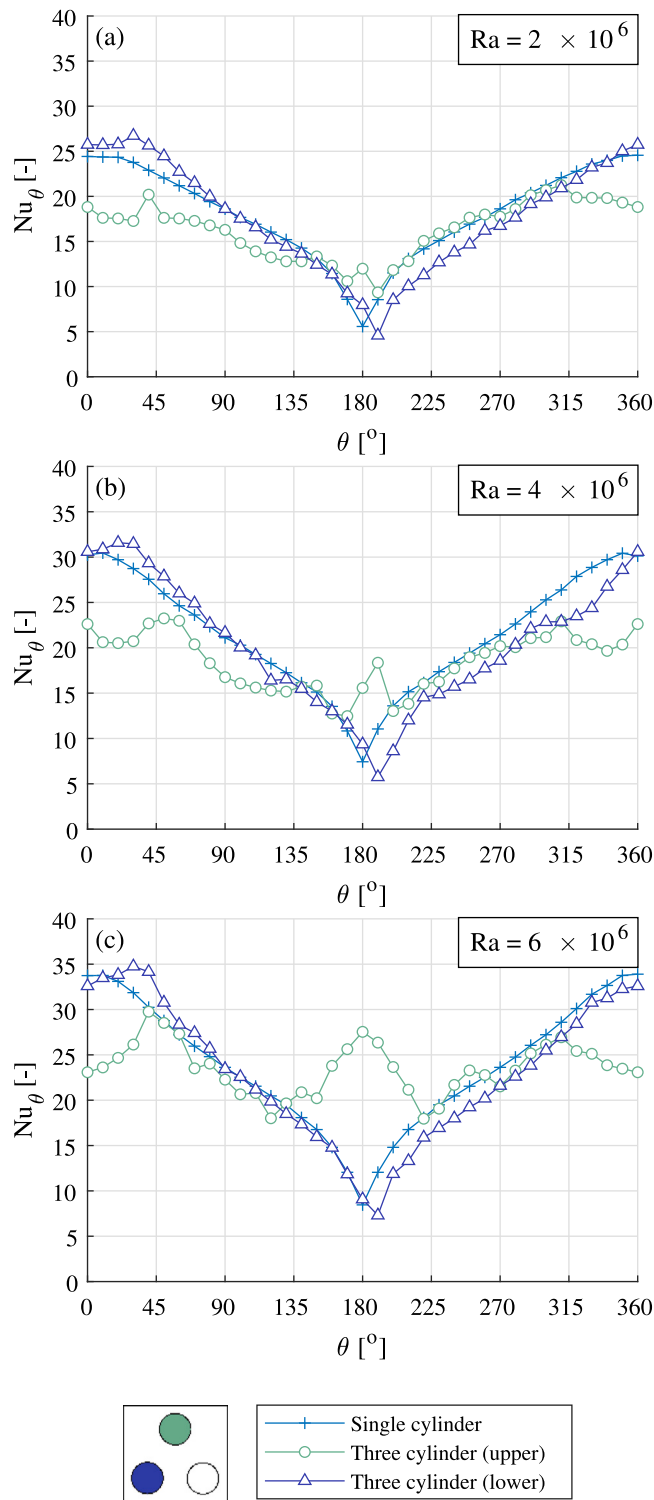


Fig. 6. Effect of Rayleigh number on the time-averaged local Nusselt number. 1:2 compared with single cylinder. $S = 2D$, (a) $Ra = 2 \times 10^6$, (b) $Ra = 4 \times 10^6$ and (c) $Ra = 6 \times 10^6$.

corresponds to a particle lag error of 0.6%. Similarly, in stationary fluid, an individual seeding particle has a negative velocity of 0.0774 mm s^{-1} . Since these differences are only slight in comparison to the actual velocities, it may be assumed that the particles represent the flow accurately.

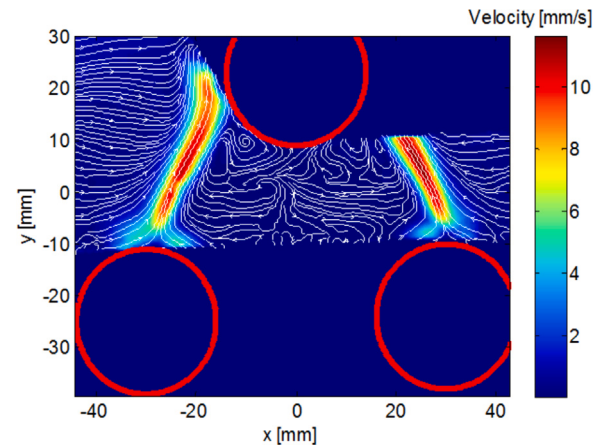


Fig. 7. Moving average streamlines and colour plot of the velocity distribution for the 1:2 setup at $S = 2D$ and $Ra = 4 \times 10^6$. View between the three cylinders.

3. Results and discussion

3.1. Heat transfer from a single cylinder

First, as a baseline for the triangular array testing, the local heat transfer results from a single cylinder are presented. These results are shown in Fig. 5. 0° is the bottom of the cylinder and 180° is the top in a clockwise direction. Peak heat transfer is observed at the bottom of the cylinder ($\theta = 0^\circ$) and monotonically decreases with increasing angle up to $\theta = 160^\circ$, with a sharper rate of decrease thereafter up to $\theta = 180^\circ$. This result is consistent with those presented by Kuehn and Goldstein [4], Merkin [19], Reymond et al. [17] and Persoons et al. [16].

3.2. Heat transfer from a 1:2 cylinder array

3.2.1. Influence of Rayleigh number

Fig. 6 shows the time-averaged local Nusselt number of the isothermal cylinders in the 1:2 configuration. Results for three Rayleigh numbers are presented: 2×10^6 , 4×10^6 and 6×10^6 for $S = 2D$. The bottom right cylinder is not presented due to the symmetric nature of the cylinder configuration. The time-averaged local Nusselt number of a single cylinder under the same experimental condition is also plotted to highlight the impact of the adjacent cylinders.

Similar to the single cylinder results, the local Nusselt number for both cylinders increases for increasing Rayleigh number. Starting with the lowest Rayleigh number, a peak local Nusselt number is noted at $\theta = 40^\circ$ and $\theta = 320^\circ$ for the upper cylinder. These peaks become more prominent as the Rayleigh number increases. This is a change from the single cylinder case. These local peak values for the upper cylinder are linked to the impingement of the generated plumes from the lower cylinders onto the upper cylinder resulting in an increased local fluid velocity which, despite the increased fluid temperature of the plume, results in a greater local heat transfer from the impinging cylinder. The presence of the two lower cylinders on the same horizontal plane causes a restriction, with the two plumes from the lower cylinders being drawn together. This behaviour is similar to that previously observed by Corcione et al. [35] in their numerical study of horizontally aligned and misaligned pairs of isothermal cylinders. As shown in Fig. 7 these plumes impinge on the upper cylinder, increasing the local fluid velocity and thus heat transfer. This is similar to the heat transfer enhancement described by Eckert and Soehngen [23] for their pair of vertically aligned cylinders.

As the Rayleigh number increases, an increase in the local Nusselt number and the region that this enhancement acts over is noted at the

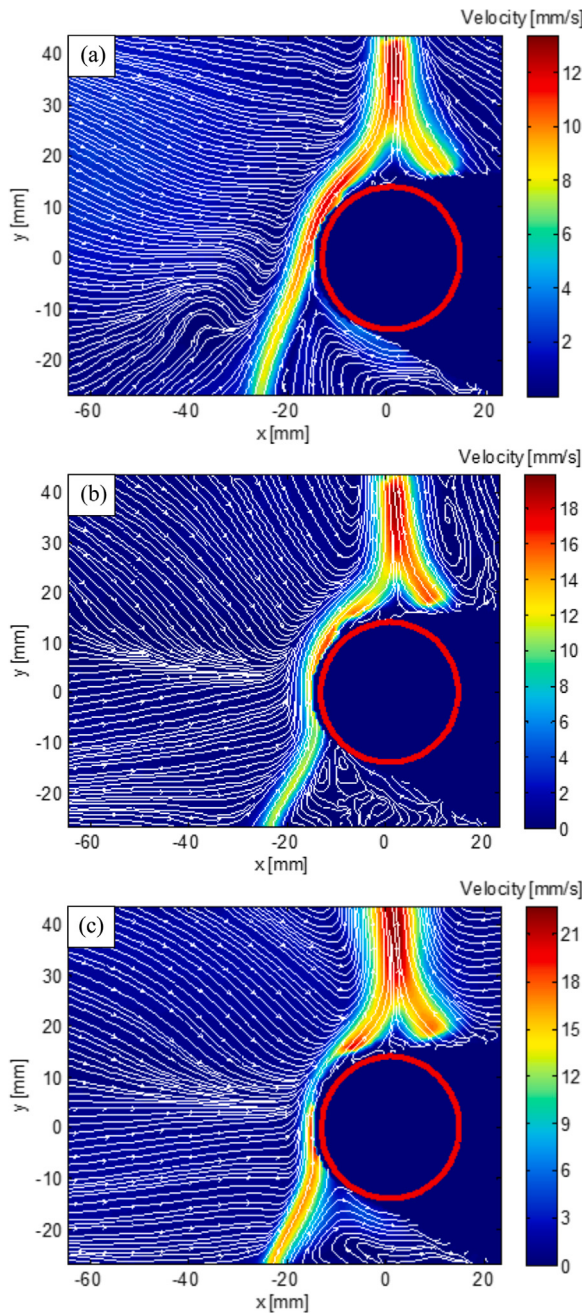


Fig. 8. Moving average streamlines and colour plot of the velocity distribution of the upper cylinder for varied Rayleigh number in the 1:2 setup at $S = 2D$. (a) $Ra = 2 \times 10^6$, (b) $Ra = 4 \times 10^6$ and (c) $Ra = 6 \times 10^6$.

top of the upper cylinder ($\theta = 180^\circ$). A threefold increase in the local Nusselt number at $\theta = 180^\circ$ for $Ra = 6 \times 10^6$ is noted in comparison to the single cylinder case. This is in contrast to the 0.5% decrease in the circumferentially averaged Nusselt number compared to the single cylinder case. This localised increase in heat transfer is due to the merging of the two generated plumes from the lower cylinders in the region just above $\theta = 180^\circ$. Fig. 8 shows a moving average streamline and colour plot of the velocity distribution for the fluid region around the upper cylinder in the 1:2 configuration. This figure highlights the increasing local fluid velocity around $\theta = 180^\circ$ due to the increasing Rayleigh number.

For the lower cylinder, a local maximum and minimum Nusselt number are observed at $\theta = 30^\circ$ and $\theta = 190^\circ$ respectively for all

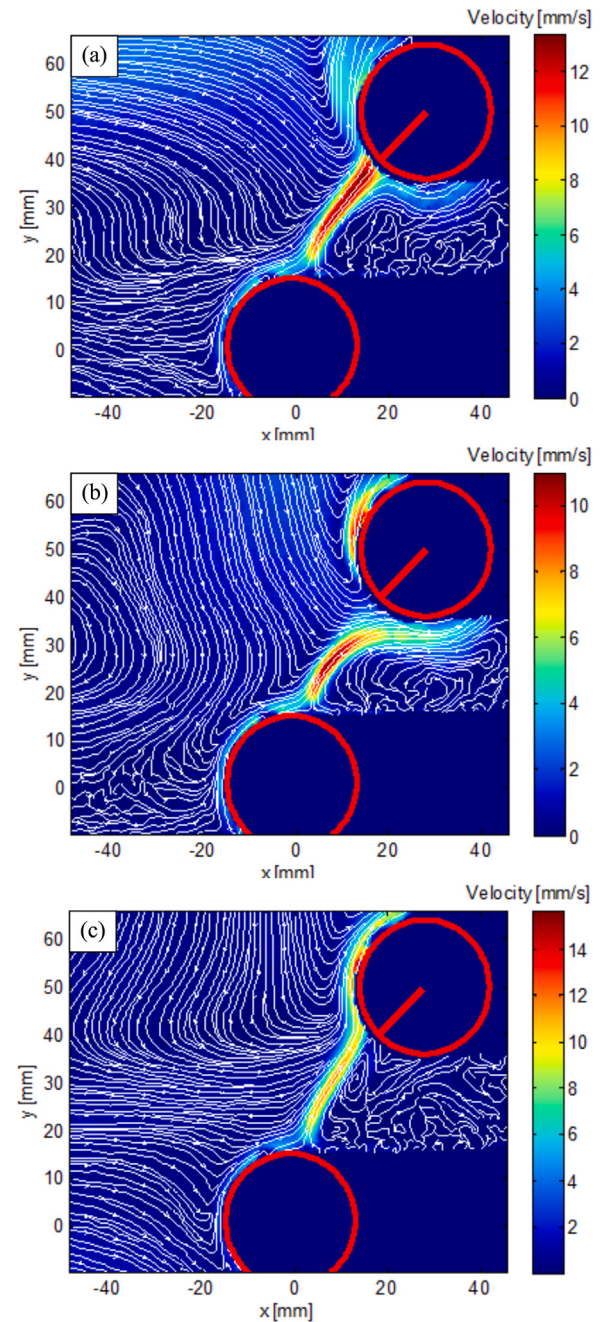


Fig. 9. Moving average streamlines and colour plot of the velocity distribution for 1:2 setup at $S = 2D$ and $Ra = 4 \times 10^6$. The shown flow fields correspond to the (a) maximum (b) minimum and (c) median local Nusselt numbers for the upper cylinder at $\theta = 40^\circ$.

Rayleigh numbers. This is due to the restriction between the two lower cylinders resulting in the buoyant plume from the lower left cylinder detaching at $\theta = 190^\circ$ (see Fig. 7). The peak heat transfer at $\theta = 30^\circ$ may be due to the thermal boundary layer thickness around the cylinder being thinnest in this region [17]. From $\theta = 60^\circ$ to 170° the local Nusselt number distribution is similar to the single cylinder case for all Rayleigh numbers, while for $\theta = 180^\circ$ to 350° the local Nusselt numbers are slightly lower than the single cylinder case for all Rayleigh numbers. This can be attributed to the plume detaching from the lower cylinder at $\theta = 190^\circ$ changing the boundary layer thickness around the cylinder compared to the single cylinder case.

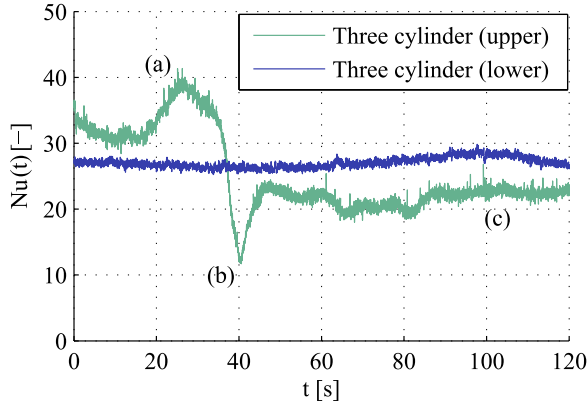


Fig. 10. Time resolved local Nusselt number at $\theta = 40^\circ$ for 1:2 setup at $S = 2D$ and $Ra = 4 \times 10^6$.

3.2.2. Simultaneous flow field and heat transfer measurement

Similar to previous investigations [15–17] for vertically aligned cylinders, the generated plumes from the lower cylinders are observed to oscillate from side to side below the upper cylinder. Simultaneous thermal and PIV measurements are taken to investigate this phenomenon and to understand the intrinsic link between heat transfer and the flow field around the cylinders. Fig. 9 demonstrates the moving average streamlines and colour plot of the velocity distribution for the 1:2 configuration with $S = 2D$ and $Ra = 4 \times 10^6$. This figure shows the flow fields around the lower left and upper cylinders. The different subplots represent the local flow field over time periods corresponding to the maximum (Fig. 9(a)), minimum (Fig. 9(b)) and median (Fig. 9(c)) local Nusselt numbers for the upper cylinder over the investigated time period. Fig. 10 plots the corresponding time resolved local Nusselt number at $\theta = 40^\circ$ for the 1:2 setup at $S = 2D$ and $Ra = 4 \times 10^6$, with the points Fig. 10(a), (b) and (c) corresponding to the plots of Fig. 9(a), (b) and (c) respectively.

A peak Nusselt number (Fig. 10(a)) is noted when the generated plume impinges directly on the heat flux sensor at $\theta = 40^\circ$ (Fig. 9(a)). This is a similar result to that first noted by Eckert and Soehngen [23], where the increased local fluid velocity results in an increase in local heat transfer. As the plume transitions to pass underneath the upper cylinder (Fig. 9(b)) a sharp decrease in the local Nusselt number is observed due to the decreasing local fluid velocity at $\theta = 40^\circ$ (Fig. 10(b)). A decrease in the overall magnitude of the plume velocity is also noted (Fig. 9(b)). Fig. 9(c) shows the buoyant plume impinging on the cylinder just above the heat flux sensor. This flow field results in the median heat transfer condition (Fig. 10(c)) and the highest overall magnitude of the plume velocity.

The local Nusselt for the lower cylinder remains steady across the investigated period. Fig. 9 demonstrates that regardless of the position of the upper portion of the generated plume from the lower cylinder, the flow around the lower cylinder and the point at which the buoyant plume detaches remains relatively constant.

3.2.3. Influence of cylinder spacing

Fig. 11 investigates the impact of cylinder spacing on the local Nusselt number for the 1:2 array. Three spacings are shown $S = 2D$, $3D$ and $4D$ for a fixed Rayleigh number of 6×10^6 . Similar trends as were previously noted in Section 3.2.1 for the local Nusselt number of the lower cylinder are observed for all spacings, with a local maximum and minimum being observed at $\theta = 30^\circ$ and $\theta = 190^\circ$ respectively. Thus, no distinct change in the local Nusselt number distribution of the lower cylinder is noted for increasing cylinder spacing, although the magnitude in relation to the single cylinder case varies moderately. In contrast, a clear impact is shown for the upper cylinder as the array spacing is increased. For the smallest spacing, local heat transfer peaks

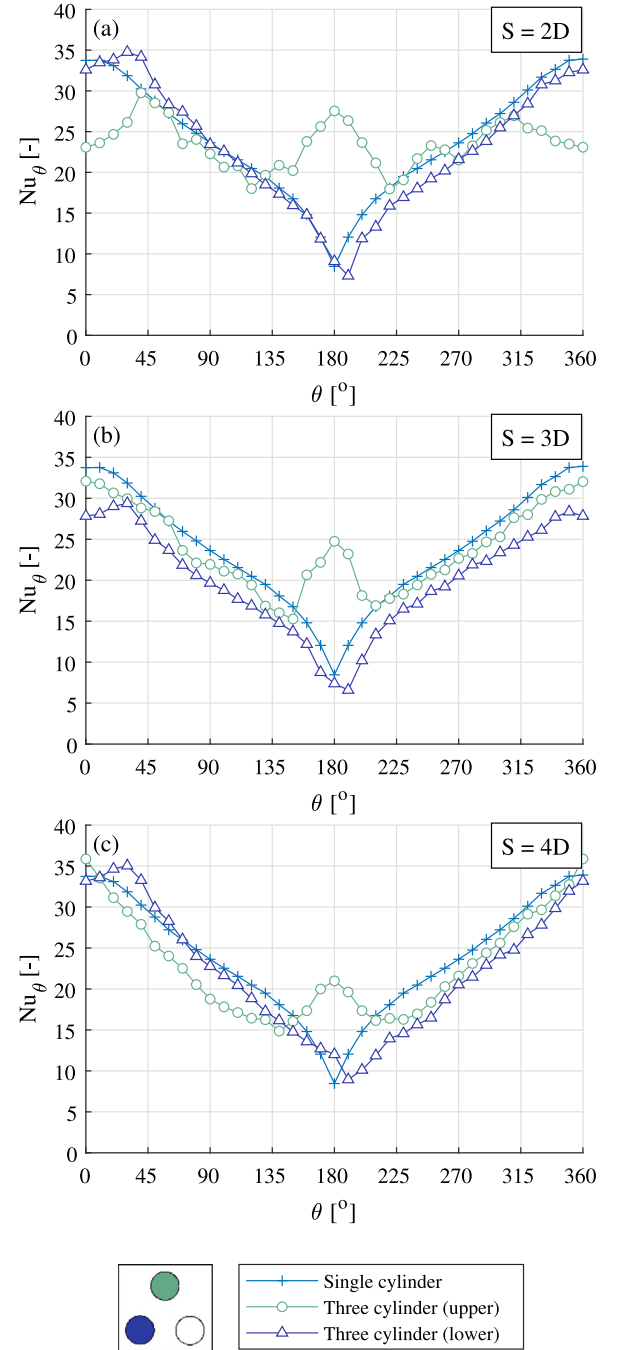


Fig. 11. Effect of cylinder spacing on the time-averaged local Nusselt number. 1:2 compared with single cylinder for $Ra = 6 \times 10^6$. (a) $S = 2D$, (b) $S = 3D$, (c) $S = 4D$.

are noted at $\theta = 45^\circ$, $\theta = 180^\circ$ and $\theta = 315^\circ$, and, as explained in connection with Figs. 7 and 8, this is due to the impingement and merging of the thermal plumes generated from the lower cylinders. As cylinder spacing increases, the effect of these plumes is diminished, with the local peaks at $\theta = 30^\circ$ and $\theta = 315^\circ$ shifting to $\theta = 0^\circ$; the local Nusselt peak at $\theta = 180^\circ$ decreases by 22%.

3.2.4. Average Nusselt number 1:2

Table 2 compares the average Nusselt number of a single cylinder and the upper and lower cylinders for the 1:2 array at $S = 2D$, $3D$ and $4D$. All results fall within the experimental uncertainty, except for the upper cylinder at $S = 3D$ and $Ra = 4 \times 10^6$ which shows a

Table 2

Average Nusselt number of a single cylinder and the upper and lower cylinders in the 1:2 arrays at $S = 2D$, $3D$ and $4D$.

	$Ra = 2 \times 10^6$	$Ra = 4 \times 10^6$	$Ra = 6 \times 10^6$
Single cylinder ($S = \infty$)	$Nu_m = 18.1$	$Nu_m = 21.4$	$Nu_m = 23.7$
Upper cylinder ($S = 2D$)	$Nu_m = 16.3$	$Nu_m = 18.4$	$Nu_m = 23.5$
Upper cylinder ($S = 3D$)	$Nu_m = 16.2$	$Nu_m = 18.1$	$Nu_m = 23.7$
Upper cylinder ($S = 4D$)	$Nu_m = 18.3$	$Nu_m = 21.6$	$Nu_m = 22.6$
Lower cylinder ($S = 2D$)	$Nu_m = 17.6$	$Nu_m = 20.2$	$Nu_m = 22.8$
Lower cylinder ($S = 3D$)	$Nu_m = 16.2$	$Nu_m = 19.2$	$Nu_m = 20.9$
Lower cylinder ($S = 4D$)	$Nu_m = 17.6$	$Nu_m = 20.6$	$Nu_m = 21.9$

Table 3

Average Nusselt number of a single cylinder and the upper and lower cylinders in the 2:1 arrays at $S = 2D$, $3D$ and $4D$.

	$Ra = 2 \times 10^6$	$Ra = 4 \times 10^6$	$Ra = 6 \times 10^6$
Single cylinder ($S = \infty$)	$Nu_m = 18.1$	$Nu_m = 21.4$	$Nu_m = 23.7$
Upper cylinder ($S = 2D$)	$Nu_m = 21.1$	$Nu_m = 22.6$	$Nu_m = 24.3$
Upper cylinder ($S = 3D$)	$Nu_m = 16.2$	$Nu_m = 19.8$	$Nu_m = 21.7$
Upper cylinder ($S = 4D$)	$Nu_m = 17.9$	$Nu_m = 20.4$	$Nu_m = 23.5$
Lower cylinder ($S = 2D$)	$Nu_m = 18.0$	$Nu_m = 20.8$	$Nu_m = 23.3$
Lower cylinder ($S = 3D$)	$Nu_m = 16.1$	$Nu_m = 20.3$	$Nu_m = 21.4$
Lower cylinder ($S = 4D$)	$Nu_m = 18.1$	$Nu_m = 20.9$	$Nu_m = 23.1$

15.7% decrease in the heat transfer performance in comparison with the single cylinder case. As the cylinder spacing and Rayleigh number are increased, the upper and lower cylinders are observed to behave more closely to their single cylinder counterpart.

3.3. Heat transfer from a 2:1 cylinder array

3.3.1. Influence of Rayleigh number

Fig. 12 plots the time-averaged local Nusselt number for the 2:1 configuration. Three Rayleigh numbers are presented (2×10^6 , 4×10^6 and 6×10^6) for the $3D$ spacing. The top right cylinder is not presented due to the symmetric nature of the cylinder configuration. The local Nusselt number of a single cylinder under the same experimental condition is again plotted as a reference case.

The trends in the time-averaged local Nusselt number for both the upper and lower cylinders are broadly similar to that of the single cylinder for all Rayleigh numbers. The maximum local heat transfer occurs at $\theta = 0^\circ$ and the minimum local heat transfer occurs at $\theta = 180^\circ$. In general, a decrease in the local Nusselt number in comparison to the single cylinder case is observed along the sides of both the upper and lower cylinders ($\theta = 45^\circ$ to 135° and $\theta = 225^\circ$ to 315°), with marginally increased heat transfer performance noted at $\theta = 180^\circ$ for all upper cylinder cases in comparison to the single cylinder case. As with the single cylinder configuration, an increase in the Rayleigh number increases the local heat transfer.

Figs. 13 and 14 show the moving average streamlines and colour plot of the velocity distribution for the lower and upper left cylinder in the 2:1 configuration respectively. These PIV results are for a cylinder spacing of $S = 3D$. The empty flow field regions in Figs. 13 in the

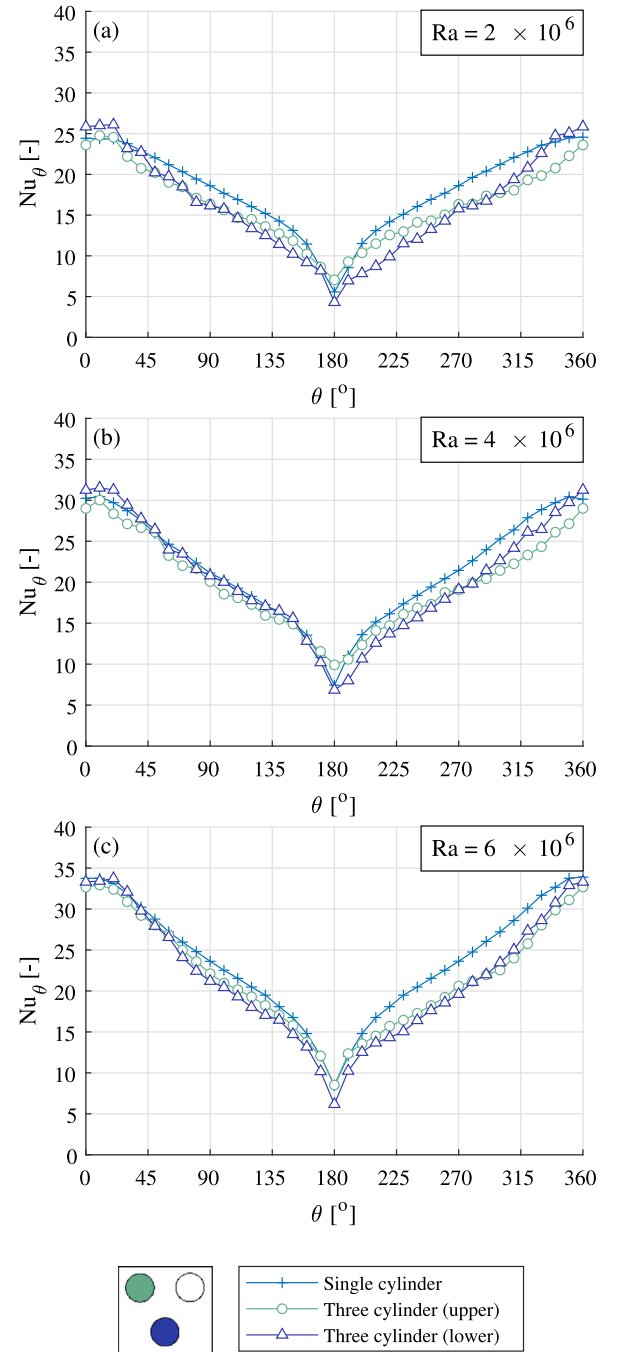


Fig. 12. Effect of Rayleigh number on the time-averaged local Nusselt number. 2:1 compared with single cylinder for $S = 3D$. (a) $Ra = 2 \times 10^6$, (b) $Ra = 4 \times 10^6$ and (c) $Ra = 6 \times 10^6$.

lower left and upper right corner of each plot are blind spots due to the PMMA faceplate design, which obscured the camera field of view. As the Rayleigh number increases so too does the magnitude of the local fluid velocity resulting in an increase in the local heat transfer as shown in Fig. 12. The time-averaged PIV results of Fig. 13 show that the generated buoyant plume detaches from the lower cylinder at the top of the cylinder ($\theta = 180^\circ$), passing upwards and through the gap between the upper cylinders. Fig. 14 demonstrates that the generated buoyant plume from the upper cylinder also detaches at $\theta = 180^\circ$ rising upward. The attraction of the adjacent plume from the upper cylinder may be impacted due to the flow from the lower cylinder. Fig. 14 highlights a

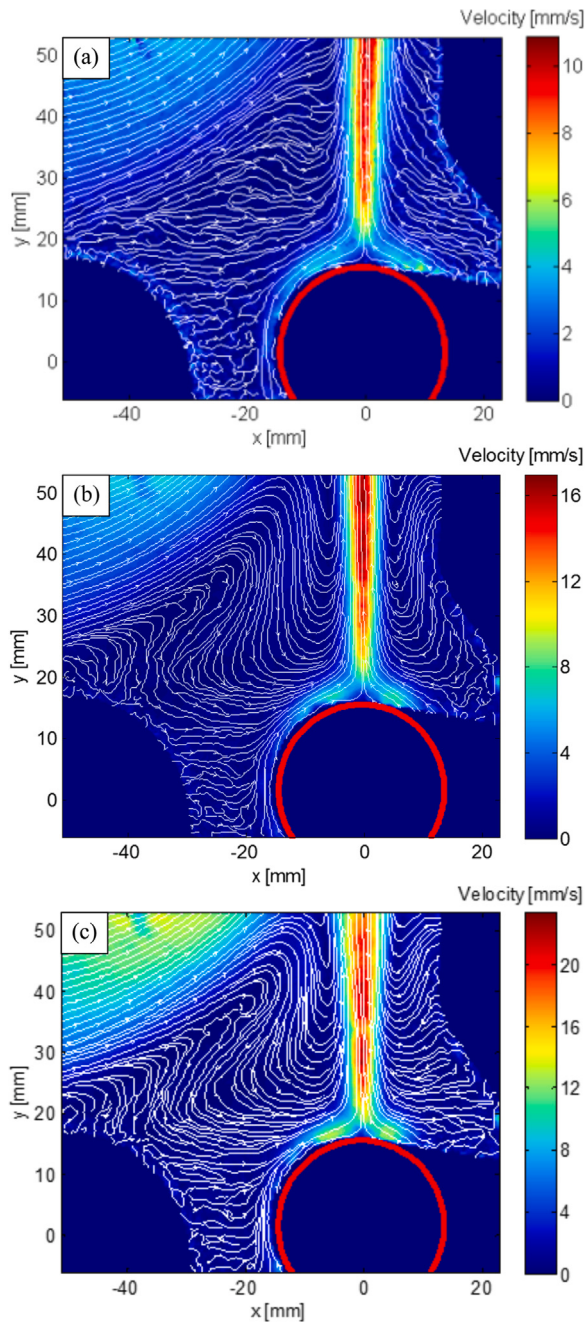


Fig. 13. Moving average streamlines and colour plot of the velocity distribution of the lower cylinder in the 2:1 setup for $S = 3D$. (a) $Ra = 2 \times 10^6$, (b) $Ra = 4 \times 10^6$ and (c) $Ra = 6 \times 10^6$.

significant flow that is observed in the region around the lower left of the upper cylinder ($\theta = 0^\circ$ to 75°), which increases in magnitude with increasing Rayleigh number. This flow can be attributed to entrainment of the flow due to the lower cylinder buoyant plume, although no significant change is observed in the time-averaged local Nusselt from the upper cylinder over this range, as seen in Fig. 12.

3.3.2. Influence of cylinder spacing

Fig. 15 shows the time-averaged local Nusselt number of the isothermal cylinders in the 2:1 configuration, for cylinder spacings of $S = 2D$, $3D$ and $4D$ for $Ra = 6 \times 10^6$. The results of the upper and lower cylinders for the $2D$ spacings are broadly similar to that of a single

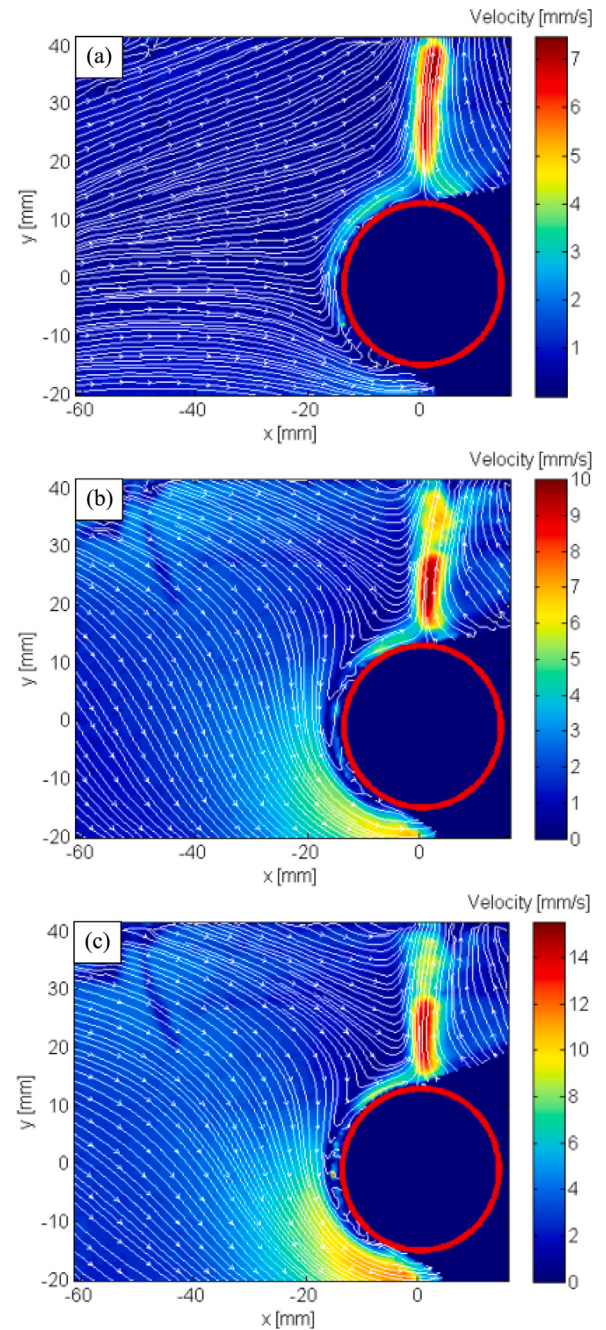


Fig. 14. Moving average streamlines and colour plot of the velocity distribution of the upper left cylinder in the 2:1 setup for $S = 3D$. (a) $Ra = 2 \times 10^6$, (b) $Ra = 4 \times 10^6$ and (c) $Ra = 6 \times 10^6$.

cylinder. A minor decrease is noted for the sides ($\theta = 0^\circ$ to 135° and 225° to 360°) of the upper and lower cylinder spacing for $S = 3D$. For the largest spacing, $S = 4D$, the lower cylinder correlated closely to the single cylinder result; however, the upper cylinder shows some significant differences. The maximum local Nusselt number occurs at $\theta = 30^\circ$, while the minimum lies at $\theta = 190^\circ$. This is more akin to the heat transfer characteristics of the two lower cylinders in the 1:2 configurations, where the restriction between the cylinders causes the plumes above the cylinders to converge. This result may be linked to the increased variation in movement of the plume from the lower cylinder allowed by the larger spacing between the cylinders.

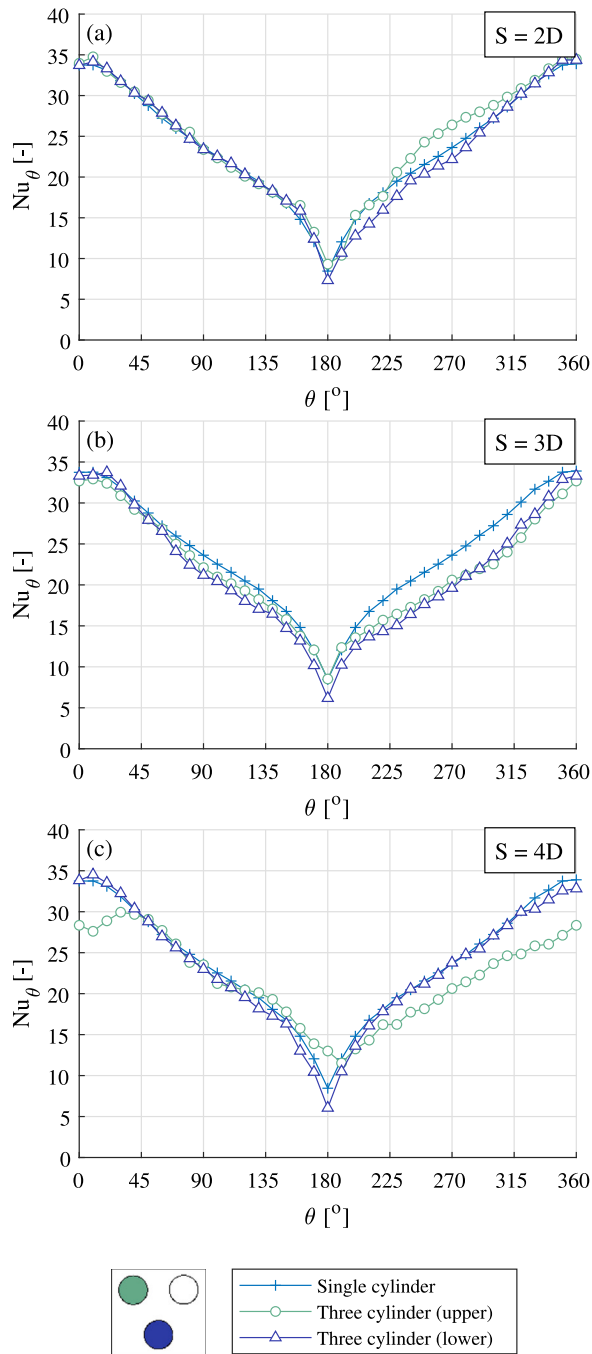


Fig. 15. Effect of cylinder spacing on the time-averaged local Nusselt number. 2:1 compared with single cylinder. $Ra = 6 \times 10^6$, (a) $S = 2D$, (b) $S = 3D$, (c) $S = 4D$.

3.3.3. Average nusselt number 2:1

Table 3 compares the average Nusselt number of a single cylinder and the upper and lower cylinders for the 2:1 array at $S = 2D$, $3D$ and $4D$. The greatest change from the single cylinder cases is a 16.9% enhancement for the upper cylinder for $S = 2D$ and $Ra = 2 \times 10^6$. This enhancement decreases monotonically for increased Rayleigh numbers. The upper cylinder for $S = 4D$ and the lower cylinder for $S = 2D$ match their single cylinder counterpart closely. The largest decrease in the average Nusselt number is noted for all Rayleigh number cases of the upper and lower cylinder at $S = 3D$.

4. Conclusion

Natural convection heat transfer from an equilateral triangular array of isothermally heated horizontal cylinders has been examined. Simultaneous flow field and convective heat flux measurements have been conducted to elucidate the governing behaviour and mechanisms of heat transfer from the array.

For the 1:2 configuration, the two lower cylinders are found to perform in a broadly similar manner to a pair of cylinders on the same horizontal plane. Fluid moving through the gap between the cylinders is restricted causing the buoyant plumes to be drawn together. This leads to a shift in the local Nusselt number distribution around the cylinders causing maximum local Nusselt number to occur at $\theta = 30^\circ$ and the minimum at approximately $\theta = 190^\circ$ for the left cylinder and at the mirrored angles for the right cylinder. These plumes change the local fluid velocities and temperature differences around the upper cylinder, altering its local heat flux distribution. The plumes are not steady, and depending on their position they can either enhance or diminish convective heat transfer from the upper cylinder.

Simultaneous PIV and heat transfer measurements identified the plume paths from the lower cylinders and linked them to observed local heat transfer effects of the upper cylinder. The effect of these plumes is dependent on both the inter-cylinder spacing and the Rayleigh number conditions. Heat transfer peaks were measured in the two diameter spacing test at $\pm 40^\circ$ from the bottom of the upper cylinder. At these locations the dominant plume positions (direct impingement for most of the time) leads to an increased time-averaged heat transfer while at other positions it leads to reduced heat transfer in comparison to the single cylinder case. The same approach also found peaks at the top of the cylinder ($\theta = 180^\circ$) for tests conducted at two diameter spacings for each of the Rayleigh number cases and for the higher Rayleigh number tested at three and four diameter spacings. These peaks are observed to reduce in magnitude and definition with decreasing Rayleigh number. The PIV testing provided no conclusive explanations for these peaks and further investigation is needed in this area. However, at some locations an increase in the local heat transfer could be linked to an increase in the local fluid velocities.

For the 2:1 setup, the lower cylinder behaves in the same manner as a single cylinder in terms of heat transfer and surrounding flow field. The time-averaged local Nusselt number results for the upper cylinder also displayed trends similar to the single cylinder for $S = 2D$ and $3D$. This was found to be due to the combined effect of the buoyant plume rising from the lower cylinder and the restriction of flow through the gap between the upper two cylinders. For $S = 4D$, the higher Rayleigh number of $Ra = 6 \times 10^6$ was found to behave differently to this trend. The upper cylinders for this case revert to performing in the same way as the two lower cylinders in 1:2 setup, with significant asymmetries in the results. PIV testing was not conducted on this spacing; however, comparisons to other results suggests that this may be caused by the increased variation in movement of the plume from the lower cylinder allowed by the larger spacing between the cylinders.

The results of this work highlight the importance of plume interaction on the local heat transfer from isothermal cylinders in a triangular array. The employed methodology of simultaneously acquiring heat transfer and flow field measurements has yielded a better insight into the governing heat transfer mechanisms of this particular case, and enables further optimisation of large scale systems in industrial applications. Future work will focus on numerical simulation of the triangular array.

CRedit authorship contribution statement

M.J. Gibbons: Data curation, Formal analysis, Visualization, Writing - original draft, Writing - review & editing. **S.G. Yates:** Methodology, Software, Investigation, Formal analysis, Validation. **S.M. O'Shaughnessy:** Writing - original draft, Writing - review &

editing, Supervision. **T. Persoons:** Methodology, Writing - review & editing, Supervision. **D.B. Murray:** Conceptualization, Methodology, Supervision, Writing - review & editing, Project administration, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The authors would like to acknowledge the financial support of the Irish Research Council, Ireland under their Government of Ireland Postdoctoral Fellowship Programme, Grant No. IRC/GOIPD/2019/666. The authors would also like to acknowledge the financial support of the Irish Research Council under their Postgraduate EMBARK Scholarship, Grant No. RS/2012/86.

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