# Implementations of Statistics, Probability, and Reliability in the Performance-Based Seismic Design of Structures

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ABSTRACT: It is well-known that the main objective of building codes is to guarantee life safety by preventing the collapse of structures in major earthquakes. However, there is a knowledge gap about controlling structural damage of buildings subjected to those dynamic demands. Without passing over what happened in other decades, by the end of the eighties and beginning of the nineties, several major earthquakes occurred around the world. Consequently, innumerable structural damages were suffered by buildings. In the aftermath of such events, a revolution in the critical thinking of structural engineers was materialized, providing the foundation of a possible shift in the basic objective of building codes. The change was oriented to control structural damage of buildings by evaluating several performance levels and achieving multiple performance objectives. As a result, the formal introduction of Performance-Based Seismic Design (PBSD) was documented in the literature. In this context, the main change proposed in the PBSD philosophy is to move from the deterministic design, based on a set of rules, to a design based on the performance of the structure subjected to seismic loading. To achieve this, the PBSD philosophy implements several methodologies based on statistics, probability, and reliability concepts. Thus, in this paper, the Author presents a technical review about several implementations of statistics and probability in the PBSD philosophy. The paper is organized as follows. First, the application of the most relevant approaches in statistics, probability, and reliability are covered in terms of meta- or surrogate-models, Bayesian approaches, improved Monte Carlo Simulation, efficient response surface method, direct differentiation methods, out-crossing approaches using importance sampling schemes, adaptive importance sampling techniques to extract multidimensional integrals, and other well-known methodologies. Then, the current challenges and future trends about the integration of the PBSD philosophy with those methods is presented. Finally, based on the current technical information reported in this paper, it is demonstrated that the use of statistics, probability, and reliability concepts in the PBSD philosophy is a step in the right direction towards the design of seismic resilient structures. Unfortunately, practical consensus between structural engineers must be achieved before the PBSD philosophy can be completely incorporated in building codes. In view of this, if PBSD is included in future buildings codes, it will be based most likely on statistics, probability, and reliability theories.

## 1. INTRODUCTION

The origin and importance of the Performance-Based Seismic Design (PBSD) must be the first item to discuss in this paper. In general terms, it is well-known that the enormous amount of property damages caused by the 1994 Northridge Earthquake prompted the profession to find an alternative to the currently accepted design criterion of life safety (LS). In this context, even though LS was not compromised during this earthquake, the structural damage caused by it was enormous. It indicated a major deficiency that existed at that time in the prescriptive design guidelines to protect life. To address the related issues, in 1995, one of the first efforts to study the seismic performance of structures was documented by the Structural Engineers Association of California (SEAOC) in two volumes titled Performance Based Seismic Engineering of Buildings (SEAOC Vision 2000, 1995). These guidelines reported a set of recommendations for engineering procedures to design buildings of predictable and defined seismic performance. The next year, in 1996, another technical advancement was documented by the Applied Technology Council (ATC) in a report titled ATC-40 Seismic Evaluation and Retrofit of Concrete Buildings (ATC-40, 1996). The ATC-40 report was particularly oriented to reinforced concrete structures providing guidance on how to investigate concrete buildings subjected to seismic shaking. One year later, in 1997, the Federal Emergency Management Agency (FEMA) documented one of the most important guidelines about the seismic rehabilitation of buildings (FEMA-273, 1997). In such guidelines, the primary purpose was to document technically sound and widely acceptable recommendations for the seismic rehabilitation of buildings. In addition, the guidelines were expected to be a tool for design engineers, a reference for building regulatory officials, and the base for the generation and implementation of building code standards. In 2000, to improve the prescriptive design guidelines, a major study funded by FEMA was conducted by the joint venture SAC (SEAOC, ATC, and California Universities for Research in Earthquake Engineering [CUREE]). The main purpose of such a study was to generate recommendations for more robust construction and seismic design of steel structures. The major findings of the research developed by SAC were documented in a series of reports (FEMA-350, 2000; FEMA-351, 2000; FEMA-352, 2000; FEMA-353, 2000; FEMA-354, 2000; FEMA-355C, 2000; FEMA-355F, 2000).

In certain way, the reports and guidelines documented in the above paragraph can be credited as the first technical documents for PBSD of structures. Such documents attempted to generate procedures to be implemented in seismic provisions of building codes. PBSD is an evolving methodology that is becoming popular among professional engineers. However, the main limitations are related to reach globally acceptable precise definitions of performance objectives, to quantify performance levels, and to estimate structural reliability considering the nonlinear behavior of the system. The recent drive of the use of PBSD for design as well as assessment of structures has led to rapid developments to enhance the attractiveness of this analytical approach (Hamburger & Hooper, 2011). The basic idea of PBSD is to design a structure so that it will perform well when subjected to different earthquake ground motions, satisfying various performance levels. In a professional perspective, the PBSD philosophy must satisfy two principal objectives. One is to couple the structural requirements with performance expectations to guarantee that hazards are treated consistently. The other is to assure financial losses associated with the damages to the implied expectations. One of the major challenges of PBSD is the unavailability of required computational tools for the proper performance assessment of structures to evaluate the structural reliability (Ghobarah, 2001).

In this paper, the author explores several implementations of statistics, probability, and reliability in the PBSD of structures, something reported as a shortcoming in the PBSD philosophy. First, the process of the PBSD is documented, then, the main challenges are reported. In addition, the application of the most relevant approaches in statistics, probability, and reliability are covered in terms of meta- or surrogate-models, Bayesian approaches, improved Monte Carlo Simulation, efficient response surface method, direct differentiation methods, out-crossing approaches using importance sampling schemes, adaptive importance sampling techniques to extract multidimensional integrals, and other well-known methodologies.

## 2. PROCESS OF PBSD

To facilitate the understanding of the process behind of the PBSD philosophy, Figure 1 is introduced in this part of the paper.

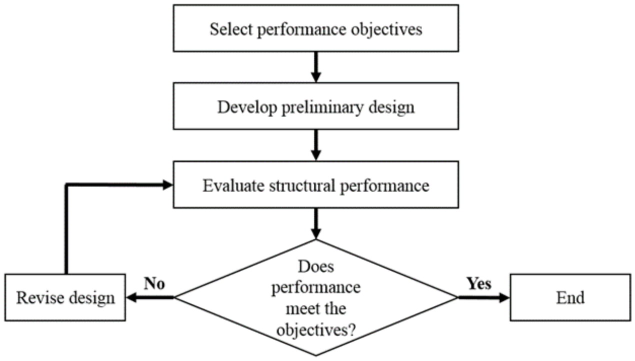


*Figure 1: Process of the PBSD philosophy.*

Based on what is presented in Figure 1, the process of the PBSD philosophy may be described as follows. First, there must be a consensus to select performance objectives between decision makers of the structure as the owner, building official, landlord, tenants, etc. Then, the structural engineer develops a preliminary design based on the idea that it will be capable of meeting the performance objectives. After that, it comes the difficult part of PBSD process where the performance of the structure must be evaluated. Finally, if the performance of the structure meets the performance objectives, the PBSD process comes to an end, if not, the structural design must be revised, and the structural performance of the revised design must be re-evaluated.

## 3. CHALLENGES IN PBSD

As previously mentioned, one of the essential building blocks of PBSD is the estimation of structural reliability corresponding to a specific performance criterion. The engineering profession must have a reliability method to estimate seismic risk, the method must satisfy underlying physics commonly used in deterministic analysis, and information must be available related to acceptable risk or target reliability levels to select different performance requirements. The basic intent is to propagate uncertainty from the variable or element level to the system level. There are three main challenges in PBSD: (1) limitations in current reliability evaluation methods for seismic loading applied in time domain, (2) reliability estimation satisfying underlying physics, and (3) target reliability levels to satisfy different performance requirements. These issued motivated the development of this paper.

## 4. APPLICATION STATISTICS, PROBABILITY, AND RELIABILITY APPROACHES IN THE PBSD PHILOSOPHY

In this section of the paper, the application of some the most relevant approaches in statistics, probability, and reliability are covered in terms of meta- or surrogate-models, Bayesian approaches, improved Monte Carlo Simulation, efficient response surface method, direct differentiation methods, out-crossing approaches using importance sampling schemes, adaptive importance sampling techniques to extract multidimensional integrals, and other well-known methodologies.

### *4.1. Meta- or surrogate-models*

The meta- or surrogate-models are required in the PBSD philosophy to obtain dramatic reduction in processing time when extracting structural risk. These models have been applied to study the performance of several structures. For example, De Pina *et al.* (2014) documented more efficient meta-models based on wavelet networks for the analysis of slender offshore structures. In this study, adequate estimations of the top tensions of mooring lines or risers were obtained by reducing computational times in the order of 20 times in comparison with other methods. It was demonstrated that the meta-models may comprise a better alternative to replace the full use of expensive Finite Element-based nonlinear dynamic analysis tools. In addition to the above-study, Garcia-Macias *et al.* (2020) developed a comparison of surrogate models for handling uncertainties in Structural Health Monitoring (SHM) of historic buildings. They documented that surrogate models or meta-models make it possible to bypass costly numerical models by establishing a black-box representation of the modal features of the structure as function of certain damage-sensitive mechanical parameters. Thus, Garcia-Macias *et al.* (2020) compared different meta-models, including the quadratic response surface method, Kriging and random sampling-high dimensional model representation models, and discussed their advantages/disadvantages for system identification through continuous model updating. The main finding of this research was the considerable amount of saved time when performing the risk evaluation of an historical building. Within this frame of reference, Hashemi *et al.* (2022) carried out sensitivity analyses to study the effect of each design variable on the performance of self-centering buckling restrained brace (SC-BRB) and the corresponding buckling restrained brace (BRB) without shape memory alloy (SMA) rods. To reduce computational cost of the simulation methods, three Meta-models including the Kriging, radial basis function (RBF), and polynomial response surface (PRSM) are utilized to construct the surrogate models. Based on the findings of this paper, it is concluded that Kriging-based Monte Carlo Simulation (MCS) gives the best performance to estimate the limit state function (LSF) of BRB and SC-BRB in the reliability analysis procedures. Very recently, Nasab & Kim (2023) developed a framework to provide an optimal distribution of energy dissipation devices in building structures. In this study, the uncertainties involved in the design parameters were modeled as fuzzy variables using the polynomial chaos expansion-Kriging model for uncertainty quantification and seismic performance evaluation. At the end, it was demonstrated that the proposed meta model was accurate and useful in the optimum seismic retrofit design of structures with uncertainties. The above-studies demonstrates that meta- or surrogate-models can be accurately implemented in the PBSD of structures, their main advantage remains in the considerable amount of computational time that can be saved if there are properly implemented.

### *4.2. Bayesian approaches*

Two are the major issues of PBSD that can be solved with the implementation of Bayesian approaches: (1) the number of ground motion records required in PBSD which generally must be eleven or more pairs of motions, and (2) the time and effort associated with performing the number of nonlinear response history analyses required to exercise PBSD. In this context, Esmaili *et al.* (2016) presented a method for the implementation of PBSD using classical linear modal analysis to develop a first estimate of probability distribution of loss, followed by utilizing Bayesian statistics to update such an estimate using approximations of loss obtained by utilizing a small number of nonlinear response history analyses of a detailed model of the building. The proposed approach was used to evaluate the distribution of monetary loss of two structures: a 4-story reinforced concrete moment-resisting frame building, and a 20-story steel moment-resisting frame building, both located in Los Angeles area. A ground motion hazard with 10% probability of exceedance in 50 years was implemented. In general, the efficiency of the proposed PBSD method was demonstrated by showing the similarity between the distribution of monetary loss at each story of case study buildings obtained from the traditional/sophisticated PBSD methodology and the proposed PBSD method. A very interesting overview about the use of Bayesian approaches was documented by Ghosh et al. (2021), in such a paper, the importance of the application of Bayesian approaches is documented with respect to seismic fragility analysis. In summary, critical observations on the developments and scope of Bayesian approaches implemented in the PBSD are widely discussed.

### *4.3. Improved Monte Carlo Simulation*

Improved Monte Carlo Simulation techniques represent a major topic in the evaluation of the risk of structures subjected to ground motions. In this context, there is a great number of investigations that are proposing the training of models to improve the sampling point selection and the efficiency of Monte Carlo as a method. In general, the main objective of these methods in the PBSD of structures is to validate alternative approaches to extract the risk of buildings subjected to ground motions (Zhang & Foschi, 2004; Gaxiola-Camacho *et al.*, 2017; Thaler *et al.*, 2022).

### 4.4. Efficient response surface method

Response surface method (RSM) represents a unique alternative to calculate the structural reliability when the performance function of the system under consideration is implicit and need to be generated (Faravelli, 1989). With the help of RSM, the author of this paper and his team have developed an alternative probabilistic approach that is able to extract the risk of structures subjected to ground motions. Such an approach integrates the finite element method, first order reliability method, and an intelligent experimental sampling point selection technique (Gaxiola-Camacho et al., 2017, 2018; Monjardin-Quevedo et al., 2022). As a result of those investigations, the author of this paper and his research team believe that they have proposed an alternative to Monte Carlos simulation technique since very few simulations are required with the novel approach.

### 4.5. Direct differentiation methods, out-crossing approaches, adaptive importance sampling techniques, and other methodologies to extract risk

The Pacific Earthquake Engineering Research (PEER) center developed a reliability approach known as Direct Differentiation Method (DDM) (Haukaas & Der Kiureghian, 2005; Der Kiureghian, Haukaas and Fujimura, 2006; Haukaas and Der Kiureghian, 2006). They assumed the structural parameters at their mean values and the loading could not be applied in time domain. They used an out-crossing approach (Koo, Der Kiureghian and Fujimura, 2005) with an importance sampling scheme (Au and Beck, 2001; Ching, Au and Beck, 2005). Au and Beck (1999) used an adaptive importance sampling methodology to compute the multidimensional integrals encountered in reliability analysis. However, multidimensional integrals are implicit for nonlinear steel structures. Other researchers (Hamburger, Foutch and Cornell, 2003; Cornell et al., 2002) studied the seismic performance of steel structures. They reported the need of reliability-based PBSD approaches. Wen (2001) proposed a reliability-based framework for PBSD. Unfortunately, a physics-based analytical approach for the structural analysis was not used. The above studies represent significant progress in the state of the art of PBSD.

## 5. CONCLUSIONS

In this paper, the author presented a complete description of the PBSD philosophy and several implementations of statistics, probability, and reliability techniques on PBSD. In general, it was demonstrated and it is expected that if PBSD is implemented as a regulatory option to perform the design and/or analysis of structures subjected to ground motions will be develop by implementing the concepts discussed in this paper. This is very important to reduce the damage of buildings when excited by ground motions and to design seismic-tolerant structures.

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