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BUBBLE GROWTH DYNAMICS INCLUDING GRAVITATIONAL AND ELECTRIC FIELD EFFECTS

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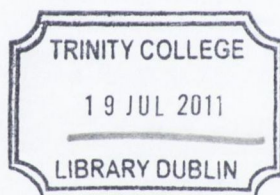
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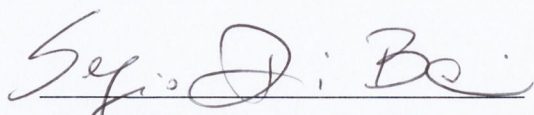


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ABSTRACT

In the present work the hydrodynamics of adiabatic gas bubble growth from a submerged orifice is investigated with the view of elucidating the interdependence of the bubble shape and the pressure field during growth and departure for different test conditions. The influence of both gravitational and electric fields around a single isolated bubble during its growth have been investigated experimentally with the aid of high speed photography and detailed image processing. To facilitate this, an uncomplicated yet versatile experimental apparatus has been commissioned which can simultaneously measure the internal gas pressure and the shape of the bubble during its lifetime. The design of the experimental rig allowed the user to modify the bubble growth conditions such as the flow rate and the orifice size. In addition to this, the effect of a uniform electric field was also studied by applying a high voltage potential across the region within which the bubbles grew.

Combining measurements of the instantaneous gas pressure, reconstruction of the local curvature and the Young-Laplace equation, the liquid pressure field acting on the gas-liquid interface, over the

entire bubble surface, was calculated and its influence discussed in context of the bubble growth characteristics for terrestrial gravity both with and without an imposed electric field.

The study was extended to focus on some aspects of varying the gravitational field on bubble growth dynamics. Since low gravity experiments are prohibitively expensive, a numerical simulation approach was adopted to achieve this goal. The commercial software TransAT was utilized and a rigorous validation study was performed against the field-free bubble growth measurements. The code was deemed validated and suitable for investigating the influence of gravity on the bubble growth dynamics. The numerical results show a very strong influence of the level of gravity on the bubble formation and departure characteristics and have been detailed in the context of the bubble shape, pressure and force evolution.

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Sergio Di Bari

"Nunc est bibendum, nunc pede libero pulsanda tellus."

Horace

Odes Book I

CONTENTS

Introduction	1
1 Literature Review	5
1.1 Experimental Bubble Formation	6
1.2 Bubble Departure Models	17
1.2.1 Quasi-Static Bubble Formation and Departure	17
1.2.2 Dynamic Bubble Formation and Departure	19
1.3 Numerical Analysis	28
1.4 EHD Effects on Adiabatic Bubble Growth and Departure . . .	39
2 Mathematical Modelling	43
2.1 Mass and Momentum Conservation	43
2.2 Constitutive Equations	46
2.3 Navier-Stokes Equations	46
2.4 Volumetric Electric Forces	47
2.5 Overall Momentum Balance	50
2.6 Theoretical Prediction of Bubble Profile	52

CONTENTS

2.7	Pressure Field on Bubble	53
2.8	Bubble Detachment	56
3	Experimental Apparatus and Data Reduction	65
3.1	Experimental Apparatus	65
3.2	Basin and Aluminium Base	68
3.3	Injector Device	69
3.4	Pressure Transducer	71
3.5	High Speed Videography	73
3.6	Electric Field and High Voltage Amplifier	75
3.7	Experiment Conditions	77
3.8	Image Processing	79
3.9	Data Analysis	85
4	Numerical Modelling	93
4.1	TransAT	93
4.1.1	Discretization of Governing Equations	94
4.1.2	Problem Formulation and Data Processing	98
4.2	COMSOL	102
5	Results and discussion: Water	105
5.1	Bubble Growth Dynamic	106
5.2	Effect of Volumetric Flow Rate	117
5.3	Effect of the Orifice Diameter	120
6	Results and Discussion: Numerical Simulations	129
6.1	Comparison with the Experimental Data	130
6.2	Flow Field and Influence of Gravity	141

CONTENTS

7 Results and Discussion: EHD	155
7.1 General Bubble Growth Behaviour	155
7.2 Finite Element Analysis	170
8 Conclusion and Future Work	181
Bibliography	187
Appendix:	
A Matlab Codes	193
B TransAT Codes	211

NOMENCLATURE

Symbol	Description	Units
a	Laplace Constant	m
D, d	Diameter	m
D_T	Tate's Diameter	m
$E, \mathbf{E}$	Electric Field Strength	V/m
$F, \mathbf{F}$	Force	N
$\mathbf{f}'_e$	Volumetric Force	N/m ³
$g, \mathbf{g}$	Earth Gravity Acceleration	m/s ²
H, h	Height	m
Ja	Jakob Number	-
L	Generic Length	m
L_p	Length Scale Coefficient	m/pixel
N_P	Number of pixels	-
P, p	Pressure	Pa
Q	Flow Rate	m
q_F	Free Electric Charge Density	C/m ³
R, r	Radius	m

NOMENCLATURE

Symbol	Description	Units
$\overline{\mathbf{T}}$	Stress Tensor	-
$\mathbf{T}_e$	Electric Stress	N/m ²
t	Time	s
U	Electric Potential	V
$v, \mathbf{v}$	Velocity	m/s
V	Volume	m ³
We	Weber Number	-
y	Axial Coordinate	m
z	Vertical Centroid Position	m

Greek

Symbol	Description	Units
β	Shape Factor	-
δ	Pressure Transducer Coefficient	Pa/Volt
ϑ	Angular Coordinate	rad
ϕ	Phase	-
ε	Permittivity	F/m
ε_0	Vacuum Permittivity	F/m
ε_r	Relative Dielectric Permittivity	-
θ	Contact Angle	rad
μ	Dynamic Viscosity	Ns/m ²
ρ	Density	Kg/m ³
σ	Surface Tension	N/m
φ	Generic Variable	-
ψ	Young Static Contact Angle	deg

NOMENCLATURE

Suffixes

Symbol	Description
0	Orifice
<i>atm</i>	atmospheric
b	Bubble
B	Buoyancy
C,c	Capillary
CP	Contact Pressure
cr	Critic
D,d	Dynamic
det	Detachment
dr	Drag
E,e	Electric
EHD	Electrohydrodynamic
eq	Equivalent
f,n,s,w,e	Facial Values
g	Gas
i	Inertia
l	Liquid
m	Momentum
n	Normal
neck	Neck
p	Pixel
P,N,S,W,E	Nodal Value
t	Tangential
T	Tip of the Bubble

INTRODUCTION

Nucleate boiling is a commonly used mode of heat transfer. It is characterized by the rapid formation of a large number of vapour bubbles on a superheated surface submersed in a liquid. The boiling phenomenon has been used as a technology since around 2000 BC when Man first boiled water to purify it. By around 500 BC, in Mesopotamia, the first alchemists were able to separate elements in a liquid mixture via the boiling process. It has been and still is one of the most common methods for cooking food. Its consistency with regard to saturation temperature and pressure allowed Fahrenheit to define his temperature scale in 1724. It had a fundamental influence on the development and impact of the industrial revolution of which the backbone was the steam engine. Currently and in the future it will play an important role in the heat exchange processes ranging from microelectronics thermal management, power generation, refrigeration to smart and small heat exchangers in space applications.

Even though it has had a great influence in the development of human technology and life in general, boiling as a science only commenced close to

the beginning of the last century. From the first boiling curves produced by Nukiyama (1934), to the parabolic flights and space gravity experiments of Lee and Merte (1996) and Di Marco and Grassi (2009), the *Science of Boiling* has and will continue to develop until the phenomenon is completely understood.

As technology has developed so has the sophistication the studies of boiling. Starting from the early simplified theoretical studies in the early to mid 1900s and passing through the wave of experimental work during the space race of the 1960s and on to the full numerical simulations of the present day, many studies have been performed which attempt to understand the boiling process and define theories and mathematical models to predict its performance. Even after one hundred years of study, global theories and robust empirical formulations seem to be elusive. This is largely due to the lack of a complete understanding of the fundamental physics of bubble dynamics, flow and heat transfer at these small time and length scales. This is complicated by the fact that bubble dynamics are sensitive to a vast array of interdependent parameters which makes exhaustive experimentation and exact numerical simulations difficult to achieve.

Due to its complexity, it has often been necessary to study a single aspect of the phenomena by limiting the number of variables, thus isolating as few parameters as possible. With regard to bubble growth dynamics, the adiabatic case facilitates a situation whereby the heat and mass transfer at the heat transfer surface as well as at the bubble interface are not present. This ostensibly separates the mechanical effects from the thermal ones and has been the focus of much work in the open literature. The primary motivation being to gain fundamental knowledge of the bubble dynamics which forms the firm stepping stone for progressing to more complex boiling

scenarios. This forms the basis for the motivation of the present work which is to gain clear insight into the influence that the force field, within which bubbles are forming, has on its growth dynamics. In the present investigation both gravitational field and electric field effects have been scrutinized.

This investigation studies the fundamental nature of adiabatic bubble growth and departure for air injected from a submerged orifice into an otherwise stagnant liquid. A very novel aspect of this study is that high speed photography and purpose built imaging and analysis software were used to determine the local curvature of the bubble over its entire surface so that the Young-Laplace equation could be implemented to allow the estimation of the net pressure difference acting locally at the bubble interface. Combining the local curvature measurements with the gas pressure measurement facilitated, for the first time, the estimation of the net liquid pressure acting locally on the bubble at a given instant in time.

Using the same approach it has been possible to analyse two different phenomena regarding the quasi-static evolution of a gas bubbles within a quiescent fluid. The first deals with the action of the dynamic pressure field around a bubble growing in water at different growth rates and orifice diameters. The second involves the evolution of the electrically induced stresses acting on bubbles growing within a dielectric fluid under the influence of a high voltage electric field. To the author's knowledge this is a unique experimental approach and previous works in this area relied on numerical approaches which were on for the most part unvalidated. Regarding this last point, an attempt has been made to simulate the bubble growth numerically. The primary goal is to develop and rigorously validate the numerical simulations against the benchmark experimental data provided by the experiments in water. To this end, the commercial software package

INTRODUCTION

TransAT has been utilized. Once the simulations were adequately validated, the numerical platform was used to study some aspects of the influence of gravity level on bubble growth dynamics.

CHAPTER 1

LITERATURE REVIEW

This section presents a review of gas injected bubble growth and departure. It will focus on the more prominent and influential works and will be organized as follows: *Experimental Bubble Formation*, *Bubble Departure Models*, *Numerical Simulations of Bubble Dynamics* and *EHD Effects on Adiabatic Bubble Growth and Departure*.

1.1 Experimental Bubble Formation

Oguz and Prosperetti (1993)

Oguz and Prosperetti (1993) presented one of the most comprehensive and seminal works on adiabatic gas bubble dynamics. This primarily experimental work considered several aspects of the growth and departure of bubbles growing from submerged needles. As will be discussed later, they also proposed a computational procedure and compared the results with experimental measurements in order to validate the model. Their study focused on a wide range of injection flow rates in order to consider both quasi-static and rapid growth.

Two strategies were adopted in their investigation, a simplified model, where the bubble was considered as spherical, and a numerical simulation, with a detailed study of the bubble shape and volume. This section will detail the simplified mechanistic model, while the numerical procedure will be explained in a later section.

In the approximate growth model the bubbles were assumed spherical throughout their lifetime. An important scale for the bubble radius was the value at which buoyancy and surface tensions were in balance, R_F , and its relative volume V_F . For spherical bubbles these are represented as follows,

$$F_B = F_\sigma \Rightarrow \rho_l V_F g = 2\pi r_0 \sigma \quad \text{with} \quad V_F = \frac{4}{3}\pi R_F^3 \quad (1.1)$$

where r_0 is the needle radius. Substituting the value of V_F in the force balance equation one obtains;

$$\rho \frac{4}{3} \pi R_F^3 g = 2\pi r_0 \sigma \Rightarrow R_F = \left(\frac{3 \sigma r_0}{2 \rho_l g} \right)^{1/3} \quad (1.2)$$

The bubble growth was considered to be subject to just buoyancy and hydrodynamic influences whilst viscous drag was considered negligible. With these simplifications the momentum balance simplified to;

$$\frac{d}{dt} \left(\frac{1}{2} \rho_l V_b \dot{z} \right) = \rho_l V_b g \quad (1.3)$$

where the surface tension term at the needle tip was neglected since it was assumed to have an important role just at the very beginning of the bubble growth and was unimportant for most of the lifetime of the bubble. To account for the motion of the surrounding liquid, an added mass was considered. For the added mass coefficient, Davison et al. (1960) used 11/16 to account, in an approximate way, for the presence of a wall tangent to the growing bubble. In their work they assumed that the needle was much longer than the bubble radius and therefore a value of 1/2 was retained as more appropriate. With the initial conditions $z(0) = 0$ and $V_b(0) = 0$ Eq.1.2 gives;

$$z(t) = \frac{1}{2} g t^2 \quad (1.4)$$

Considering the time evolution of the radius of a spherical bubble;

$$R(t) = \left(\frac{3 Q t}{4 \pi} \right)^{1/3} \quad (1.5)$$

and calling $\bar{t}$ the time at which the centre of gravity of the bubble has grown

to an amount equal to its radius, this is equal to:

$$R(\bar{t}) = z(\bar{t}) \Rightarrow \left(\frac{3}{4} \frac{Q\bar{t}}{\pi} \right)^{1/3} = \frac{1}{2} g \bar{t}^2 \Rightarrow \bar{t} = \left(\frac{6Q}{\pi g^3} \right)^{1/3} \quad (1.6)$$

The radius and the vertical position at are thus equal to;

$$\bar{R} = \bar{z} = \left(\frac{9}{8} \frac{Q^2}{g\pi^2} \right)^{1/5} \quad (1.7)$$

A major difference in the growth behaviour was expected to depend on whether $\bar{R}$ was reached before or after R_F . For small flow rates the bubble detaches when $R_F \approx \bar{R}$. On the other hand, for high flow rates, the radius of detachment is proportional to $Q^{2/5}$. The threshold value of the gas flow rate at which $\bar{R} = R_F$ is given by;

$$\bar{R} = R_F \Rightarrow \left(\frac{9}{8} \frac{Q_{cr}^2}{g\pi^2} \right)^{1/5} = \left(\frac{3}{2} \frac{\sigma r_0}{\rho_l g} \right)^{1/3} \Rightarrow Q_{cr} = \pi \left(\frac{16}{3g^2} \right)^{\frac{1}{6}} \left(\frac{\sigma r_0}{\rho} \right)^{\frac{5}{6}} \quad (1.8)$$

Such an analysis was used to differentiate quasi-static bubble growth from a dynamic bubble growth regime. For flow rates where $Q < Q_{cr}$, the bubble was expected to grow quasi-statically and detach with a volume close to V_F . On the other hand, for $Q > Q_{cr}$, the faster growing bubbles expected to detach with a volume proportional to $Q^{6/5}$.

Based purely on observation, bubble detachment was considered to occur when the position of the centre of gravity was approximately one needle radius larger than its radius;

$$z(t) = R(t) + r_0 \quad (1.9)$$

Since, for small flow rates, this condition was reached in the early stage

of the growth process, when the buoyancy effect was not sufficiently large to overcome the surface tension, a surface tension term was added in the calculation just when the position of the centre of gravity was higher than that of the equivalent radius of the bubble,

$$-2\pi r_0 \sigma \tag{1.10}$$

From the results, Oguz and Prosperetti determined how, for flow rates higher than the critical value, $Q > Q_{cr}$, surface tension had a negligible effect on the detachment process and z reached the value of $R + r_0$ when the radius of the bubble was already greater than R_F . However, for slow flow rates, the motion of the centre of gravity became oscillatory due to the alternate action of the surface tension force in the direction opposing gravity.

Comparing the results obtained from the simple method with the predicted ones they found that the spherical bubble approximation was not particularly representative of reality for the slow-growth regime where the lack of inertial influences, which tend to promote sphericity, results in an interplay between surface tension and gravity which deform the bubble.

Gaddis and Vogelpohl (1986)

Gaddis and Vogelpohl (1986) proposed a simple model for bubble formation in quiescent liquids under constant injection flow condition. The prediction of the detachment volume was obtained by releasing the constraints of a spherical bubble shape and including a cylindrical neck. The model proposes two different approaches:

- In the first, the equation of motion was applied to the neck at the moment of detachment and then, with the help of a few assumptions,

simplified equations were obtained, which were later solved analytically.

- In the second approach, the total volume of the bubble just before detachment was calculated as the sum of the volume of the bubble at the end of detachment stage and the volume of gas in the neck during detachment, where the volume of the bubble at the end of the detachment stage was determinate by a force balance.

Since the second approach involves a number of errors including the correct neck volume, the first approach is generally used in the investigation of the dynamics of bubble formation.

For the force balance several simplifications were introduced. As with Oguz and Prosperetti (1993), they considered bubbles growing with a spherical shape, considering them completely submerged in the liquid phase. On the contrary, the presence of a surface tension term component was considered with no restriction on the position of the centre of gravity. Moreover, the consideration of a difference of pressure between the bubble and the liquid resulted in an upward force proportional to the overpressure and to the contact area of the bubble. Further, terms were introduced to take into account the inertia effects, depending on the acceleration of the bubble, and drag and momentum forces.

Balancing these forces gives;

$$F_B + F_{m,g} + F_p = F_\sigma + F_{dr} + F_i \quad (1.11)$$

where on the LHS F_B is the buoyancy component, $F_{m,g}$ the gas momentum force, F_p is the force due to the bubble overpressure while on the RHS, F_σ is the surface tension action, F_{dr} the drag effect and F_i the inertia force. The

forces are defined as;

$$\begin{aligned}
 F_b &= \frac{\pi}{6} d^3 (\rho_l - \rho_g) & F_p &= \frac{\pi}{4} d_0^2 (p_g - p_l) & F_{m,g} &= \frac{\pi}{4} d_0^2 \rho_g w^2 \\
 F_{dr} &= \frac{\pi}{4} d^2 C_D \frac{\rho_l w_l^2}{2} & F_i &= (\rho_g V_b - \rho_l V_b) \alpha & F_\sigma &= \pi d_0 \sigma
 \end{aligned}$$

where C_D and w are the drag coefficient and the bubble rise velocity respectively.

From their results they determined that a ratio existed between the maximum neck length and the bubble's diameter, $h_{neck}/d = 0.25$, and also between the displacement of the centre of gravity and the diameter, $z/d = 0.75$. Comparison between theoretical predictions and experimental measurements made by a large number of investigators has shown that the model is valid over a wide range of viscosity and gas flow rates, and hence seems to be relatively reliable.

Buyevich and Webbon (1996)

Buyevich and Webbon (1996) have discussed the effect of gravity on bubble growth and detachment with a two stage bubble formation model tested both for terrestrial and reduced gravity. At normal gravity and at relatively low gas flow rates, the bubble is modelled as a spherical segment and the bubble foot remains fixed to the orifice rim during its entire life (Fig.1.1a). If the growth rate exceeded a threshold value, a second stage of detachment was introduced. Similar to Gaddis and Vogelpohl (1986), in this second stage a nearly cylindrical stem forms at the orifice and below the bubble dome that lengthens as the bubble rises above the surface (Fig.1.1b). This second stage

always develops when gravity is sufficiently low, irrespective of the gas flow rate. At high gas flow rates the bubble shape resembles that of a mushroom, and its upper dome continues to grow until the gas supplied through the stem is completely cut off. In order to predict the motion of the bubble

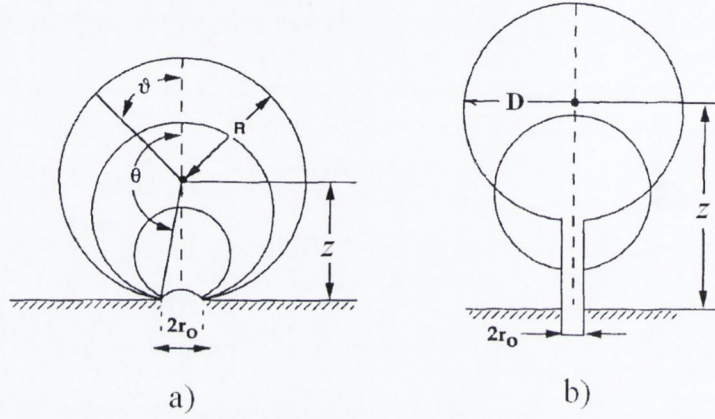


Figure 1.1: Schematic of bubble growth in (a) expansion and (b) detachment stages, Buyevich (1996).

an equation based on the Newtons second law was derived by equating the sum of the forces acting on the bubble to the rate of change for the upward momentum of a bubble of varying volume V ;

$$\frac{d}{dt} \left[\frac{k(\theta)}{2} \rho_l V \frac{dz}{dt} \right] = F_u + F_l + F_c \quad (1.12)$$

where the LHS is the inertia effect of the liquid, with $k(\theta)$ defining an effective virtual mass coefficient defined in such a way as to equal unity for a sphere ($\theta = \pi$). On the RHS, F_u , F_l and F_c where respectively: the force due to the normal stress in the liquid, the force due to of momentum transfer to the bubble and the contact force. These can be written formally as;

$$F_u = -2\pi R^2 \int_0^\theta p(\theta) \cos \theta \sin \theta d\theta \quad (1.13)$$

$$F_l = \pi r_0^2 \left(p_g + \frac{1}{2} \rho_g u^2 \right) \quad (1.14)$$

$$F_c = -2\pi r_0 \sigma \sin \theta \quad (1.15)$$

Here, r_0 is the orifice radius, ρ_g the gas density, p_g the uniform pressure inside the bubble, p is approximated as the sum of hydrostatic pressure far away from the bubble and a dynamic pressure contribution due to the bubble's expansion, ϑ is the angular coordinate and u the gas velocity at the orifice. Two cases of bubble growth were considered. In one case the flow rate from the orifice was considered as constant, while in the second the pressure inside the chamber below the orifice was held constant.

The simulations from their model showed that the duration of bubble expansion in normal gravity conditions slightly decrease with the gas flow rate whereas, the final bubble volume is roughly proportional to it. However, at sufficiently low gravity, the bubble formation frequency is proportional to gas flow rate whereas the bubble detachment volume does not depend on it. Their results also show how the bubble volume continuously increases at low gravity conditions with a slow lengthening of the neck. In the case of the constant pressure condition, the authors have reported a different kind of mechanism of bubble formation with modifications in the dependencies of bubble size on orifice radius and gas density.

Duhar and Colin (2006)

More recently, Duhar and Colin (2006) experimentally investigated quasi-static injection of air bubbles into silicon oil both with and without a viscous shear flow. Using high-speed videography and detailed image processing, they determined the bubble radius and centre of gravity throughout the bubble growth period. Using the experimental results they validated a force balance model during the bubble life time. Their study corresponded with the quasi-static regime as the gas injection rates adopted were lower than the critical value suggested by Oguz and Prosperetti (1993).

In this investigation it was noticed that the radius at detachment, R_{det} , increases with the gas flow rate and decreases with the shear rate while the formation frequency increases with the gas flow rate and with the shear rate. In their experiments they considered the bubble as perfectly spherical and the resulting force balance was given by;

$$\frac{4}{3}\pi R^3 \rho_g \mathbf{g} + \mathbf{F}_c - \int_S (p_l - \rho_l g z) \mathbf{n} dS - \int_A p_g \mathbf{n} dS + \int_S \boldsymbol{\tau}_l \mathbf{n} dS = 0 \quad (1.16)$$

where the first term is the bubble weight, F_c is the capillary force, p_l the pressure in the liquid, τ_l is the deviatoric stress tensor due to viscous effects, p_g is the pressure inside the bubble, S the bubble surface exposed to liquid flow, and A the surface of the bubble foot in contact with the orifice of radius, r_0 . Also, R , is the bubble equivalent radius based on the gas volume limited by A and S . Expressing the third and the forth terms in terms of a reference pressure, p_c , in the liquid at the wall, expression were defined for the various

forces, in particular; a buoyancy forces component given by;

$$F_B = (\rho_g - \rho_l) \frac{4}{3} \pi R^3 g \quad (1.17)$$

and a contact pressure term defined as;

$$F_{CP} \approx \frac{2\sigma\pi r_0^2}{R} \quad (1.18)$$

In their analysis they considered both the growth in a quiescent liquid and in a shear flow. For the first case the force balance was written only for the vertical direction;

$$F_B - 2\pi\sigma r_o \sin \theta + F_{CP} - 4\pi\mu_l R v_b k_z = 0 \quad (1.19)$$

The second term is the capillary action while the last is the drag effect. From the image processing the value of the bubble radius and the velocity of the centre of gravity were calculated in order to solve the equation in terms of the contact angle, θ , at the bubble foot. These values were compared with those experimentally obtained from image processing and showed adequate agreement.

Possibly for the first time, the authors noted that the radius at detachment increased with the gas flow rate and decreases with the shear rate. The formation frequency, however, increased with both the gas flow rate and with the shear rate. The comparison of the calculated contact angle showed good agreement with the experimental results since the spherical shape assumption was approximately valid for these circumstances. Moreover, their study provided information regarding the evolution of the forces acting on

the bubble. These were plotted with respect to the time and provided a base knowledge of the force evolutions around the bubble during its lifetime.

1.2 Bubble Departure Models

Bubble departure is another fundamental aspect of crucial importance in single bubble events, in particular for nucleate boiling. The diameter at which a bubble departs from a surface during this process is generally thought to depend on buoyancy and inertia forces, aiding the detachment of the bubble from the surface, and surface tension and hydrodynamic forces which tend to resist departure. Furthermore, the shape of the bubble may deviate significantly from the idealized spherical shape. While bubbles growing quasi-statically tend to be more spherical, rapidly growing bubbles can be deformed significantly due to the interaction with the surrounding liquid.

1.2.1 Quasi-Static Bubble Formation and Departure

Perhaps the most uncomplicated case to analyse is that of a large, slowly growing bubble on a flat surface facing upward, for which the hydrodynamic and inertia forces are negligible. It is generally accepted that bubble departure occurs when the buoyancy force, acting to lift the bubble, overcomes the surface tension force trying to hold it back. The surface tension force also depends on the contact angle whereby a contact angle approaching 90° increases the surface tension force and hence the bubble departure diameter.

Fritz (1935)

The first model of bubble detachment was introduced by Fritz (1935). The Fritz equation, utilizing the contact angle, θ , and surface tension, σ , gives the bubble departure diameter as the diameter that satisfies the condition whereby the buoyancy force is exactly balanced by the capillary force;

$$\frac{\pi D_{eq}}{6} (\rho_l - \rho_g) g = \pi D_0 \sigma \sin \theta_0 \quad (1.20)$$

where;

$$D_{eq} = \sqrt[3]{\frac{6V_b}{\pi}} \quad (1.21)$$

Here, it is supposed that the diameter of the bubble base is proportional to the equivalent diameter of the bubble

$$D_0 = C D_{eq} \quad (1.22)$$

Then, by substituting 1.22 into 1.20 one obtains;

$$\frac{D_{eq}^2}{6} (\rho_l - \rho_g) g = C \sigma \sin \theta_0 \quad (1.23)$$

which results in the definition of the detachment diameter;

$$D_{det} = \sqrt{6C \sin \theta_0} \sqrt{\frac{\sigma}{g(\rho_l - \rho_g)}} \cong C \theta_0 \sqrt{\frac{\sigma}{g(\rho_l - \rho_g)}} \quad (1.24)$$

where, C is taken as 0.0208 and θ_0 is in degrees.

Chesters (1977)

A similar method to predict the value of the bubble diameter at detachment was proposed by Chesters (1997). Using the same physical condition as Fritz (1935), he showed that for small values of the shape factor, defined as,

$$\beta = \frac{(\rho_l - \rho_g) g R_T^2}{\sigma} \quad (1.25)$$

where R_T is the bubble radius at the apex.

Bubble detachment takes place when,

$$D_{det} = k \sqrt[3]{\frac{6 D_0 \sigma}{(\rho_l - \rho_g) g}} = k D_T \quad (1.26)$$

where D_T is the so called *Tate's diameter*, Tate (1864). The correction factor, $k < 1$, was added to account for the overpressure inside the bubble.

1.2.2 *Dynamic Bubble Formation and Departure*

While quasi-static detachment of a bubble has been treated with some relative success, the case of detachment in dynamic conditions is somewhat open and a definitive detachment criterion has yet to be identified. Although the prevailing theories approach the problem as a competition between attaching forces and detaching ones, the force balance is complicated by new components due to the inertia of the gas and the hydrodynamics of the surrounding liquid, Di Marco (2008).

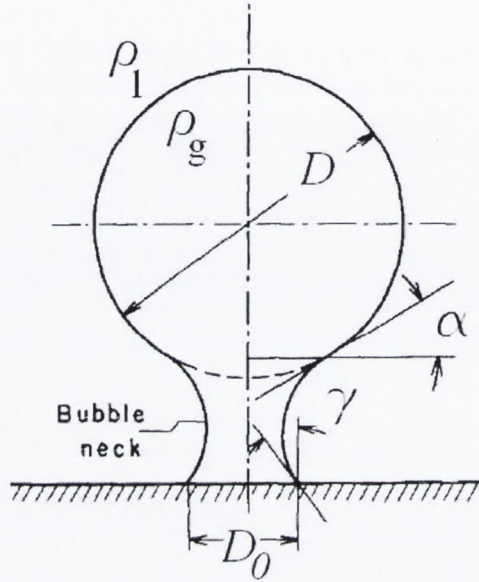


Figure 1.2: Geometry of the growing bubble, Kiper (1970).

Kiper (1970)

Kiper (1971) described a method for determining the minimum bubble departure diameter in saturated nucleate pool boiling, proposing that the minimum departure size varies with Jakob number only. In his work, the bubble is considered to be of spherical form ending in a small neck which connects the bubble to the heated surface. He neglected both the drag force and the vapour inertia force, though he considered the inertia of the fluid. As with the previous models, detachment occurred when the force balance equation is satisfied with the forces acting on the bubble being defined as:

The *buoyancy force*,

$$F_B = \frac{\pi}{6} g \frac{(\rho_l - \rho_g)}{g_\zeta} D^3 \quad (1.27)$$

where g_ζ is a conversion constant, considering the contact area is small with respect to the bubble and that the overpressure effect is negligible;.

The *capillary force* was defined as;

$$F_C = -\sigma\pi D_0 \cos \gamma \quad (1.28)$$

Furthermore, if the geometry of the neck is known then the term $D_0 \cos \gamma$ can be expressed as;

$$F_C = -\sigma\pi G(\alpha)D \quad (1.29)$$

where $G(\alpha)$ is related to geometry of the bubble neck.

The *liquid inertia force* was defined as;

$$F_{i,l} = \frac{11}{384} \frac{\pi \rho_l}{g_\zeta} \varepsilon^4 \quad (1.30)$$

where ε is the bubble growth parameter and is given by $D = \varepsilon \cdot t^{1/2}$

The *pressure restraining force* was defined as;

$$F_R = -0.0181\pi \frac{\rho_l}{g_\zeta} \varepsilon^4 \quad (1.31)$$

This force is due to the specific pressure distribution on the bubble surface which tends to flatten the bubble and hold it against the wall.

The force balance can then be rewritten as;

$$F_B + F_{i,l} = F_c + F_R \quad (1.32)$$

This can be recast as;

$$aD_d^3 + bD_d + c = 0 \quad (1.33)$$

where

$$a = \frac{1}{6}g\frac{\rho_l - \rho_g}{g_\zeta} \quad b = -\sigma G(\alpha)_d \quad c = 0.0105\frac{\rho_l}{g_\zeta}\varepsilon^4 \quad (1.34)$$

The equation has several solutions though just one was found to predict the experimental result which was;

$$D_{det} = 2.7Ja \quad (1.35)$$

where Ja is the *Jakob number*.

Gaddis and Vogelpohl (1986)

Gaddis and Vogelpohl (1986) developed an uncomplicated expression to predict the bubble detachment diameter in quiescent liquids under constant volumetric gas flow rate conditions. They took into account not only the buoyancy, surface tension and inertia effects, but also the effect of the drag force, F_{dr} , and the gas momentum, F_m , as well. The force balance to predict the bubble detachment was given by;

$$F_B + F_m = F_\sigma + F_{dr} + F_i \quad (1.36)$$

From this it is possible to obtain a generic expression for the bubble diameter, d ;

$$d^3 = A + \frac{B}{d} + \frac{T}{d^2} \quad (1.37)$$

with

$$\begin{aligned} A &= \frac{6d_0\sigma(4 - We)}{4\Delta\rho g} & B &= \frac{81\eta V}{\pi\Delta\rho g} \\ T &= \left(\frac{135}{4\pi^2} + \frac{27\rho_g}{\pi^2\rho_l} \right) \frac{\rho_l V^2}{\Delta\rho g} & We &= \frac{16\rho_g V^2}{\pi^2 d_h^3 \sigma} \end{aligned}$$

where We is the Weber number. Three different conditions can be applied to control the detachment diameter to yield a generalized equation of D_{det} as;

$$D_{det} = \left[\left(\frac{6d_0\sigma}{\rho_l g} \right)^{4/3} + \left(\frac{81uQ}{\pi g} \right) + \left(\frac{135Q^2}{4\pi^2 g} \right)^{4/5} \right]^{1/4} \quad (1.38)$$

At the transition to the jetting regime, drag and the inertial force due to liquid acceleration differ greatly compared with those calculated by the formulas above. This being the case, the above equation can be considered valid only up to the point of the transition to the jetting regime. A jet is formed only when the force due to gas momentum exceeds the surface tension force, $F_m > F_\sigma$, and surface tension is not capable of forming the spherical or quasi-spherical bubble. This occurs at a critical Weber number which is approximately equal to 4, and thus the force balance changes to $F_m + F_p = F_\sigma$.

Zeng (1993)

Zeng et al. (1993) proposed a somewhat different model considering the bubble to be subject to just the buoyancy and liquid inertia effects neglecting the surface tension term, which was included in almost all of the previous

theories. Under these circumstance the detaching condition occurs when;

$$F_B + F_{duy} \geq 0 \quad (1.39)$$

where the first term is the buoyancy effect which is equal to;

$$F_B = V_b (\rho_l - \rho_g) g \quad (1.40)$$

The second term is the unsteady growth force in the vertical direction. The author modelled this force by considering a hemispherical bubble expanding in an inviscid liquid. An empirical constant, C , was introduced to account for the presence of a wall;

$$F_{duy} = \rho_l \pi r^2 \left(\frac{3}{2} C \dot{r}^2 + r \ddot{r} \right) \quad (1.41)$$

The value of C was determined experimentally and its value was $C = 20/3$. The evaluation of the velocity and acceleration stemmed from knowledge of the bubble growth rate $r(t)$. In general this follows a power law;

$$r(t) = K t^n \quad (1.42)$$

where K and n were determined empirically. The unsteady growth force can then be defined as;

$$F_{duy} = -\rho_l \pi K^{\frac{2}{n}} \left[\frac{3}{2} C n^2 + n(n-1) \right] r^{4-(2/n)} \quad (1.43)$$

By the balancing of the buoyancy and unsteady growth forces the value of the detachment diameter was obtained;

$$D_{det} = 2 \left\{ \frac{3}{4} \frac{K^{2/n}}{g} \left[\frac{3}{2} C n^2 + n(n-1) \right] \right\}^{n/(2-n)} \quad (1.44)$$

From this equation it is possible to see that for gravity tending to zero the bubbles will not depart from the heating surface unless there is some external mechanism to induce detachment. Experiments made in reduced gravity tend to support this analysis. However, this model requires a priori knowledge of the growth characteristic as an input parameter which limits its utility.

Buyevich et al. (1995)

Buyevich and Webbon (1996) developed a model for bubble formation at a submerged orifice under normal and reduced gravity. Their study showed how in these different scenarios the bubble is subjected to a completely new set of forces. They found that there are two major factors that determine the moment of bubble detachment: the buoyancy force and a force due to the momentum flowing into the bubble from the injected gas. The buoyancy force dominates the process at normal gravity whereas the inflowing momentum force plays a key role under negligible gravity conditions. In microgravity, the force due to the injection of gas into the bubble is the only force which pushes the bubble upward and the momentum arising out of it is small and compensated by the inertial forces arising from the increasing bubble volume.

During the detachment stage the inertia force caused by the bubble expansion was neglected. Moreover, for small contact perimeters, both the capillary effect and contact pressure effect were ignored. The inertia effect of the liquid was thus the only force which opposed the bubble detachment. The bubble is thus pushed away from the orifice by the buoyancy force effect and the upward force due to the initial momentum of the injected gas being

forced into the bubble. A balance of these forces gives;

$$\frac{\rho_l}{2} \frac{d}{dt} \left[k(\pi - \delta) V \frac{dz}{dt} \right] = V \rho_l g + \frac{1}{2} \pi r_0^2 \rho_g u^2 \quad (1.45)$$

where k is a coefficient which is introduced in order to take into account the wall effect. This was found to be equal to $k = 11/8$ when $\delta = \pi - \theta \ll \pi$. For constant flow rate, when the flow rate is equal to $Q = \pi r_0^2 u$, Eq.1.45 can be written as;

$$\frac{11}{16} \frac{d}{dt} \left(t \frac{dz}{dt} \right) = gt + \frac{vQ}{2\pi r_0^2} \quad (1.46)$$

where $v = \rho_g/\rho_l$ is the density ratio. With the initial condition of $z = 0$ at $t = 0$, is it possible to predict the evolution of the distance of the centre of gravity with respect to time;

$$z = \frac{4}{11} gt^2 + \frac{8}{11} \frac{\kappa Q}{\pi r_0^2} t \quad (1.47)$$

Bubble growth stops when the value of z equals the bubble radius such that;

$$R(t) = \left(\frac{3}{4} \frac{Qt}{\pi} \right)^{1/3} \quad (1.48)$$

The solution of Eq.1.47 and 1.48 gives the value of the detachment time t_{det} , when both z and R have the same value. The detachment diameter is thus given by;

$$D_{det} = \frac{8}{11} gt_{det}^2 + \frac{16}{11} \frac{\kappa Q}{\pi r_0^2} t_{det} \quad (1.49)$$

Detachment Models Comparison

Table 1.1 summarizes the different formulations of the detachment diameters considered thus far and represents a cross section of the work which exists in the open literature.

Integrators	Detachment Diameter
Fritz (1935)	$D_{det} \cong C\alpha_0 \sqrt{\frac{\sigma}{g(\rho_l - \rho_g)}}$
Kiper (1970)	$D_{det} = 2.7Ja$
Chesters (1977)	$D_{det} = k \sqrt[3]{\frac{6D_0\sigma}{(\rho_l - \rho_g)g}} = kD_T$
Gaddis (1986)	$D_{det} = \left[\left(\frac{6d_0\sigma}{\rho_l g} \right)^{4/3} + \left(\frac{81uQ}{\pi g} \right) + \left(\frac{135Q^2}{4\pi^2 g} \right)^{4/5} \right]^{1/4}$
Zeng (1992)	$D_{det} = 2 \left\{ \frac{3}{4} \frac{K^{2/n}}{g} \left[\frac{3}{2} Cn^2 + n(n-1) \right] \right\}^{n/(2-n)}$
Buyevich (1996)	$D_{det} = \frac{8}{11}gt_{det}^2 + \frac{16}{11}\frac{\kappa Q}{\pi r_0^2}t_{det}$

Table 1.1: Bubble detachment models

1.3 Numerical Analysis

The mechanics and dynamics of fluids are governed by a set of equations that define the conservation of mass, momentum and energy. From these equations it is possible to define the Navier-Stokes equations which describe the motion of fluid substances. These equations are nonlinear partial differential equations for which exact solution exist for only very simple scenarios. For most scenarios no analytic solution exists. The evolution of a bubble is not one that cannot be solved analytically. Even when one describes the evolution of the bubble interface under quasi-static conditions, the system of differential equations is still impossible to solve. As a result, numerical approximations have been developed to simulate the bubble growth dynamics. Even though the solutions are an approximation of reality, with valid simplifying assumptions and accurate numerical techniques, a numerical simulation can give useful results. Also, such calculations allow the investigator to study parameters which are difficult to study experimentally, such as microgravity.

In the next section some examples of numerical approaches are discussed. Tsuge (1990), Prosperetti (1993) and Gerlach (2005) attempted to predict the evolution of the bubble by focussing on the gas/liquid interface; where the flow field around the bubble was not solved. On the contrary, Quann (2008) and Das and Das (2009) recently performed CFD simulations which solved the entire fluid domain and implemented interface tracking techniques. The interface of the bubble could thus move through the computational domain and the governing equations were solved for both fluid and gas phases.

Tsuge et al. (1990)

Tsuge et al. (1997) proposed a non-spherical model that described the shape of the bubble for bubble injection from an orifice. In their model, the gas-liquid interface of the bubble was viewed as a continuous curve. This was divided into a number of two-dimensional axisymmetric elements which were characterized by two principal radii of curvature R_1 and R_2 (Fig.1.3). R_1 is the radius of the circle which has the centre at O and passes through the elements $j-1, j$ and $j+1$. The other radius, R_2 , is the distance from the bubble's symmetrical axis to the element j through the point O .

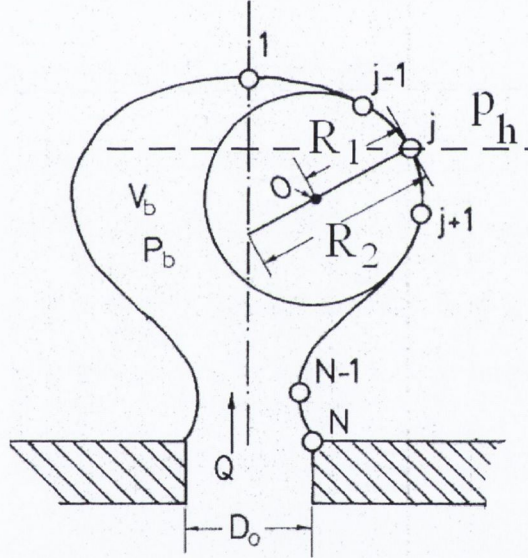


Figure 1.3: Schematic diagram of non spherical bubble formation model, Tsuge (1990).

An equivalent radius was introduced in the form;

$$\frac{1}{R} = \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (1.50)$$

This radius takes in to account the curvature of the bubble and was used only for the calculation of the surface tension, while for the other

calculations such as the bubble volume, the value of R was used. The bubble was considered axisymmetric with respect the vertical axis and completely isolated.

On a generic element j , the bubble was considered as a section of a sphere with radius equal to R_1 . The expansion of the gas-liquid-gas was defined by the modified Rayleigh equation;

$$p_b - p_h = \rho_l \left[R \frac{d^2 R}{dt^2} + \frac{3}{2} \left(\frac{dR}{dt} \right)^2 \right] + \frac{2\sigma}{R} + \frac{4\mu dR}{Rdt} \quad (1.51)$$

where p_h is the hydrostatic pressure term at the element j and the three terms on the RHS represent inertial, surface tension and the normal component of the viscous forces respectively. The equation of a rising bubble is described by a balance between inertia, buoyancy, drag and momentum such that;

$$\frac{d}{dt} \left(M' \frac{dz}{dt} \right) = (\rho_l - \rho_g) V_b g - \frac{1}{8} C_D \rho_l \left(\frac{dz}{dt} \right)^2 \pi D_m^2 + \frac{4\rho_g Q_0^2}{\pi D_0^2} \quad (1.52)$$

The initial bubble shape was considered to be hemispherical, with a radius equal to the orifice radius r_0 and the fluid around the interface was considered quiescent. The bubble pressure was presumed equal at the hydrostatic pressure acting on the flat plate plus the surface tension term $2\sigma/r_0$.

The element $j = 1$ was constrained to move just along the vertical axis of symmetry, while the one $j = N$ was stationary. At every step, Δt , a finite-difference procedure calculated the change of the pressure in the bubble, the radial displacement of each element with its velocity and acceleration. Furthermore, the bubble volume was obtained by integrating the coordinates of all the elements. This procedure was repeated several times until detachment of the bubble. Since with growth of the bubble the

total length of the interface increases, additional elements were introduced in order to better simulate the bubble shape.

The influence of viscosity was also investigated and it was found that the bubble formation time was much longer in highly viscous liquids than in low viscosity liquids, so that the bubble volumes were much larger. The bubble volume, shape and pressure of the internal gas chamber calculated by this model were compared with experimental results for a wide range of viscosity showing good agreement.

Oguz and Prosperetti (1993)

In Oguz and Prosperetti (1993) the first principal assumption was related to the flow field around the bubble. This can be viewed as a vector field, $\bar{\mathbf{u}} = f(\bar{x}, \bar{t})$, where $\bar{x}$ is the position vector and f a general function. If the fluid flow is assumed to be irrotational, a velocity potential $\Omega = \Omega(\bar{x}, t)$ exists. Ω is a scalar field whose gradients are related to the velocity field;

$$\bar{\mathbf{u}} = \nabla\Omega \quad \text{with} \quad \nabla^2\Omega = 0 \quad (1.53)$$

Starting from the Young-Laplace equation, they used Bernoulli's equation $p + \rho v^2/2 + \rho gh = \text{const}$ to express p_l , developing the following expression for the rate of change of the potential of a fluid particle on the bubble surface;

$$\frac{d\Omega}{dt} + \frac{1}{2} |\nabla\Omega|^2 + \frac{\sigma}{\rho} \vartheta + \mathbf{g}\bar{x} - \frac{p_b - p_\infty}{\rho} \quad (1.54)$$

where $\bar{x}$ is the position vector and p_∞ is the pressure at the needle tip under stagnant conditions. Imposing the normal velocity on the surface of the

needle in contact with the liquid to be null;

$$\mathbf{n} \cdot \nabla \Omega = 0 \quad (1.55)$$

the authors created a mixed boundary layer problem constituting the Laplace equation with Dirichlet (Eq.1.54) and Neuman (Eq.1.55) boundary conditions imposed on the previous equations. The system was solved numerically by a boundary integral method.

Once the potential vector field was defined the position of the interface was calculated by integrating with respect to time;

$$\frac{dx}{dt} = \nabla \Omega(\bar{x}, t) \quad (1.56)$$

for each surface particle.

On the bubble surface the potential was determined from the time integration of Eq.1.56, and differentiation along the surface gave the tangential component of the liquid velocity. The normal component of the bubble surface was given by the use of the Greens identity function and the volume growth rate was obtained by the integral;

$$Q_b = \int_{S_b} \Omega_n r ds \quad (1.57)$$

where S_B is the bubble surface, Ω_n defines the normal derivate of the potential respect the normal of the surface and ds is the surface coordinate. The computational procedure was validated by comparison with experimental results. The simulation was set in order to reproduce as much as possible the experimental test conditions. The agreement of the computed and observed bubble shapes was remarkably good. Simulations were performed for both

stagnant liquid and liquid with an imposed flow. For the second case, an upward flow parallel to the needle where the bubble is growing resulted in a premature detachment of the bubble.

Gerlach et al. (2005)

Gerlach et al. (2005) studied the influence of the Young contact angle by considering two modes of growth. In the first the bubble base remains attached to the orifice rim and in the second spreading of the bubble base on the horizontal surface is allowed. The two modes were called, respectively, A and B (Fig.1.4).

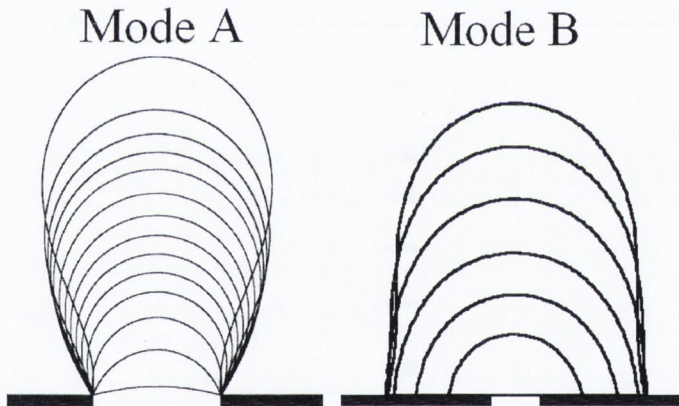


Figure 1.4: Development of the bubble shape for Mode A and B, Gerlach et al (2005).

The influence of the wettability of the solid surface was thus investigated. This effect is defined by the Young contact angle, ψ , which is the equilibrium angle formed at the contact line of three phases on a horizontal and perfectly smooth surface.

Mode A was not affected by the value of ψ , and the bubble foot remained attached to the orifice rim. On the contrary, in mode B, the contact perimeter

could spread if the instantaneous contact angle exceeded the Young contact angle. As long as the contact angle remained below ψ , the growth followed the conditions of mode A.

The simulation deals with a quasi-static bubble injection from a submerged horizontal orifice. The profile of the bubble was described by the Laplace equation and the equilibrium surface, pressure and gravitational forces acting on the bubble surface. The bubble interface was described by the expression (which will be discussed in the next Chapter),

$$\frac{\bar{y}''}{(1 + \bar{y}'^2)^{3/2}} + \frac{\bar{y}'}{\bar{x}(1 + \bar{y}'^2)^{1/2}} = \frac{2}{\bar{R}_T} - 2\bar{y} \quad (1.58)$$

upon which substituting,

$$w = \frac{y'}{(1 + y'^2)^{1/2}} \quad (1.59)$$

gives two first order ordinary differential equations,

$$\frac{dw}{dx} = -\frac{w}{x} + 2 \left(\frac{1}{R_0} - y \right) \quad (1.60)$$

$$\frac{1}{2} \frac{w}{(1 - w^2)} \quad (1.61)$$

These could be integrated with the initial condition: $y = 0$ and $w = 0$ at $x = 0$. The set of equations were solved by employing a fourth-order Runge-Kutta scheme and, for a given orifice radius r_0 , a sequence of bubble shapes were found by varying the parameter R_0 . Furthermore, at the beginning of growth, the gas/liquid interface was assumed flat at the orifice mouth, such that $R_0 \rightarrow \infty$ and the height of the apex being zero. R_0 was decreased in small steps and the computations searched for a new bubble shape with a slightly increased volume and height.

The increase of the bubble volume from step to step was restricted to a small value since more than one solution existed if r_0 and R_0 were fixed. In order to avoid a jump in the bubble volume, V , in the apex radius R_0 and the bubble height, the computations only allowed changes of V and R_0 to be extremely small. Following this procedure, the computations resulted in a sequence of bubble characteristics, which describe the stable stages of a bubble formation at an underwater orifice. The method was continued as long as no further solution could be found. At this final stage any perturbation leads inevitably to the release of the bubble.

The simulations were in good agreement with the results observed experimentally, for both modes of growth. The mode A formation was found to be dependent only on the non-dimensional orifice radius, $\bar{r} = r/a$ where a is the Laplace constant, while the maximum bubble size of mode B formation was influenced by the Young contact angle. Moreover, the study confirmed the strong influence of the plate material where the bubble was growing with the observation of large difference in the two modes of formation for the same orifice radius.

Quann and Hua (2008)

Quan and Hua (2008) describe the dynamics of bubble growth and pinch-off by the integration of the full Navier-Stokes equations for both gas and liquid phases. The equations were solved with a finite-volume method based on the SIMPLE scheme coupled with a front tracking method to locate the interface between the two phases. The two fluids were assumed to be immiscible and incompressible and the problem was assumed to be axisymmetric as the bubble grows at a very slow rates (about 10 s for release of one bubble) to minimize any inertial effects so that the problem was buoyancy dominated.

The initial flow fields for both liquid and gas phases were assumed to be quiescent. The initial position of the gas-liquid interface was assumed flat at the capillary nozzle outlet. A no-slip boundary condition was applied to the outer walls.

Both the conservation of mass and momentum were solved for the entire computational domain. Both the equations were made non-dimensional and the numerical method solved one set by treating the two phases as one fluid with variable material properties. A uniform mesh grid of 360x90 was used to discretize the simulation domain for the axisymmetric model and the radius of the nozzle outlet was resolved with 30 grid points.

The authors intention was to analyse the effect of liquid viscosity, surface tension and gas density on the bubble growth and pinch-off. Although these effects were always coupled in the experiment, in their simulations they were investigated separately in order to describe their influence. Their simulations showed how the bubble pinch-off depended on both the viscosity and surface tension coefficient. Higher values of these coefficients tend to retard the necking process, and thus create a larger bubble. Moreover, it was found that the density of the gas phase plays a minimal role on the bubble breakup.

Buwa et al. (2007)

Buwa et al. (2007), numerically investigated bubble formation from a submerged orifice for relatively high flow rates. The influence of inertia was included in their model with the intent of analysing the influence of the flow rate on the interaction of the detached bubble on the subsequent one. In particular, by combining the level set method with the volume of fluid method for interface tracking (LSVOF), they analysed the transition from single isolated bubble formation to the *interacting* bubble regimes. The

first, defined as the PERIOD-1 bubbling regime, occurred for lower gas flow rate and was characterized by a periodic formation of single bubbles. Upon increasing the flow rate two distinct bubble formation periods that repeat regularly were observed and their interaction could result in coalescence between two bubbles before the separation from the orifice signifying the PERIOD-2 bubbling regime.

The influence of the orifice diameter was also studied. From their analysis it was observed that bubbles growing from a larger orifices created a strong vortex which could influence the succeeding bubble passing from PERIOD-1 to PERIOD-2. The results were compared with experimental measurements and satisfactory agreement was obtained for larger orifices, while for smaller orifices the formation periods were over predicted.

Das and Das (2009)

Das and Das (2009), simulated the evolution of a bubble in a liquid pool using the smoothed particle hydrodynamic method (SPH). With this approach the fluid domain is not described by a computational grid, but as a set of discrete elements. The particles, which can be imagined as the fluid molecules, have coordinates which move with the fluid (i.e. a mesh-free Lagrangian method).

The initial distribution of the particles is generally uniform around the fluid domain, and the particles present no interconnectivity between them. Unlike the previous numerical method, where the value of a variable in a computational element depends on its neighbours, in the SPH method the relevant physical quantities associated with the particle are calculated by considering only the surrounding particles within a specific distance, h , from the particle of interest, see Fig.1.5.

Each particle has so spatial distance over which their properties are

smoothed by a kernel function. The contribution of the particles within the kernel range, the grey area of Fig.1.5, depends to their distance from central particle. The model was adapted in order to include the interaction between two immiscible fluids. A no penetration force along the interface was integrated in order to avoid particle exchange between the two fluids domains.

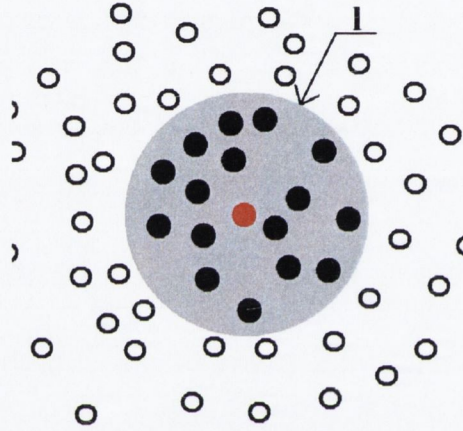


Figure 1.5: Interested particle and its support domain.

The model demonstrated capability in predicting the bubble growth, neck formation and pinch-off by comparing the numerical results with experimental ones. Some parameters, such as the bubble departure diameter or frequency were investigated with respect to the flow rate and good agreement with previous experiments was observed.

1.4 EHD Effects on Adiabatic Bubble Growth and Departure

Pool boiling heat transfer can be influenced by a great number of factors. Different studies have been performed in order to increase the amount of heat exchange during the boiling process. The evolution of the vapour bubble is one of the key parameter which affects the boiling heat transfer coefficient and thus the performance associated with thermal systems and devices.

The influence of an external electric field on heat transfer was observed near the beginning of last century by Chubb (1916). Experiments performed during the ensuing years discovered a remarkable influence of the electric field on the bubble evolution. In particular, the great part of the experimental results converged to a common observation regarding the deformation of the bubble. Within an electric field, bubbles were found to be elongated along the parallel direction of the electric field which resulted in an increase of the aspect ratio of the bubble, Chen et al. (2007), Dong et al. (2006).

The overall effect on the bubble growth parameters, such as the departure time or the detachment volume, can be influenced by a great number of factors. Kweon et al. (1998) investigated the effects of an electric field on the evolution of a bubble by changing the electrode configuration. The experiments showed that the electric field uniformity had an important influence. In particular, in a non-uniform electric field, the bubble departure volume was observed to decrease with the electric field strength, while for a uniform electric field it remained nearly constant.

Cosentino et al. (2005) performed a series of experiments analysing the influence of a uniform electric field on bubbles growing in two different fluids. Even if the bubbles tended to be elongated in both of the cases, an

earlier detachment was observed only for the liquid with the higher dielectric permittivity. The bubble growing within the lower permittivity fluid tended to be less influenced by the electric field strength and its detachment volume did not undergo any remarkable increase or decrease. This is consistent with the recent work of Siedel et al. (2010) who observed no notable change in the departure volume for bubbles growing from an artificial nucleation site in n-pentane; a liquid with a low dielectric permittivity.

The bubble growth dynamics depends strongly by the experimental test conditions. The effect of the external electric field has indeed two principal components: an increase of the bubble pressure and a generation of the Maxwell stress applied to the bubble surface. Their combined effect influences the bubble shape and depends strongly on the experimental conditions, such as electrode configuration or fluid proprieties, Cattide et al. (2007). Moreover, their overall effect can result in promoting, retarding or in fact having very little effect on the detachment stage.

Experiments performed in microgravity have shown how, under certain conditions, the electric field can completely replace the influence of gravity. Di Marco et al. (2003) performed a set of tests analysing the influence of the electric field on gas bubble growth and detachment under varying gravity conditions. From their experiments a constant detachment volume was observed for higher applied voltages regardless of the level of gravity.

Herman et al. (2002) performed similar experiments using a fluid with similar characteristics but with a larger orifice than the one tested by Di Marco et al. (2003). From their experiments, even if a high voltage electric field managed to detach the bubble in microgravity, the bubble volume was still ten times larger than the corresponding one at earth gravity. They concluded that: *the electric forces are not strong enough to substitute the*

buoyancy force.

As is evident from the above discussion the experimental test condition plays a very important role on the evaluation of the experiment results and have lead to some discrepancies and misunderstanding with regards to the general influence of electric fields on the bubble growth and departure dynamics.

Even though there have been investigations aimed at elucidating the influence of the electric field on bubble growth and departure, the entire phenomenon is still not completely understood, although significant progress has been made. In order to progress this area of thermal fluid science it is envisaged that in the short to mid term hybrid experimental and numerical techniques, such as in Dong et al. (2006) and Chen et al. (2007), will be implemented. In particular, Chen et al. (2007) were able to solve the electric field distribution around one of their experimentally measured bubble profiles and from that deduce the electrically induced pressure acting on the bubble over the entire surface. In the longer term, full numerical simulations, following on from the recent work of Zhang et al. (2010), seem like the plausible route.

CHAPTER 2

MATHEMATICAL MODELLING

2.1 Mass and Momentum Conservation

Roughly speaking, fluid dynamics is an area of study concerned with the effects of forces generated within or on bounding regions of fluids. One of the fundamental aspects of this discipline is the analysis of *transport phenomena*. This involves mass, momentum and energy exchange between a confined system and the surrounding space. In particular, the *balance equations* evaluate the variation of a physical quantity in a control volume within a time interval. Its general statement is that in a control volume, the variation of any physical quantity depends on the incoming and outgoing flux as well as on that generated or destroyed inside the control volume.

$$\textit{Variation} = \textit{Input} - \textit{Output} + \textit{Generation} - \textit{Consumption}$$

In the present study, the two principal physical quantities which define the

system's motion and dynamics are the conservation of mass and momentum.

Conservation of mass balance simply states that the variation of mass in a control volume (Fig.2.1a) equals the difference between the inlet and outlet mass flow rates:

$$\frac{dM}{dt} = \sum_i \dot{m}_i = \dot{m}|_{in} - \dot{m}|_{out} \quad (2.1)$$

In differential form, the generalized form of conservation of mass, for an unsteady compressible fluid is;

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (2.2)$$

For an incompressible fluid the relation simplifies to;

$$\nabla \cdot (\rho \mathbf{v}) = 0 \quad (2.3)$$

The momentum balance, on the other hand, is a generalization of Newton's Second Law ($\mathbf{F} = m\mathbf{a}$): in a finite volume, the variation of the momentum is related to the overall effect of the forces that act on the volume and the difference between the incoming and outgoing momentum (Fig.2.1b):

$$\frac{dM\mathbf{v}}{dt} = \mathbf{v}\dot{m}|_{in} - \mathbf{v}\dot{m}|_{out} + \sum_n \mathbf{F}_n \quad (2.4)$$

The forces acting on the control volume can be classified in two different groups: body forces and surface forces:

$$\sum_n \mathbf{F}_n = \sum_j \mathbf{F}_{V,j} + \sum_k \mathbf{F}_{S,k} \quad (2.5)$$

The body forces, $\mathbf{F}_{V,j}$, act throughout the volume of a body (e.g. gravity or centrifugal) the surface forces, $\sum_k \mathbf{F}_{S,k}$, act across internal or external

surfaces which defines the control volume (e.g. normal pressure or shear stresses).

The effect of the surface forces on the control volume can be represented by the terms ∇p and $\nabla \cdot \bar{\mathbf{T}}$. The first term is the pressure gradient and arises from the isotropic part of the stress tensor. This part is given by normal stresses that appear in almost all situations, dynamic or otherwise. The anisotropic part of the stress tensor gives rise to $\nabla \cdot \bar{\mathbf{T}}$, which conventionally describes viscous forces. Thus, $\bar{\mathbf{T}}$ is the deviatoric stress tensor, and the stress tensor can be cast as:

$$\delta = -p\bar{\mathbf{I}} + \bar{\mathbf{T}} \quad (2.6)$$

where $\bar{\mathbf{I}}$ is the 3x3 identity matrix. Combining the equation for each component the momentum conservation can be expressed as;

$$\rho \frac{D\mathbf{v}}{Dt} - \rho \mathbf{g} + \nabla p - \nabla \cdot \bar{\mathbf{T}} = 0 \quad (2.7)$$

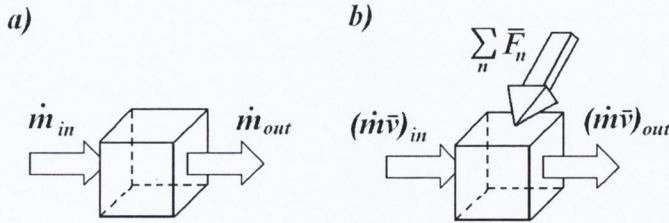


Figure 2.1: Mass and Momentum Balance on a control volume.

2.2 Constitutive Equations

In order to solve Eq.2.7 it is necessary to know the stress terms and how those interact with the fluid motion. A constitutive equation is a relation between two physical quantities that is specific to a material or substance, and approximates the response of that material to external forces. Simplifying assumptions on the specific behaviour of a fluid are needed (based on natural observations) and applied in order to specify the stresses in terms of the other flow variables, such as velocity and density. For example Navier and Stokes made the following assumptions with regard to the deviatoric stress tensor;

$$\nabla \cdot \bar{\mathbf{T}} = \mu \nabla^2 \mathbf{v} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{v}) \quad (2.8)$$

2.3 Navier-Stokes Equations

Is it now possible to substitute Eq.2.8 into 2.7 resulting in;

$$\rho \frac{D\mathbf{v}}{Dt} - \rho \mathbf{g} + \nabla p - \mu \nabla^2 \mathbf{v} - \frac{1}{3} \nabla (\nabla \cdot \mathbf{v}) = 0 \quad (2.9)$$

For an incompressible fluid $\nabla \cdot \mathbf{u} = 0$ such that;

$$\rho \frac{D\mathbf{v}}{Dt} = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{v} \quad (2.10)$$

This is the classic form of the Navier-Stokes equation, which defines a set of non-linear differential equations that describe the flow of a fluid whose stress depends linearly on velocity and on pressure.

2.4 Volumetric Electric Forces

Of particular interest in this work is the interaction of electric fields on the bubble growth dynamics. It is thus prudent to discuss some of the underlying physics associated with the relevant electrostatics and how this can influence two phase flows.

Generally speaking, a force field is a particular vector field which generates a force which is dependent on its position in space and/or time. Like a gravitational field, an electric field interacts with a physical domain exerting a force. In particular, for a fluid domain, the body force generated by an electric field $\mathbf{E}$ is given by, Landau and Lifshitz (1986):

$$\mathbf{f}'_e = q_F \mathbf{E} - \frac{\varepsilon_0 E^2}{2} \nabla \varepsilon_r - \frac{\varepsilon_0}{2} \nabla \left[\rho E^2 \left(\frac{\partial \varepsilon_r}{\partial \rho} \right)_T \right] \quad (2.11)$$

where q_F is the free electric charge density, ρ is the density of the medium, ε_0 and ε_r are, respectively, the vacuum and relative dielectric permittivity. The first term on RHS of Eq. 2.11 represents the *Culoumb* stress, which is generated on the free charges and is the only term which depends on the sign of the electric field. The second term is the *Dielectrophoretic* stress which arises from the spatial gradient of the medium permittivity. The third term on the RHS is the *Electrostriction* stress which is due to the variation of the permittivity with the mass density and non uniformity of the electric field.

In a two phase-flow a principal area of interest is the effect that the electric force has on the gas-liquid interface. Introducing the Maxwell stress tensor, Durand (1966), Eq. 2.11 can be rewritten as:

$$\mathbf{T}_{e,j} = \varepsilon_j (\mathbf{n} \cdot \mathbf{E})_j \cdot \mathbf{E}_j - \frac{\varepsilon_j}{2} \mathbf{E}_j^2 \left(1 - \frac{\rho}{\varepsilon} \frac{d\varepsilon}{d\rho} \right)_j \cdot \mathbf{n}_j \quad (2.12)$$

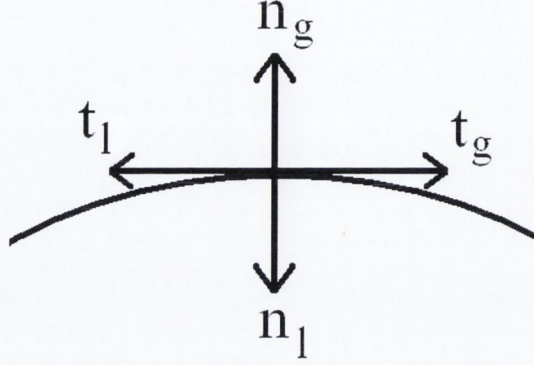


Figure 2.2: Unit vector directions on a gas-liquid interface.

where j represents the gas or the liquid phase while $\mathbf{n}_j$ is the unit normal component in each phase. The overall stress is given by the resultant of the two phase stresses, Zaghdoudi and Lallemand (2000):

$$\mathbf{T}_e = \mathbf{T}_{e,l} + \mathbf{T}_{e,g} \quad (2.13)$$

This can be expressed through the normal and tangential components;

$$\mathbf{T}_{e,n} = \frac{1}{2} \left[\varepsilon_l (E_{ln}^2 - E_{lt}^2) - \varepsilon_g (E_{gn}^2 - E_{gt}^2) + \rho_l \frac{d\varepsilon_l}{d\rho_l} E_l^2 - \rho_g \frac{d\varepsilon_g}{d\rho_g} E_g^2 \right] \cdot \mathbf{n}_l \quad (2.14)$$

$$\mathbf{T}_{e,t} = (\varepsilon_l E_{ln} - \varepsilon_g E_{gn}) E_{gt} \cdot \mathbf{t}_l \quad (2.15)$$

where the unit vector directions are shown in Fig.2.2

Considering the gas and the liquid as a dielectric medium, the interface between two different dielectric regions has to satisfy two boundary conditions, the continuity of the tangential component of the electric field and of the normal vector component of the electric flux density across the

interface ,Chen et al. (2007):

$$\mathbf{E}_{lt} = \mathbf{E}_{gt} \quad \varepsilon_l \mathbf{E}_{ln} = \varepsilon_g \mathbf{E}_{gn} \quad (2.16)$$

Under these assumptions the stress tensor only has a normal component acting on the bubble gas-liquid interface. Moreover, for non polar fluids, the variation of the fluid dielectric permittivity can be expressed by the Clasuius-Massotti law, Cattide et al. (2007):

$$\rho \frac{d\varepsilon}{d\rho} = \frac{(\varepsilon - 1)(\varepsilon + 2)}{3} \quad (2.17)$$

By substituting Eq. 2.17 with the boundary condition in Eq. 2.14 one obtains the expression for the electrohydrodynamic stresses acting on a gas-liquid interface

$$\mathbf{T}_{EHD} = -\varepsilon_0 \frac{(\varepsilon_{rl} - 1)^2}{6} (2E_{ln}^2 - E_{lt}^2) \cdot \mathbf{n}_l \quad (2.18)$$

In order to evaluate Eq. 2.18 one must know the relevant physical properties as well as electric field distribution.

2.5 Overall Momentum Balance

A growing bubble can be viewed as a body sitting atop a horizontal surface, as depicted in Fig. 3. It is possible to divide the bubble surface into a dry area, A , and a wet area, S , which defines the gas-liquid interface. The bubble interacts with the wall along at the contact line, CL .

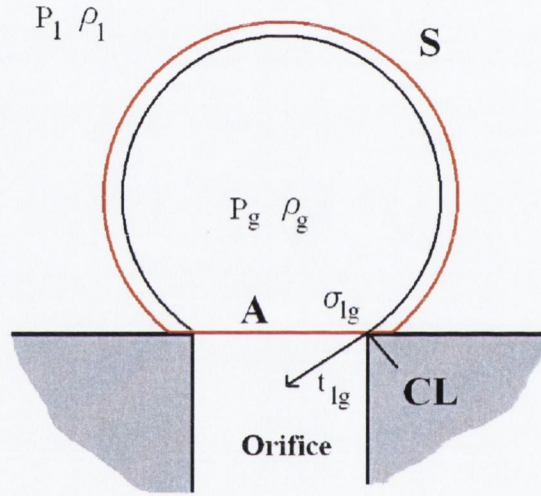


Figure 2.3: : Application of momentum balance on a growing bubble.

Once the control volume has been defined is it possible to solve the momentum balance equation. The variation of the momentum inside the control volume is given by the, LHS of Eq.2.4 as;

$$\frac{d}{dt}(M\mathbf{v}) = \frac{d}{dt} \int_V \rho_g \mathbf{v}_g dV \quad (2.19)$$

where V defines the volume of the bubble while ρ_g and ρ_l are the density of the gas and its velocity, respectively.

The overall momentum flux across the surfaces A and S is equal to:

$$\dot{m}\mathbf{v}|_{in} - \dot{m}\mathbf{v}|_{out} = \int_A [\rho_g \mathbf{v}_g (\mathbf{v}_g - \mathbf{v}_s)] \cdot \mathbf{n} dA + \int_S [\rho_l \mathbf{v}_l (\mathbf{v}_l - \mathbf{v}_s)] \cdot \mathbf{n} dS \quad (2.20)$$

where $\mathbf{n}$ is the outward unit vector, $\mathbf{v}$ is the velocity and the suffixes g and l refer to gas and liquid phases, respectively.

Finally, the sum of the forces acting on the control volume is given by:

$$\begin{aligned} \sum_n \mathbf{F}_n &= \int_V \rho_g \mathbf{g} dV + \int_{CL} \sigma_{lg} \mathbf{t}_{tg} dL + \\ &+ \int_S [\overline{\mathbf{T}}_{v,l} - p_l \bar{\mathbf{I}} + \overline{\mathbf{T}}_{e,l}] \cdot \mathbf{n} dS \\ &+ \int_{S+A} [\overline{\mathbf{T}}_{v,g} - p_g \bar{\mathbf{I}} + \overline{\mathbf{T}}_{e,g}] \cdot \mathbf{n} dS \end{aligned} \quad (2.21)$$

where p is the pressure, $\mathbf{g}$ is the gravity acceleration, $\mathbf{n}$ and $\mathbf{t}$ are the outward normal and tangent (in the meridian plane) unit vectors respectively, σ_{lg} is the surface tension and $\overline{\mathbf{T}}_v$ and $\overline{\mathbf{T}}_e$ are the viscous and the Maxwell stress tensors.

In Eq. 2.21 the first term on the RHS is the weight of the gas bubble, the second term defines the interaction between the bubble and the solid surface while the last two terms account for the effect of the static, dynamic and electric effects on the bubble surface.

Combining Eqs. 2.19, 2.20, 2.21 gives:

$$\begin{aligned}
\frac{d}{dt} \int_V \rho_g \mathbf{v}_g dV &= \int_V \rho_g \mathbf{g} dV + \\
&+ \int_S [\bar{\mathbf{T}}_{v,l} - p_l \bar{\mathbf{I}} + \bar{\mathbf{T}}_{e,l} + \rho_l \mathbf{v}_l (\mathbf{v}_l - \mathbf{v}_s)] \cdot \mathbf{n} dS + \\
&+ \int_{S+A} [\bar{\mathbf{T}}_{v,g} - p_g \bar{\mathbf{I}} + \bar{\mathbf{T}}_{e,g} + \rho_g \mathbf{v}_g (\mathbf{v}_g - \mathbf{v}_s)] \cdot \mathbf{n} dS + \\
&+ \int_{CL} \sigma_{lg} \mathbf{t}_{tg} dL
\end{aligned} \tag{2.22}$$

which represents the generic overall momentum balance on a bubble attached to a surface.

2.6 Theoretical Prediction of Bubble Profile

One of the principal aims of the current study is to gain a fundamental understanding of the relation between the bubble shape and the forces acting on it for quasi-static bubble growth. Inertial and viscous effects are in most practical situations negligible as supposed by Fritz (1935) or Chesters (1997). Another simplifying assumption is that the bubble will be considered as axis-symmetric with respect to the vertical axis.

For convenience, the origin of the coordinate system is placed at the tip of the bubble with the y coordinate directed downwards in the direction of the gravitational force, as depicted in Fig.2.4.

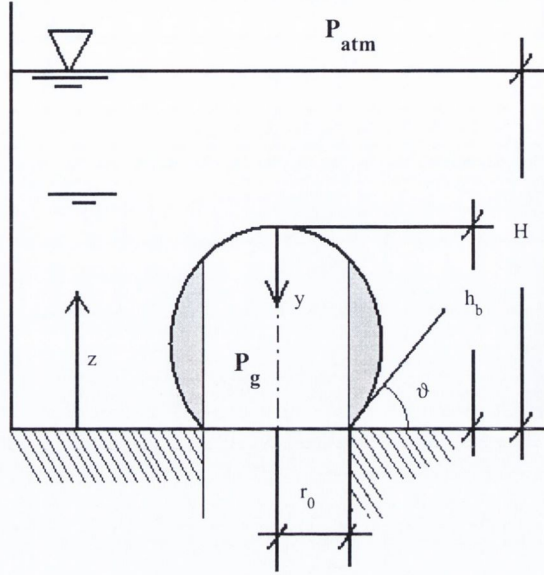


Figure 2.4: Schematic of a gas bubble issuing from an orifice into a liquid pool.

2.7 Pressure Field on Bubble

Pressure Balance in absence of electric field

In the absence of an electric field the pressures acting on the interface can be defined in terms of the gas pressure p_g , the stress due to the surface tension p_σ and the liquid pressure p_l . The resultant pressure balance is given by;

$$p_g = p_\sigma + p_l \quad (2.23)$$

For a stationary liquid the pressure can be divided into hydrostatic pressure $p_h = \rho_l g(h_b + y)$ and ambient pressure p_{atm} components, Fig.2.4. The surface tension stress is related to the surface tension and the curvature of the gas-liquid interface;

$$p_\sigma = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (2.24)$$

where R_1 and R_2 are the radii of curvature of the interface at the point (x, y) . Substitution into Eq.2.23 gives.

$$p_g = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) + p_{atm} + \rho_l g(h_b + y) \quad (2.25)$$

For the axis-symmetric case the two radii of curvature are equal at the tip of the bubble, R_T , such that the first term on the RHS of the Eq.2.25 can be written as;

$$\sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \Big|_{y=0} = \frac{2\sigma}{R_T} \quad (2.26)$$

Defining $p_{g,T}$ as the value of the pressure at the apex of the bubble, the bubble pressure balance can be rewritten as;

$$p_g = p_{g,T} + \rho_g g y = \frac{2\sigma}{R_T} + p_{atm} + \rho_l g h_b + \rho_l g y \quad (2.27)$$

Substituting Eq. 2.27 into 2.25 gives;

$$\frac{2\sigma}{R_T} = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) + (\rho_l - \rho_g) g y \quad (2.28)$$

which is also known as the *Capillary Equation*, Bashforth and Adams (1892).

For the solution of the above equation one must apply the following relations for the radii of curvature;

$$R_1 = \frac{(1 + y'^2)^{3/2}}{y''} \quad R_2 = \frac{x(1 + y'^2)^{1/2}}{y'} \quad (2.29)$$

where the symbol ' indicates the derivative respect to x . Upon substitution

one obtains the following;

$$\frac{y''}{(1+y'^2)^{3/2}} + \frac{y'}{x(1+y'^2)^{1/2}} = \frac{2}{R_T} - \frac{2y}{a^2} \quad (2.30)$$

where;

$$a^2 = \frac{2\sigma}{(\rho_l + \rho_g)g} \quad (2.31)$$

is called the *Laplace constant*. By normalizing the equation with respect to a , such that $\bar{x} = x/a$, $\bar{y} = y/a$, $\bar{R}_T = R_T/a$, a non-dimensional form of the previous equation is obtained;

$$\frac{\bar{y}''}{(1+\bar{y}'^2)^{3/2}} + \frac{\bar{y}'}{\bar{x}(1+\bar{y}'^2)^{1/2}} = \frac{2}{\bar{R}_T} - 2\bar{y} \quad (2.32)$$

Even for quasi- static bubble formation, no closed-form solution appears to exist for this ordinary differential equation in which the radius of curvature at the vertex remains as a parameter, Di Marco et al. (2005).

Pressure Balance in presence of electric field

As discussed, the electric field generates a deviatoric stress component and this is well known to influence the bubble shape and departure characteristics, Dong et al. (2006). In absence of electrical charges on the bubble surface, only the normal component of the electric stress acts on the bubble interface and Eq. 2.23 can be rewritten as:

$$p_g + p_{eg} = p_\sigma + p_l + p_{el} = p_l + p_{el} + \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (2.33)$$

which is the equivalent of the Young-Laplace equation in the presence of an electric field. From Eq. 2.33 and Eq. 2.28 it is possible to note an

important interaction between the electrodynamic stress and the bubble shape. If the electrically induced pressures are large enough the principal radii of curvature will be different, compared with that of no electric field, in order to establish mechanical equilibrium. Interestingly, and what makes this problem particularly problematic, is that the bubble shape itself changes the electric field distribution and ensuing electrical stresses. To the author's knowledge, no attempt has been made to solve Eq. 2.33 such that the study of the bubble shape in the presence of an electric field is still a wide open subject in fluid-dynamics research.

2.8 Bubble Detachment

The departure of a bubble from the surface upon which it is growing is one of the most relevant subjects in the study of the boiling phenomena. The investigation of the underlying physics of this event and the development of methods to predict it has been the topic of a great number of studies and debates in the academic community. In this section the author does not attempt to develop a new method to predict bubble departure, but aims at discussing the influence of external force fields on bubble growth and detachment in static and adiabatic conditions.

In absence of external forces a bubble, or equivalently a liquid drop, tends to assume the minimal surface configuration, which correspond to a spherical shape (Fig.2.5a). The molecules on the interface with the other phase have a higher value of potential energy. This makes it necessary to impose mechanical work in order to extend the surface of the bubble by an external perturbation (Fig.2.5b). Under this condition the potential energy of the molecules of the interface increases proportionally with the increment of

the bubble surface and with the surface tension of the liquid. If the external perturbation is removed (Fig.2.5c) the bubble surface does not coincide with the minimal one, and the molecules on the interface are able to perform work due to their higher potential energy. Thus, the bubble returns so to its previous configuration.

The previous simple example illustrates how a bubble, initially in equilibrium, increases its potential energy under the effect of an external perturbation and once the perturbation is removed the bubble goes back to the minimal potential state.

This is a condition of *stable equilibrium* (Fig.2.6a). The curve of the energy potential presents a minimum and the system, looking for the condition of minimum potential, remains at point A. A small mechanical perturbation increases the potential energy of the system, moving the system to the point B. Removing the perturbation the system goes back to A.

In the case of *neutral equilibrium* the curve of the potential is defined by a horizontal line. The system maintains the minimal potential possible regardless the position on the curve. A simple example is a sphere on a flat horizontal table. Moving the system from the configuration A to B does not require any energy supply into the sphere, and configuration B is maintained when the perturbation is taken away.

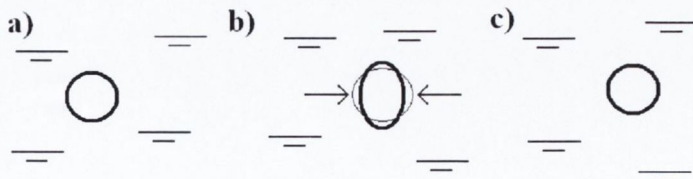


Figure 2.5: Bubble configuration prior the external perturbation (a), with the external perturbation (b), after the perturbation has been removed (c)

If the curve of the potential presents a maximum, as shown in Fig.2.6c, the system is in a condition of *unstable equilibrium*. The passage from the configuration A to B is in accordance with the tendency of the system to reduce its potential energy. The passage from A to B does not require energy but releases energy. Upon removing the perturbation, the system is still able to decrease its potential and does not return to the initial equilibrium state.

If a bubble grows from an orifice, gravity tends to deviate the bubble shape from spherical. Experiments in microgravity have shown how a bubble growing under this condition maintains a spherical shape, Di Marco et al. (2003). If the growth is quasi-static during the first stages the bubble configuration coincides with the stable equilibrium state (Fig.2.6a). All the external forces acting on the bubble have a resultant equal to zero and if the air injection supply were shut down the bubble would retain its shape and position indefinitely. This is, however, unless an external action, like a small mechanical perturbation from the orifice where is growing from, increases the potential energy on its surface. This condition is equivalent to a hemisphere lying with its curved surface on a table. After tipping the hemisphere and letting it go, the original position will eventually be re-established. Likewise

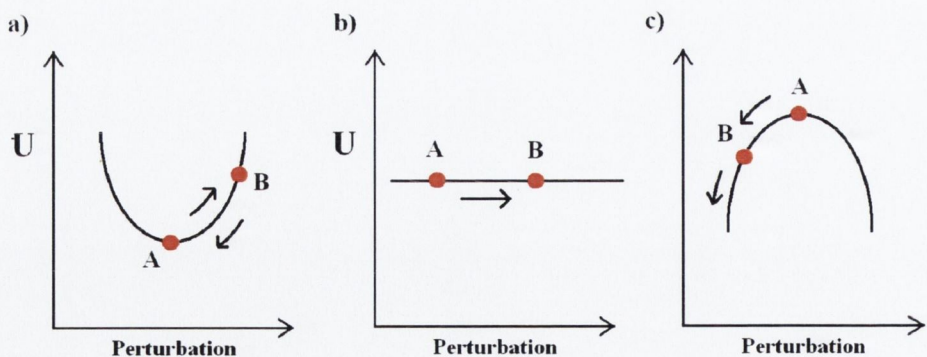


Figure 2.6: Stable (a), neutral (b) and unstable (c) static equilibrium configuration.

for the bubble, once the cause which deforms the bubble is removed it will return to its initial equilibrium state. Even still, if the perturbation is large enough it is possible to cause the bubble to detach from the surface. As one can imagine, the magnitude of the perturbation needed to detach the bubble from the surface decreases with the bubble growth, until it reaches a configuration of unstable equilibrium (Fig.2.7b) whereby the perturbations is infinitesimal. Under this circumstance the bubble coincides with a cone in equilibrium on its tip on a table. Any minimal external action causes the bubble to move from its equilibrium point in order to reach a lower potential state (Fig.2.6c).

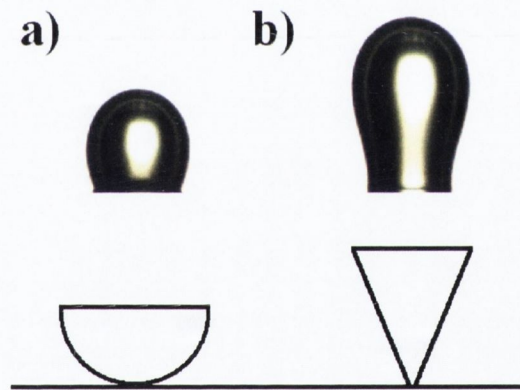


Figure 2.7: Stable (a) and unstable (b) equilibrium condition of a bubble growing from an orifice.

It is hypothesised that the detachment of a bubble occurs when this unstable equilibrium point is reached. This depends on a great number of factors. The force field acting on the bubble can promote or postpone the event. However, the complexity of the system makes it very difficult to define the *point of no return*.

The bubble indeed is not a rigid body, as a hemisphere or a cone, but can be considered as a continuous deformable body subject to different kinds

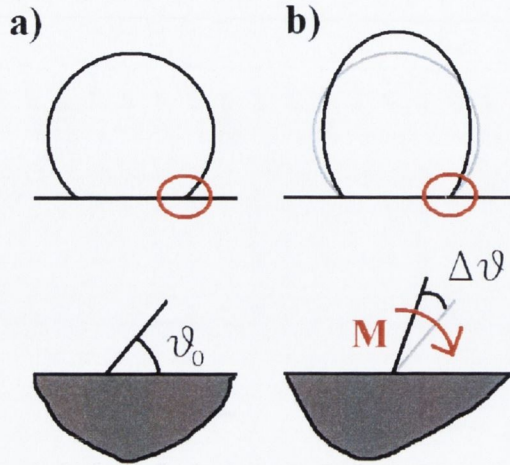


Figure 2.8: Mechanical analogy of a bubble

of constraints. In the absence of gravity, a bubble attached to a surface is subject to two principal constraints:

- Its tendency to reach the minimal surface area possible
- Its interaction with the wall, which forces it to assume a constant contact angle.

An analogous case in mechanics is a deformable ring with its ends fixed on the wall with the angle equal to, say, the static contact angle of the fluid with the material of the wall, Fig.2.8a. If the ring is deformed by an external force field, as shown in Fig.2.8b, it tends to change its equilibrium configuration. In particular, the enlargements in Fig.2.8, show how the deformation produces a variation of the contact angle in opposition with the fixed end constraint. This opposes further rotation by applying a moment proportional to the rotation itself. Once the external force field is removed the ring goes back to its initial state.

This simple example gives only a general idea about the interaction of a bubble with an external perturbation, but does not explain why a condition of unstable equilibrium is reached during the bubble growth. This is probably due to an oversimplified definition of the constraint representing the triple contact line with the wall.

Fig.2.9a shows a different mechanical representation of the triple contact line of a bubble with a surface. It consists of a device composed of a rocker arm and a spring hinged together. The rotation imposed to the rocker arm, $\Delta\vartheta$, produces a deformation of the spring, Δs , which reacts with a force proportional to the deformation, Fig.2.9b. A moment is thus applied to the rocker arm and tends to drive the arm back to its initial position. It is easy to see how this is a condition corresponding with a state of stable equilibrium.

When the arm reaches a 90° angle, Fig.2.9c, the device is now in a configuration of unstable equilibrium. The three centres of rotations of the connection wall-spring-arm are on a common line and the force of the spring does not produce a moment. Any small rotation of the arm will create a moment which tends to bring the system to a lower potential condition which, this time, corresponds with a contact angle of ϑ_0 or $180^\circ - \vartheta_0$.

The example represented in Fig.2.8 can now be modified considering this

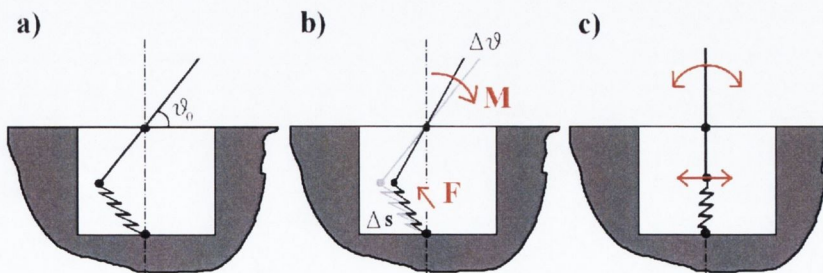


Figure 2.9: Contact angle constraint for: its stable equilibrium configuration (a), under the effect of a deformation (b), its unstable equilibrium configuration (c).

new approach to define the connection of the ring with the wall, Fig.2.9. For simplicity in the figure the constraint is not represented diagrammatically. As long as the contact angle is lower than 90° , the ring is in a configuration of stable equilibrium, Fig.2.10a. Once the contact angle reaches 90° , Fig.2.10b, the ring presents an unstable equilibrium condition on its base but its upper dome section still wants to assume a circular shape. This can thus force the contact angle to become obtuse and moves the connection from stable to unstable.

Once the ring passes this instability it tries to assume a new condition of minimal potential. In this case, even if the force that produced the deformation of the ring is removed, the ring will not go back to its initial shape (Fig.2.6a) but it will look for a lower potential configuration (Fig.2.6c) where the tendency of the upper part to assume a circular shape equilibrates the tendency of the connection constraint to achieve a static contact angle.

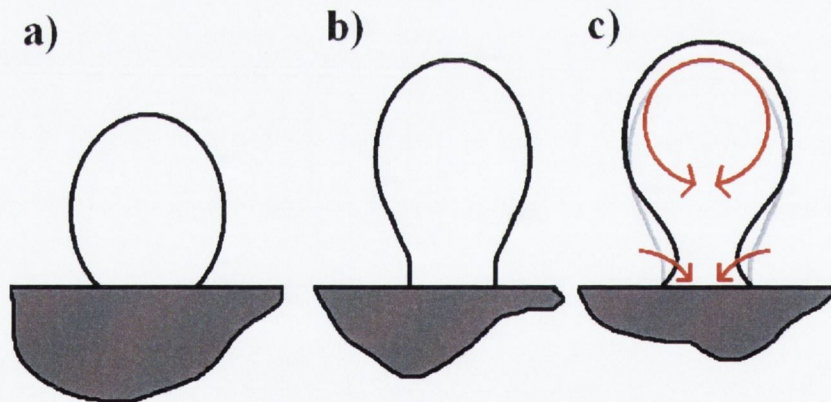


Figure 2.10: Stable (a) and Unstable (b) equilibrium configuration

The mechanical equivalences adopted in this section is admittedly simplified. They are presented only to highlight the importance of the local equilibrium of a deformable system. The example of the ring does not consider the effect of the internal pressure or buoyancy which would be

influential for a bubble. Moreover, in reality, the triple contact line is able to move and is not fixed as the rocker arm constraint is, and the ring is not able to detach. Also, the force field acting on a bubble can change with time and is related to the bubble shape. For this reason, the definition of an unambiguous unstable equilibrium configuration is very hard to obtain and still needs to be analysed in more detail, though is outside the scope of this work. Even still, this approach is quite different than conventional theories which tend to apply a bubble detachment criterion based on a balance of forces. The obvious paradox with this type of approach is that in many cases, such as the ones in this study, the static forces are quite obviously balanced throughout the bubble's lifetime.

CHAPTER 3

EXPERIMENTAL APPARATUS AND DATA REDUCTION

3.1 Experimental Apparatus

An experimental apparatus has been commissioned which can simultaneously measure the internal gas pressure and the shape evolution of the bubble during its growth and departure from a submerged orifice. This was facilitated by injecting air bubbles through submerged orifices and utilising a high speed camera such that it was possible to take video footage of the bubble growth and store each frame for subsequent analysis. The test rig could also impose an electric field around the bubble. As shown in Fig.3.1, the bubble grows in a small basin made of Perspex. This material was suitable for this type of experiment due to its high transparency, ease of manufacture and compatibility with the fluids tested.

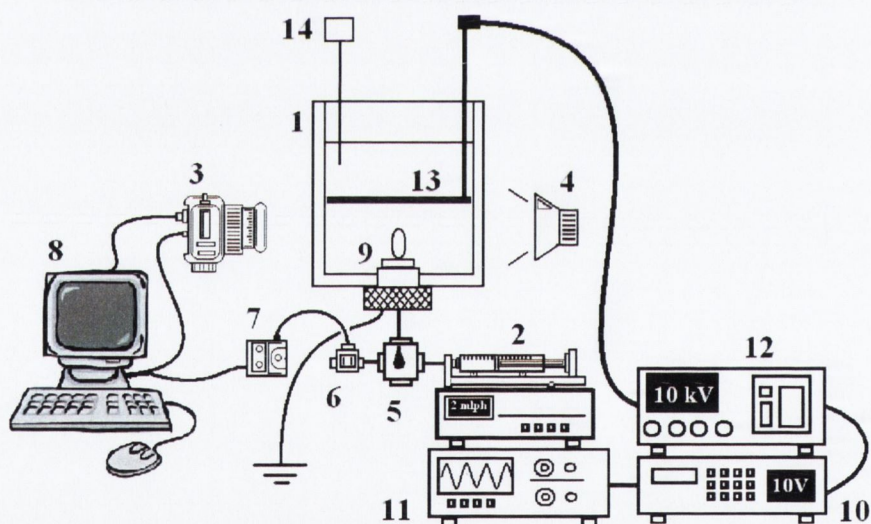
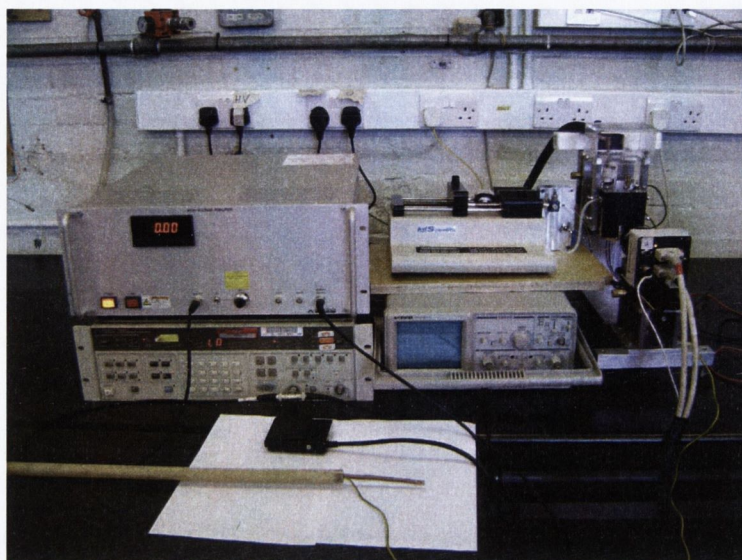


Figure 3.1: Schematic of test facility experimental apparatus used for the experiments (1) Liquid Vessel (2) Syringe Pump (3) High Speed Camera (4) Light Source (5) 4 Prot T-Valve (6) Pressure Transducer (7) A/D converter (8) Data Acquisition Computer (9) Drilled Aluminium Cylinder (Ground) (10) Signal Generator (11) Oscilloscope (12) High Voltage Amplifier (13) High Voltage Electrode (Anode) (14) Thermocouple.

The top of the vessel is open to the atmosphere and the bottom was flat and sealed. The bubbles grow from an orifices drilled through aluminium cylinders which were sealed to the bottom of the vessel. The aluminium bases were interchangeable to facilitate testing with different orifice diameters. An axis-symmetric electric field could be generated by a metal grid electrode fixed above the aluminium cylinder which was connected to a high voltage amplifier with the aluminium base being connected to ground.

3.2 Basin and Aluminium Base

As depicted in Fig.3.2 a the base dimension of the basin is 60 mm x 60 mm and is sufficiently large compared with the bubble sizes that the influence of the side walls could be considered negligible. A through hole was drilled into the base where the aluminium section was inserted. The aluminium base insert consists of a succession of cylindrical sections. Fig.3.2b shows that a somewhat oversized circular cavity was drilled through the centre of the insert for the greater part of its length whilst the small orifice hole was drilled in the last cylindrical section. Three different orifice sizes were studied such that three different aluminium bases were manufactured.

The aluminium insert was connected to the basin structure through a threaded section. In this way the two components were rigidly joined. Also, this type of connection avoided possible leaks of including a rubber O-ring between the base and the basin.

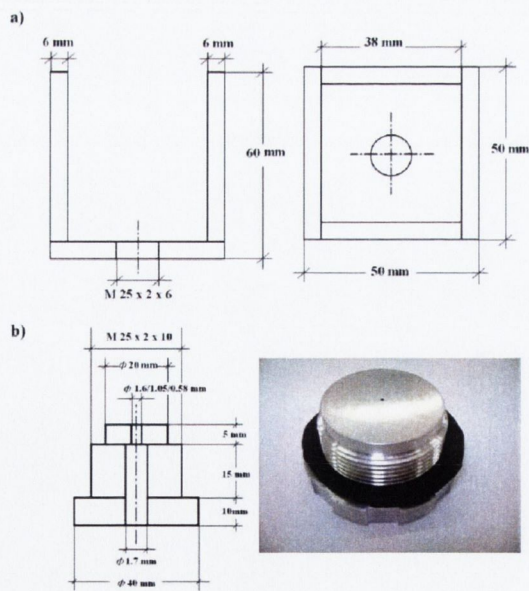


Figure 3.2: Working drawing of (a) the basin and (b) the aluminium base.

3.3 Injector Device

The main component of the bubble injector system is a *Kd Scientific Model 200 series* syringe pump (Fig 3.3a). This device converts the rotational motion of a motor to a translational motion which subsequently pushes the arm of an air-filled syringe. The linear motion of the syringe pump allows for accurate control of the air volumetric flow rate which was varied between 10 mlph and 100 mlph for this study. The syringe adopted is an *Hamilton 1750 CX 500 lw/Stop(1/4-28)* (ref A81262) with an inner volume of $500\mu\text{l}$ (Fig 3.3b).

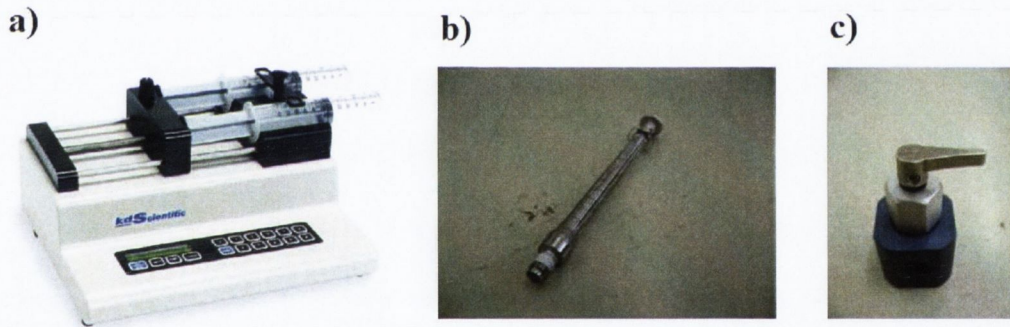


Figure 3.3: Syringe pump (a), $500\mu\text{l}$ syringe (b) and T- flow path 4-ports valve (c).

The syringe is connected to the orifice and to the pressure transducer through a T flow path 4-ports *Hamilton HV 4-3* (Ref A86730) (Fig 3.3b). In this way it is possible to keep the syringe connected to the basin and the pressure transducer during the phase of air injection (i.e. bubble growth) when the plunger is being pushed by the plate of the pump. Also, during the drawing back of the syringe, it is possible refill the syringe with air without having to disconnect it. Figure 3.4 give a better depiction of the valve in use.

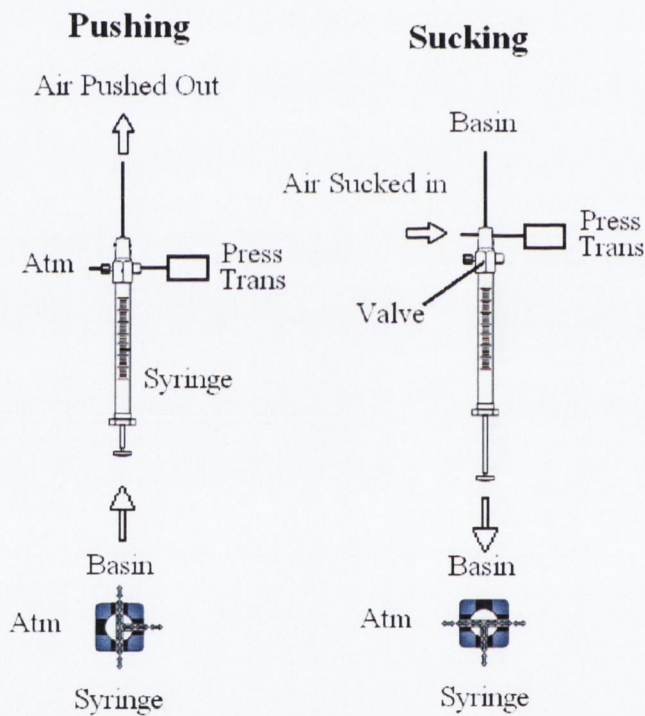


Figure 3.4: Scheme of the (a) pushing and (b) sucking stages.

3.4 Pressure Transducer

In order to measure the pressure history inside the bubble as it grows and departs, a fast response ($\sim 1ms$) *Honeywell 164PC* differential pressure sensor was joined to the T valve. The device is shown in Fig.3.5.

The pressure transducer converts the differential pressure between the inside of the syringe and atmospheric pressure into an analog electrical signal. As it is evident from Fig.3.5, the device has two connections; one connected to the valve, the other to the atmosphere. The output voltage signal had to be amplified through an electrical network and was subsequently inserted into an enclosure, as shown in Fig. 3.6. The analog signal is subsequently converted into a digital signal by a 12-Bit (NI USB-6008) A/D converter and logged and analysed using LabVIEW and a dedicated computer.

Before commencing the experiments it was necessary to calibrate the pressure transducer in order to determine the relationship between the pressure differences applied at one side of it and its output signal. Utilizing a U-tube manometer as the reference pressure a series of measurements were performed and the output voltage obtained from the sensor was matched with the value of the known pressure from the manometer scale. The calibration results are shown in Fig.3.7. As it is possible to see from the graph there is

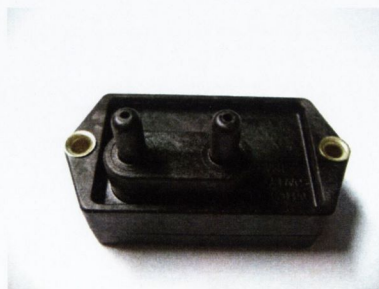


Figure 3.5: Honeywell 164PC pressure transducer.

EXPERIMENTAL APPARATUS AND DATA REDUCTION

a clear linear relationship between the output signal and the magnitude of the differential pressure. A linear regression of the data gives a conversion constant of, $\delta = 502 \text{ Pa/Volt}$.

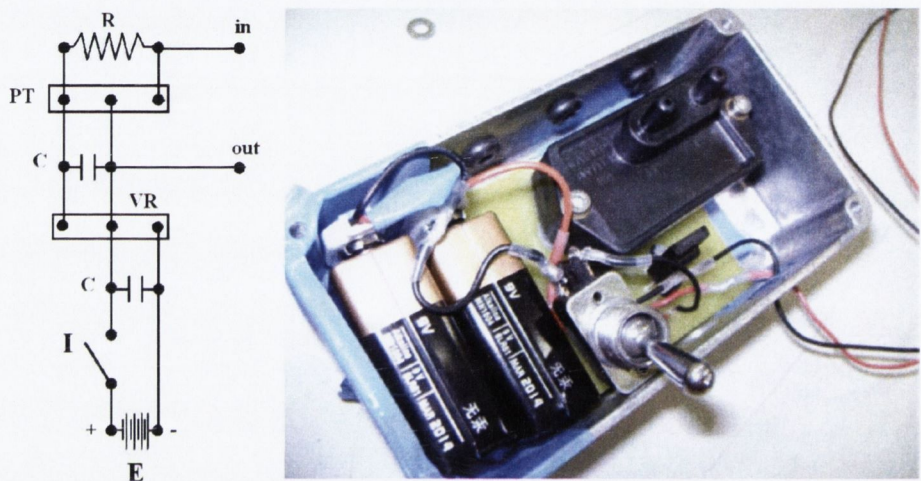


Figure 3.6: Electrical circuit (left) and photograph of pressure measurement system(right). (E) Energy Supply (I) Switch (C) Condenser (VR) Voltage Regulator (PT) Pressure Transducer (R) Resistance.

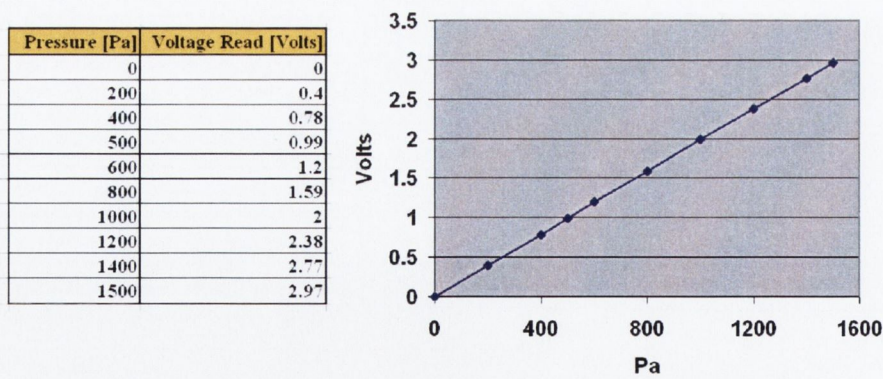


Figure 3.7: Raw calibration data (left) Pressure calibration curve (right)

3.5 High Speed Videography

The bubble shape during its growth was obtained by digital processing of high speed video images. The images were captured with NAC Hi-Dcam II high speed camera at frame rates varying between 60 fps and 1000 fps depending on the air injection rate. The maximum recording time of the camera depends on the frame rate and the area of recording and specialized software allows the user to store the pictures for a chosen time interval. The principal aim was to select a smallest possible recording area that was still large enough to capture the complete bubble growth cycle. In this way:

- (a) the recordable time period increases;
- (b) the file size of a singular frame decreases.

The necessity to limit the recording area was more important for the high flow rates and the smaller orifice diameters, where the bubbles grew relatively fast.

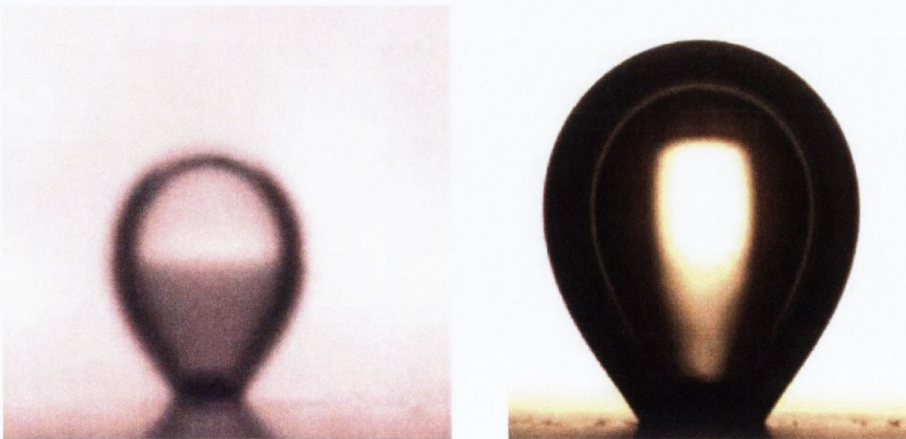


Figure 3.8: (left) Low quality image (320x250 pixel) and (right) High quality image (310x421 pixel).

Having very high quality images is fundamental to this work. As it will be explained in the next sections, this study is focused not only on the principal geometric characteristics of the bubble, such as volume and equivalent area, but also on the bubble curvature. This required careful attention to the effect of lighting and reflection and the positioning of the experiment and the light source in such a way that the captured picture defines, as close as possible, the air/liquid interface (Fig.3.8).

3.6 Electric Field and High Voltage Amplifier

An electric field was generated by a flat metal mesh electrode fixed above the grounded aluminium cylinder at a fixed distance L through a copper support (Fig.3.9a and Fig.3.10) and connected to a 20 kV High Voltage Amplifier (Matsusada Precision AMS/AMT series). A constant DC signal was generated by a Hewlett Packard 3325B function generator and subsequently amplified by the High Voltage Amplifier (Fig.3.9b). The metal mesh electrode was selected since it is electrically conductive and allowed the bubbles to pass through. This is important since bubble agglomeration on the electrode can distort the electric field. The aluminium cylinder was connected to the ground as shown in Fig.3.10.

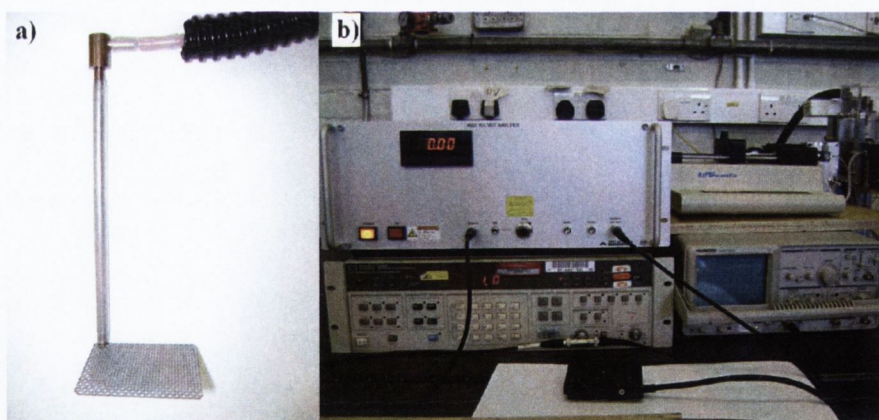


Figure 3.9: (a) Metal mesh electrode and (b) Function Generator and High Voltage Amplifier.

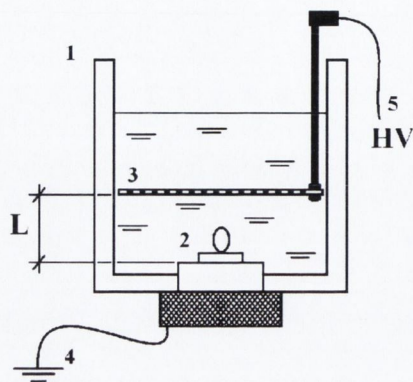


Figure 3.10: Schematic detail of the experimental apparatus (1) Liquid Vessel (2) Aluminium Cylinder (3) Metal Mesh Electrode (4) Ground Connection (5) High Voltage Connection

In order to satisfy the basic safety requirements the entire experimental rig was connected to the ground (Fig.3.11a) and a Perspex shield was fixed to the top of the basin as shown in Fig.3.11b. This prevented direct contact by the user with the high voltage support. As a further precaution, the High Voltage Amplifier could be activated only by a foot pedal switch placed a safe distance from the experiment and activated by the user.

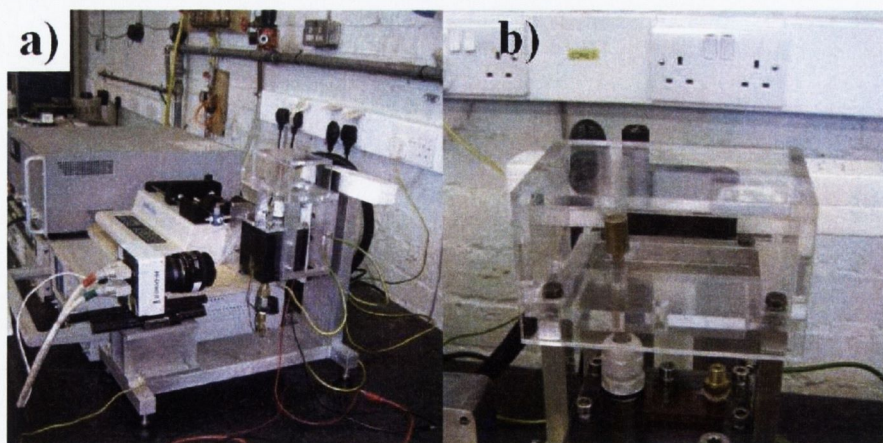


Figure 3.11: (a) Grounded experimental rig (b) Security box

3.7 Experiment Conditions

As already mentioned in the previous sections, for this investigation the bubble grows and departs from a small orifice drilled in a flat plate submerged in a basin filled with otherwise quiescent liquid. A constant injection volumetric flow rate is ensured by utilizing a controllable syringe pump. Orifice sizes of 1.6 mm, 1 mm, and 0.58 where drilled into their respective aluminium base. The uncertainty of the orifice diameter is estimated to be $\pm 0.05mm$.

The experiments were conducted at atmospheric pressure and terrestrial gravity. Two different working fluids where tested: pure degassed water and the dielectric fluid HFE-7000. Due to its insulating properties only the HFE-7000 could be used for the electric field tests. Moreover due to the solubility of the air in the dielectric fluid, before each experiment it has been necessary to wait few minutes until the liquid was saturated of air. The table on the next page lists the principal experimental parameters and relevant liquid proprieties.

Description	Water	HFE-7000	Units
Flow Rates	10-20-50-100	2	mL/h
Orifice Diameters	1.6-1.0-0.58	1	mm
Liquid Level	21	26	mm
Frame Rate	60 ÷ 1000	500	fps
Liquid Density	998	1400	kg/m ³
Kinematic Viscosity	1	0.32	cSt
Boiling Point at 1 atm	100	34	°C
Surface Tension	0.073	0.0124	N/m
Dielectric constant	80	7.4	-
Voltage Applied	-	0-2-4-6-8	kV
Distance Grid	-	10	mm

Table 3.1: Experimental conditions.

3.8 Image Processing

A piece of purpose-built Matlab code was written in order to define the relevant bubble characteristics such as the position of the centre of gravity, the coordinates of the bubble interface and the instantaneous contact angle. The image processing consists of;

- (a) selection of the area of interest (Fig.3.12a);
- (b) application of a median filter in order to smooth the bubble boundaries;
- (c) the determination of a gray scale image (Fig.3.12b);
- (d) the choice of a threshold for the gray levels;
- (e) creation of a B/W picture through the threshold value (Fig.3.12c);
- (f) determinations of the bubble centroid and interface (Fig.3.12d).

The first step of the procedure is to convert from a RGB format picture to a BW format. The RGB colour model is an additive colour model in which

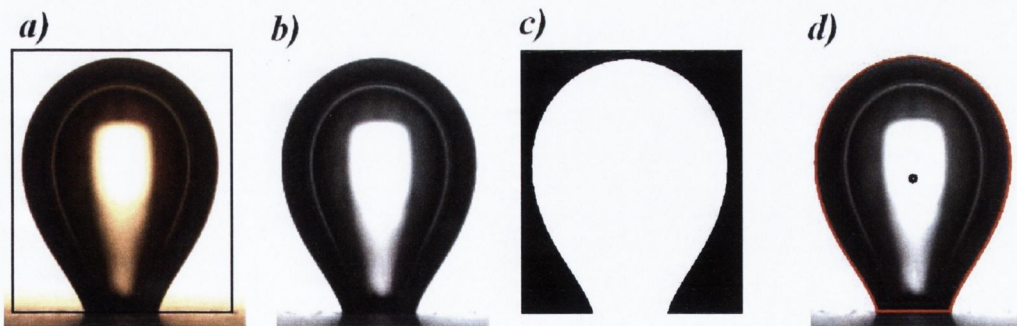


Figure 3.12: Sequence of image processing: (a) Selection of the area of interest (b) Conversion in a grey scale format (c) Creation of a binary picture (d) Definition of the boundaries and centre of gravity.

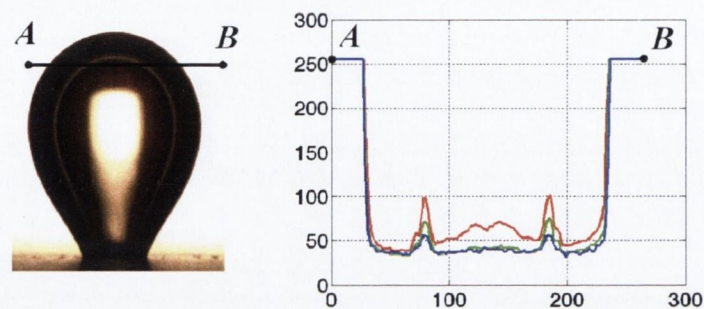


Figure 3.13: RGB format (left), evolution of the three primary colours along the segment A-B (right).

red, green and blue colour are added together in various ways to reproduce a broad array of colours. An RGB image is then analysed in Matlab as a $N \times M \times 3$ matrix, where N and M define the pixels of the height and width of the picture. Fig.3.13 represents the evolution of the three colours along the line A-B. The RGB scale is then converted to grey scale, also analysed by Matlab as a $N \times M$ matrix. In this way it is possible to choose a threshold value that is used to determine if a pixel is inside or outside the bubble as shown in Fig.3.14.

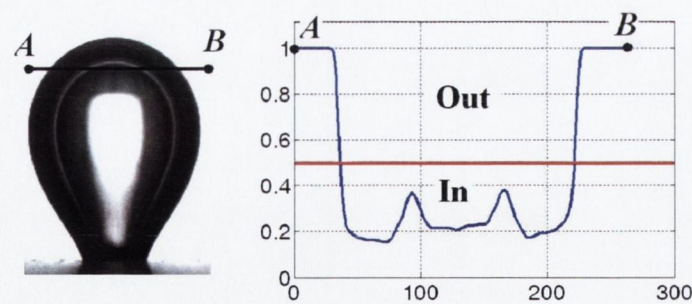


Figure 3.14: Grey scale format (left) and evolution of the intensity along the segment A-B (right)

The code creates a binary picture where every pixel assumes a value of 1 (inside the bubble) or 0 (outside the bubble). The bubble characteristic dimensions and interface are subsequently determined using the 'regionprops' and 'bwboundaries' utilities in Matlab. The image is then converted from a pixel scale to a physical scale using a pixel to length conversion coefficient L which is determined by comparing a measured pixel distance to an already know physical length.

Further post-processing of the bubble image is required in order to adequately quantify the two principal radii of curvature at a given point on the gas-liquid interface. The assumption of axisymmetry facilitates the formulation of the second radii of curvature. These are depicted in Fig. 3.15 for a bubble near departure.

The two circles (C_1 and C_2) which approximate the bubble surface at a generic point j lie on two orthogonal planes Ω_1 and Ω_2 (Fig.3.16). Ω_1 is the meridian plane which contains the vertical axis of the bubble while Ω_2 , orthogonal to Ω_1 , contains the normal to the bubble surface at the generic point considered.

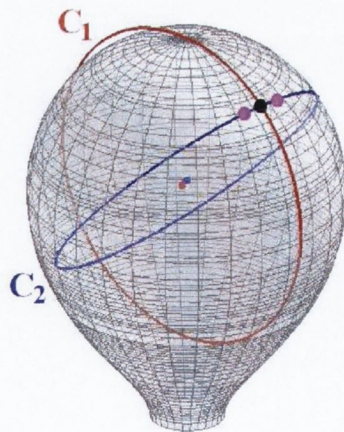


Figure 3.15: A 3D reconstruction of a bubble showing both radii of curvature for three selected points.

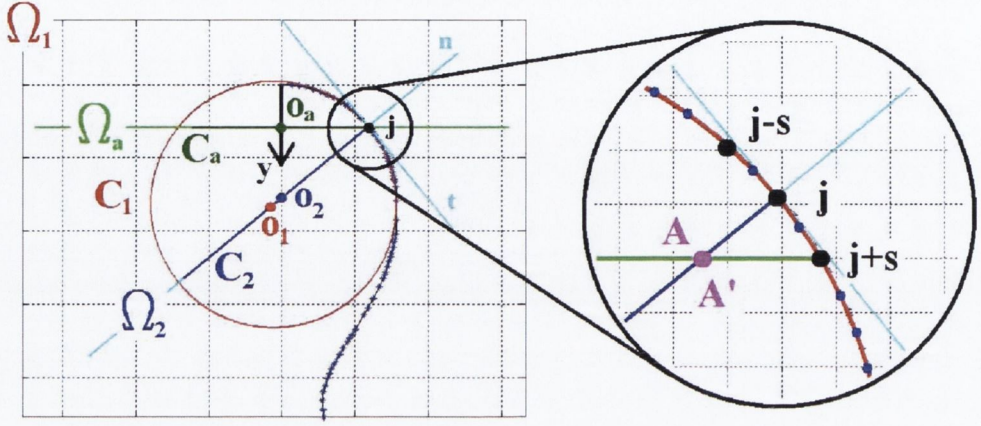


Figure 3.16: Circle that defines the bubble at the j -point (left) detail on its determination (right)

A simple routine was written in order to define the equation of the meridian circumference (C_1) passing through the generic point j and its neighbouring points, $j - s$ and $j + s$, where s defines the step (Fig.3.16). An auxiliary circumference (C_a) normal to the bubble axis is generated. This passes through one of the two neighbouring points and its centre, O_a , lies on the bubble's axis. The intersection between the auxiliary circumference and Ω_2 defines two auxiliary points A and A' , those are represented with violet points in Fig.3.15 and Fig.3.16. The second circumference (C_2) passes through j and the two auxiliary points A and A' (Fig.3.15).

The contact angle ϑ is defined considering the tangential of the circumference passing through the bubble's base. o and two previous points $o - s$ and $o - 2s$ (Fig.3.17).

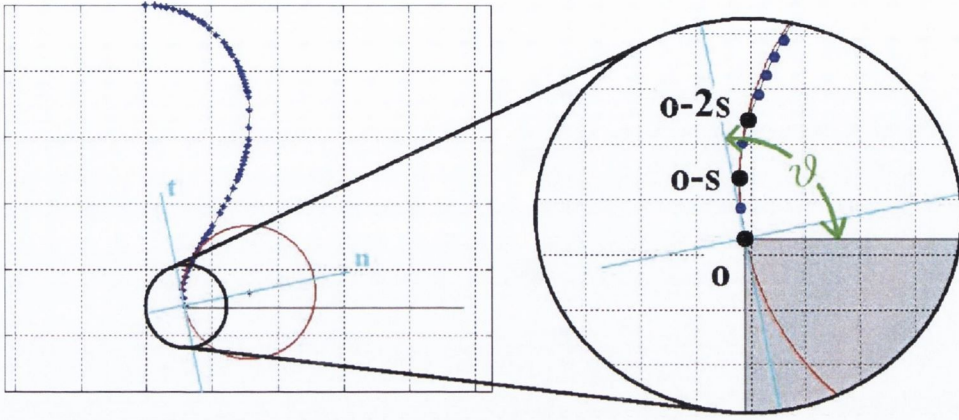


Figure 3.17: Circle that defines the bubble in the proximity of the bubble base (left) detail on its determination (right)

Since the bubble interface is approximated by a large number of discrete and closely spaced points, the definition of the two curvatures by using only a step size causes inevitable discontinuities (blue lines in Fig.3.18). It was thus decided to use more than one step size and smooth each trend using a simple filter to define the average curve (red lines in Fig.3.18).

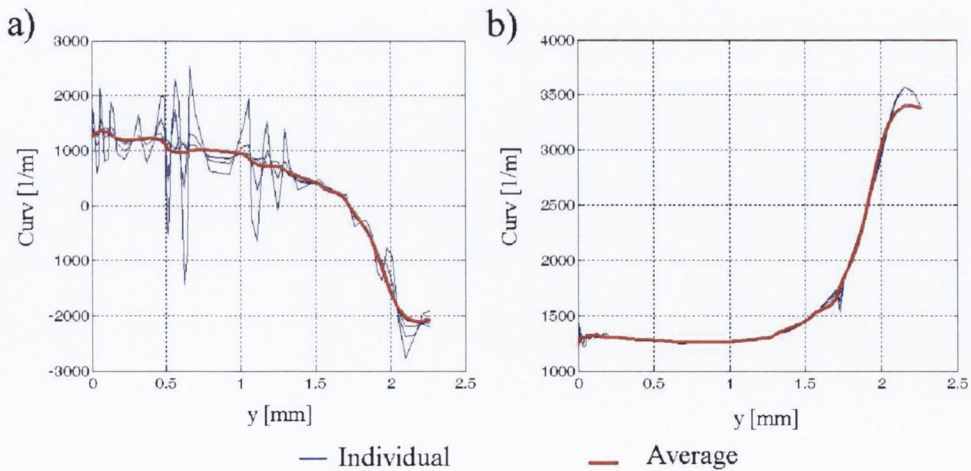


Figure 3.18: Evolution of the two principal radii of curvature: $1/R_1$ (a) and $1/R_2$ (b)

Since the test facility did not permit visual access into the orifice it was not possible to video the bubble when it was beneath the surface. As depicted in Fig.3.19, measurement of the bubble geometry and dynamics is thus impossible during this growth phase and is only possible when the bubble has emerged completely from the channel with the bubble foot attached to the rim of the orifice. For convenience, in order to make comparisons with different bubbles, a common $t = 0$ start condition is chosen when the bubble has a hemispherical shape with a contact angle of approximately 90° .

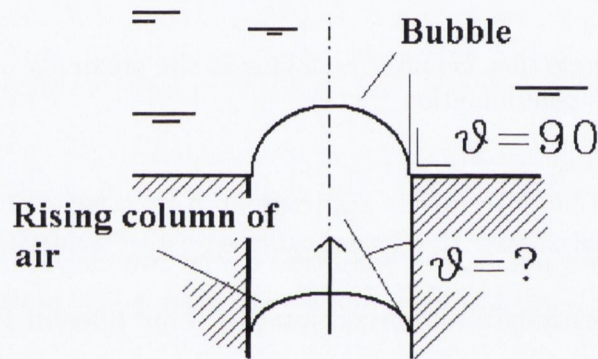


Figure 3.19: Rising bubble column to the orifice mouth.

3.9 Data Analysis

The integral form of the momentum equation on a control volume surrounding the bubble which is bounded by the liquid side of the gas-liquid interface and by the gas side of the bottom dry area is written as;

$$\begin{aligned}
 \frac{d}{dt} \int_V \rho_g \mathbf{v}_g dV &= \int_V \rho_g \mathbf{g} dV + \\
 &+ \int_S [\bar{\mathbf{T}}_{v,l} - p_l \bar{\mathbf{I}} + \bar{\mathbf{T}}_{e,l} + \rho_l \mathbf{v}_l (\mathbf{v}_l - \mathbf{v}_s)] \cdot \mathbf{n} dS + \\
 &+ \int_{S+A} [\bar{\mathbf{T}}_{v,g} - p_g \bar{\mathbf{I}} + \bar{\mathbf{T}}_{e,g} + \rho_g \mathbf{v}_g (\mathbf{v}_g - \mathbf{v}_s)] \cdot \mathbf{n} dS + \\
 &+ \int_{CL} \sigma_{lg} \mathbf{t}_{lg} dL
 \end{aligned} \tag{3.1}$$

where V defines the volume of the bubble, $\mathbf{n}$ and $\mathbf{t}$ are the outward normal and tangent unit vectors respectively, p is the pressure, $\mathbf{u}$ is the velocity, $\mathbf{g}$ is the acceleration due to gravity, $\bar{\mathbf{T}}_v$ and $\bar{\mathbf{T}}_e$ are the viscous and the Maxwell stress tensors and σ_{lg} is the surface tension. The suffixes g and l refer to gas and liquid phases, respectively. The dynamic effects are accounted for in the normal and tangential stress distribution around the bubble, which must be evaluated by taking into consideration the surrounding liquid motion.

In the ensuing analysis of the bubble dynamics several simplifying assumptions have been made in order to quantify the forces. These are listed as follows;

- (a) All of the fluid properties are considered constant and evaluated at room temperature.
- (b) Bubble growth occurs adiabatically and axisymmetrically.

- (c) The momentum associated with the vapour bubble is negligible compared with the magnitude of the external forces.
- (d) The gas pressure is uniform within the bubble and the gas viscous, inertia, electric stress and momentum flux influences are negligible.
- (e) The orifice pressure coincides with the pressure inside the bubble.
- (f) The contact line remains fixed to the rim of the orifice.

The axisymmetry of the problem simplifies the solution of the integrals, reducing the force balance to the vertical component only. Introducing the vertical unit vector $\mathbf{j}$, the last term of the RHS can be expressed as;

$$F_C = \int_{CL} \mathbf{j} \cdot \sigma_{lg} \mathbf{t}_{lg} dL = 2\pi \sigma r_0 \sin \theta \quad (3.2)$$

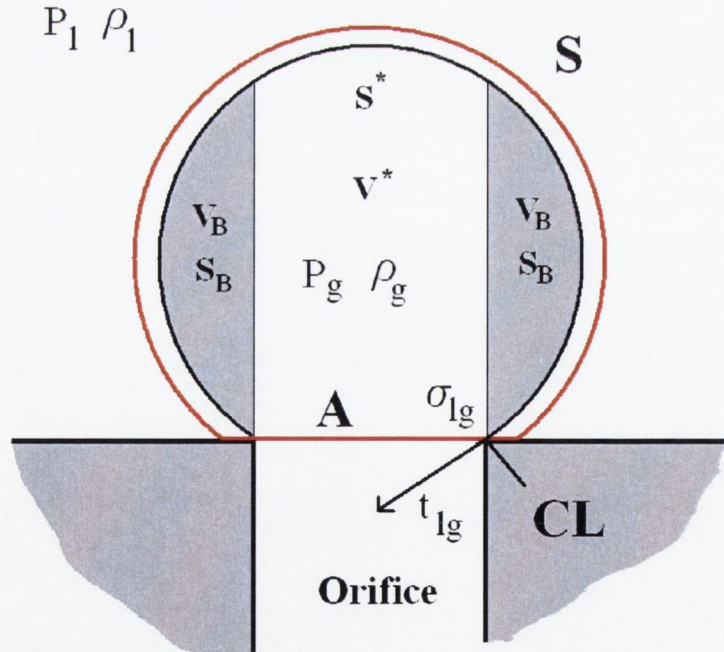


Figure 3.20: Application of momentum balance on a growing bubble

This is the capillary force which is a downwardly directed force that tends to keep the bubble attached to the wall. It arises due to capillary action at the triple interface and is related to the contact perimeter of the bubble and contact angle, θ . Dynamic and static effects of the liquid are included in the normal and tangential stress distribution in the second integral of the RHS of Eq.3.1:

$$\int_S [\bar{\mathbf{T}}_{v,l} - p_l \bar{\mathbf{I}} + \rho_l \mathbf{v}_l (\mathbf{v}_l - \mathbf{v}_s)] \cdot d\mathbf{S} = \int_S p_{ls} \cdot \mathbf{n} dS + \int_S p_{ld} \cdot \mathbf{n} dS \quad (3.3)$$

where p_{ls} and p_{ld} are the static and the dynamic component of the liquid pressure, respectively.

Since the bubble presents also dried zones, the buoyancy force is not calculated considering the entire volume of the bubble but only the portion completely surrounded by liquid (grey area in Fig.3.20). This method differs from the classical one where a body is subject to the Archimedes principle which states that *any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object itself*. The surface area and volume of the bubble can be so divided in two regions: the body completely embedded in the liquid, defined by the grey area in Fig.3.20, and the region above the orifice, defined by the white region. The effect of the static liquid pressure acting on the surface, S , and the weight of the bubble (first term of the RHS of Eq.3.1.) can be written as:

$$\int_S p_{ls} \cdot \mathbf{n} dS = \int_{S_B} p_{ls} \cdot \mathbf{n} dS + \int_{S^*} p_{ls} \cdot \mathbf{n} dS \quad (3.4)$$

$$\int_V \rho_g \cdot \mathbf{g} dV = \int_{V_B} \rho_g \cdot \mathbf{g} dV + \int_{V^*} \rho_g \cdot \mathbf{g} dV \quad (3.5)$$

Where the suffixes B and $*$ refer to the body completely surrounded by liquid and that part which is not, respectively.

The buoyancy force is thus given as;

$$F_B = \int_{V_B} \rho_g \cdot \mathbf{g} dV + \int_{S_B} \mathbf{j} \cdot p_l \mathbf{n} dS = (\rho_l - \rho_g) g V_B \quad (3.6)$$

Where V_B is the volume of the bubble subject to the buoyancy force.

The third term of the RHS of Eq.3.1 accounts for the effect of the internal pressure of the control volume. Due to the homogeneous gas pressure inside the bubble and in the orifice, the net force acting on the dry area A is equal to zero. The vertical net force acting on the surface completely embedded in the liquid is also zero. Under these conditions the internal pressure of the bubble acts only on the top surface S^* generating an upward force which pushes the bubble away from the orifice. On the other side of S^* the liquid pressure interacts with the surface generating a downwardly directed force which opposes the overpressure of the bubble. The overall effect is a vertical force directed upward called the Contact Pressure force:

$$F_{CP} = \int_{S^*} \mathbf{j} \cdot p_g \mathbf{n} dS - \int_{S^*} \mathbf{j} \cdot p_l \mathbf{n} dS \quad (3.7)$$

where the vertical projection of the top surface S^* coincides with the horizontal bottom area A . Moreover, for the axisymmetry condition and for the assumption of the contact line CL coincident with the orifice rim, Eq.3.7 can be simplified to:

$$F_{CP} = \int_A (p_g - p_l) dA = \pi r_0^2 (p_g - p_T) \quad (3.8)$$

where p_T is the liquid pressure at the tip of the bubble and r_0 is the contact radius. Choosing to use p_T in Eq.3.8 simplifies the calculation of the integral

of the liquid pressure acting on the upper region of the bubble.

Finally, the dynamic and electrically induced vertical forces are given by the surface integrals:

$$F_D = \int_S \mathbf{j} \cdot p_{ld} \cdot \mathbf{n} dS \quad (3.9)$$

$$F_E = \int_S \mathbf{j} \cdot \bar{\mathbf{T}}_{e,l} \cdot \mathbf{n} dS = \int_S \mathbf{j} \cdot \bar{\mathbf{E}}_{EHD} dS \quad (3.10)$$

For quasi-static bubble growth the momentum equation simplifies to a balance of the above mentioned forces, such that,

$$\sum F = F_B + F_{CP} + F_C + F_D + F_E = 0 \quad (3.11)$$

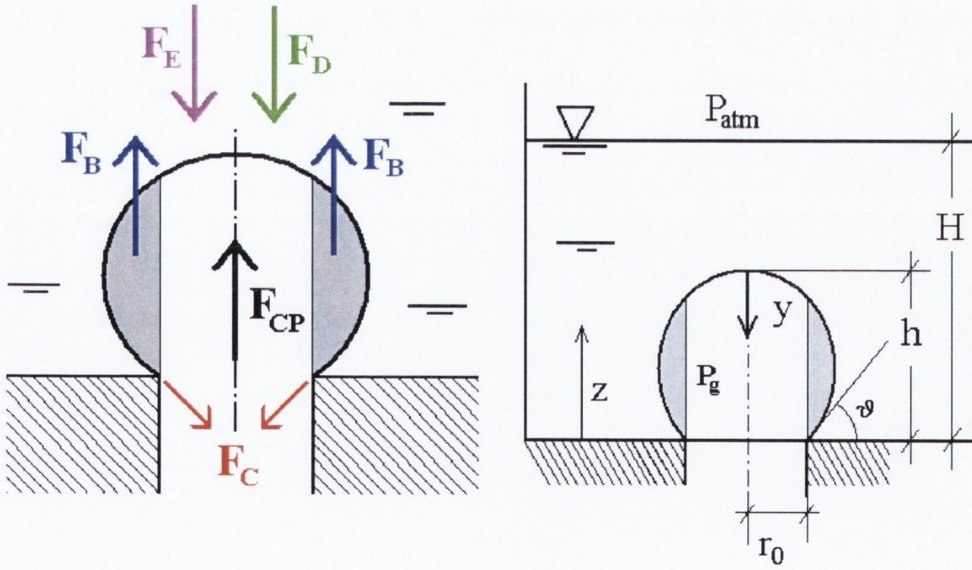


Figure 3.21: Forces field acting on a growing (left) and schematic of gas bubble issuing from an orifice into a liquid pool (right).

The assumption of an axisymmetric bubble facilitates a straightforward estimation of the bubble volume. This is achieved by a numerical integration described by the expression;

$$V_b = L_p^3 \cdot \sum^{N_p} \pi \cdot r_{p,j}^2 \quad (3.12)$$

where $r_{p,j}$ is the pixel to pixel length between pixel j of the bubble region and the symmetry axis, N_P is the number of pixels inside the bubble and L is the pixel to length conversion coefficient which is determined by comparing a measured pixel distance to a known physical length. The vertical position of the centre of gravity was calculated by considering the bubble as a solid of revolution.

A primary aim of this investigation is to combine the measurement of the instantaneous gas pressure with the local curvature of the bubble to estimate the magnitude of the local dynamic and electric pressures. The gas pressure inside the bubble was measured and is given as;

$$P_g = \Delta P_m + P_{atm} \quad (3.13)$$

where ΔP_m is the measured differential pressure and P_{atm} is the atmospheric pressure. The pressure of the liquid at a vertical distance y from the bubble tip is;

$$P_l = P_{atm} + \rho_l g (H - h) + \rho_l g y + P_{ld} \quad (3.14)$$

where the first two terms account for the hydrostatic head between the free liquid surface and the tip of the bubble, (Fig.3.21). By substituting Eq.3.13 and Eq.3.14 into the Young-Laplace equation in the presence of an electric field (Eq.2.33) one obtains:

$$\Delta P_m + P_{eg} = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) + \rho_l g (H - h) + \rho_l g y + P_{el} + P_{ld} \quad (3.15)$$

In Eq.3.15 the electric pressure of the liquid and gas can be grouped together by a common term and the sum of the dynamic liquid pressure and net electric pressure term can be so estimated as follows:

$$P_d + P_e = \Delta P_m - \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) - \rho_l g (H - h) - \rho_l g y \quad (3.16)$$

In absence of an electric field, Eq.3.16 gives an estimation of the liquid dynamic pressure acting on the bubble interface. For low gas injection rates where the bubble is expected to grow quasi-statically the dynamic pressure is expected to be negligible compared with the other pressure components. A significant deviation from $P_d \sim 0$ Pa could suggest the presence of a dynamic effect on the definite area of the bubble interface.

When the bubble grows in an electric field the electric and dynamic pressures are impossible to determine independently. However, for low gas injection rates where the bubble growth is quasi-static, the dynamic force may be negligibly small and Eq.3.16 can be used to determine the electric pressure. To the author's knowledge, this technique for experimentally determining the electrically induced pressure field is the first of its kind as others have relied on numerical techniques, Chen et al. (2007) or Cattide et al. (2007).

CHAPTER 4

NUMERICAL MODELLING

4.1 TransAT

Fluid mechanics is governed by a set of equations which define the conservation of mass, momentum and energy. These equations are *non-linear partial differential equations* for which analytic solutions can be realized for only very simple scenarios. The non-linearity makes most problems difficult or impossible to solve. The evolution of a bubble is not one that, at present, can be solved analytically. Even when one describes the evolution of the bubble interface under simplified quasi- static conditions, the system of differential equations is still impossible to solve. As a result, solution methods must be deployed to simulate the bubble growth dynamics. Even though the solutions are an approximation of reality, with valid simplifying assumptions, accurate numerical techniques and validation against experiments, a numerical simulation can give results very close to

reality. Moreover, such numerical treatment of the problem allows the investigator to study the influence parameters which are difficult to study experimentally; an obvious one being reduced gravity.

In the present work, the full set of Navier-Stokes equations, which describe the dynamic of bubble growth, have been solved by the finite-volume method package TransAT (Transport Phenomena Analysis Tools). This code has been developed by ASCOMP GmbH, a technology development company founded in 2003 specialized in the simulation of industrial fluid dynamics and heat and mass transfer.

4.1.1 Discretization of Governing Equations

In this section the principal governing equations, simplifying assumptions and numerical treatment of the problem are outlined

The liquid and gas phases are both treated as incompressible. Thus, as discussed in Chapter 2 the conservation of mass and momentum can thus be expressed as,

$$\nabla \mathbf{v} = 0 \quad (4.1)$$

$$\rho \frac{D\mathbf{v}}{Dt} - \rho \mathbf{g} + \nabla p + \nabla \bar{\mathbf{T}} = 0 \quad (4.2)$$

Moreover, for quasi-static bubble growth the flow is laminar such that momentum conservation can be expressed as in Eq.2.10:

$$\rho \frac{D\mathbf{v}}{Dt} = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{v} \quad (4.3)$$

In the numerical treatment of the problem the partial differential equations are converted in to a system of ordinary differential equations which

are then numerically integrated in a discrete domain.

The physical domain is divided into a finite number of quadrilateral cell elements, as shown in Fig.4.1. Each cell represents a computational node on its geometric centre, while the grid lines define its face. A further computational node is placed on the faces and corners of the cells which define the boundaries of the computational grid.

Fig.4.1 shows a two dimension control volume centred at node P. As it is possible to see, the nodes that do not belong to the boundaries are surrounded by four other computational nodes and faces.

In absence of internal sources or sinks, the flux balance of a variable on a computational cell centred at P is given by;

$$I_e - I_w + I_n - I_s = 0 \quad (4.4)$$

where I represents the total flux of a variable across the cell face and the suffixes e, w, n and s refer to the faces east, west north and south, respectively.

The terms defining the total flux have two components, one being convective I_f^C , the other being diffusive I_f^D ;

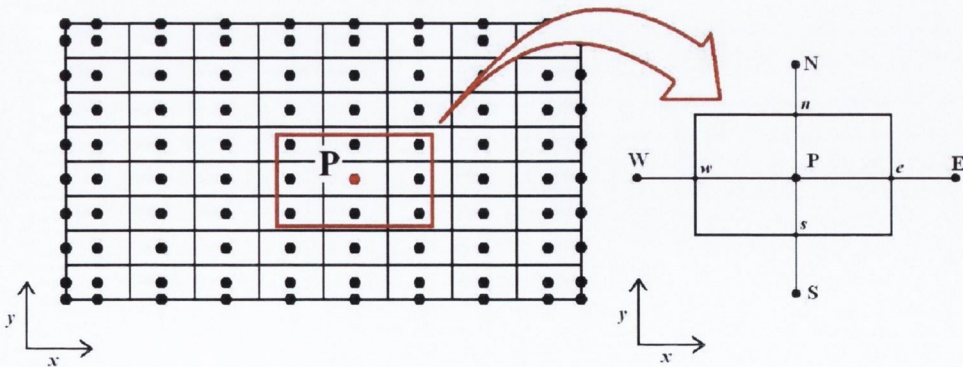


Figure 4.1: Grid definition (left) and control volume centred at node P (right).

$$I_f = I_f^C + I_f^D \quad (4.5)$$

where f refers to the face. The first term on the RHS of Eq. 4.5 can be written as:

$$I_f^C = C_f \varphi_f \quad (4.6)$$

where the term C_f represents the mass flux across the cell face f while φ_f defines the face value of the variable φ across the face. The determination of this value is very important for the stability and the accuracy of the numerical solution. The code presents a list of different solution methods and for the present work two schemes were adopted: the third order unbounded **QUICK**, Leonard (1979), for the interface tracking and the second order bounded **HPLA**, Zhu (1991), for the velocity and density advection.

Using the **QUICK** approach the value of φ_f is approximated by fitting a parabolic curve through the three nodal values P, W and WW , whose positions on the computational domain are shown in Fig. 4.2. A face-value such as φ_w is so given by the following expression:

$$\varphi_w = \frac{3}{8}\varphi_P + \frac{3}{4}\varphi_W - \frac{1}{8}\varphi_{WW} \quad (4.7)$$

The **HPLA** scheme can be written as:

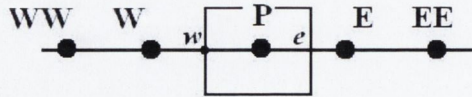


Figure 4.2: Nodes definition for a cell centred in P

$$\varphi_w = \varphi_W + \gamma_w (\varphi_P - \varphi_W) \frac{\varphi_W - \varphi_{WW}}{\varphi_P - \varphi_{WW}} \quad (4.8)$$

where the term γ_w is equal to:

$$\gamma_w = \begin{cases} 1 & \text{if } |\hat{\varphi}_W - 0.5| < 0.5 \\ 0 & \text{otherwise} \end{cases} \quad (4.9)$$

and where;

$$\hat{\varphi}_W = (\varphi_W - \varphi_{WW}) / (\varphi_P - \varphi_{WW}) \quad (4.10)$$

The diffusive contribution I_f^D can be split in two parts:

$$I_f^D = I_f^{DN} + I_f^{DC} \quad (4.11)$$

The first term on the RHS contains only one term which has the first derivate of the variable in the normal direction of the generic cell face f and the second part contains all the other terms.

Replacing all the terms in Eq. 4.4 with their discrete equivalent one obtains:

$$A_p \varphi_P = \sum_{nb} A_{nb} \varphi_{nb} \quad nb = W, E, S, N \quad (4.12)$$

the terms A_p and A_{nb} are related through:

$$A_P = \sum_{nb} A_{nb} \quad (4.13)$$

where the coefficient A_{nb} that relates the unknown variable, φ_P , to its neighbouring nodes, φ_{nb} , contains the contribution from convection and normal diffusion terms.

4.1.2 Problem Formulation and Data Processing

Fig. 3 shows a schematic of an injected gas bubble growing from an orifice into a quiescent liquid with a representation of the grid adopted. The growth from two different orifice diameters, 1.6 and 1.0 mm, has been solved numerically and compared with the experimental results for flow rates of 10, 20, 50 and 100 mlph.

The effect of gravity has also been analysed by changing the magnitude of the gravity acceleration. In particular two different conditions have been reproduced numerically, 0.1, 0.5, 1 and 1.5 g, for the 1 mm orifice diameter at the highest flow rate. The results obtained have then been compared with the normal gravity results in order to study the influence of the gravity on the bubble growth.

Several assumptions have been made to simplify the analysis and reduce the computational cost of the simulation without having a significant impact on the underlying physical phenomena. These are listed below:

- (a) Bubble growth occurs adiabatically;
- (b) The bubble remains axis-symmetric;
- (c) The inlet flow rate is constant with time;
- (d) All fluid properties are constant and evaluated at room temperature;
- (e) The pressure at the outflow boundary is constant with time.

A structured mesh grid of 160X110 cell nodes has been used to define the experimental domain. The relevant boundary conditions are also shown in Fig. 4.3.

The injected air from the orifice has been represented with an inflow boundary condition using a laminar velocity profile which is constant with time.

The effect of the extra column of fluid not represented in the grid has been included with a static pressure value given by:

$$P_{out} = \rho_L g (H - H_m) = \rho_L g \Delta H \quad (4.14)$$

and applied at the outflow boundary condition.

The corner of the orifice is represented in the simulations as a sharp edge. This can create an issue for the calculation of the contact line in the proximity of the sharp corner which causes the contact line to move away from the orifice. Under this condition, the best approach to define the triple contact line is by applying a thin film boundary condition on the bottom wall so the contact line doesn't touch the wall and remains pinned at the sharp corner. The viscosity at the thin film boundary wall is set in order to satisfy the continuity of the shear stress on the interface:

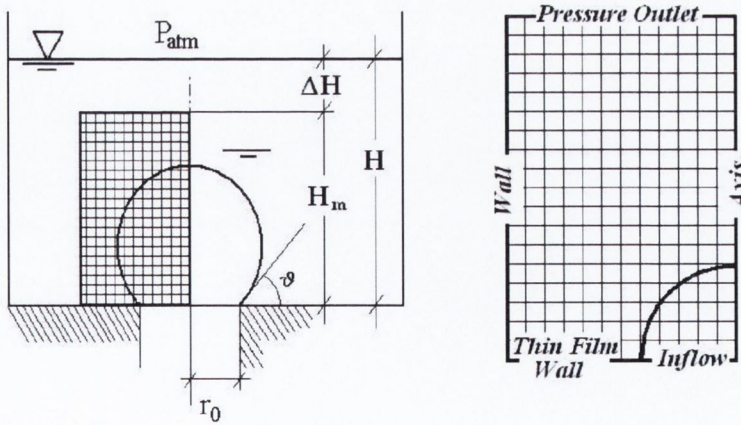


Figure 4.3: Schematic of gas bubble issuing from an orifice (left) and grid with boundaries condition adopted (right).

$$\tau_{GI} = \tau_{LI} \Rightarrow \mu_G \frac{(u_P - u_I)}{\delta_G} = \mu_L \frac{(u_I - u_W)}{\delta_L} \quad (4.15)$$

Fig. 4.4 represents a volume of fluid in proximity to the wall; it is clear how the film thickness should be smaller than the distance between the wall and the first grid node. From Eq. 4.15 it is possible to derive the velocity of the interface u_I :

$$u_I = \frac{u_P \frac{\mu_G}{\delta_G} + u_W \frac{\mu_L}{\delta_L}}{\frac{\mu_G}{\delta_G} + \frac{\mu_L}{\delta_L}} \quad (4.16)$$

An effective viscosity is introduced to satisfy the following equation:

$$(u_P - u_W) \frac{\mu_{eff}}{\delta_G - \delta_L} = (u_I - u_W) \frac{\mu_L}{\delta_L} \quad (4.17)$$

which gives the value of the effective viscosity:

$$\mu_{eff} = \frac{\mu_L \mu_G (\delta_G + \delta_L)}{\mu_L \delta_G + \mu_G \delta_L} \quad (4.18)$$

The inherent difficulty in simulating the physics of the triple contact line behaviour at the sharp corner made it impossible to study the bubble interface beneath the orifice rim.

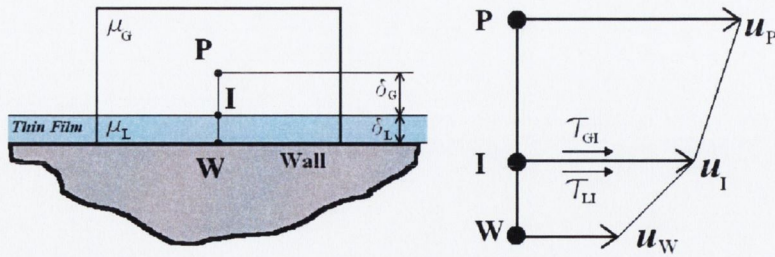


Figure 4.4: Sketch of the thin film boundary condition (left) and definition of the nodal velocities (right).

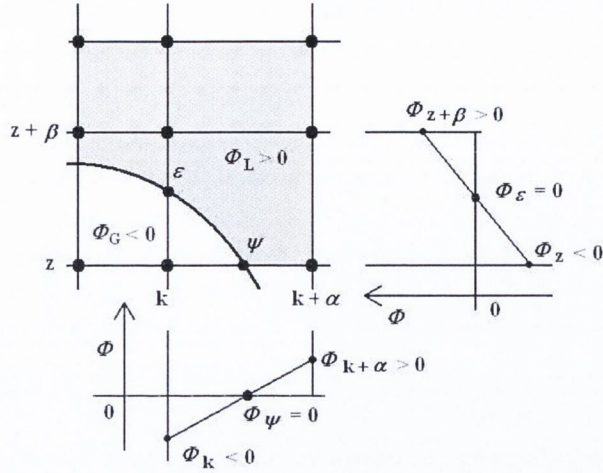


Figure 4.5: Interface coordinates definitions.

This being the case, the initial shape of the bubble was assumed to be hemispherical, as illustrated in Fig: 4.3. It should be stressed that this assumption will not be valid for situations in which the volumetric flow rate is high enough that the gas and/or liquid inertia influence the bubble dynamics.

In the CFD package TransAT the distinction between the phases is made through a variable ϕ which defines the value of the phase on each node. This is set to be negative for the less dense phase and positive for the denser one. A simple user routine was written in order to define the gas/liquid interface coordinates together with the internal pressure of the bubble and the physical time associated with the relevant time step. The routine detects the nodal value of the phase through two simple loops along the horizontal and vertical direction and defines the transition from one phase to the other when a change of sign is detected, see Fig.4.5. A linear interpolation between two subsequent nodes with opposite signs is then performed in order to calculate the coordinate of the interface where the phase becomes null. Once the coordinates of the interface have been defined they are processed

in MATLAB using the same code developed to analyse the experimental results. This facilitated the comparison between the numerical solutions and the experimental results which will be discussed in a later chapter.

4.2 COMSOL

The calculation of the electric stresses acting on the bubble surface is complicated by the strong inter-relationship between the bubble deformation and the deformation of the electric field. The integration of Eq.2.33 requires a coupled solution which, to the author's knowledge, still has to be found. A numerical approach can also be adopted in order to analyse the interaction between the bubble shape and an electric field and calculate the electric stresses.

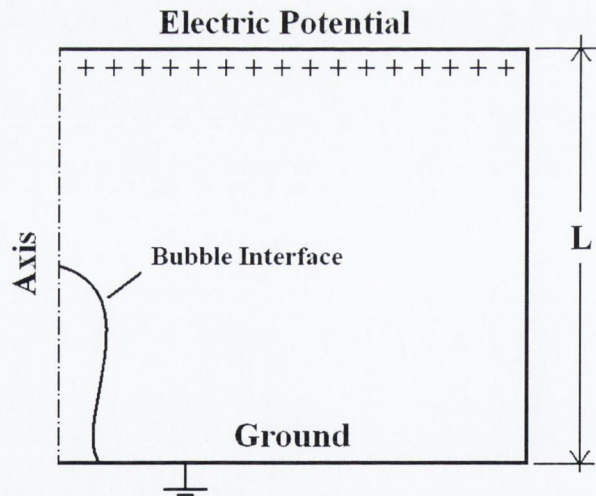


Figure 4.6: Schematic of the numerical approach.

In the present work the coordinates of the bubble interface have been exported into a finite element software package, *COMSOL Multiphysics*. A simple set of simulations have been set up in order to represent the

experimental conditions (Fig.4.6). A constant electric potential value has been applied on the top edge of the numerical domain in order to reproduce the effect of the charged electrode. A ground boundary condition ($V = 0$) was imposed on the lower surface. The software package then calculates the electric field distribution by solving the Laplace equation for the voltage field over the entire domain:

$$\nabla^2 U = 0 \quad (4.19)$$

and then differentiating Eq.4.19 to obtain

$$E = -\nabla U \quad (4.20)$$

where E is the electric field distribution.

The post processor utility allowed the user to evaluate the magnitude of the electrohydrodynamic stress acting on the bubble interface and compare the numerical results with the ones evaluated experimentally in order to validate the two methodologies proposed.

CHAPTER 5

RESULTS AND DISCUSSION: WATER

This chapter focuses on the hydrodynamic of an adiabatic gas bubble growing from submerged orifices in a basin partially filled with pure degassed water. The discussion of the results has been divided in three main sections. The first section analyses the dynamic of a bubble growing from an orifice of 1.6 mm diameter with a flow rate of 100 mlph with the view of elucidating the evolution of the geometric and dynamic parameters. The influence of the air injection rate is discussed in the second section for the same orifice diameter as above and for flow rates of 10, 20, 50 and 100 mlph. In the final section the influence of orifice size is discussed.

Bubble Growth Dynamic

Bubble Evolution

Fig.5.1 shows a photographic sequence of a bubble growing from the largest orifice tested for the highest air injection rate. From Fig.5.2, which represents the variation of the volume with time, is it possible to see how the entire growth occurs with a fairly constant flow rate.

The bubble evolution consists of three principal stages as is clear in Fig.5.3 which is the time history of the centre of gravity.

During the early stage, ($t < 0.1s$), the bubble is emerging from the orifice

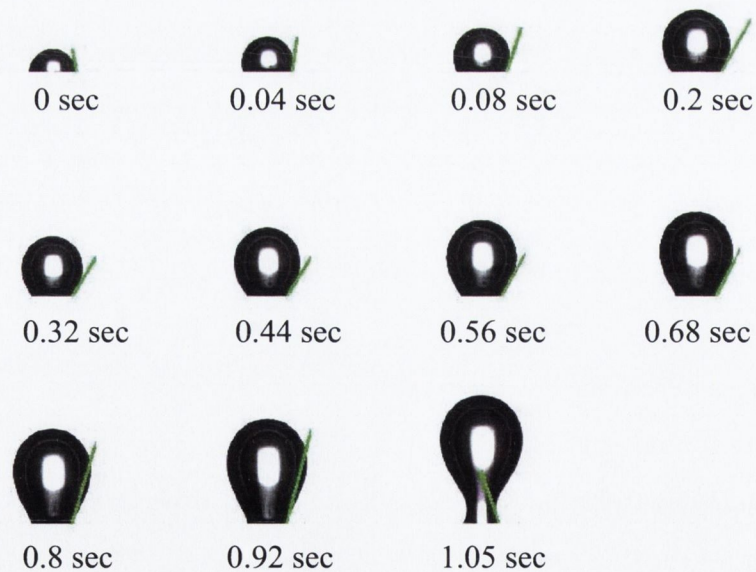


Figure 5.1: Video Sequence of bubble growth at 100 mlph for 1.6 mm orifice diameter.

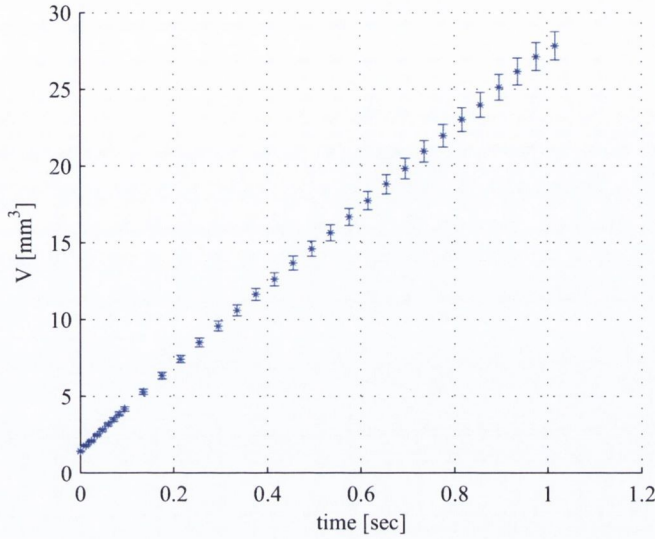


Figure 5.2: Volume of the bubble at 100 mlph for 1.6 mm orifice diameter.

and its shape is that of a nearly a perfect truncated sphere.

Although the volumetric flow rate is constant, the evolution of the vertical position of the centre of gravity, as reported in Fig.5.3, shows that it is decelerating.

As the bubble volume increases, a larger portion of the bubble is acted upon by buoyancy. When this portion becomes large enough the bubble becomes progressively more elongated as the force tends to lift the upper portion of the bubble with the bubble foot remaining fixed to the orifice mouth. During this middle stage ($0.1s < t < 0.95s$) the centre of gravity increases linearly with time as shown in Fig.5.3. The final stage ($t > 0.95$) coincides with the departure of the bubble from the orifice by forming a neck near the bubble base. Now, the influence of buoyancy is enhanced as the curving inward of the neck increases the volume being acted on by buoyancy. This causes the upper region to accelerate upward, as depicted in Fig.5.3, where it is noticed that the centre of gravity accelerates upward

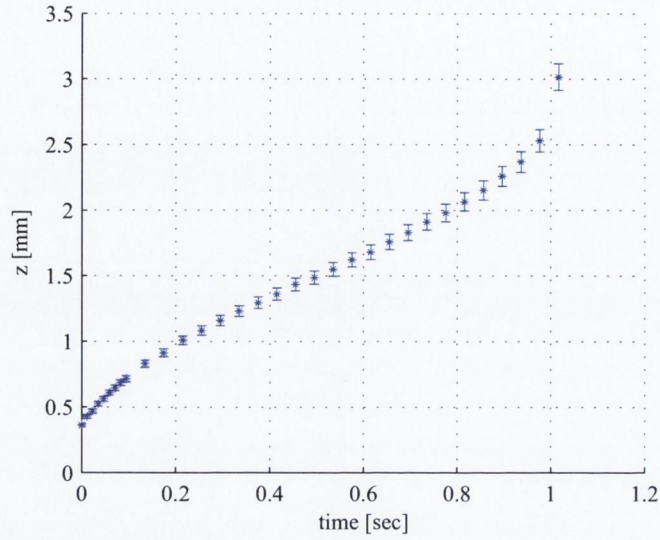


Figure 5.3: Motion of the centre of gravity at 100 mlph for 1.6 mm orifice diameter

prior to departure. The acceleration is caused by what is essentially a positive feedback effect whereby the necking of the bubble is responsible for increasing the rate of the necking processes by increasing the magnitude of the buoyancy force.

When the minimum diameter of the neck is comparable with the size of the molecules of the liquid the neck pinches completely and the bubble detaches and rises upward in the bulk liquid. Each frame in Fig.5.1 also has included a line drawn tangent to the bubble on its base with the view showing the value of the contact angle during bubble formation.

The time evolution of the contact angle is given in Fig.5.4. From this figure is it possible to clearly identify the three stages of the bubble growth defined by a different behaviour of the contact angle. As the bubble emerges from the orifice the contact angle decreases rapidly and changes from an obtuse angle to an acute angle. Since at the beginning of its growth the bubble has a hemispherical shape, with a contact angle equal to 90° , for the

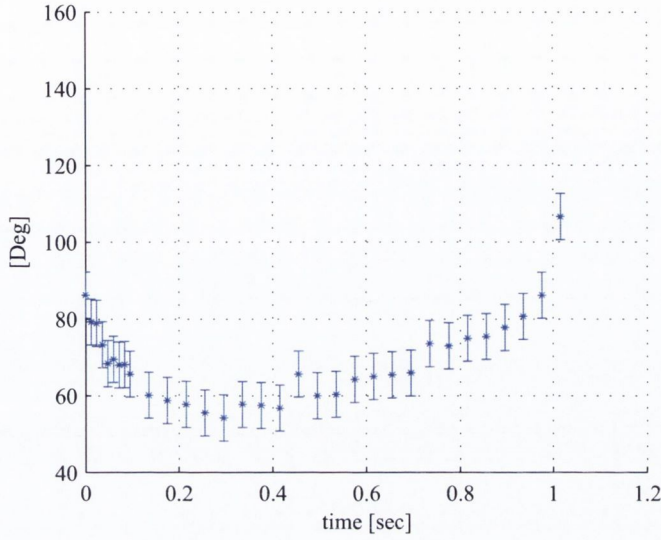


Figure 5.4: Contact angle evolution at 100 mlph for 1.6 mm orifice diameter.

approach adopted in this work (see Chap: 2) the bubble is not affected by the Buoyancy effect. Beyond 90° the Buoyancy force increases in magnitude until the contact angle reaches a minimum constant value of approximately 60° . Once the action of buoyancy starts to elongate the bubble, the contact angle starts to increase. When the contact angle reaches a right angle the bubble departure phase commences. Here, the neck forms and the contact angle increases suddenly and once again becomes obtuse as it begins to pinch off, indicating bubble break-off.

The differential pressure transducer facilitates the measurement of the gas pressure within the bubble while it is growing. The differential pressure history is given in Fig.5.5. In agreement with the Young-Laplace equation, the maximum pressure value corresponds with the minimum radius which occurs when the bubble is hemispherical with its radius of curvature equal to the orifice radius. By measuring the difference of pressure between the bubble and the pressure of the liquid it has been possible to calculate the

value of the surface tension given by the Young-Laplace equation:

$$\sigma = \frac{\Delta P \cdot R}{2} = \frac{179.42 \cdot 0.8}{2000} = 0.0718 N/m \quad (5.1)$$

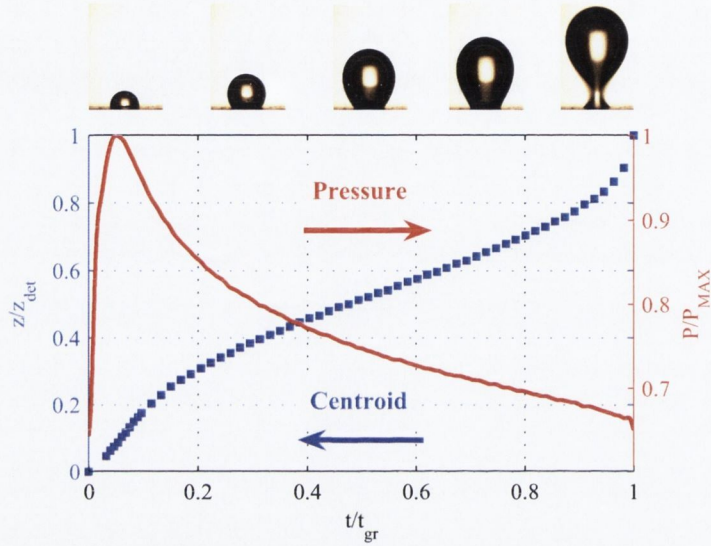


Figure 5.5: Normalized position of the centre of gravity position and differential pressure at 100 mlph for 1.6 mm orifice diameter.

This calculated value is within 1.6% of the tabulated value for water ($\sim \sigma_{H_2O} = 0.073 N/m$) which partially validates the calibrated measurement accuracy.

Fig.5.5 shows that as the bubble expands pasts the hemispherical shape the equivalent radius of the bubble increases and thus the pressure difference decreases monotonically to departure.

Forces Acting on a Growing Bubble

Once the geometric characteristics of the bubble are known it is possible to define all the vertical forces acting on the bubble and also calculate their resultant force. The evolution of these forces is given in Fig.5.6. The

upwardly directed contact pressure force, F_{CP} , due to its dependency on the differential pressure, has its maximum near the beginning of the bubble growth when it is hemispherical and the pressure difference is maximum (Fig.5.5). Subsequent to this point in time this force monotonically decreases, generally following the trend of the pressure evolution.

As to be expected, the upwardly directed buoyancy force, F_B , is equal to zero for approximately $t < 0.1s$ as there is no liquid underneath the bubble until the contact angle drops below 90° and the bubble resembles a truncated sphere. From this point onward the upward buoyant force acts on the region of the bubble that exists radially outward from the triple contact line and increases at a nearly constant rate. It is clear that as the bubble expands and grows the buoyancy force changes from having a negligible influence during early spherical growth to a predominant one during the bubble elongation and departure phases. At the end of the growth it becomes of the same order of the contact pressure and becomes the main upward force.

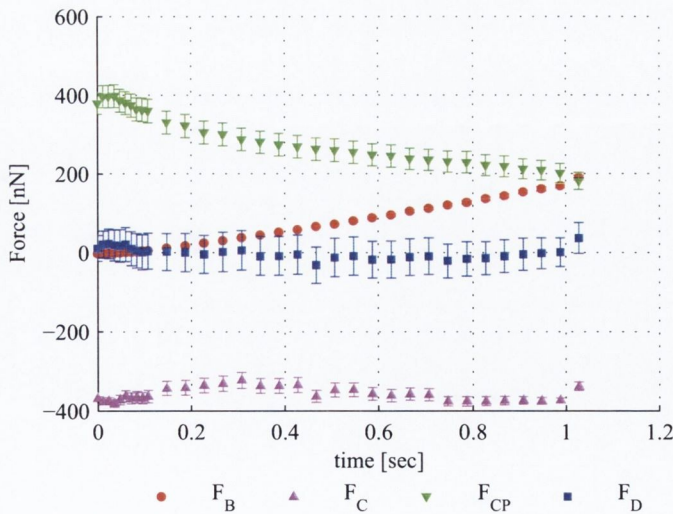


Figure 5.6: Vertical forces acting on the bubble at 100 mlph for 1.6 mm orifice diameter.

The capillary force, F_C , is downwardly directed and acts to keep the bubble fixed to the wall. It has its absolute maximum when the bubble is hemispherical in shape as the sine function is unity. Its magnitude is not however characterized by major transitions during the different growth stages since the minimum contact angle reached here is about of 60° and $\sin 60^\circ$ is only 13% smaller than unity.

The dynamic force, F_D , is calculated from the resulting force balance, Eq.3.11. Due to error propagation, the uncertainty associated with the dynamic force is the highest compared with others. With this in mind, Fig.5.6 reveals the quasi- static nature of the bubble growth as the dynamic influences remain essentially close to zero during the entire lifetime of the bubble.

Bubble Curvature and Local Pressure

The image processing facilitated the calculation of the values of the two radii of curvature along the entire bubble interface. For convenience, positive curvatures correspond with the situation in which the centre of the circle, which defines the principle radius, is situated at the inner side of the bubble.

Fig.5.7 shows the distribution of the inverse of the principle radii, $1/R_1$ and $1/R_2$, with dimensionless distance y/h where the vertical coordinate, y , has its origin at the apex of the bubble and increases positively in the direction of gravity, and h is the instantaneous bubble height. A positive curvature corresponds with the situation in which the centre of the circle which defines the principle radius is situated to the gas side of the interface. Further to this it is worth clarifying that R_1 is the local radius with respect to a vertical plane cutting the bubble along its vertical axis, whilst R_2 lies on the orthogonal plane to that of R_1 .

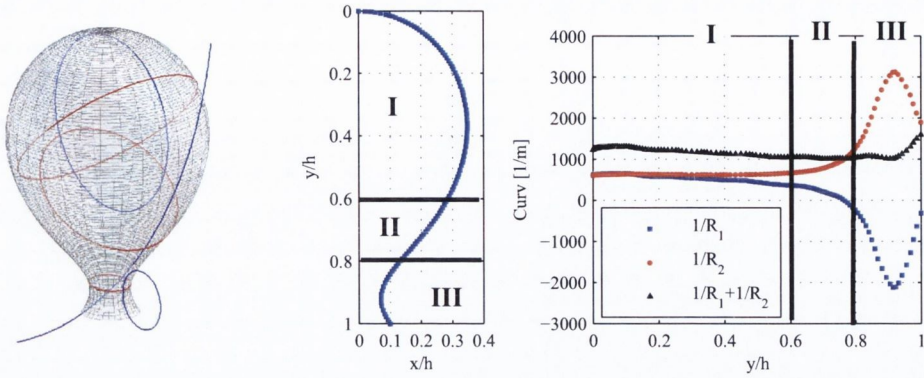


Figure 5.7: Vertical shape profile and curvature characteristics of a bubble near departure.

Depending on the growth stage it is possible to divide the bubble into three distinct sections. This is exemplified in Fig.5.7 for a bubble near departure where the neck has formed near the bubble base. A top surface, Region I, which for this instance is approximately in the range $0 \leq y/h \leq 0.6$, is generally the largest section of the bubble. It has an elongated spherical shape in which R_2 remains approximately constant. The magnitude of R_1 is close to that of R_2 so it can be said that Region I is defined by a nearly spherical segment. Even still, R_1 tends to increase slightly in this region causing the curvature to decrease as well. The net effect is that Region I exhibits a moderate decrease in the overall curvature which is of course related to a decrease in the Laplace pressure drop across the interface.

Region II can be considered a transition region between the upper spherical dome (Region I) and the bubble neck (Region III). It is characterized by concaveness with respect to R_1 and a notable drop in R_2 as the bubble begins to pinch-off near the neck. In this section the term $1/R_1$ drops to zero as the shape changes from convex to concave. Since the two curvatures act in opposite directions they have opposing influences on the Laplace pressure drop which tends to remain constant.

Region III only appears near bubble departure where the neck is formed and bubble pinch-off is occurring i.e. when the centre of gravity is accelerating. It is important to note that due to the diminishing size and rapid motion of the necking phenomena, the surface reconstruction in this region tended to be more influenced by the image processing quality. Even still, it is evident that $1/R_1$ and $1/R_2$ reach local minimum and maximum values respectively. In the first part of this region the Laplace pressure drop remains somewhat constant due to the offsetting effects of the two principal radii of curvature while is subject to a visible increase in the proximity of the bubble base.

The evolution of the curvature and the effects on the pressure field on the bubble interfaces are plotted in Fig.5.8 for three different dimensionless times equal to $t/t_{gr} \sim 0.1, 0.5$ and 1 . These are chosen to be indicative of the different stages of bubble growth. The sign convention chosen is that positive values correspond with pressure directed external to the bubble (i.e. bubble pressure) while negative values correspond with pressure directed internally (i.e. liquid pressure).

During the earlier stage, the influence of the gravity is not large enough to force the bubble to deviate visibly from the spherical shape as is evident from the two curvature values being very close in magnitude. The term $1/R_1$ tends to decrease slightly while $1/R_2$ remains essentially constant; the sum of the two terms is so subject to a small yet notable constant decrease.

For a mid-growth bubble the bubble remains essentially spherical in the region of the bubble apex. R_1 starts to increase with a higher rate making its relative curvature decrease. Approaching the triple contact line, the drop in $1/R_1$ becomes more pronounced while $1/R_2$ starts to visibly increase. The two effects have an opposite influence and the evolution of their sum still

maintains a globally linear-like trend. For the late stage the curvature profiles depicted in Fig.5.7 have already been discussed.

At a given location of the surface the pressures must satisfy

$$\Delta P_m = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) + [\rho_l g (H - h) + \rho_l g y] + P_d = P_{YP} + P_l + P_D \quad (5.2)$$

The quasi-static nature of this test case is confirmed by the negligible value of the dynamic pressure acting on the bubble interface for all the three non dimensional times analysed. The linear decrease in the magnitude of the Young-Laplace drop, P_{YP} , is in fact balanced by the linear increase of the magnitude of the liquid pressure. For $t/t_d = 0.1$ the curvature is maximum so that P_{YP} is close to the liquid pressure P_l . This being the case, ΔP_m , which is representative of the gas pressure, is maximum. As the bubble expands the increased volume incurs a drop in the overall curvature with a corresponding drop in the gas pressure. The liquid pressure changes somewhat as the bubble height and associated hydrostatic head changes.

As mentioned, the dynamic pressure, P_D tends to be negligible over the lifetime of the bubble. However, a possible exception is notable in the proximity of the neck at $t/t_d \sim 1$ where the dynamic pressure is subject to a possible sharp increase due to the sharp constriction of the neck at departure. However, this is sensitive to the image quality which tends to diminish during the departure event. Even still, there is a plausible explanation for this observation. During a very short interval the concave radius R_2 decreases whereas the convex radius R_1 increases having a net effect of increasing the Laplace pressure. Assuming the liquid in the nearby region was nearly stagnant, the collapsing neck accelerates liquid inwards; the dynamic reaction is so orientated in the opposite direction and tends to decelerate the flowing

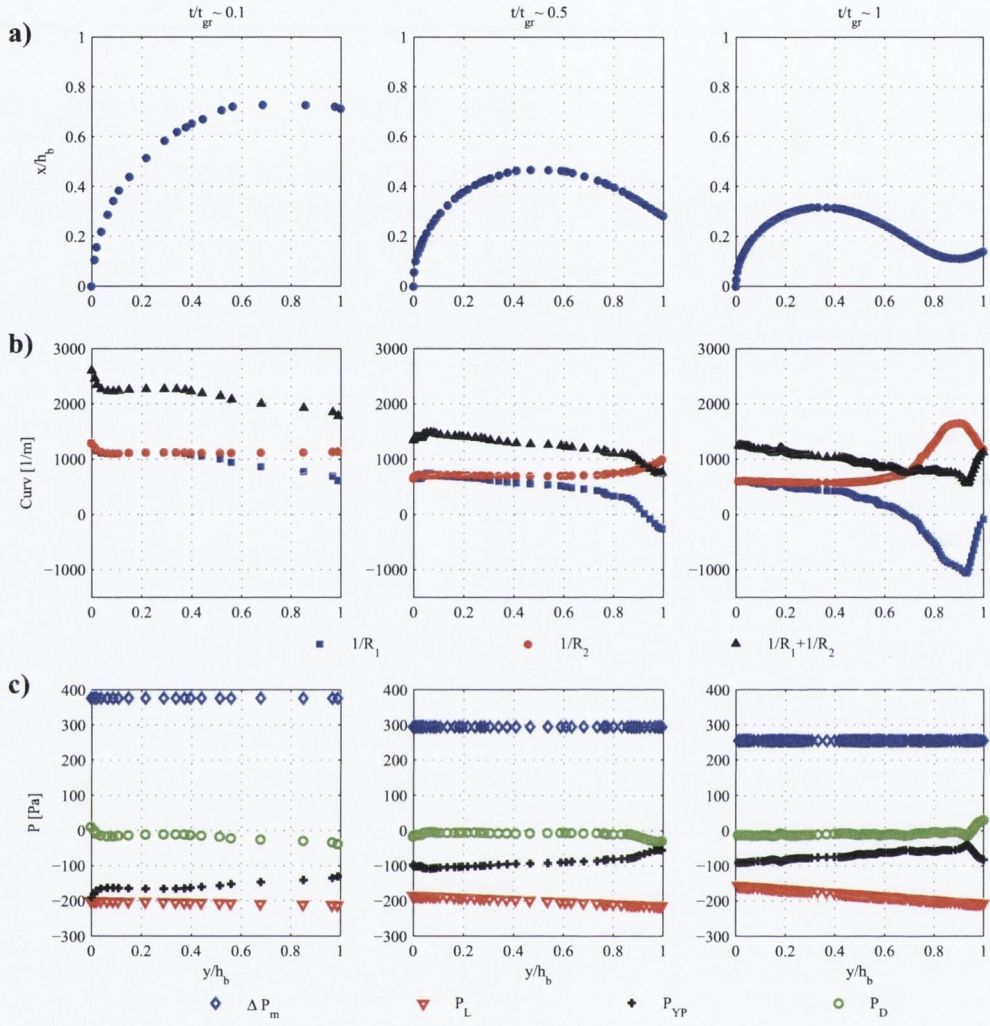


Figure 5.8: Bubble shape (a), curvature (b) and local pressure (c) evolution for different dimensionless growth time bubble at 100 mlph for 1.6 mm orifice diameter.

liquid in that vicinity.

5.2 Effect of Volumetric Flow Rate

The effect of the flow rate on the position of the centre of gravity and on the growth rate for the largest orifice tested is evident in Fig.5.9. The use of a non- dimensional scale facilitated the comparison between the different cases. In the graphs the error-bars have not been included to facilitate the comparison.

As expected the increase of the bubble volume is linear for all the cases. It is also evident that the measured bubble detachment volume does not depend strongly on the volumetric growth rate as has been observed in previous investigations, Duhar and Colin (2006) or Siedel et al. (2008). The reason

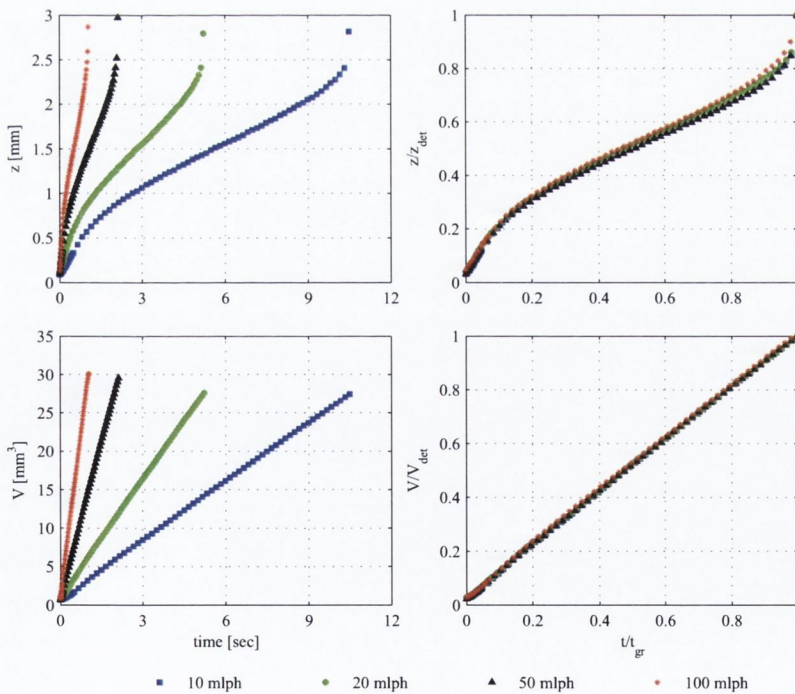


Figure 5.9: Dimensional and non-dimensional plots of the centre of gravity (top) and volumetric growth rate (bottom).

for this is that the volumetric flow rates of this study are below the critical value roughly defined as, Oguz and Prosperetti (1993);

$$Q_{cr} = \pi \left(\frac{16}{3g^2} \right)^{\frac{1}{6}} \left(\frac{\sigma r_0}{\rho} \right)^{\frac{5}{6}} \quad (5.3)$$

As depicted in Fig.5.10 the flow rates tested are well below $Q/Q_{cr} = 1$ indicating that all tests can be considered quasi-static. The bubbles depart with a volume slightly smaller than V_F , defined as the volume at which Buoyancy and surface tension are in balance, Oguz and Prosperetti (1993), and defined as:

$$V_F = \frac{2\pi r_0 \sigma}{\rho_l g} \quad (5.4)$$

where r_0 is the needle radius.

The higher value of V_F respective to the experimental observation arises from the absence of the Contact Pressure component in Eq:5.4 which

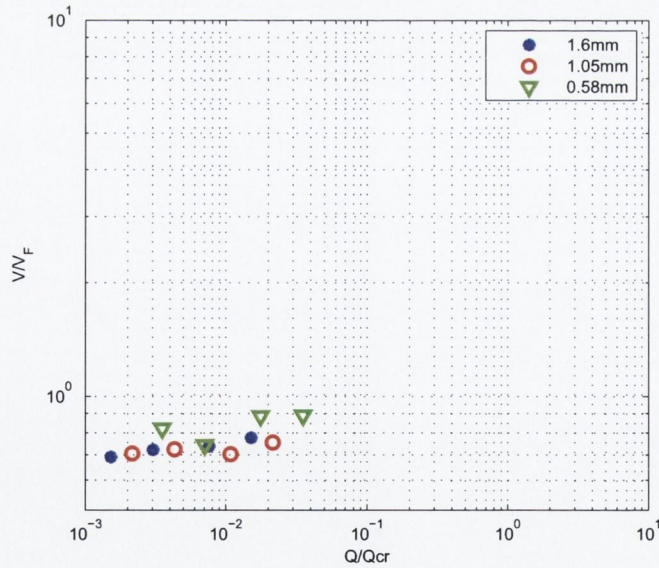


Figure 5.10: Normalized bubble detachment volume versus gas volumetric flow rate normalized respect the critical flow rate Q_{cr}

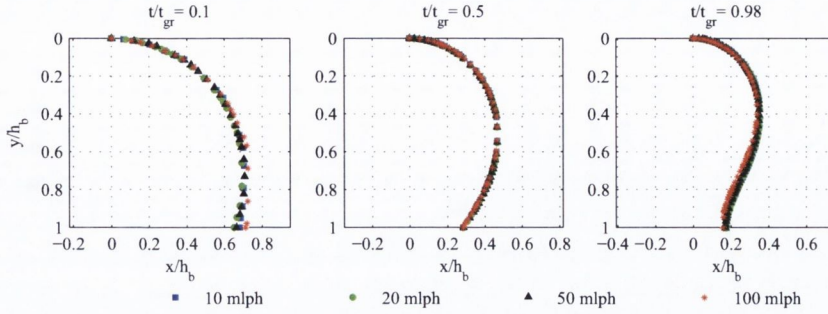


Figure 5.11: Comparison of bubble shape at different dimensionless times.

promotes the departure of the bubble. This is more evident for a larger diameter where the contact area is bigger and therefore so is the Contact Pressure force. Bubbles emerging from smaller orifices assume a more spherical shape (see next section). This decreases the Contact Pressure influence on the overall force balance while increasing the Buoyancy effect. Under these conditions Eq:5.4 results to be more accurate and the bubble departs with a volume closer to V_F .

With regard to the bubble's centre of gravity for varying gas injection rates a similar observation is evident. From the dimensional plots it is observed that each curve has the three growth regions discussed earlier though the lower gas flow rates result in longer bubble growth times. However, the centre of gravity at departure is nearly constant such that the dimensionless plots collapse onto a single curve indicating geometrical similarity of the bubble which is independent of the gas injection rate for the range of rates studied in this investigation. This is in fact confirmed in Fig.5.11 where the bubble profiles for early, mid and late growth are plotted for each gas injection rate.

5.3 Effect of the Orifice Diameter

The experimental apparatus facilitated the interchanging of the cylinder bases with different orifice dimensions in order to study their influence on the bubble growth. Three different orifice diameters of 0.58, 1 and 1.6 mm have been tested. Unlike the flow rate, the size of the orifice has a strong influence on bubble growth and departure characteristics.

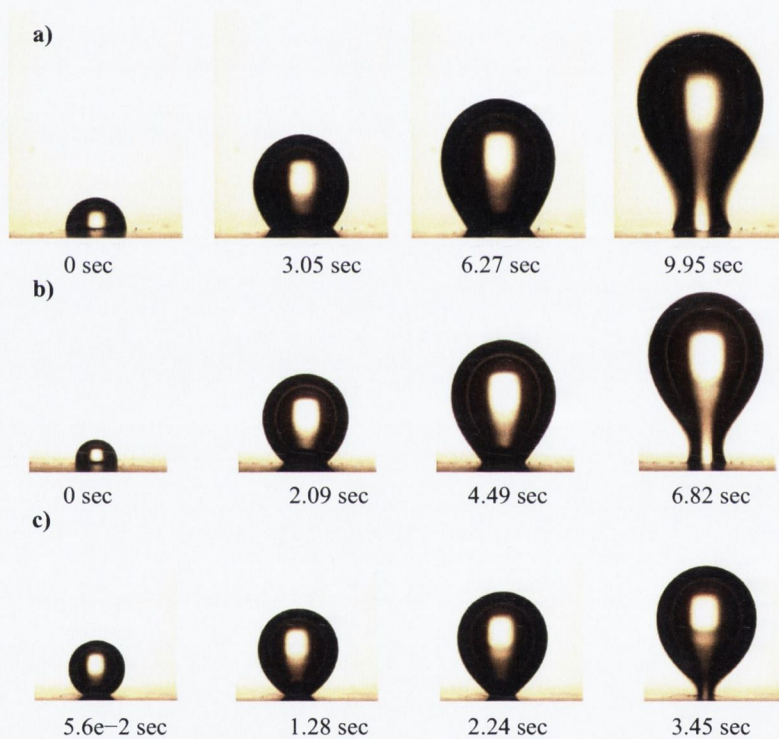


Figure 5.12: Photographic sequence of bubble formation for a gas injection rate of 10 mlph and orifice diameter of (a) 1.6 mm (b) 1.05 mm and (c) 0.58 mm.

Fig.5.12 shows a photographic sequence of the bubble growth from the orifice adopted at the lowest flow rate. The sequence shows that the bubble growing from the smallest orifice tends to be smaller and assume a more spherical shape. On the other hand, the increased strength of the buoyancy

force tends to elongate the bubble growing from the largest orifice.

The orifice diameter is essential in determining the final bubble volume and position of the centre of gravity before bubble departure. Fig.5.13 illustrates that, for a fixed gas injection rate, both the departure size and subsequent bubble growth time decrease with decreasing orifice diameter, as has been observed in previous studies [Kasimsetty et al (2007)].

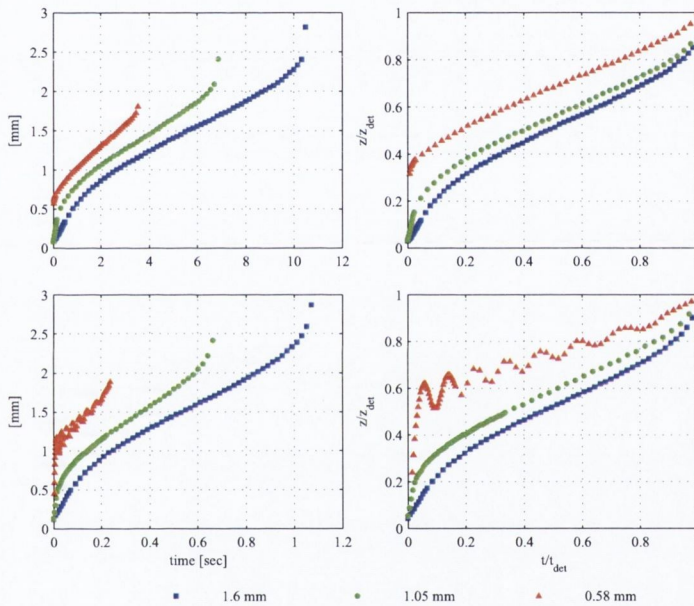


Figure 5.13: Dimensional and non-dimensional motion of the centre of gravity for varying orifice diameter at 10 mlph (top) and 100 mlph (bottom).

The three stages of the bubble growth discussed earlier are evident in all cases. However, as the orifice diameter is decreased the velocity of the centre of mass increases, in particular during early growth.

This is more evident during early growth of the smallest orifice at the highest injection rate. During a very short period of time the bubble grows very quickly from the orifice mouth and decelerates rapidly once the triple contact line reaches the rim of the orifice and remains fixed, essentially

pinning the bubble foot in place. The growth then becomes oscillatory due to interplay between capillary and dynamic liquid forces.

As the bubble decelerates the liquid inertial forces pull the bubble away from the size that would exist in the absence of this force. As the inertia forces diminish, surface tension stresses cause the bubble to contract rapidly ostensibly reversing the phenomena such that the liquid inertia now compresses the bubble beyond its quasi-static shape. Once again the inertia forces diminish and the bubble recoils thus restarting the period of oscillation. As is expected the magnitude of the oscillations diminish with time.

Fig.5.14 shows the evolution of the vertical forces for the three orifices at the highest flow rate. From the graphs is it possible to observe a decrease of both the contact pressure force and the capillary force due to the smaller area and perimeter of the orifice. The buoyancy force is considerably higher for the largest orifice compared with the smallest and is sufficient enough to elongate the bubble somewhat away from a spherical shape thus increasing the contact angle, see in Fig.5.13. In essence, the buoyancy force acts in such a manner to increase a force which opposes it. Also of interest is the observation that for the largest orifice, the buoyancy force is less than the contact pressure force and only equalizes it at the very end of the growth period. With decreasing orifice size, the crossover point moves to an earlier times making buoyancy the larger of the two until bubble detachment.

The evolution of the dynamic force allow one to assume a quasi- static growth for the 1.6 mm and 1.05 mm orifice diameter cases as the magnitude is much smaller than the other forces. For the smallest orifice the dynamic behaviour of the inertial force is evident along with the oscillating capillary force due to the changing contact angle as reported in Fig.5.15. Consistent with the discussion above, F_C and F_D oscillate in phase.

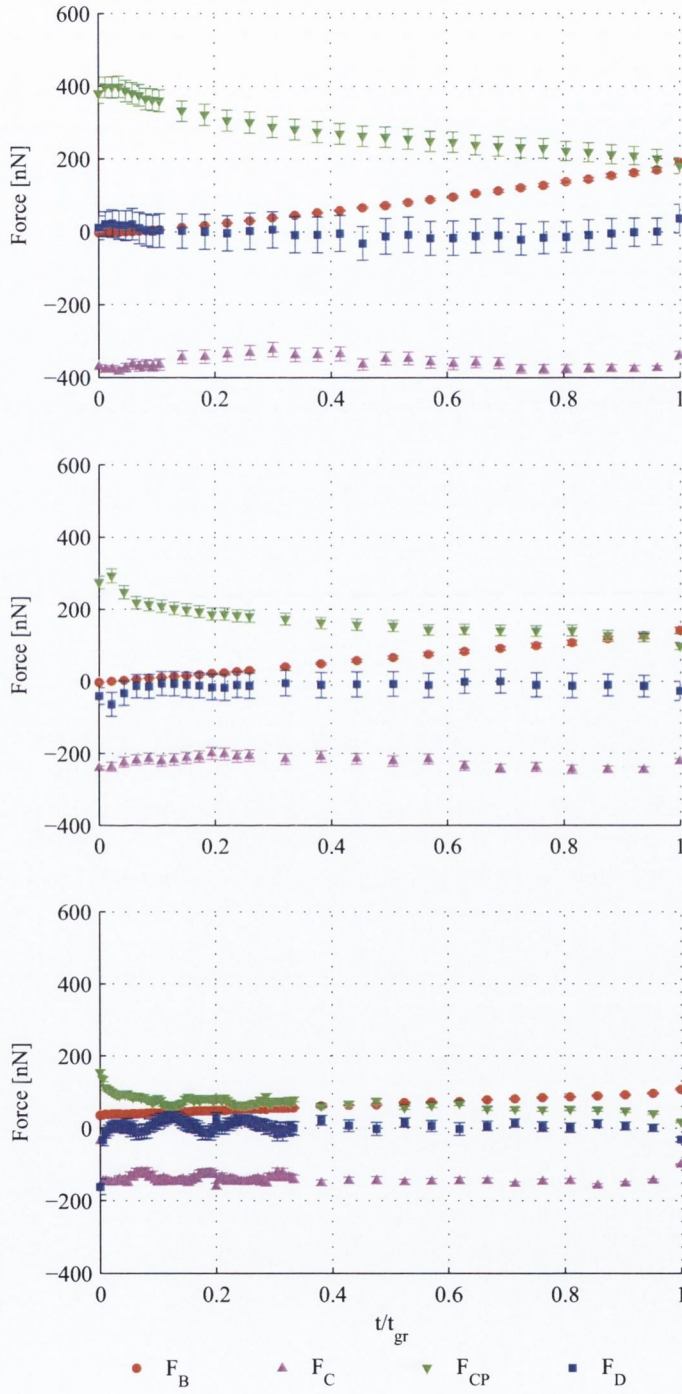


Figure 5.14: Vertical forces acting on the bubble at 100 mlph (a) 1.6 mm (b) 1.0 mm (c) 0.58 mm.

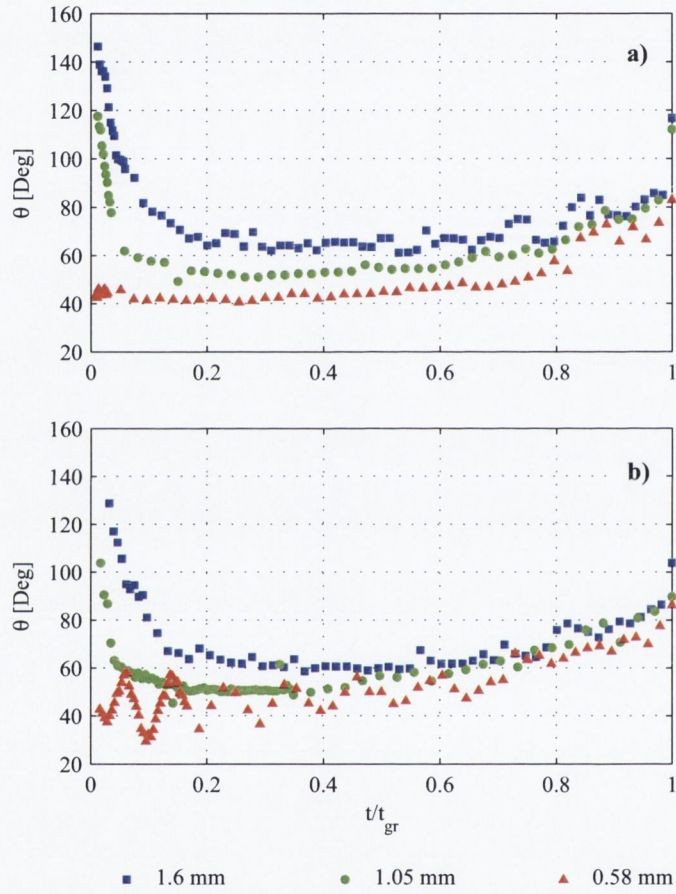


Figure 5.15: Contact angle histories for varying orifice diameter at (a) 10 mlph and (b) 100 mlph.

The influence of the orifice size on the bubble shape is more evident in Fig.5.16. The evolution of the bubble profiles for the largest and smallest orifice is compared at the highest flow rate. As was done previously, non-dimensional times of $t/t_{gr} \sim 0.1, 0.5$ and 1 have been chosen to be indicative of the principal growth stages.

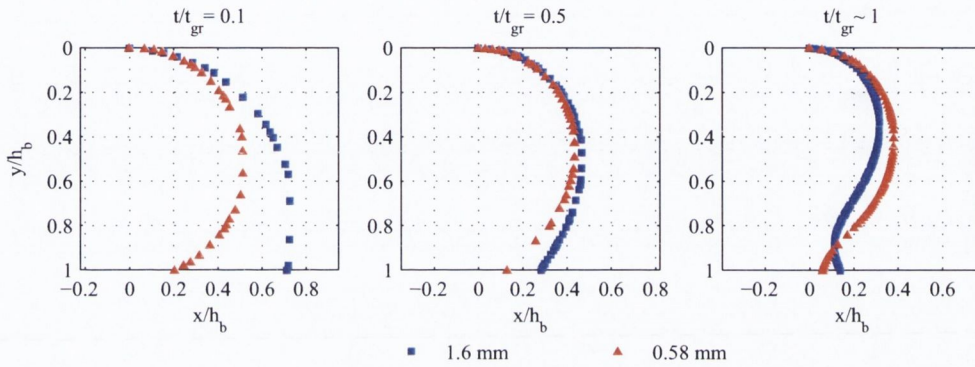


Figure 5.16: Comparison of bubble shape for the largest and smallest orifice diameter.

As it is evident, the shapes of the bubbles are clearly different at the beginning of the growth cycle. The bubble growing from the 1.6 mm orifice has just passed the hemispherical shape, but the buoyancy force is still too weak to stretch the bubble upward. Its shape is completely inside Region I, as shown in Fig.5.17. Moving from the tip of the bubble both the curvatures, $1/R_1$ and $1/R_2$ remain more or less constant. A definite spherical contour is indeed observable for the bubble growing from the smallest orifice, under these circumstances the buoyancy force is already somewhat important at this stage of the bubble's growth, (see Fig.5.14c).

For the mid-growth bubbles the two shapes becomes more elongated due to the increasing effect of buoyancy. Region II begins to form in proximity to the bubble base as it is possible to see from the increase of R_1 and the decrease of R_2 . For the smaller orifice a notable increase in $1/R_2$ occurs as

the bubble tends to remain more spherical compared with the larger bubble owing to an increase in the Young-Laplace pressure.

For the late stage bubble the curvature profiles depicted in Fig.5.7 have already been discussed. A strong influence of the orifice size on the neck profile is evident. From the evolution of the two curvatures it is possible to associate a larger orifice with a longer neck. Region III begins at approximately $y/h_b \sim 0.8$ for the biggest orifice, and is squeezed to $y/h_b \sim 1$ for the smallest one. The evolutions of the curvatures show how the neck of the bubble growing from the smallest orifice coincides with the bubble foot where R_2 reach its maximum value. It is also evident that the two curvatures are close in magnitude over the majority of the bubble's surface indicating a dome which is quite spherical.

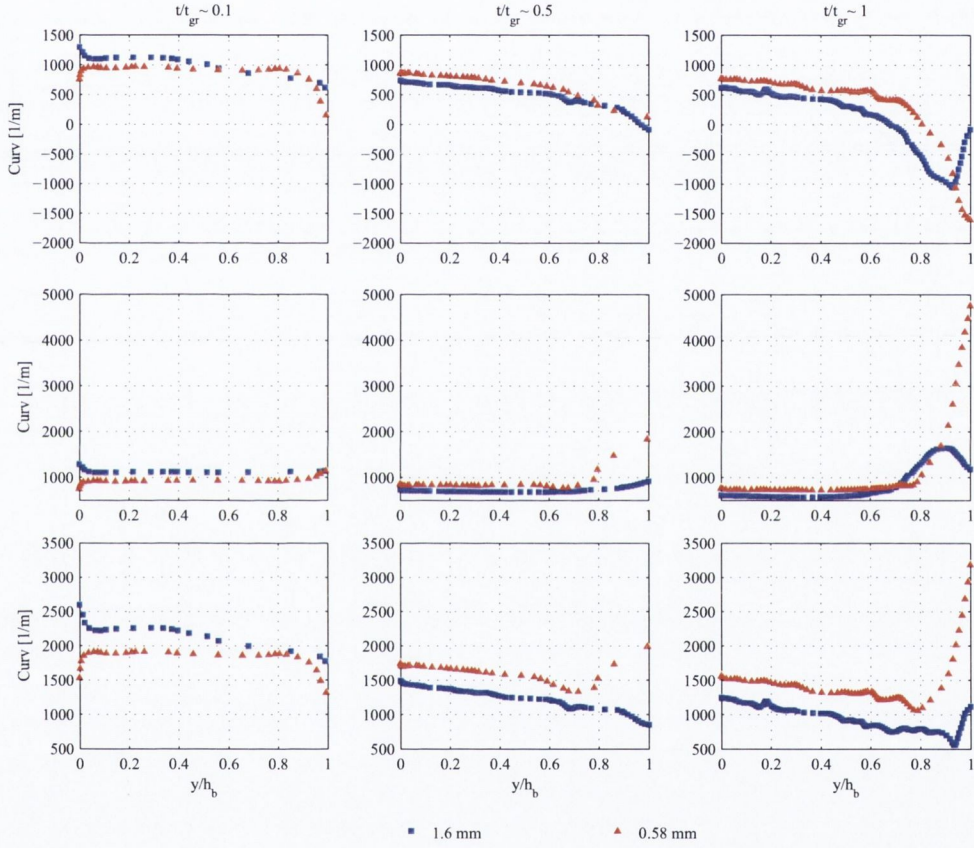


Figure 5.17: Bubble curvatures of $1/R_1$ (top), $1/R_2$ (middle) and $1/R_1 + 1/R_2$ (bottom).

Fig.5.18 shows the evolution of the different pressure components acting on the bubble interface. A value of the dynamic pressure close to zero or positive is present in almost all the graphs, meaning that the dynamic effects are still not very important. Of particular interest is the last stage of the bubble growth in the proximity of the neck where a stronger influence on the dynamic pressure is observed with the decrease of the diameter.

For the larger orifice there tends to be a more pronounced change in the Laplace pressure and associated internal gas pressure. Due to the relaxed surface tension stress the bubble is more susceptible to deformation as a result of the non uniform pressure field. For the smaller orifice the sum of the two curvatures, and therefore also the Laplace pressure is higher resulting in generally higher relative surface tension stress and, associated with the smaller buoyancy force, a more spherical bubble.

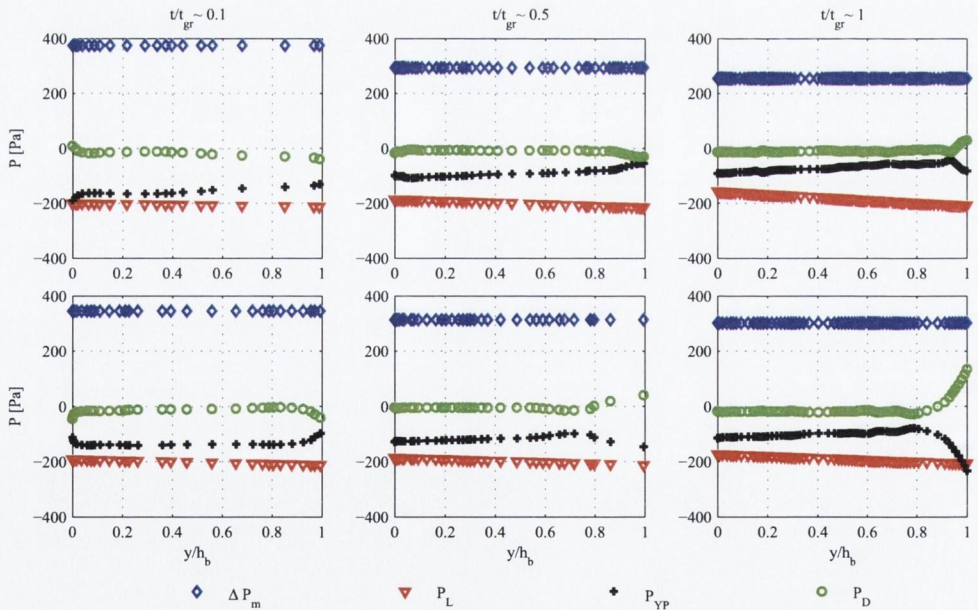


Figure 5.18: Local pressures acting on the bubble interface for 1.6 mm (top) 0.58 mm (bottom) orifice diameters.

CHAPTER 6

RESULTS AND DISCUSSION: NUMERICAL SIMULATIONS

The evolution of a bubble growing from a submerged orifice has been investigated numerically with the aid of the finite volume method software TransAT. The simulations have been set with the intent to reproduce the experimental conditions as faithfully as possible. The results obtained from the numerical approach have been subsequently compared with the experimental measurements in order to confirm the validity of the CFD software.

The time needed for the code to reproduce the entire bubble growth, from its initial hemispherical profile to its detachment, was related to the orifice diameter and flow rate adopted. The simulation time varied from 8h, in the case of a 1mm orifice diameter at 100 mlph, up to 120h for the 1.6 mm diameter at the 10 mlph case studied.

Once the the code was validated, the influence of gravity on bubble growth dynamics was studied. Four different gravity levels have been simulated and studied, 1.5, 1.0, 0.5 and 0.1 g. The results thus obtained are reported in the last section of the present chapter.

6.1 Comparison with the Experimental Data

Bubble Evolution

In Fig.6.1 the bubble evolution observed in the experiments for the lowest flow rate is compared with the respective numerical simulation. The photographic sequence shows how the bubble profile predicted in the simulation matched the experimental results very closely. Although the code tends to underestimate the bubble size during the beginning of its growth, it is clear from Fig.6.2 that the experimental volumetric flow rate was not perfectly constant and was slightly higher than the fixed flow rate input into the CFD simulation.

The three stages of the bubble growth are clearly visible. In both the

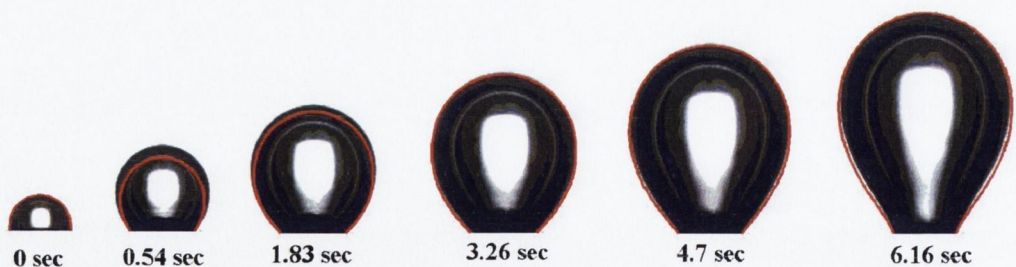


Figure 6.1: Photographic sequence of bubble growth in water from a 1 mm orifice diameter compared with numerical simulation (red line). The sequence refers to a flow rate of 10 mlph.

experiments and the simulation, the early stage of the bubble growth is characterised by a bubble shape quite close to a truncated sphere with a volume completely surrounded by water, though it is still not large enough to generate a significant buoyancy force.

As the bubble emerges from the orifice the buoyancy effect increases and the bubble becomes progressively more elongated as the force tends to lift the upper portion of the bubble with the bubble foot remaining fixed to the orifice mouth. In the final frames the bubbles are in the process of detaching by

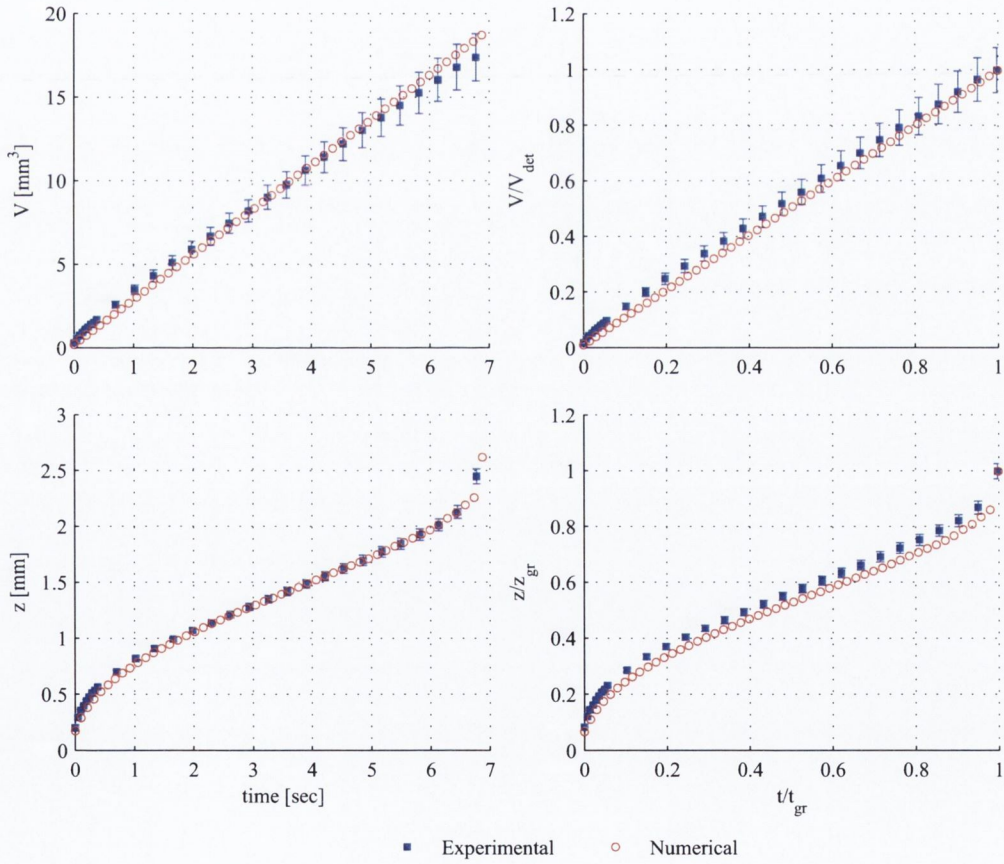


Figure 6.2: Dimensional and dimensionless volumetric growth rate (top) and motion of the centre of gravity (bottom) at 10 mlph.

forming a neck near the bubble base. The influence of buoyancy is enhanced as the curving inward of the neck increases the volume being acted on by buoyancy.

In Fig.6.2 the evolution of the centre of gravity and volume are compared in both dimensional and dimensionless scales and very close agreement is observed. The constant growth rate proves the validity of a constant velocity boundary condition at the orifice mouth.

The code predicted the vertical position of the centre of gravity with a very narrow margin of disagreement and is generally within experimental uncertainty. The succession of the three growth stages is clear. During the initial stage the velocity of the centre of gravity decreases due to the passage from a hemispherical profile to a truncated spherical bubble. Subsequent to this, the bubble becomes more elongated and the vertical coordinate increases linearly with time. In the final stage the neck forms and the centre of gravity increases its velocity until the detachment of the bubble.

Good agreement of the contact angle evolution is observable in Fig.6.3. Even though the code tends to underestimate the experimental measurement slightly, the validity of the thin film boundary condition approach is confirmed. The contact angle is subject to a rapid decrease during the initial growth where the bubble passes from the hemispherical profile to a truncated spherical one. Afterwards, an approximate minimum constant value is maintained during the truncated spherical phase. As the bubble becomes more elongated under the effect of the buoyancy force it begins to increase with an increasing rate. In the later stage the formation of the neck forces the contact angle to become obtuse as the neck pinches off.

As it is possible to see, the numerical prediction presents less irregularity compared with the experimental data since the profile of the bubble can be

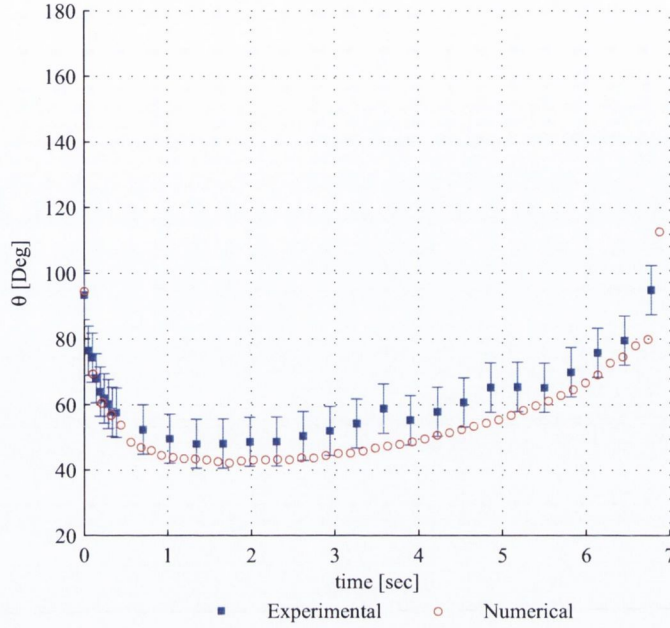


Figure 6.3: Contact angle comparison.

defined very accurately.

Fig.6.4 reports the comparison of the bubble shapes at three different non-dimensional times. They have been chosen to be indicative of the principal growth stages and are $t/t_{gr} \sim 0.1, 0.5$ and 1. The comparison between the experimental and numerical results is facilitated by normalizing

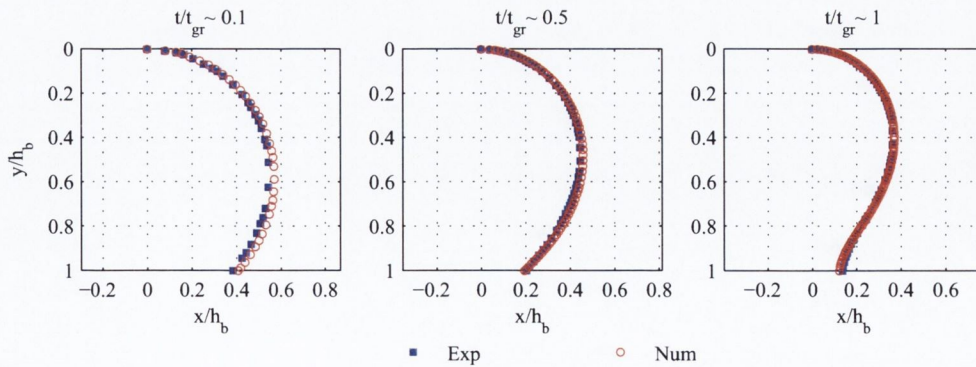


Figure 6.4: Comparison of bubble shape at different dimensionless times.

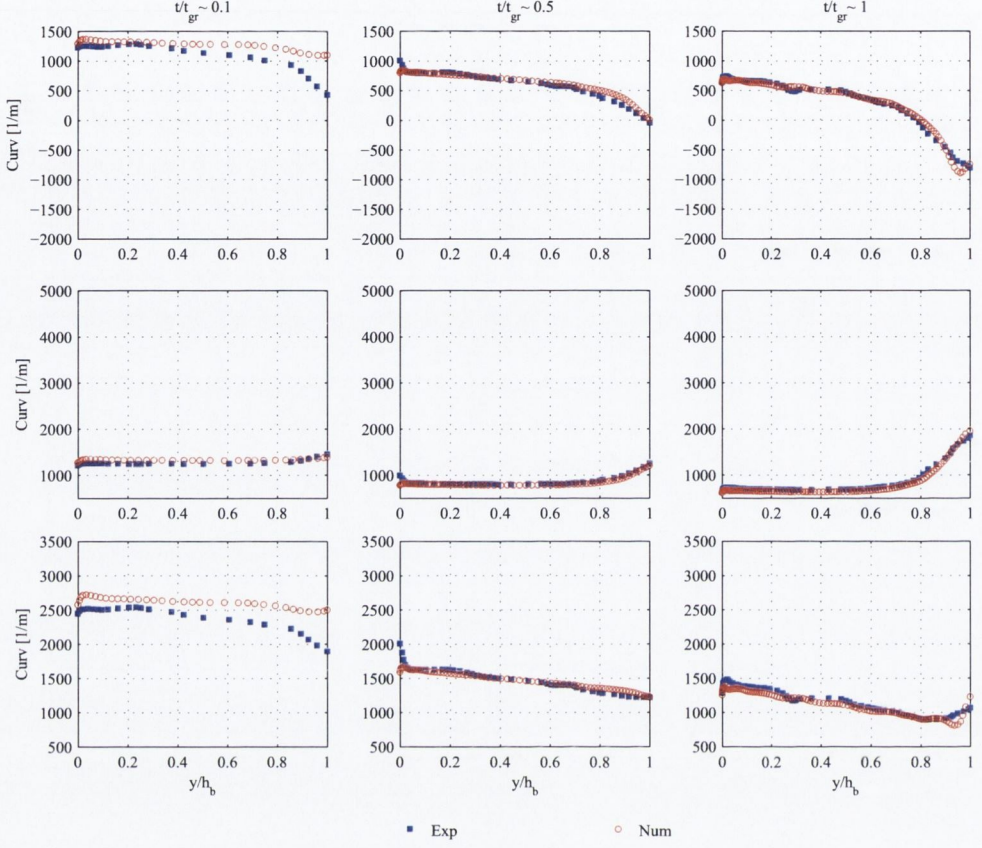


Figure 6.5: Bubble curvatures of $1/R_1$ (top), $1/R_2$ (middle) and $1/R_1 + 1/R_2$ (bottom).

each shape with the respective bubble height. The shape predicted numerically matches very well with the experimental measurements. The validity of the code is also confirmed by the satisfactory agreement of the two radii of curvature evolutions and their sum reported in Fig.6.5. The discrepancy at the beginning of the growth is due to the slight mismatch between the experimental and simulated volumetric flow rates.

For the final stage the distribution of the inverse of the main radii, $1/R_1$ and $1/R_2$, with their sums are compared and represented in Fig.6.6.

The development of the three regions discussed in Chapter 5 is also evident. The similarity of the three curves confirms the capability of the

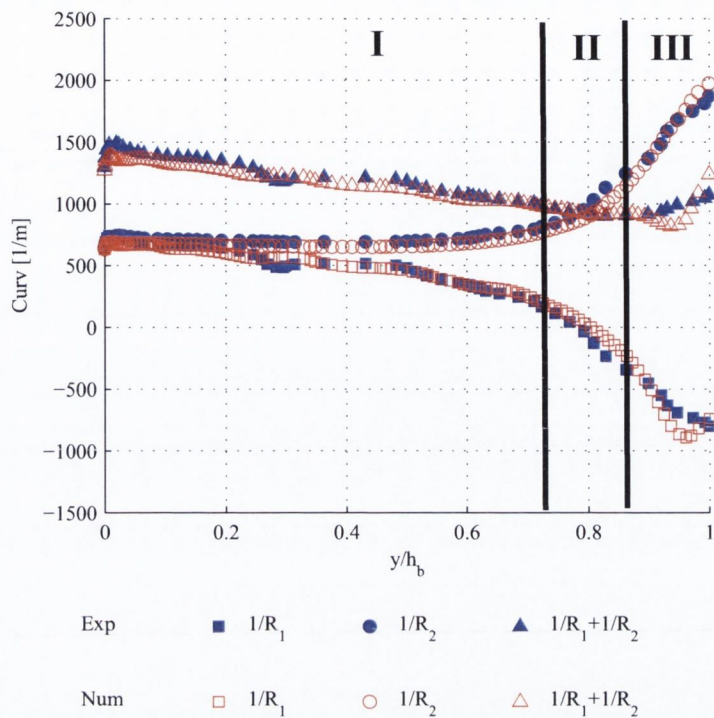


Figure 6.6: Curvature comparison during the detachment stage.

code to model the bubble profile in order to satisfy the mass and momentum balance in the presence of an external force field, in this case the earth's.

Effect of Volumetric Flow Rate

The influence of the magnitude of the volumetric flow rate has also been tested. Numerically this is achieved by varying the magnitude of the velocity at the inlet boundary. The same flow rates tested in the experiments (10, 20, 50 and 100 mlph) have been simulated numerically. The results thus obtained have been compared with the experimental measurements.

In Fig.6.7 the experimentally measured volume and the centre of gravity evolutions at different flow rate are compared with the respective numerical prediction. In order to facilitate a clear comparison, the error-bars for the

experimental trends are not represented. As it is possible to see, the curves obtained from the numerical approach well represent the experimental ones. The difference at the end of the evolutions is due to the difficulty in obtaining high quality pictures during the rapid necking stage where the bubble is subject to a sudden change of its profile. For the lowest volumetric flow rate,

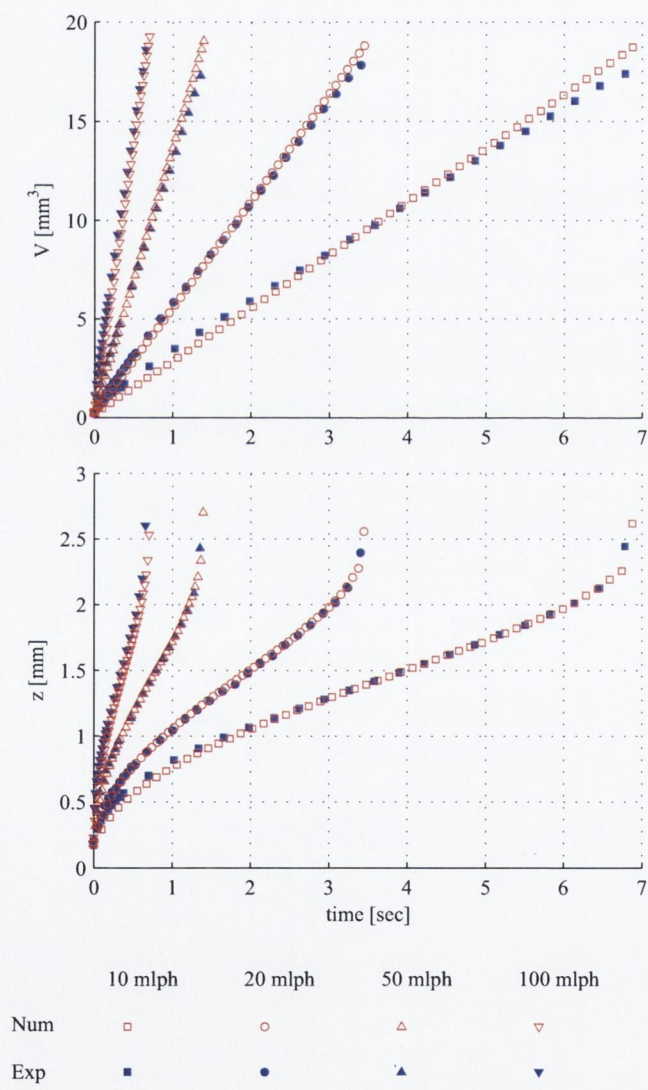


Figure 6.7: Comparison of volume (top) and centre of gravity (bottom) evolution of a bubble growing from a 1 mm orifice diameter at different flow rates.

the experimental curve tends to dip somewhat causing some discrepancy there.

The development of the volume shows how the volumetric growth is nearly linear in all cases indicating approximately constant injection rate. Moreover, it is evident that, as already pointed in a previous chapter, the measured bubble detachment volume does not depend strongly on the volumetric growth rate.

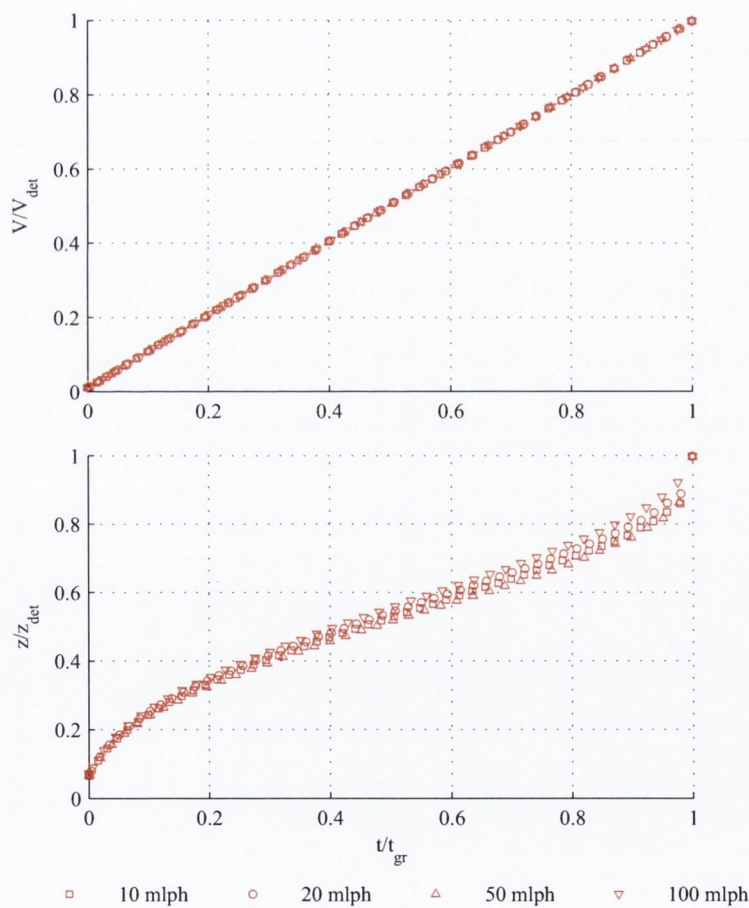


Figure 6.8: Numerical prediction of the volume (top) and centre of gravity (bottom) in non -dimensional scale at different flow rate.

The numerical approach also predicted the weak influence of the flow rate on the bubble evolution for these quasi-static cases. This is evident in Fig.6.8 and Fig.6.9. The figures report the non- dimensional comparison of the evolution of the volume, centre of gravity and contact angle at different flow rates. From the dimensionless graphs a very good collapse of all the curves is observed. The code demonstrated the ability to predict not only the growth of a bubble, but also the influence of an essential parameter such as the growth rate on its evolution.

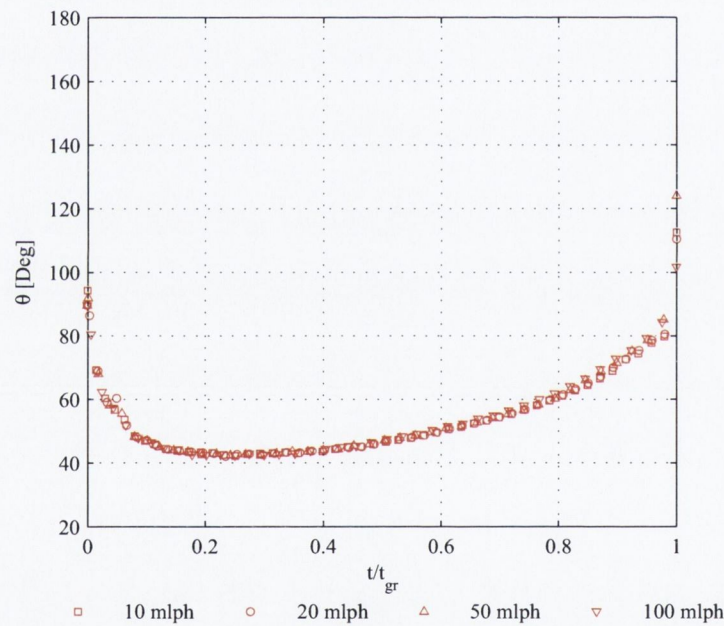


Figure 6.9: Numerical prediction of the contact angle in non dimensional time scale at different flow rate

Comparison with a larger orifice

In order to further confirm the validity of the code, the same study of the previous two sections has been performed for a larger orifice diameter. The evolution of a bubble growing from the 1.6 mm orifice at different growth rates was simulated numerically and the results obtained have been compared with the experimental ones.

In Fig.6.10 the bubble evolution observed in the experiments for the lowest flow rate is compared with the respective numerical calculation. The code confirmed its capacity to represent the shape of the bubble during its growth including the effect of gravity which elongates the bubble in the vertical direction.

The validity of the code is confirmed in Fig.6.11 where, as in Fig.6.7, the growth rate and the vertical position of the centre of gravity calculated numerically are compared with the relative experimental measurements. The numerical predictions are in good agreement with the experimental results. The weak influence of the flow rate is also evident from the constant value of the detachment volume and centroid position.

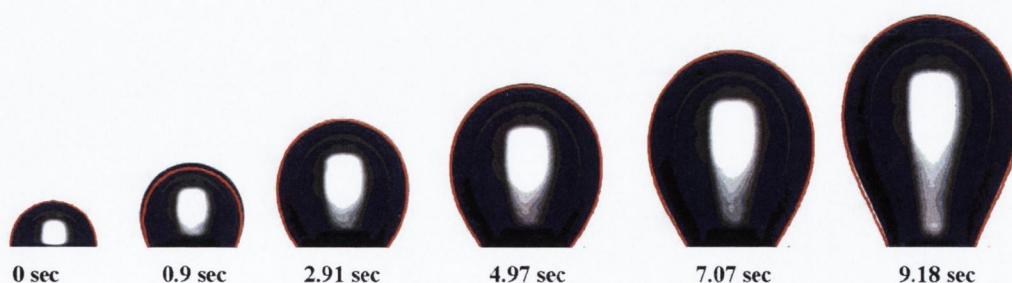


Figure 6.10: Photographic sequence of bubble growth from a 1.6 mm orifice diameter and numerical prediction (red line). The sequence refers to a flow rate of 10 mlph.

Considering the above discussion it can be concluded that for the range of parameters tested, the CFD package TransAT and the numerical treatment of the problem are sufficiently validated for quasi-static and adiabatic bubble formation at submerged orifices.

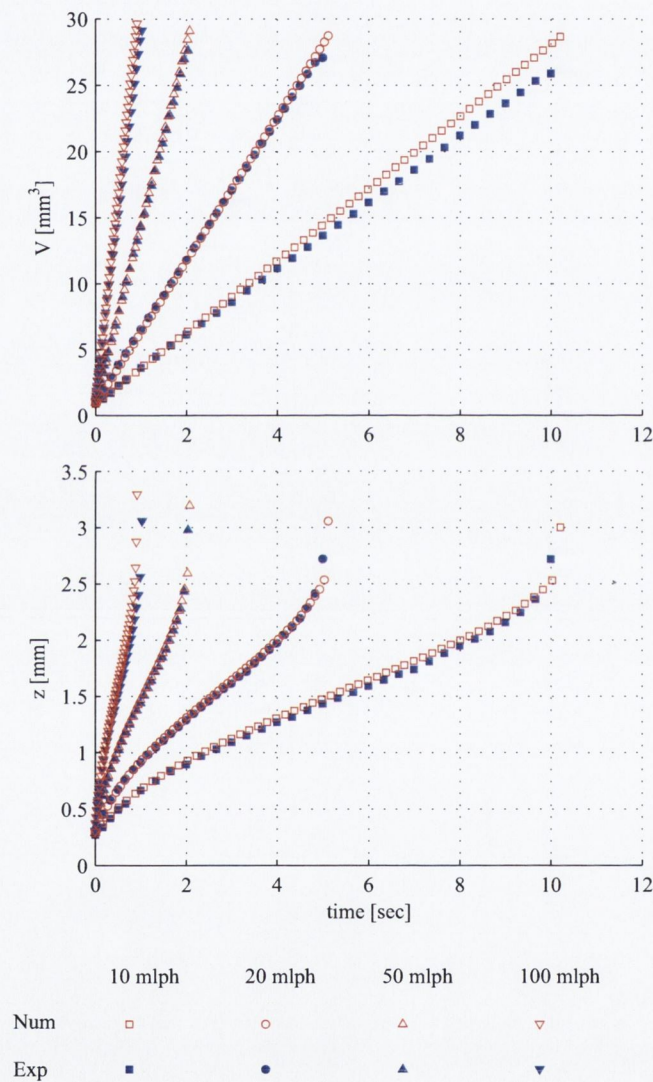


Figure 6.11: Comparison of volume (top) and centre of gravity (bottom) evolution of a bubble growing from a 1.6 mm orifice diameter at different flow rates.

6.2 Flow Field and Influence of Gravity

Flow field around a bubble growing from a submerged orifice

The pathlines and velocity vector fields around and inside the bubble are presented in Fig.6.12 for the highest flow rate tested. Four different dimensional times have been chosen in order to include all the stages of the bubble growth, including the phase by which it rises upward through the bulk liquid subsequent to departure.

As the bubble grows the its volume increases inside the domain and occupies the space previously occupied by liquid. As such, the liquid is displaced and the characteristic of the flow field depends strongly on the bubble growth stage.

As is evident in Fig.6.12, as the bubble passes from its original hemispherical shape and into a slightly elongated truncated sphere, the fluid in the upper region moves outward in the radial direction. A notable recirculation region is observed in the proximity of the bubble foot since the bubble is not only expanding but is translating upward which draws liquid inward in order to conserve mass.

As the bubble becomes larger the centre of the main vortex is dragged upward and it extends several diameters from the orifice influencing the flow field over a substantial portion of the domain. At $t = 0.52s$ it is possible to see the pathlines collapsing in the region of the bubble foot creating a sharp corner. This represents an unstable configuration and a second vortex appears in this proximity of the neck as seen for $t = 0.7s$. This second vortex collapses during the detachment stage and its effect disappears completely as the liquid primarily moves upward beneath the rising free bubble. Finally, the free rising bubble creates a large recirculation region since the liquid is

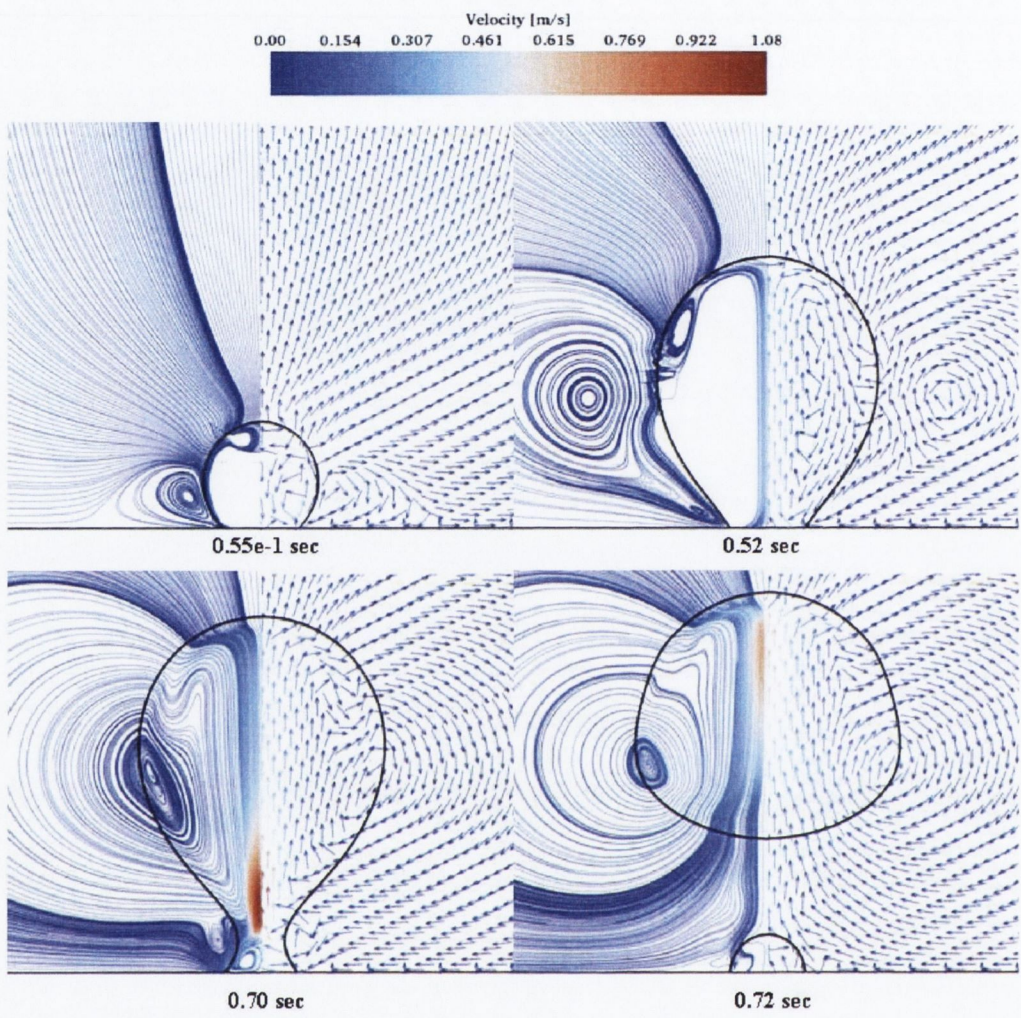


Figure 6.12: Pathlines and velocity vectors around the bubble false-coloured by velocity magnitude.

forced upward in the upper region and drawn inward in the lower region so as to replace the missing gas volume as the bubble translates upward.

Influence of gravity on bubble growth dynamics

The primary focus of this work is to investigate the influence that various forces have on bubble formation and departure. As such it is logical to study the influence of the magnitude of gravitational acceleration on bubble growth dynamics. The analysis of the contribution of the gravity field has drawn the attention of a great number of researchers, largely due to the potential of two phase thermal management in space applications. Experiments performed in reduced gravity, Di Marco et al. (2003), have shown how a gas bubble grows whilst keeping a near perfect spherical shape and, for low enough injection rates, remains attached on the surface on which it is growing. As one can imagine, these experiments are very sophisticated and expensive. However, the same analysis can be performed quite easily and inexpensively with the aid of CFD by simply changing the magnitude and/or direction of the gravity vector during the pre-processing phase. This is, however, if sufficient care has been taken to ensure the software has been adequately validated.

In this part of the investigation a set of four different simulations have been performed using the same grid used for the terrestrial gravity simulation of the previous section. Four different gravity magnitudes have been tested for the 1 mm orifice diameter at the highest flow rate:

Orifice Diameter: 1 mm

Flow Rate: 100 mlph

Gravity: 0.1, 0.5, 1.5 g where $g=9.81$

With the exception of the lowest gravity level, all the simulations ended with the detachment of the bubble from the orifice. The results obtained for the 0.1 g case are reported at the end of this section with an aim only to analyse the behaviour of a bubble in a very low gravity field. The simulations

were interrupted due to the computational cost required to reach the bubble detachment which would require a very large computational domain and involve computations over a very long time interval.

The evolutions of the bubble volume for the three gravity levels tested are reported in Fig.6.13, both for dimensional and non-dimensional scales. From the graphs is clear that the bubble volume increases linearly and is the same for the three simulations. In the non-dimensional scale the three lines collapse to a common line as is to be expected.

From Fig.6.13 and Fig.6.14 is clear how the growth time, the detachment volume and diameter depend on the gravity level. An earlier detachment is observed for the highest gravity level tested. On the other hand, halving earth gravity extends the growth time and increases the departure volume. The departure of the bubble is thus driven by a balance between Buoyancy and Surface Tension. From Eq.1.26 and Eq.5.4 is clear that:

$$V_{det} \propto \frac{1}{g} \quad \text{and} \quad D_{det} \propto \frac{1}{\sqrt[3]{g}}$$

Fig.6.14 illustrates how the detachment volume and diameter predicted are inversely proportional to the gravity field and to its cubic root, respectively. The theoretical values (dotted line) are calculated using the earth gravity level as reference.

Due to the lower level of gravity, in order to detach the bubble, the buoyancy force requires a larger volume in order to counter the capillary force. For this reason, the growth is extended until the volume completely surrounded by liquid creates a buoyancy force that is large enough to form the neck and initiate the departure phase.

The vertical coordinate of the centre of gravity is given in Fig.6.15. It is clear that gravity plays an important role on the evolution of the centre

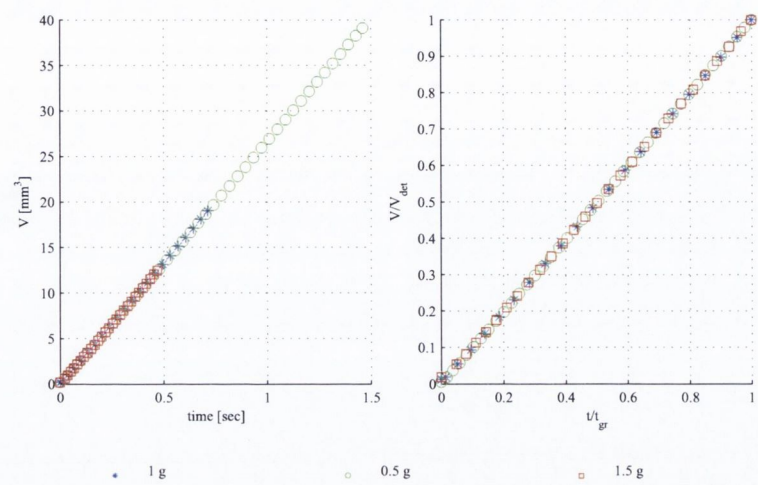


Figure 6.13: Dimensional (left) and non dimensional (right) bubble growth rate at three different gravity levels.

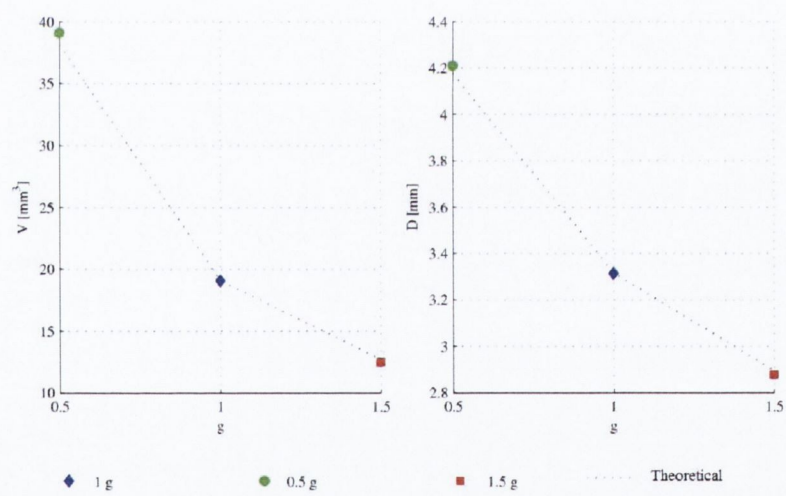


Figure 6.14: Detachment volume (left) and diameter (right) versus the gravity level.

of gravity. During the initial stage, since the volume acted on by buoyancy is still relatively small, the influence of this force on the bubble dynamics is not significant and the three curves coincide. When the volume affected by buoyancy increases the force starts to elongate the bubble and the centre

of gravity begins to move upward with a velocity that increases with the magnitude of gravity. The position of the centre of gravity at detachment decreases with the increase in gravity field. Unlike the volume evolution, the curves do not collapse onto a common line when represented using a non-dimensional scale.

The effect of gravity on the bubble shape is evident in Fig.6.16. Two different non-dimensional times, $t/t_{gr} \sim 0.5$ and $t/t_{gr} \sim 1$, have been chosen in order to show the main difference on the last two bubble growth stage since the early growth stage is nearly identical for all gravity levels tested and has been discussed in earlier sections.

The shape evolutions observed in Fig.6.13 and Fig.6.15 for the volume and centre of gravity are well explained by comparing the shapes of the different bubble. From the dimensional comparison it is clear that the bubble growing in low gravity is the largest since the growth time is longer. Gravity plays an important role not only on the bubble size, but also on its shape. This

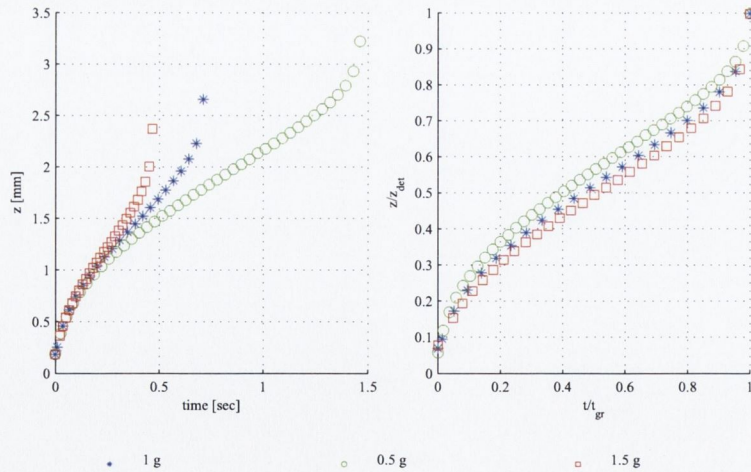


Figure 6.15: Dimensional (left) and non dimensional (right) motion of the centre of gravity at three different gravity levels.

is clearer with the aid of the non-dimensional comparison. The shape of the bubble growing in low-gravity is more spherical than the other two. The buoyancy force is indeed not strong enough to counteract surface tensions stresses which tend to keep a spherical-like shape in order to minimize its surface area. For this case the bubble is not notably elongated along the vertical direction. Interestingly, the volume acted on by buoyancy is similar for each case tested, as evident in Fig.6.17. Thus, for the highest gravity level, the buoyancy effect succeeds in stretching the bubble and the neck tends to be longer. This is what accounts for the observation in Fig.6.13, whereby beyond approximately $t = 0.2s$, the bubbles have the same overall volume, but the centres of gravity are larger for the higher gravity cases.

As mentioned and shown in Fig.6.17, the evolution of the volume affected by buoyancy is similar for each gravity level and follows a linear trend common to the volume evolution. From the non-dimensional comparison, where the buoyancy volume is normalized by the instantaneous volume, the initial growth stage is characterized by a sharp increase of this ratio. The curves show how the bubble growing at the lowest gravity level has the tendency to have a larger proportion of its volume completely immersed in the liquid compared the other two; this is due to its more spherical shape. For the bubble growing at 0.5 g, $V_B/V_b \sim 80\%$ for the non-dimensional time of $t/t_{gr} \sim 0.4$. For the same non-dimensional time this value decreases to 70% for the normal gravity level and around 60% for the 1.5 g case. At the end of growth is it possible to see how for the lowest gravity the volume of the bubble prior to the detachment is nearly completely surrounded by liquid, representing almost 90% of its entire volume. In contrast, for the highest gravity level this decreases down to about 75% since this bubble is more elongated.

The tendency for lower gravity to produce more spherical bubbles also influences the evolution of the contact angle along the triple contact line. A lower value for the contact angle for the lower gravity simulation is evident in Fig.6.18. At lower gravity, the capillary action has to counteract a weaker buoyancy force, the bubble maintains a more spherical like shape and the value of the contact angle decreases. For the high gravity level, the effect

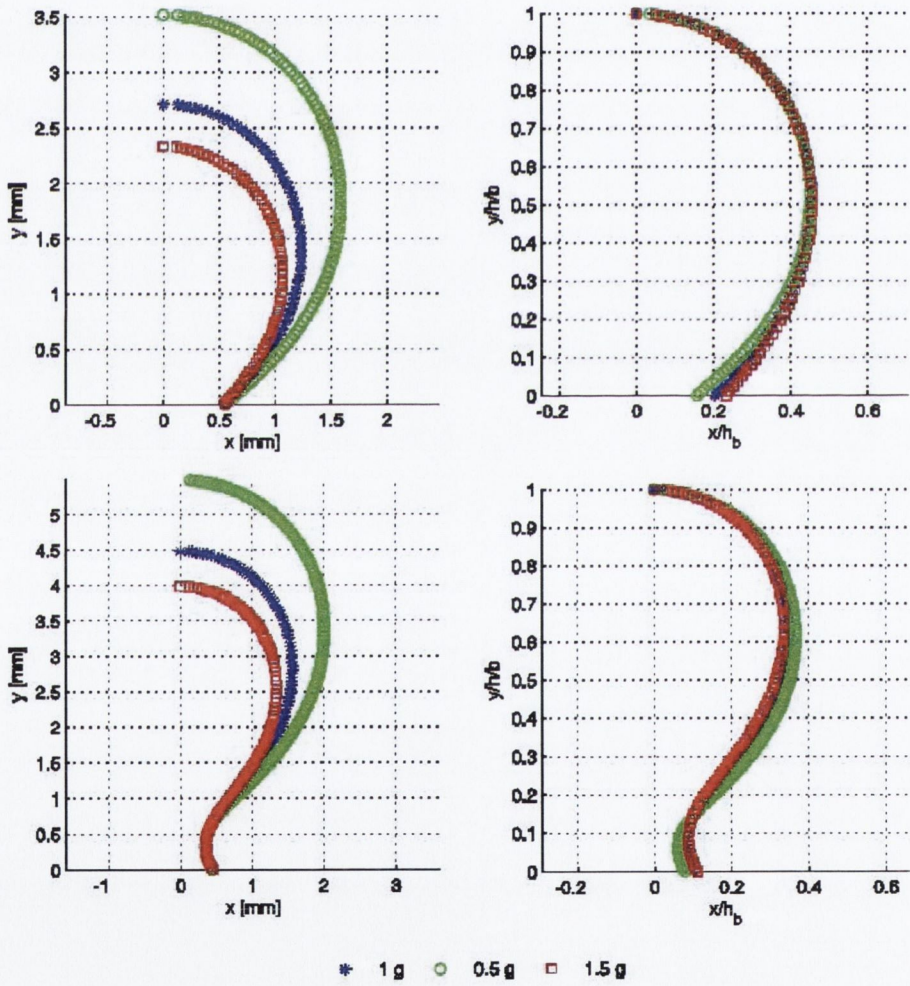


Figure 6.16: Dimensional (left) and non-dimensional (right) shape comparison at two different non dimensional times, $t/t_{gr} \sim 0.5$ (top) and $t/t_{gr} \sim 0.9$ (bottom)

of the buoyancy force elongates the bubble and the contact angle tends to be notably larger. The differences between the three curves are large at the end of the first growth stage, when the bubble centroid has just finished decelerating and the bubble starts to be more influenced by the buoyancy force. Moving to higher non-dimensional times, the differences between the contact angles decreases until, near departure, they are nearly the same.

Figs.6.19 and 6.20 show the evolution of the vertical forces acting on the bubble for the three gravity levels tested. As evident in Fig.6.19 the net force has a negligible influence with respect the other forces indicating quasi-static growth. Fig.6.20 shows that the magnitudes of the buoyancy force are similar though they tends to decrease with the gravity level due to the decreased buoyancy volume discussed earlier. The more notable increase in the contact pressure with increasing gravity level is due to the higher value of the bubble gas pressure at the beginning of growth. The difference of pressure for the hemispherical bubble can be calculated by the Young-Laplace equation:

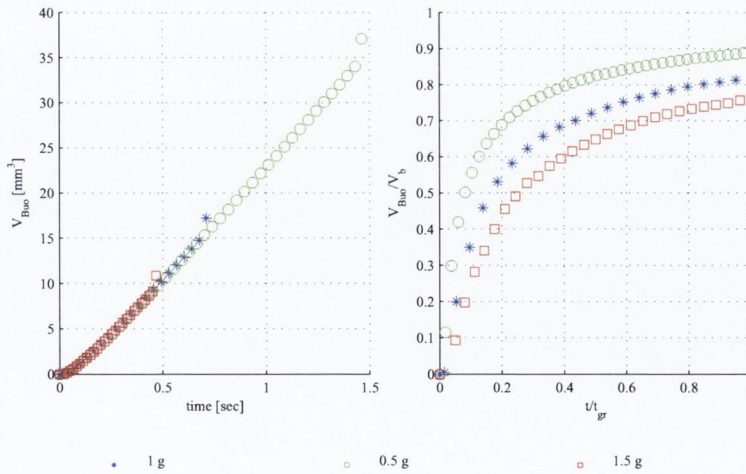


Figure 6.17: Dimensional (left) and non dimensional (right) buoyancy volume at three different gravity values

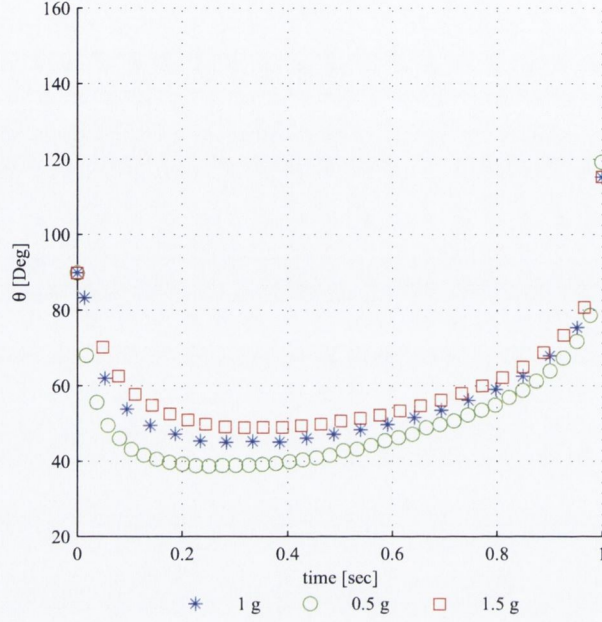


Figure 6.18: Contact angle evolution for the three gravity levels tested

$$p_g - p_l = \frac{2\sigma}{R_0} \quad (6.1)$$

Since the orifice radius, R_0 , is constant, the RHS of Eq.6.1 remains constant. On the other hand, the liquid hydrostatic pressure around the bubble increases with increased gravity. For this reason, in order to retain equilibrium, the internal gas pressure of the bubble and the subsequent contact pressure force also increase.

The increase of the gravity level also tends to marginally increase the capillary force due to the higher value of the contact angle as previously discussed.

Of particular interest is the influence of the gravity level on the evolution of the forces. As it is possible to see in Fig.6.19, and was discussed in Chapter 5, the cross-over point where the buoyancy force exceeds the capillary force

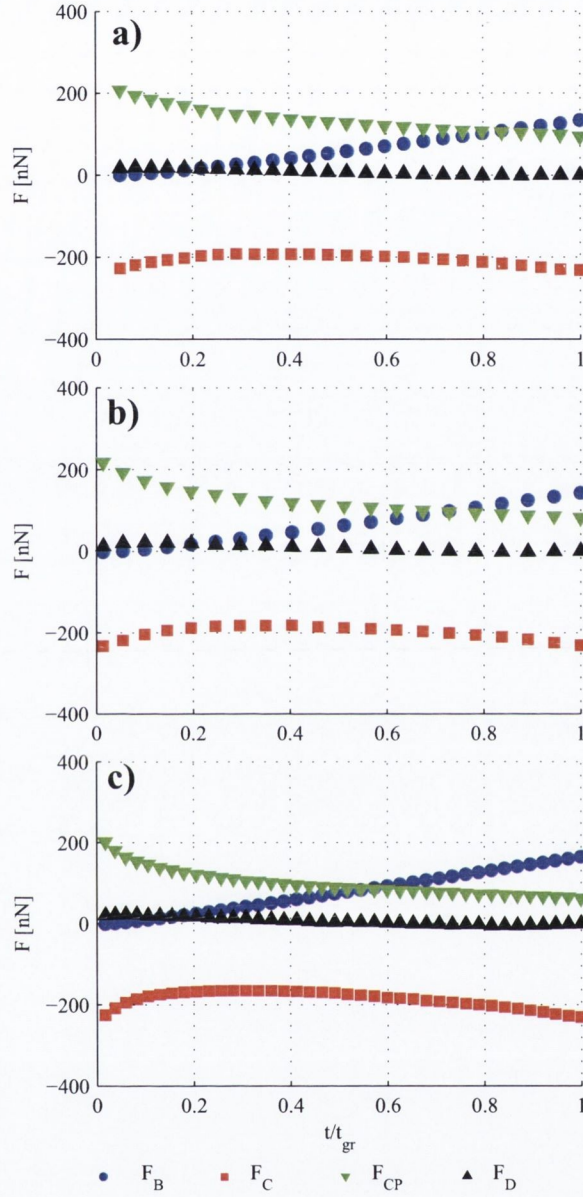


Figure 6.19: Vertical forces acting on the bubble:(a) 1.5 g, (b) 1 g and (c) 0.5 g.

happens near the end of the bubble's life of the highest gravity level tested. Interestingly, reducing gravity moves this cross-over point to notably earlier in the bubble's life due to the combined influence of increased relative buoyancy and decreased relative capillary forces.

The results obtained from the simulation at the lowest gravity level tested, 0.1 g, have been compared with the results obtained for terrestrial gravity. Since the simulations had to be interrupted prior to bubble departure, for reasons discussed earlier, the values of the various parameters at detachment are unknown. For this reason it has been decided to perform only dimensional comparisons in order to elucidate the differences in the bubble growth dynamics.

Fig.6.21 shows the evolution of the bubble shapes for the two cases considered. It is clear how the bubble for the lowest gravity maintains a nearly perfect spherical shape during its growth. The growth rate continues to remain constant and equal to the imposed one, as depicted in Fig.6.22. Due to the nearly negligible buoyancy force the bubble is no longer drawn vertically upward and the motion of the centre of gravity tends to be asymptotic, in stark contrast to the terrestrial gravity case.

The spherical-like shape of the bubble notably reduces the contact angle. The simulation was interrupted when the contact angle appeared to reach its

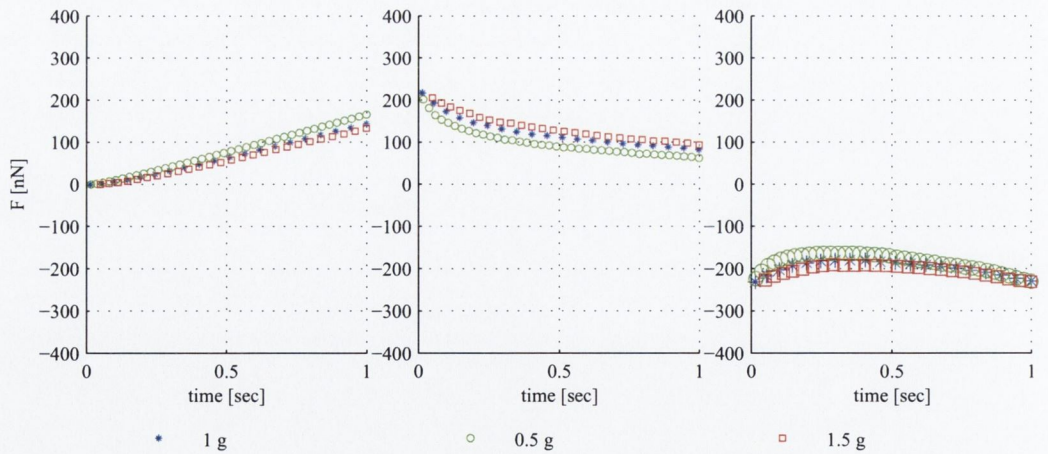


Figure 6.20: Vertical component of Buoyancy Force (left), Contact Pressure force (middle) and Capillary Force (right) acting on the bubble.

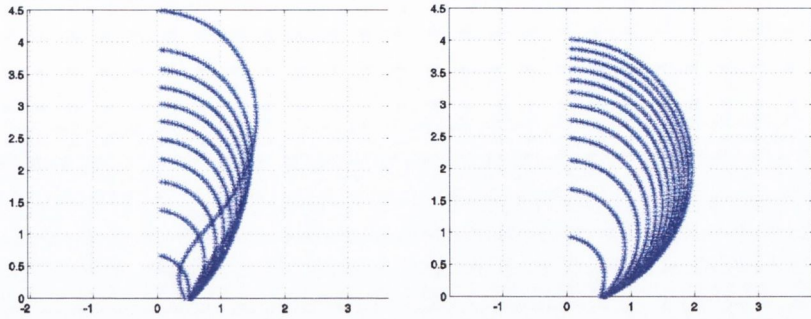


Figure 6.21: Bubble shape evolution at 1 g (left) and 0.1 g (right)

minimum value. A reduction of the minimum value of about 40% has been observed between 1 g and 0.1 g. With regard to the evolution of the forces it is quite obvious that the lowering of the contact angle is necessary for the capillary force to counter changes in the only remaining downwardly directed contact pressure force.

The influence of the gravity field on bubble growth and departure has been analysed. These preliminary findings show that the gravity field plays an

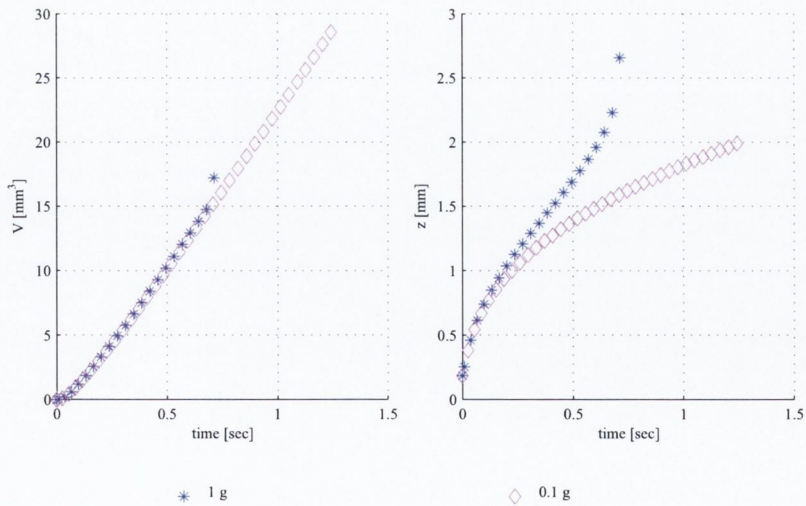


Figure 6.22: Volume (left) and centre of gravity (right) evolution at 1 and 0.1 g

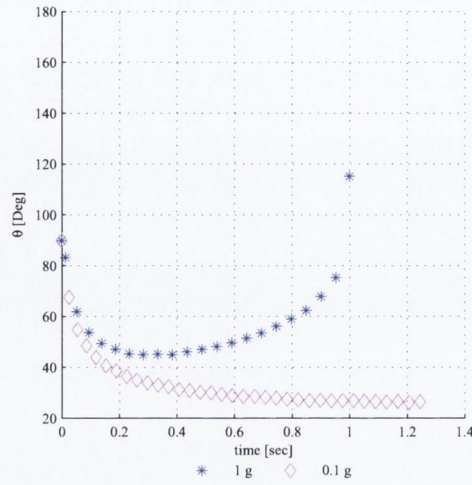


Figure 6.23: Contact angle evolution in normal and microgravity

important role in the growth evolution. Higher values of gravity corresponds with shorter growth times and smaller bubbles at departure. On the contrary, a reduction of the magnitude of gravity delays bubble departure and these bubbles detach from the surface with a larger volume. Not unrelated, the bubble shape is also influenced by the gravitational field. In the extreme case of the lowest gravity level tested, the evolution predicted numerically is consistent with previous experimental observation. The bubble, in the near absence of an upward directed force, maintains a truncated sphere in agreement with its natural disposition to assume the minimal surface area possible.

CHAPTER 7

RESULTS AND DISCUSSION: EHD

7.1 General Bubble Growth Behaviour

The evolution of gas bubbles growing from a 1 mm orifice in a basin filled with HFE-7000 is given in Fig.7.1 for different electric field strengths. In the absence of an electric field, which will be called the field-free case, Fig.7.1a shows that the growth generally follows the behaviour already discussed in the previous chapter. During early growth the bubble is almost a truncated sphere, for mid growth it has an elongated shape and near departure a neck forms. As it is shown in Figs.7.1b and 7.1c, the effect of an electric field influences the bubble growth and shape in a significant way. From Fig.7.1b and Fig.7.1c it is evident how the bubbles are stretched in the vertical direction and squeezed in the horizontal one. A 3D reconstruction of the field-free bubble and a bubble in the highest imposed electric field prior to departure is shown in Fig.7.2.

The electric field strength affects the bubble shape from the time it emerges from the orifice as is clear in the first image of Fig.7.1c where the hemispherical contour is no longer evident. In the mid growth stage the elongated characteristic is due solely to the influence of the electric field. It is clear that the bubble has no liquid volume under the bubble dome so that, in contrast with the field-free case, there is no buoyancy force pushing the



Figure 7.1: Photographic sequence of bubble growth under the influence of an electric field strength of (a) 0 (b) 0.4 and (c) 0.8 kV/mm.

bubble upward. Here the bubble can be divided in two main sections: a top section where the bubble has an ellipsoidal shape and a lower section with a more cylindrical geometry. Finally, the necking stage is considerably influenced by the electric field. As will be discussed later, the electric field creates a compressive pressure around the bubble interface which tends to reduce its cross section. The effect of this external pressure is more evident on the lower bubble region which reduces its section until the detachment of the bubble.

It should be noted that for the highest electric field strength the bubbles tended to vibrate slightly, probably due to fluid impurities and a not perfectly axis-symmetric electric field. The effect of these vibrations was analysed by comparing the growth behaviour of many different bubbles. From the comparison it was concluded that the small vibrations had a negligible random effect on the overall bubble growth behaviour.

An uncomplicated method for quantifying the influence of the electric field strength is by measuring the major axis length, d , and the minor axis length, w , as shown in Fig.7.3, and comparing the bubble Aspect Ratio, defined as

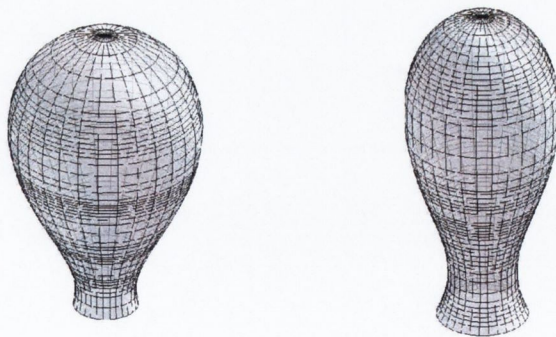


Figure 7.2: Three dimensional reconstruction of a bubble prior to detachment for electric field strength of 0 kV/mm (left) and 0.8 kV/mm (right).

$$AR = \frac{d}{w} \tag{7.1}$$



Figure 7.3: Definition of the bubble minor and major axis

Some results are plotted in Fig.7.4 which shows the bubble aspect ratio versus dimensionless time. For convenience a common $t = 0$ start condition

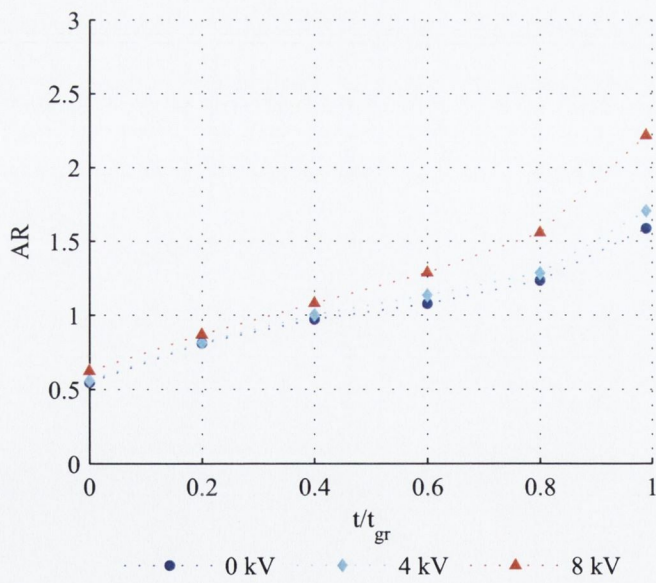


Figure 7.4: Bubble aspect ratio versus dimensionless time for different electric field strengths.

is chosen when the bubble volume equals the hemispherical shape volume. It is clear that increasing the electric field strength tends to increase the bubble aspect ratio over the entire life of the bubble. Also, the increased slope of the electric field cases indicates that the influence tends to increase as the bubble gets larger and is largest near departure.

As suggested by Chen et al. (2007), the aspect ratio of a bubble at departure in the presence of electric field (AR_e) can be compared with the one in absence of electric field (AR_0) by calculating the relative aspect ratio (AE_e/AE_0). This is plotted in Fig.7.5a and Fig.7.5b against the electric field strength and the electrical Weber number. The electrical Weber number, which represents the ratio between the electric and the surface tension stresses, is defined as;

$$We = \varepsilon_0 \varepsilon_{rl} E_0^2 \frac{r_d}{\sigma_{HFE}} \quad (7.2)$$

where r_d is the bubble equivalent radius at departure, defined by Chen et al. (2007) as:

$$r_d = (dw^2)^{1/3} / 2 \quad (7.3)$$

while E_0 is the electric field strength, $E_0 = U/L$ where U is the voltage applied on the top electrode and L is the distance between the two electrodes.

Fig.7.5 shows that the relative aspect ratio increases with the electric field strength and electrical Weber number, meaning that for the same major axis length, the minor one decreases. Moreover, Fig.7.5b indicates a linear relationship between the relative aspect ratio and the electrical Weber number, as was observed by Chen et al. (2007). Furthermore, very close agreement between the data of this work and that of Chen et al. (2007) is

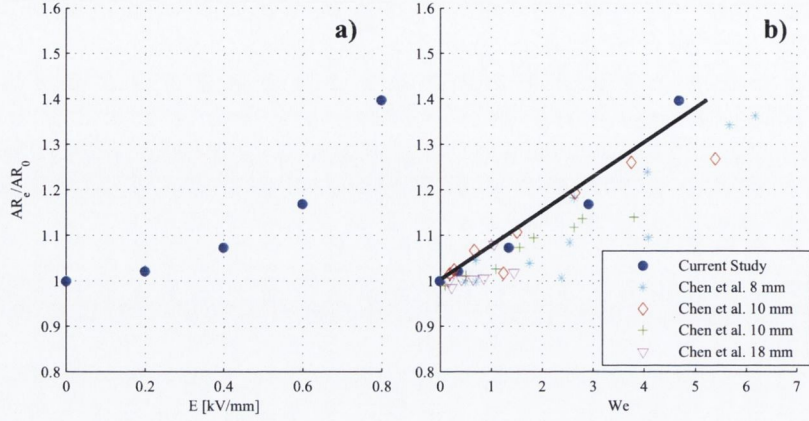


Figure 7.5: Evolution of the relative Aspect Ratio with the electric field strength (a) and with the electrical Weber number (b).

observed, in particular for electrode spacing close to that of this study. The linear regression fit to the experimental data is given by:

$$AE_e/AE_0 = 1 + 0.0759 \cdot We \quad (7.4)$$

The slope is somewhat higher than that calculated by Chen et al. (2007) and it is believed that this is because it is dependent on the experimental conditions such as the orifice diameter or the electrode spacing.

Fig.7.6 shows the evolution of the bubble centre of gravity for each high voltage tested for both dimensional and dimensionless scales. It is evident that the electric field reduces the growth time of the bubble even though it does not have a strong influence on the position of the centre of gravity prior to the departure. For the greatest part of the growth period the vertical coordinate of the centre of gravity increases almost linearly with time. When the bubble passes the unstable equilibrium point the necking stage begins and the centre of gravity increases sharply as the bubble leaves the surface. The non-dimensional plot shows how the trends are generally quite similar,

though there seems to be some deviation at the higher electric field strengths.

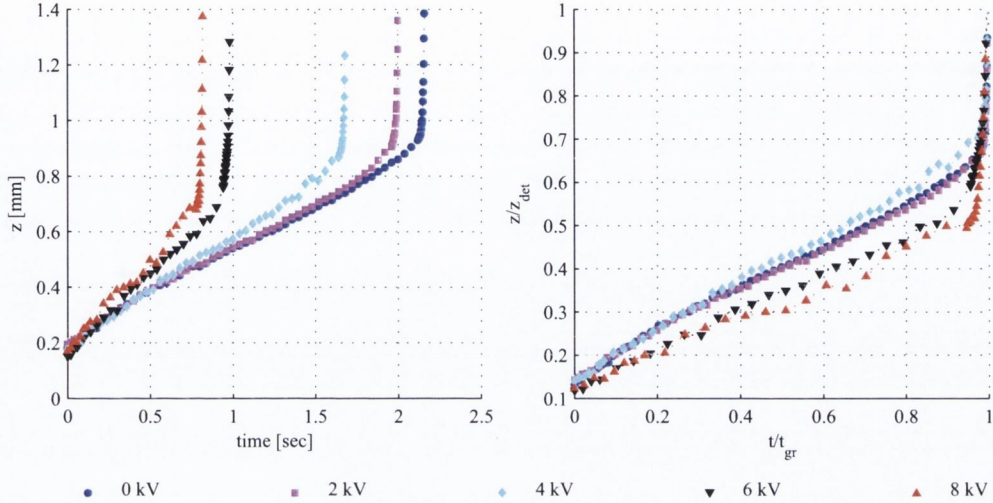


Figure 7.6: Vertical position of the centre of gravity in dimensional (left) and non dimensional (right) scales.

The total bubble volume as well as the volume acted on by buoyancy are given in Fig.7.7 for both dimensional and non-dimensional scales. It is clear that the volumetric growth is nearly linear in all cases indicating an approximately constant injection rate. Furthermore, unlike the centre of gravity, the bubble departure volume decreased notably with increased electric field strength. Such behaviour is consistent with previous work, Cosentino et al. (2005), who also observed a premature detachment stage caused by the effect of the electric field.

Unlike what was observed for the water experiments, a notable increase of the bubble volume with time is evident at the end of the bubble growth during the necking stage. At first this seemed quite perplexing. However, upon closer scrutiny of the triple contact line it was found that the inflection point corresponded with the contact line detaching from the orifice rim and sinking into the orifice. This is illustrated in Fig.7.8. As the liquid starts to

penetrate into the orifice it displaces an equal volume of air directly into the bubble, increasing the bubble growth rate. Unfortunately, the rig does not facilitate visual access into the orifice so the shape and motion of the triple contact line could not be measured once it was below the rim of the orifice.

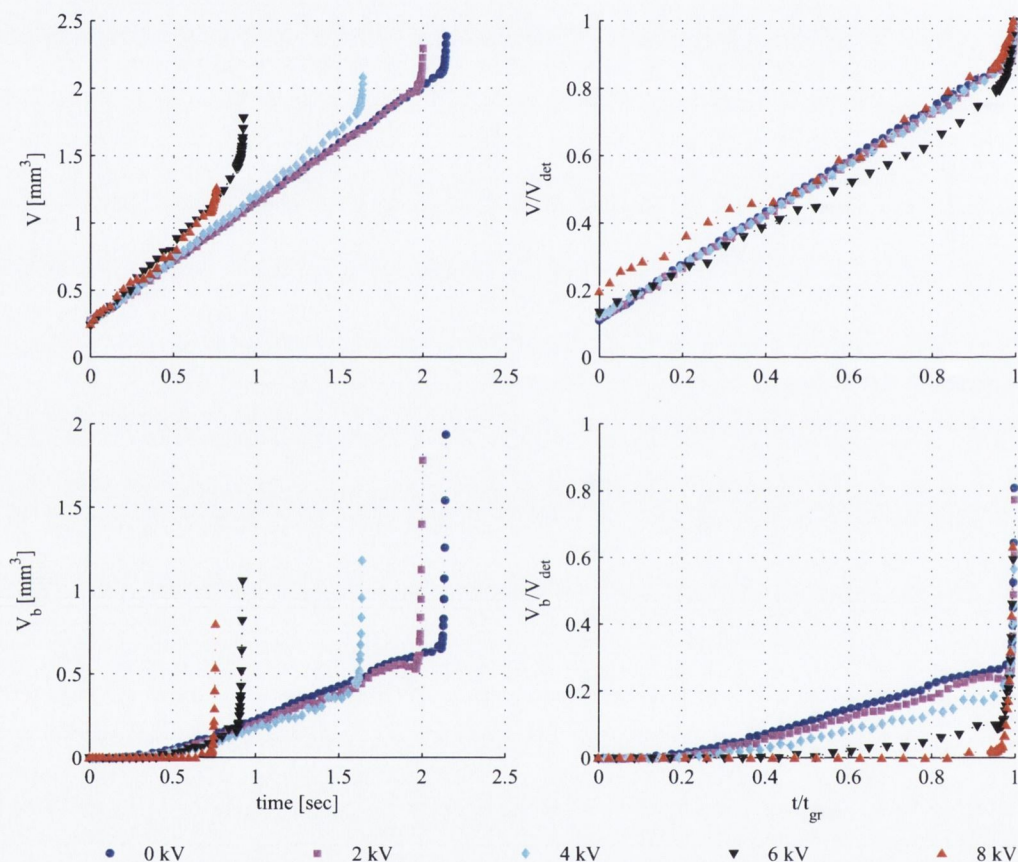


Figure 7.7: Bubble volume (top) and Buoyancy volume (bottom) in dimensional (left) and non dimensional (right) scale.

From the buoyancy volume graphs it is clear that for the field-free case it increases linearly with time, as does the bubble volume. The final sharp increase at the end of growth coincides with the necking stage. Here, the pinching of the neck considerably increases the volume of the bubble

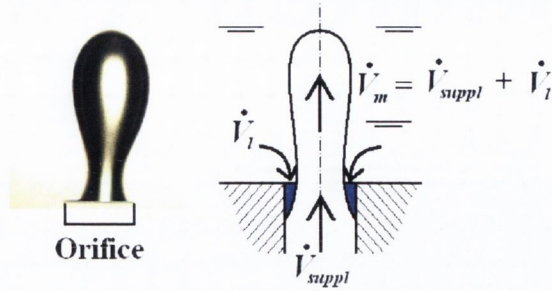


Figure 7.8: Increase of the measured growth rate at detachment.

completely surrounded by the liquid as was discussed in the previous chapter.

In the non-dimensional comparison the value of the bubble volume at detachment has been used as the characteristic scale. In this way is it possible to roughly quantify the fraction of the bubble which generates the upward buoyancy force near departure. The figure shows the influence of the electric field on the buoyancy volume evolution. It decreases with the electric field strength and, for the highest voltage it is zero for the greatest part of the bubble growth. For the field-free case, when the bubble reaches its unstable equilibrium configuration, the buoyancy volume is about 30% of the entire bubble volume. For a highest voltage of 8 kV it is only about 5% which is a significant reduction.

Fig.7.9 shows the evolution of the contact angle for different electric field strengths. It is clear that the contact angle increases with the voltage and does so for the entire lifetime of the bubble. It must be noted, however, that after the point at which the triple contact line sinks into the orifice it was not possible to measure the true contact angle and the values shown are likely overestimated for the short time interval during the necking stage.

For the field-free case the bubble grows from a hemispherical shape to a truncated spherical shape and the contact angle changes from an acute angle to an obtuse angle, dropping to about 70°. This is maintained during

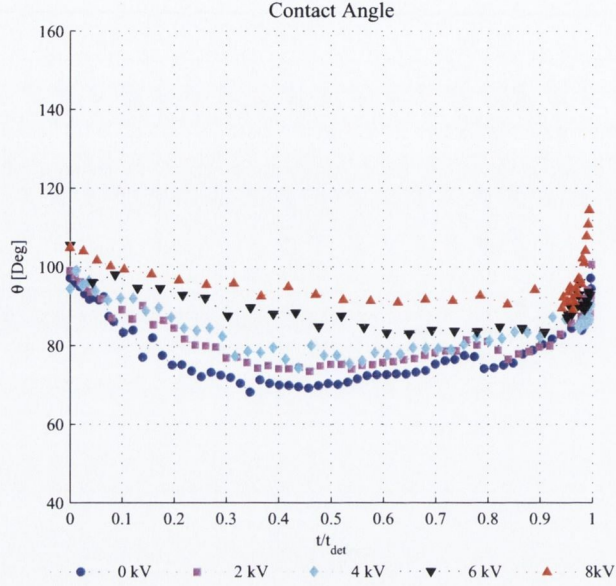


Figure 7.9: Contact angle evolution in dimensionless time scale.

the bubble elongation phase as the buoyancy volume portion becomes large enough to stretch the bubble. Here the contact angle starts to increase mildly. During the detachment stage, when the neck is forming at the bubble base, the contact angle quickly increases. This type of behaviour was discussed in detail in the last chapter.

In contrast, considering the highest electric field strength, the contact angle decreases but only marginally. In fact, the contact angle remains obtuse for the entire growth phase. This is of course related to the above discussion regarding the buoyancy volume. During the mid growth phase it remains essentially constant at around 95° until the bubble departure phase begins and it increases dramatically.

Fig.7.10 shows the evolutions of the vertical forces, calculated experimentally, acting on the bubble for electric field strengths of 0, 0.4 and 0.8 kV/mm.

For the field-free case it is clear that the magnitudes of the forces

are smaller than those associated with water. This is essentially due to the different surface tension of the dielectric fluid which is approximately six times smaller than that of water. Also, consistent with the water experiments, the net force for this case is, within experimental uncertainty, close to zero confirming the absence of dynamic forces.

For an electric field strength of 0.4 kV/mm the electrohydrodynamic force, F_E , tends to become slightly negative, possibly indicating a retarding force. This is more evident for the highest electric field tested. Here the electrohydrodynamic force is clearly in opposition to bubble translation. However, its magnitude tends to decrease with time and is nearly zero at detachment.

Fig.7.10 also illustrates the manner in which the electric field influences the buoyancy force acting on the bubble, F_B . Clearly it decreases with increasing electric field strength and is related to the buoyancy volume evolution previously discussed. In fact, for the highest electric field strength the buoyancy force is equal to zero for the most part of the bubble formation process, only increasing at the very end during the detachment stage.

The capillary force, F_C , is generally constant for the higher electric field strengths since, as discussed, the contact angle only changes marginally. For these cases the capillary force and contact pressure force, F_{CP} , are in balance since there is no bubble volume for buoyancy to act. Previous experiments performed in terrestrial gravity and microgravity discovered a minor effect of the gravity on bubbles growing in electric fields and stated that *'the application of electric field showed effective in providing a force able to remove the bubbles away from the orifice and in promoting bubble departure at values of diameter greater but of the same order of magnitude as in normal gravity'*, Di Marco et al. (2003). This observation is consistent with this work and leads

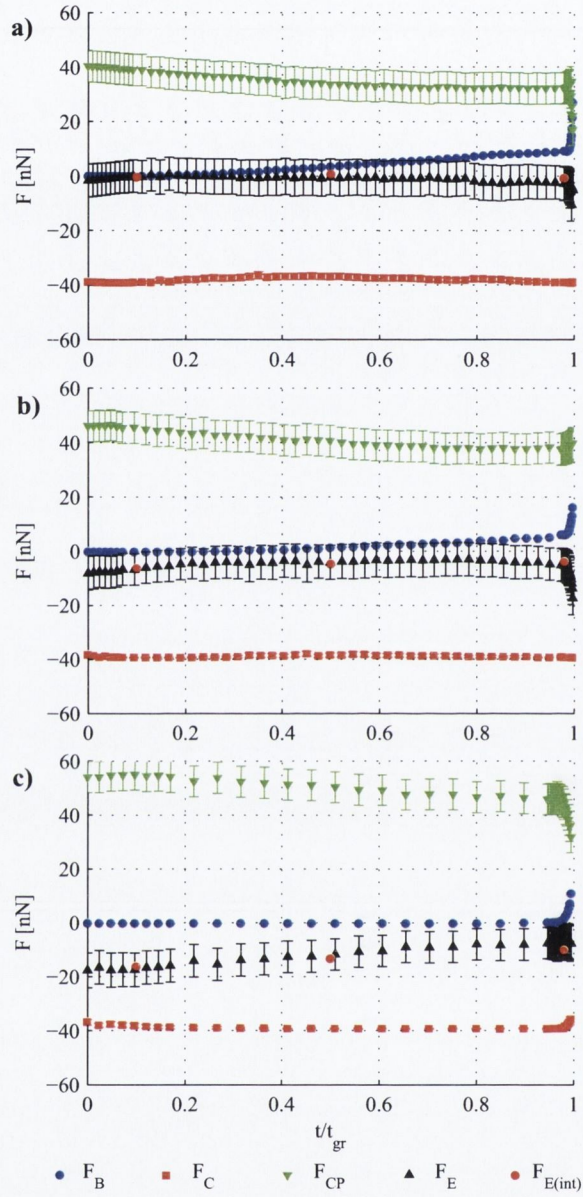


Figure 7.10: Vertical forces acting on the bubble for electric field strengths of 0 (a), 0.4 (b) and 0.8 (c) kV/mm

to the conclusion that, for high enough electric field strengths, the electric field plays a predominant role in the bubble growth dynamics compared with gravity which is, for the most part, unimportant. Interestingly, this is mostly due to the influence that the electric field has on the bubble shape since the

magnitude of the electrically induced vertical force is still small compared with the magnitudes of the capillary and contact pressure forces. Also, the electric field has a strong influence on bubble detachment as it is seen to decrease with increasing field strength. As will be discussed, the earlier detachment is due to the effect of the electric field on the bubble neck.

The bubble shapes in absence of electric field and for the highest electric field strength are reported in Fig.7.11 for three different non-dimensional times. These times have been chosen to be representative of early growth, mid growth and near detachment. In Fig.7.12 and Fig.7.13 the bubble curvatures and pressure distributions are compared for the latest non-dimensional time only.

As expected, Fig.7.11 shows that a bubble growing in the presence of an electric field is subject to a constrictive action along the horizontal direction. During its growth, the bubble does not assume the truncated spherical configuration like the field-free bubble, and its elongation is due only to the electrohydrodynamic influence.

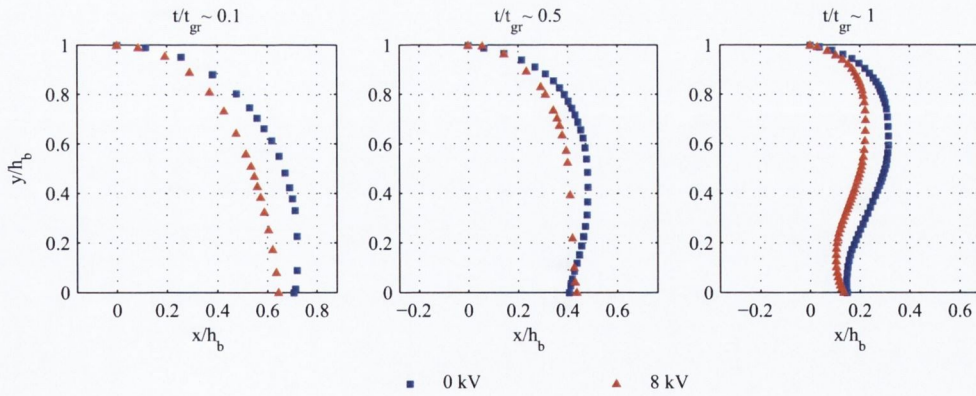


Figure 7.11: Non dimensional comparison of bubble shape in absence and for the highest electric field strength tested.

The net vertical electric field force acting on the bubble has also been calculated by integrating the local net pressure over the entire bubble surface. The results obtained for the three principal non-dimensional times are represented in Fig.7.10 with red markers, $F_{E(int)}$. The good agreement between the value calculated through force balance with the one calculated with the integration process confirms the validity of the experimental analysis.

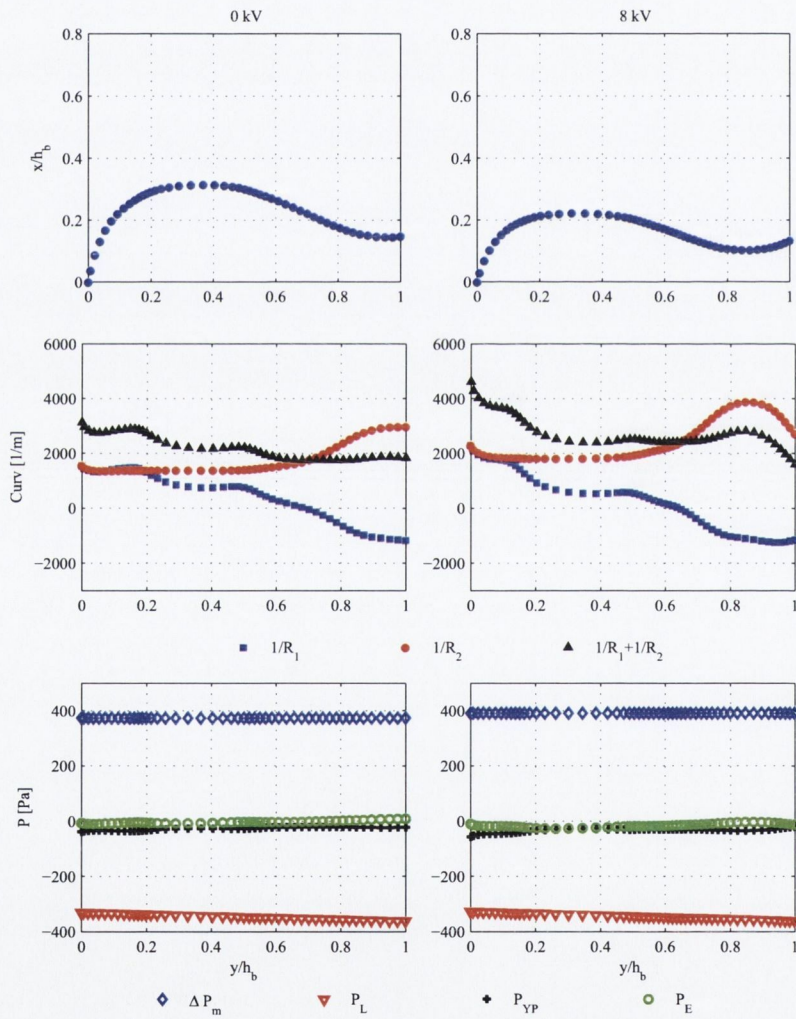


Figure 7.12: Effect of the electric field on the bubble curvatures (middle) and pressure distribution (bottom).

As is evident the two methods give a consistent result with a low margin of difference. The validity of the force balance approach adopted to evaluate the net force acting on the bubble has thus been confirmed.

In order to better understand the influence of the electric field on the bubble during the different stages of growth requires more detailed information about the electric field. Since it is not feasible to measure the electric field distribution a numerical approach, similar to that utilized by Dong et al. (2006) and Chen et al. (2007) will be employed and discussed in the next section.

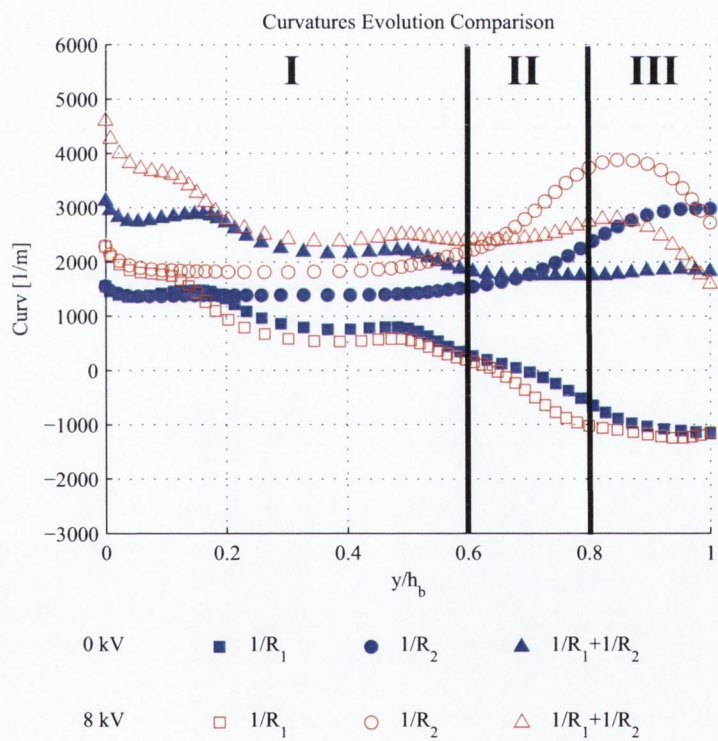


Figure 7.13: Curvature comparison during the detachment stage.

7.2 Finite Element Analysis

The analysis of the influence of the electric field on the bubble growth evolution has been studied in more details using the finite element software package COMSOL. Back details to this have been given in Sec:4.2. Fig.7.14 shows the evolution of the equipotential lines and electric field vectors of a bubble during its detachment stage. The figure refers to the highest electric field strength tested.

As it is possible to see, the effect of a body with a step change in dielectric permittivity is influences the electric field in the region next to the body itself. The presence of the bubble has a negligible influence on the electric field further out from the interface as is clear from the parallel equipotential lines and electric field arrows.

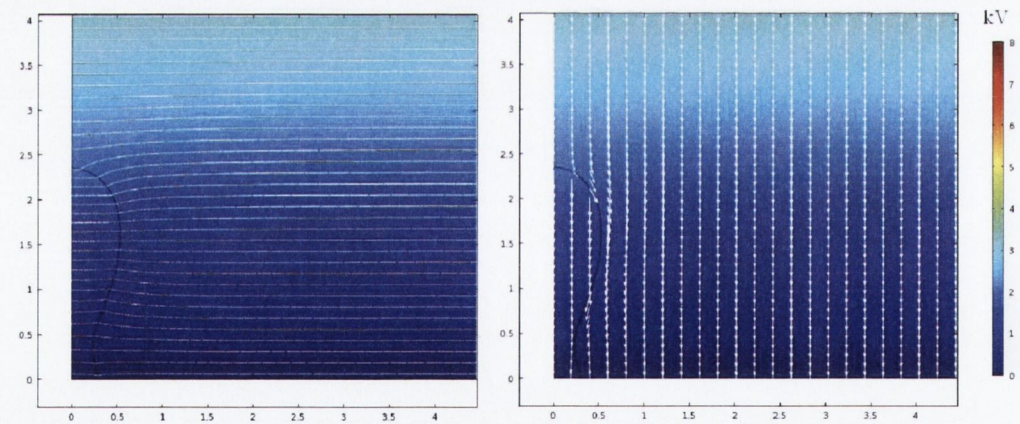


Figure 7.14: Equipotential lines (left) and electric field arrows (right) of a bubble next to the departure for the highest electric field strength.

The software package facilitates the calculation of both the vertical and horizontal component of the electric field. Those are reported in Fig.7.15 and 7.16, respectively, whilst Fig.7.17 shows the magnitude of the electric

field strength. The figures refer to the cases of 0.4 kV/mm and 0.8 kV/mm for the three different non-dimensional times. For comparison purposes the same false colour scale has been applied to both field strengths so that relative changes in colour qualitatively represent changes in the field strength. Perhaps clearer than in Fig.7.14 it is evident that the presence of the bubble affects only the region next to the bubble, while the rest of the domain remains undisturbed.

From Fig.7.15 a considerable variation of the vertical component of the electric field is clearly visible on the top region of the bubble for each non-dimensional time. Approaching the interface along the vertical axis from the liquid phase the vertical component decreases in absolute magnitude reaching a minimum in the proximity of the interface. The passage to the gas phase is characterised by a sudden increase calculated to be of the same order of the electric field strength (440 kV/m for the 0.4 kV/mm case and 814 kV/m for the 0.8 kV/mm case, for all the non-dimensional times). Moving downward along the bubble interface the discontinuity between the two phases becomes less pronounced.

As illustrated in Fig.7.16, the horizontal component of the electric field presents its maximum variation on the top external side of the bubble. This corresponds with the region where the streamlines, as the electric vectors, show the largest distortion (see Fig.7.14). The intensity of the maximum variation of the horizontal component has been compared by calculating the difference between the gas and liquid phases of the vertical horizontal component in the proximity of the maximum discontinuity region. For each dimensionless time it has been found that for the 0.4 kV/mm case the increase is about 0.53 times the electric field strength (210 kV/m). This decreases for the 0.8 kV/mm cases up to 0.47 times the electric field strength (380 kV/m).

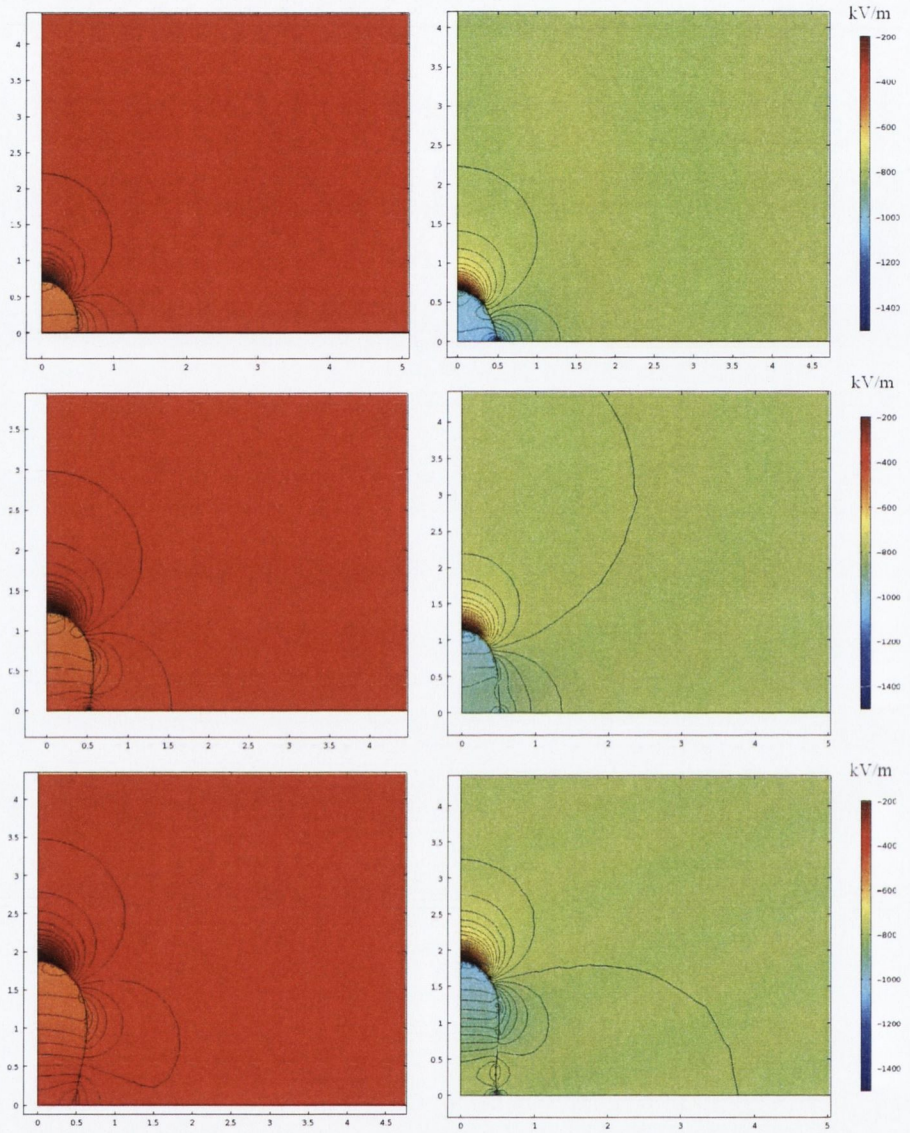


Figure 7.15: Vertical component of the electric field at three dimensionless times for an electric field strength of 0.4 kV/mm (left) and 0.8 kV/mm (right).

As it is possible to imagine the magnitude of the deviation is related to the shape of the bubble. The bubble growing with the highest electric field tends to be more elongated along the vertical direction compared with the lower and null electric field cases due to the increased electrically induced

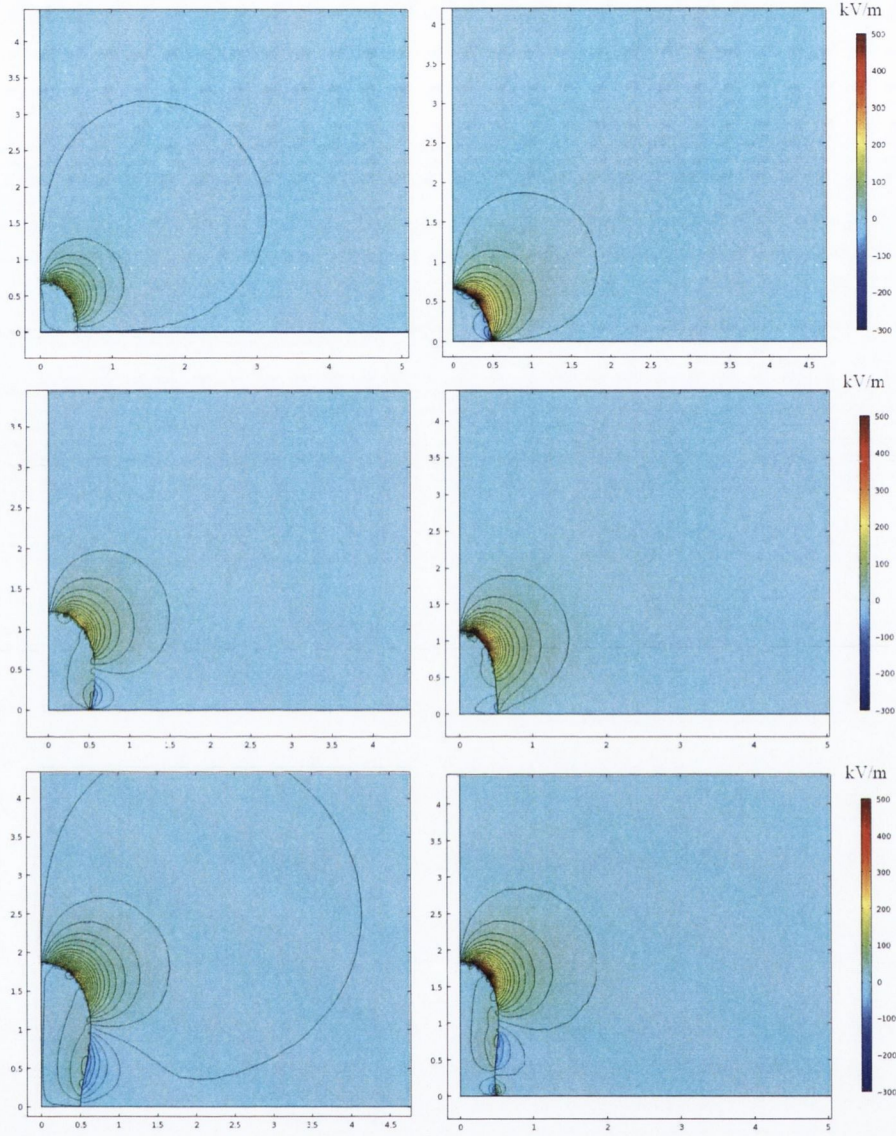


Figure 7.16: Horizontal component of the electric field at three dimensionless times for an electric field strength of 0.4 kV/mm (left) and 0.8 kV/mm (right).

stresses pushing on the bubble in this region.

The intensity of the electric field is shown in Fig.7.17. This, as well as the vertical component, presents a visible decrease in the fluid phase in the proximity of the top region of the bubble. An increase of the same order of

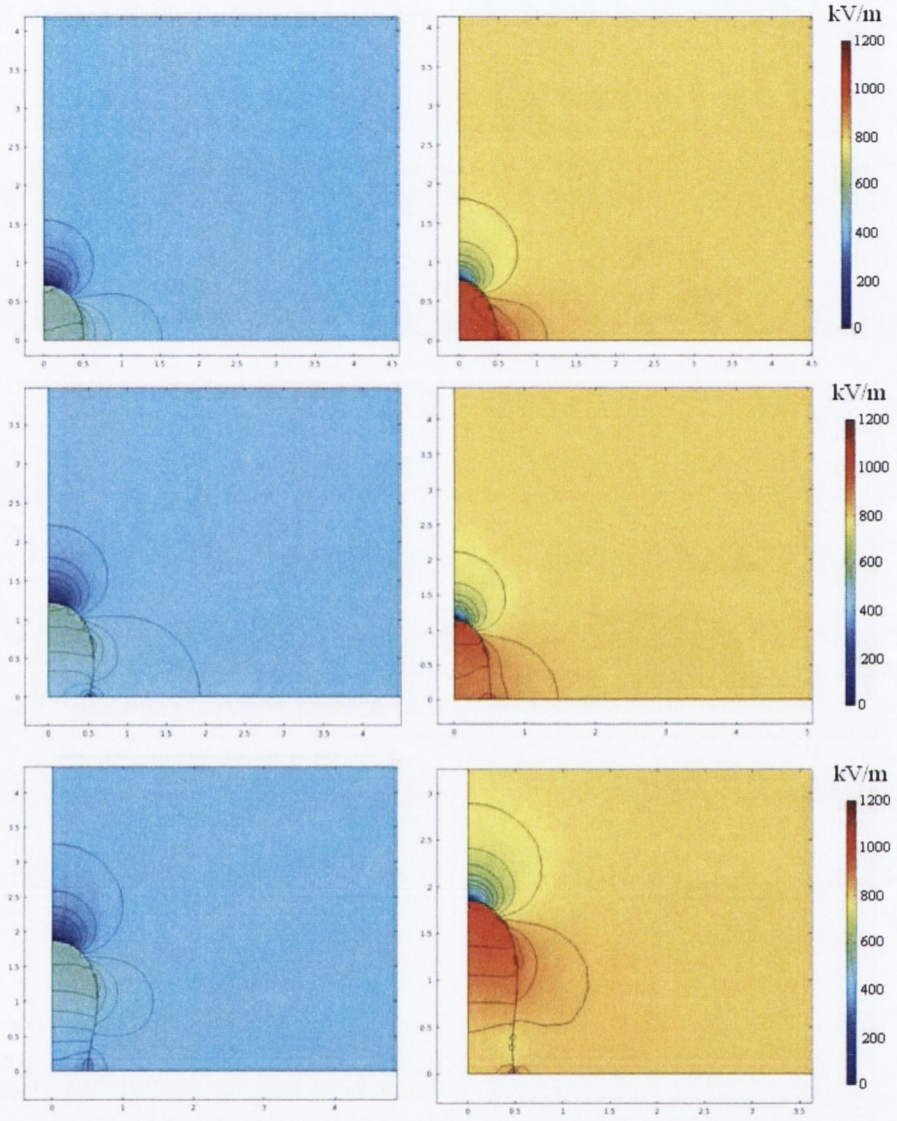


Figure 7.17: Electric field magnitude at three dimensionless times for an electric field strength of 0.4 kV/mm (left) and 0.8 kV/mm (right).

the electric field strength is observable across the gas-liquid interface. This is not surprising since the horizontal component is very low or equal to zero in the proximity of the vertical axis.

The presence of the bubble deforms the electric field from what would be

uniform if the bubble were not present. From the action-reaction principle the electric field generates a stress field around the bubble proportional to the deformation and intensity of the electric field. The bubble being a deformable body, changes its shape until an equilibrium condition between the bubble shape and the surrounding electric field is reached.

Fig.7.18, Fig.7.19 and Fig.7.20 show the evolutions of the electric field stress, calculated both numerically and experimentally, together with the Young-Laplace deviatoric stress. The pictures refer to electric field strengths of 0.4, 0.6 and 0.8 kV/mm , respectively. Adequate agreement is observed between the measured and simulated electrically induced pressure curves for all cases. The curves follow similar trends and are generally close in magnitude. This is true for all the three stages confirming the validity of both the experimental and numerical approaches. The differences are likely due to limits in the image processing.

Figs.7.18 to 7.20 show that there is a considerable interaction between the bubble shape and the electric field stress acting on it. The deformation of the bubble is related to the electric field strength, as already shown in Fig.7.4 where the aspect ratio of the bubble increases with increasing electric field strength and tends to be accentuated at larger bubble volumes. For the lower electric field strength, Fig.7.18 shows that the electric stress is about one order smaller than the Young-Laplace deviatoric stress. Thus, it is not large enough to deform considerably the bubble shape. Increasing the electric field increases the intensity of the electric field and thus the magnitude of the electrically induced stress to the point that it influence the bubble shape considerably. This is clear in Fig.7.20 for the highest electric field tested. For this case the electric field stress tends to be very small at the bubble tip. Moving downward along the bubble surface it increases in magnitude,

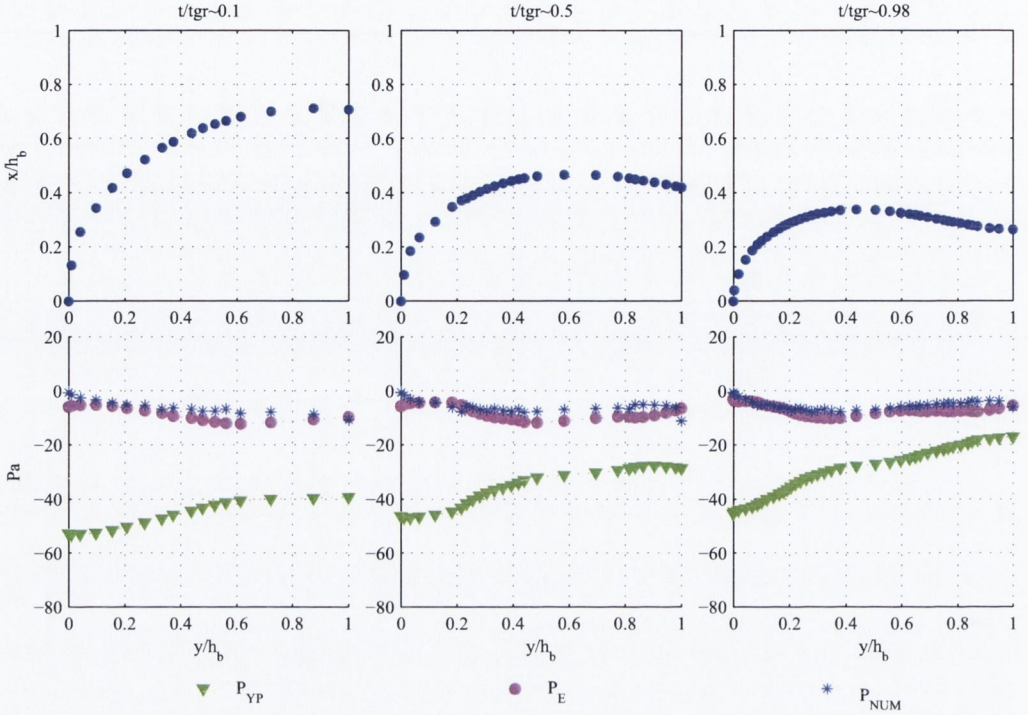


Figure 7.18: Comparison of electric stresses evaluated experimentally (P_E) and numerically (P_{NUM}) for an electric field strength of 0.4 kV/mm.

consistent with the electric field simulations, and becomes the same order of the Young-Laplace component. This being the case, the bubble shape is considerably different than the field-free case. During the departure stage its influence is more prominent in the horizontal direction compared with the vertical one and the bubble thus tends to be squeezed along the horizontal direction.

The negative value of the net vertical electrically-induced force reported in Fig.7.10 can also be explained in the context of the pressure field. By integrating the net pressure around the bubble surface it is possible to evaluate the vertical and horizontal component of the net force acting on the bubble. Its magnitude is proportional to the electric strength while its direction depends on the bubble shape

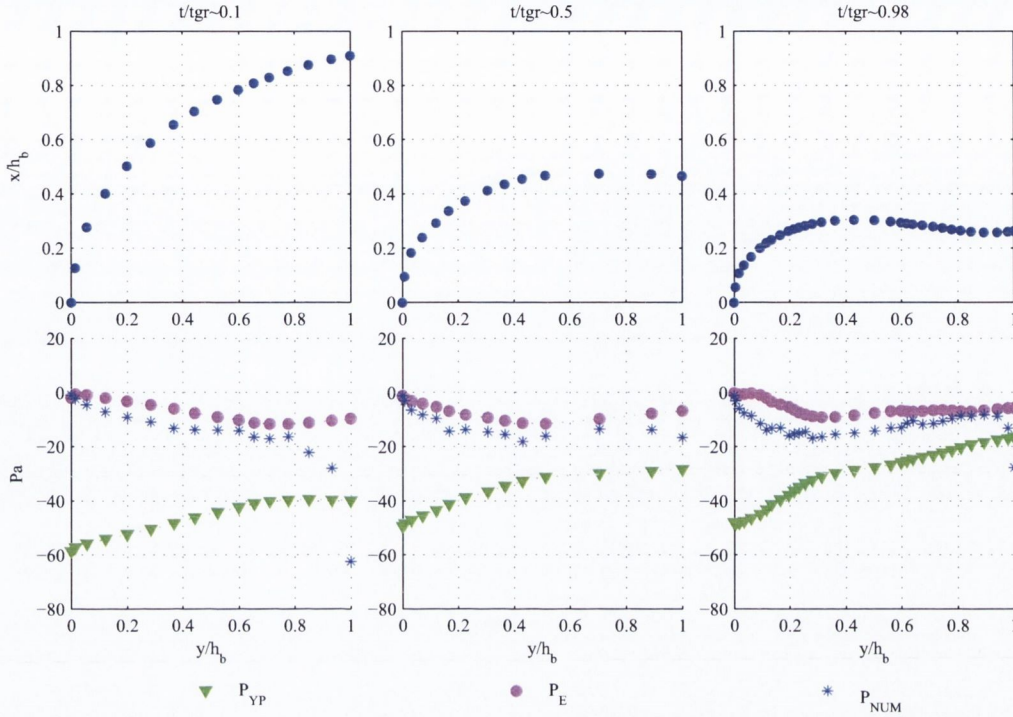


Figure 7.19: Comparison of electric stresses evaluated experimentally (P_E) and numerically (P_{NUM}) for an electric field strength of 0.6 kV/mm.

Fig.7.21 shows the comparison of the measured and simulated electric stresses for a bubble next the detachment for the highest electric field strength. Again good agreement is observed between the experimental and the numerical calculation. The magnitude of the stress is at its minimum at the bubble tip. Moving toward the base it increases its absolute magnitude and reaches an absolute maximum around the dimensionless coordinate $y/y_b=0.3$. This roughly corresponds with the maximum width of the bubble. After this point its magnitude begins to decrease and reaches a minimum at the neck after which it begins to increase once again to the bubble foot.

Fig.7.21 thus shows how the shape of the bubble is essentially due to a constriction effect of the electrohydrodynamic stress. There is very little buoyancy force pulling upward on the bubble and the elongated shape is

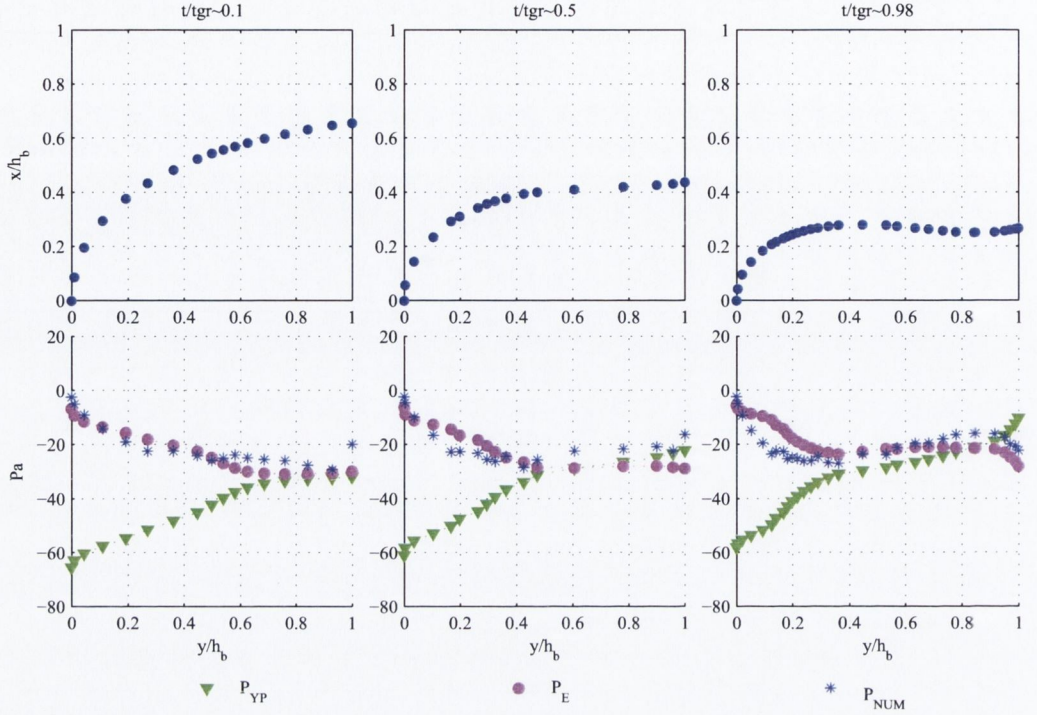


Figure 7.20: Comparison of electric stresses evaluated experimentally (P_E) and numerically (P_{NUM}) for an electric field strength of 0.8 kV/mm.

primarily due to the electric field. As already stated for Fig.7.10, for the highest electric field strength the buoyancy volume was zero for the most of the bubble growth. When the bubble was near detachment its value was only a small fraction of the entire bubble volume. It can thus be said that buoyancy plays only a small role in the bubble departure event. Thus, in contrast with the field free case where the bubble departure and rise events are governed by the interplay of vertical forces, the bubble departure event in the highest electric field is governed by stresses directed horizontally while the bubble rise is still governed by the buoyancy action, Pamperin and Rath (1995).

Thus, for high enough voltages the premature departure is more due to the action of the electric field which acts on the bubble neck encouraging

it to reach the point of instability which causes the pinching off process. The electric field literally tends to '*throttle*' the bubble at the neck until it deforms to the extent that it must detach. For the case here where gravity is present, once the neck stage begins the feedback action of the buoyancy force accelerates this process. However, even buoyancy plays a late role during the detachment for these tests, is it not correct to think that that is the cause of the bubble departure event. As already mentioned, experiments conducted in microgravity, Di Marco et al. (2003), showed that even in the absence of a gravitational field, the electric field is able to cause the bubbles to detach from the surface with a volume of the same order of the one observed in normal gravity. This observation is consistent with the mechanism discussed above.

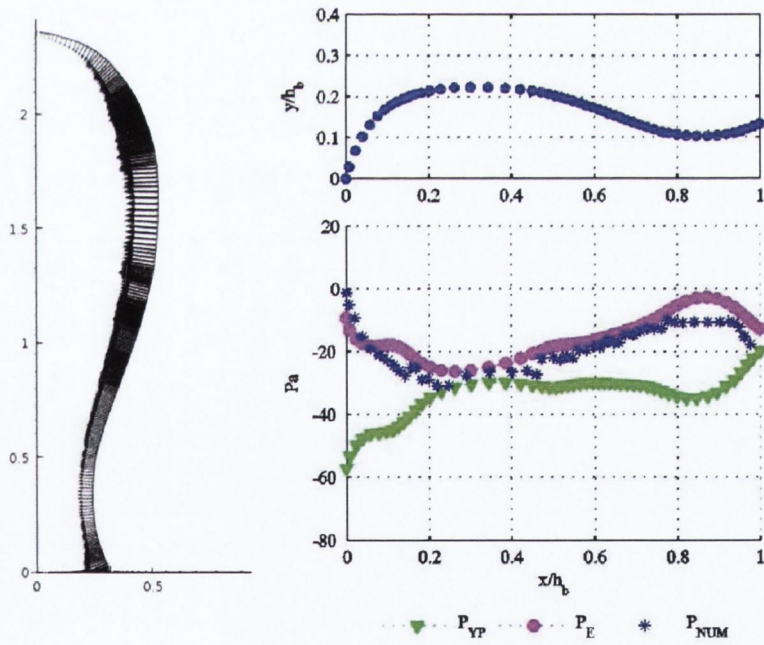


Figure 7.21: Evolution of the normal electro-dynamic stresses on the bubble interface (left) and comparison between experimental data and numerical calculation (right) for a bubble next to its departure for the highest electric field strength.

CHAPTER 8

CONCLUSION AND FUTURE WORK

The hydrodynamics of adiabatic gas bubble growth from a submerged orifice have been analysed with the view to elucidating the interdependence of the bubble shape and pressure field during its growth. By performing a pressure balance of all the components acting on the bubble it has been possible to evaluate the local net pressure acting on the gas/liquid interface. The influence of the dynamic pressure and Maxwell stress tensor has been also experimentally investigated. A set of numerical simulations have been also performed and compared with the experimental observations.

Water experimental results

The growth showed a remarkable dependence on the size of the orifice from which the bubble was growing. Dimensionless analysis displayed a weak effect of the flow rate on the evolution of the bubble. Profiles for the same non-dimensional time were found to be relatively similar for the same orifice at different flow rates while presenting remarkable differences for different orifice diameters. The evolution of the curvature and thus of the Laplace stress are affected by the dimension of the rim. Relatively similar behaviour was found for the largest and medium orifice, while the smallest was characterized by a clear dynamic evolution.

EHD experimental results

Thanks to the image processing capability it has been possible to observe and quantify the deformation of a bubble under the influence of an electric field. The bubble become elongated vertically, and its aspect ratio increased proportionally with the electric field strength. The deformation of the bubble also affected the force field acting on the bubble itself. In particular a drop in the buoyancy volume at high voltages was related to a proportional decrease of the buoyancy force and thus gravitational contribution on the bubble growth.

The effect of the electric field did not influence the growth rate of the bubble but rather induced earlier bubble detachment and subsequently caused the final volume of the bubble to decrease.

The use of commercial finite element package software allowed the interaction between the growing bubble and the electric field to be studied. A visible influence between the bubble and the electric field, and vice versa, was observed and the calculation of the electric stress tensor was in good

agreement with the experimental analysis.

The study showed how the electric field acts in reducing the maximum width of the bubble, in particular a correlation between the perturbation on the bubble base and the earlier necking stage was observed

Numerical results

The bubble profile predicted by the numerical approach has been compared with experimental results. The code correctly described the evolution of the main bubble proprieties such as the vertical position of the centre of gravity, the bubble volume, the contact angle and the internal pressure. The weak influence of the flow rate on the bubble evolution, already known from the experimental experience, was also observable in the numerical study using a non-dimensional scale.

The validity of the code was confirmed by comparing the bubble shape calculated numerically with the experimental ones for the same non-dimensional time. The comparison showed how the profiles and the evolutions of the main curvatures, and thus of the Laplace stress, match very well.

The use of a numerical method allowed analysis of the flow field around the bubble while it was growing and rising within the liquid domain. The bubble perturbs the fluid domain by creating a recirculation region in its proximity. The low influence of the liquid inertia has been confirmed by the low value of the liquid velocity around the bubble.

The influence of gravity was also analysed by changing the magnitude of the gravitational component. The simulations showed a considerable influence of gravitational level on all the bubble proprieties such as bubble frequency, departure volume etc. It has been observed that the bubble

detaching volume and departure time decreases at higher gravity levels.

The change in aspect ratio of the bubble modified the evolution of the force components. In particular, the contact pressure and capillary forces increased with the gravity level, the opposite behaviour was observed for the buoyancy component. A perfectly spherical bubble was observed for very low gravity levels showing very good agreement with previous works performed for similar conditions.

Future work

The validation of the software package encourages further application of the numerical approach in analysing the bubble evolution under different flow conditions. The study of vapour bubbles with the inclusion of heat exchange can be now investigated. Inertia effects can also be studied by increasing the velocity magnitude on the inlet boundary condition.

The study of the effect of the electric field on the bubble growth can be improved by increasing the quality of the measurement such as the pressure inside the bubble, or refining the image analysis in order to improve the experimental analysis. Moreover, another possible development includes the study of the electric field when it influences only a limited portion of the bubble. In particular the effect on the bubble base can be studied with the experimental apparatus scheme showed in Fig.8.1. In the experiment the bubble grows from an insulating material within a dielectric fluid. Four electrodes create a horizontal electric field which affects the shape of the bubble in the proximity of its neck. Preliminary simulations using *COMSOL* showed that for a particular electrode charge configuration, a non-uniform electric field can be created in the proximity of the orifice. Insulating material fixed at the top of the electrode can decrease the expansion of the electric

CONCLUSION AND FUTURE WORK

field on the entire fluid domain. The effect of the electric field in retarding or anticipating the bubble detachment can be studied by modifying the electrode configuration, and testing both DC/AD electric fields.

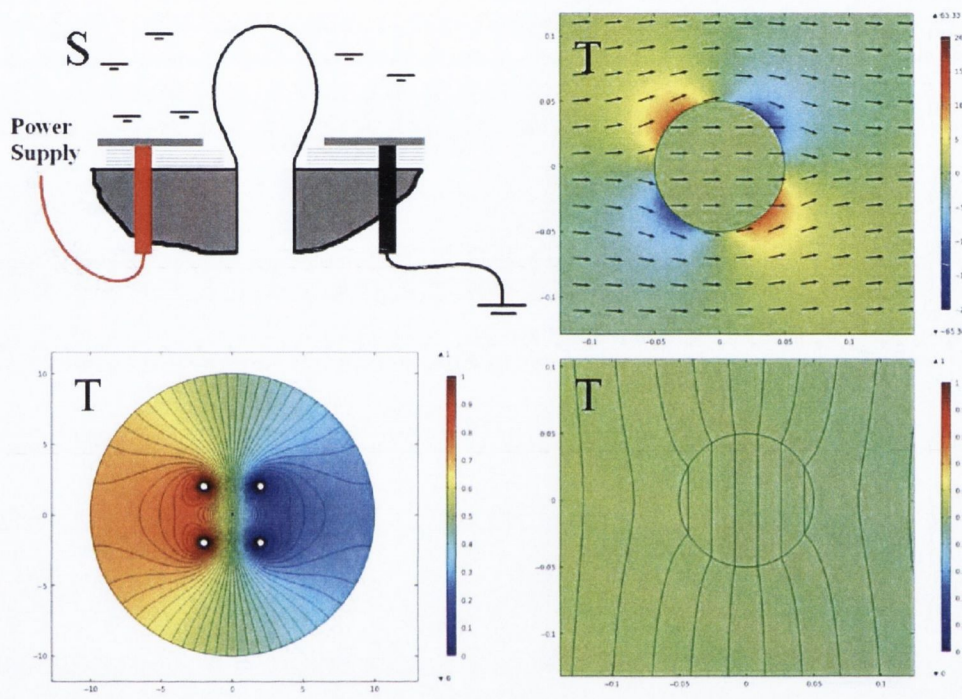


Figure 8.1: Possible experimental apparatus development (S) side view (T) top view.

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APPENDIX A

MATLAB CODES

```
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% STUDY OF A SINGLE BUBBLE %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
%% %% The study of a single bubble is done by uploading                %%%  
%% %% the frame that defines the bubble, a frame as background, a frame %%%  
%% %% used to define the scale and the .txt file of the voltage %%%  
%% %% evolution which defines the pressure magnitude.                    %%%  
%% %% The experimental condition, together with the value of the liquid %%%  
%% %% density and surface tension, are included in the files %%%  
%% %% Experimental_Data_namefluid. %%%  
%% %% The evolution of the bubble is obtained by a sequence of .BMP %%%  
%% %% files with a common initial name "Bubble_namefile_" and a final %%%  
%% %% number which defines the frame. %%%  
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
clc  
close all  
clear all  
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Input Experimental Data %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
Hole_Diameter      = 1.6,1,0.5; % [mm]  
ActualDist         = 1.6,1,0.5; % [mm]  
Vac_perm           = 8.854*10-12; % [N/V2]
```

MATLAB CODES

```

dist_mm                = 10;                                % [mm]
dis_m                  = dist_mm/1000;
%% %%%%%%%%%% Definition of the path where the picture are %%%%%%%%%%
%% %%%%%%%%%% stored and the bubble base name.                %%%%%%%%%%
direct=('C:\path\');
base_name=('Bubble_namefile_');
load([direct,'\Frame_Data.txt']);
%% %%%%%%%%%% Frame Definition %%%%%%%%%%
FrameRate              = Frame_Data(1);
FrameZero              = Frame_Data(2);
LastFrame              = Frame_Data(3);
Beg_BackFrame          = Frame_Data(4);
ScaleFrame             = Frame_Data(5);
StartFrame             = Frame_Data(6);
MiddleFrame            = Frame_Data(7);
MiddleFrame_II         = Frame_Data(8);
EndFrame               = Frame_Data(9);
t1                     = Frame_Data(10);
t2                     = Frame_Data(11);
t3                     = Frame_Data(12);
%% %%%%%%%%%% Definition of the liquid and experiment conditions %%%%%%%%%%
Experimental_Data_HFE;
%% %%%%%%%%%% Or alternatively %%%%%%%%%%
Experimental_Data_Water;
%% %%%%%%%%%%
HV_k                   = Frame_Data(14);
HV_V                   = HV_k*1000;
E_kV_mm                = HV_k/dist_mm;
E_V_m                  = HV_V/dis_m;
%% %%%%%%%%%%
%% %%%%%%%%%% Definition of the frame the user intents %%%%%%%%%%
%% %%%%%%%%%% to analyse, calculation of the detaching %%%%%%%%%%
%% %%%%%%%%%% time of the bubble and calculation of %%%%%%%%%%
%% %%%%%%%%%% the own time of the frame analysed %%%%%%%%%%
%% %%%%%%%%%%
Frame=EndFrame;
time_det=abs(LastFrame-FrameZero)/FrameRate;
single_time=abs(Frame-FrameZero)/FrameRate;
%% %%%%%%%%%% Importing the .BMP files %%%%%%%%%%
BackgroundBubble = imread ([direct,'\',base_name,'',...
num2str(Beg_BackFrame),'.BMP']);

```



```

Bubble          = imread ([direct,'\ ',base_name,'\ ',...
num2str(Frame),'.BMP']);
BubbleScaling    = imread ([direct,'\ ',base_name,'\ ',...
num2str(ScaleFrame),'.BMP']);
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Importing the .txt file with the trend %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% of the electric potential obtained from %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% the pressure transducer %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Pressure         = load    ([direct,'\P_volts.txt']);
Pressure(:,1)=Pressure(:,1)*time_det;
Pressure(:,2)=Pressure(:,2)*502;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Converting images to HSV %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
BackgroundBubbleHSV = rgb2hsv(BackgroundBubble);
BubbleHSV           = rgb2hsv(Bubble);
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Separating Channels %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
BackgroundBubbleH   = BackgroundBubbleHSV(:, :,3);
BubbleH             = BubbleHSV(:, :,3);
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Filtering %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
h                   = ones(5,5) / 25;
FiltBackgroundBubbleH = imfilter(BackgroundBubbleH,h);
FiltBubbleH         = imfilter(BubbleH,h);
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Select region of interest %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure('Units','centimeters','Position',[5 5 20 20])
imshow(Bubble);
title('Select area of interest (Top L- Bottom R) Note second'...
      'point should be at Plate level','FontSize',10,'fontname','times')
axis equal;
axis tight;
axis ij;
[X,Y]=ginput(2);
close;
X = int16(X);
Y = int16(Y);
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Subtracting Background from Bubble %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Difference         = (FiltBackgroundBubbleH-FiltBubbleH);
ClippedDifference   = Difference((Y(1):(Y(2)),(X(1):(X(2)))));
Enhancement        = (FiltBackgroundBubbleH./FiltBubbleH);
ClippedEnhancement = Enhancement((Y(1):(Y(2)),(X(1):(X(2)))));
ClippedBubble      = Bubble((Y(1):(Y(2)),(X(1):(X(2)))));
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Converting selected image to binary manually %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% by using a threshold value. %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

W          = find (ClippedEnhancement >= 1.5);
B          = find (ClippedEnhancement < 1.5);
BWClippedEnhancement = ClippedEnhancement;
BWClippedEnhancement(W) = 1;
BWClippedEnhancement(B) = 0;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Scale The Bubble %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
scale_matrix=zeros(1,2);
q=1;
for q=1:2
Scale_the_bubble
end
scale_min=min(scale_matrix);
scale_Max=max(scale_matrix);
scale=(scale_Max+scale_min)/2;
Delta_scale=abs(scale_Max-scale_min)/2;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Ghigliottin %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
BWClippedEnhancement=gigliottin(BWClippedEnhancement);
%% %%% Fills holes in the image,making it bubble and background only %%%
BWClippedEnhancement = imfill(BWClippedEnhancement);
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Finds boundaries in a b&w image %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Bound          = bwboundaries (BWClippedEnhancement,8);
bound          = Bound{1};
s              = regionprops(BWClippedEnhancement, 'all');
centroid       = cat(1, s.Centroid);
centroid       = int16 (centroid);
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Interface average between the left %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% and the right side of the bubble %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
symm_bound=make_bubble_symmetric(bound);
symm_bound=sortrows(symm_bound);
symm_bound(:,1)=symm_bound(:,1)-symm_bound(1,1);
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Smoothing the interface %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
fa=3;
fb=0.3;
bred7=Smooth_bondaries_symm(symm_bound,fa,fb);
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Calculation of bubble height and foot %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Bubble_Height=bred7(end,1);
Bubble_Height_mm=Bubble_Height*scale;
Bubble_Height_m=Bubble_Height_mm/1000;
Bubble_Foot=2*bred7(end,2);
Bubble_Foot_mm=Bubble_Foot*scale;
Bubble_Foot_m=Bubble_Foot_mm/1000;

```


MATLAB CODES

```
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%% Definition Bubble volume and centre of gravity %%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Xg_Vol=COG_Vol(bred7);
Centroid_Level=Bubble_Height-Xg_Vol(1);
Centroid_Level_mm=Centroid_Level*scale;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Volume=Xg_Vol(2);
Volume_mm=Volume*scale^3;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Volume_Bouy=Vol_Bouy(bred7,Volume);
Volume_Bouy_mm=Volume_Bouy*scale^3;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%% Find the pressure inside the bubble %%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Bubble_Pressure=Find_Pressure(Pressure,single_time);
%% %%%%%%%%%% Find the liquid pressure at the top of the bubble %%%%%%%%%%
Tip_Pressure = Base_Pressure - Bubble_Height_m*9.81*liquid_density;
%% %%%%%%%%%% FIND THE CONTACT ANGLE %%%%%%%%%%
Contact_Angle_Deg=Find_the_contact_angle(bred7);
Contact_Angle_Rad=pi*Contact_Angle_Deg/180;
%% %%%%%%%%%% Contact Perimeter and CP Area %%%%%%%%%%
Contact_Area_m=pi*(Hole_Diameter_m^2)/4;
Contact_Perimeter_m=pi*Hole_Diameter_m;
%% %%%%%%%%%% FORCES %%%%%%%%%%
%% %%%%%%%%%% BUOYANCY %%%%%%%%%%
Buoy_Force_Newton=Volume_Bouy_mm*(liquid_density_mill-air_density_mill )*9.81;
Buoy_Force_microNewton=Buoy_Force_Newton*1e6;
%% %%%%%%%%%% CAPILLARY %%%%%%%%%%
Capillary_Force_Newton =Contact_Perimeter_m*surface_tension*...
sin(Contact_Angle_Rad);
Capillary_Force_microNewton=Capillary_Force_Newton*1e6;
%% %%%%%%%%%%CONTACT PRESSURE %%%%%%%%%%
Upward_Force_Newton=Bubble_Pressure*Contact_Area_m;
Downward_Force_Newton=Tip_Pressure*Contact_Area_m;
%% %%%%%%%%%%
Contact_Pressure_Force_Newton=Upward_Force_Newton-Downward_Force_Newton;
Contact_Pressure_Force_microNewton=Contact_Pressure_Force_Newton*1e6;
%% %%%%%%%%%% DYNAMIC %%%%%%%%%%
Net_Force_Newton=Capillary_Force_Newton-(Contact_Pressure_Force_Newton...
```

```
+Buoy_Force_Newton);
Net_Force_microNewton=Net_Force_Newton*1e6;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Bubble_Prop(1,1)=single_time;
Bubble_Prop(2,1)=Volume_mm;
Bubble_Prop(3,1)=Centroid_Level_mm;
Bubble_Prop(4,1)=Volume_Bouy_mm;
Bubble_Prop(6,1)=Bubble_Height_mm;
Bubble_Prop(7,1)=Contact_Angle_Deg;
Bubble_Prop(8,1)=Bubble_Pressure;
Bubble_Prop(9,1)=Buoy_Force_microNewton;
Bubble_Prop(10,1)=Capillary_Force_microNewton;
Bubble_Prop(11,1)=Contact_Pressure_Force_microNewton;
Bubble_Prop(12,1)=Net_Force_microNewton;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Bubble_Propietes.Time=single_time;
Bubble_Propietes.Volume=Volume_mm;
Bubble_Propietes.Centroid_Level=Centroid_Level_mm;
Bubble_Propietes.Interface=bred7*scale;
Bubble_Propietes.Volume_Bouy=Volume_Bouy_mm;
Bubble_Propietes.Bubble_Height=Bubble_Height_mm;
Bubble_Propietes.Contact_Angle=Contact_Angle_Deg;
Bubble_Propietes.Bubble_Pressure=Bubble_Pressure;
Bubble_Propietes.Buoy_Force=Buoy_Force_microNewton;
Bubble_Propietes.Capillary_Force=Capillary_Force_microNewton;
Bubble_Propietes.Contact_Pressure=Contact_Pressure_Force_microNewton;
Bubble_Propietes.Net_Force=Net_Force_microNewton;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
close all
figure('Units','centimeters','Position',[5 5 10 10])
plot(bred7(:,2),bred7(:,1),'b*')
axis ij
axis equal
grid on
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
disp('And the bubble Proprietes are:')
Bubble_Propietes
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```



```

%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% STUDY OF THE BUBBLE EVOLUTION %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% The following program analyse the growth of the %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% bubble. The study is divided in tree parts, the %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% user has the possibility to change the frame step %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% of each part in order to focus the analysis on a %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% particula growth stage. %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% All the data of the bubble growth are stored in %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% the matrix "Total" which can be saved in the %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% user working directory. %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc, close all
clearex scale Delta_scale
Single_bubble_2010
close all
clc
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Numbers of iterations %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
First_N_FrameRate = abs(MiddleFrame - StartFrame);
First_N_Iteration = abs((MiddleFrame-StartFrame))/t1;
Second_N_FrameRate = abs(MiddleFrame_II - MiddleFrame);
Second_N_Iteration = abs((MiddleFrame_II-MiddleFrame))/t2;
Third_N_FrameRate = abs(EndFrame - MiddleFrame_II);
Third_N_Iteration = abs((EndFrame-MiddleFrame_II))/t3;
Number_FrameRate = abs((EndFrame-StartFrame));
clc
Number_Iteration = First_N_Iteration + Second_N_Iteration...
+ Third_N_Iteration + 1
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
total=zeros(Number_Iteration+2,11);
Error=zeros(size(total));
total(2,1)=abs(Beg_BackFrame-FrameZero)/FrameRate;
Error(2,1)=total(2,1);
k=2;
time= abs(StartFrame-FrameZero)/FrameRate;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% First Part %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

t=-t1;
first=StartFrame;
last=MiddleFrame;
Growing_Bubble_Final;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Middle Part %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
time = time-abs(t)/FrameRate;
t=-t2;
time = time+abs(t)/FrameRate;
first=MiddleFrame+t;
last=MiddleFrame_II;
Growing_Bubble_Final;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Final Part %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
time = time-abs(t)/FrameRate;
t=-t3;
time = time+abs(t)/FrameRate;
first=MiddleFrame_II+t;
last=EndFrame;
Growing_Bubble_Final;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
total(end+1,1)=time_growth;
Error(end+1,1)=time_growth;
[e1 e2]=size(total)
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%% Calculation of the bubble proprieties for %%%%%%%%%
%% %%%%%%%%% a particular time step. %%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for frame=first:t:last
    k=k+1;
    clear bound
    single_time=abs(frame-FrameZero)/FrameRate;
    Bubble = imread([direct,'\',base_name,'\',num2str(abs(frame)),'.BMP']);
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    BubbleHSV = rgb2hsv(Bubble);
    BubbleH = BubbleHSV(:, :,3);
    FiltBubbleH = imfilter(BubbleH,h);

```


MATLAB CODES

```

Difference = (FiltBackgroundBubbleH-FiltBubbleH);
ClippedDifference = Difference((Y(1):(Y(2)),(X(1):(X(2)))));
Enhancement = (FiltBackgroundBubbleH./FiltBubbleH);
ClippedEnhancement = Enhancement((Y(1):(Y(2)),(X(1):(X(2)))));
ClippedBubble = Bubble((Y(1):(Y(2)),(X(1):(X(2)))));
%% %%%%%%%%%% Converting selected image to binary manually %%%%%%%%%%
B = find (ClippedEnhancement >= 1.7);
W = find (ClippedEnhancement < 1.7);
BWClippedEnhancement = ClippedEnhancement;
BWClippedEnhancement(B) = 1;
BWClippedEnhancement(W) = 0;
%% %%%%%%%%%% Ghigliottin %%%%%%%%%%
BWClippedEnhancement=gigliottin(BWClippedEnhancement);
%% %%%% Fills holes in the image,making it bubble and background only %%%%
BWClippedEnhancement = imfill(BWClippedEnhancement);
%% %%%%%%%%%% Boundaries of the bubble %%%%%%%%%%
Bound = bwboundaries (BWClippedEnhancement,8);
bound = Bound{1};
%% %%%%%%%%%% Finds centroid of the bubble %%%%%%%%%%
s = regionprops(BWClippedEnhancement, 'all');
centroid = cat(1, s.Centroid);
centroid = int16 (centroid);
%% %%%%%%%%%% %%%%%%%%%%
symm_bound=make_bubble_symmetric(bound);
symm_bound=sortrows(symm_bound);
symm_bound(:,1)=symm_bound(:,1)-symm_bound(1,1);
%% %%%%%%%%%% %%%%%%%%%%
fa=2;
fb=0.3;
bred7=Smooth_bondaries_symm(symm_bound,fa,fb);
%% %%%%%%%%%% %%%%%%%%%%
Bubble_Height=bred7(end,1);
Bubble_Foot=2*bred7(end,2);
Bubble_Foot_mm=Bubble_Foot*scale;
Bubble_Foot_m=Bubble_Foot_mm/1000;
%% %%%%%%%%%% %%%%%%%%%% Find Centroid and Volume %%%%%%%%%%
Xg_Vol=Center_Of_Gravity(bred7);
Centroid_Level=Bubble_Height-Xg_Vol(1);
Volume=Xg_Vol(2);
%% %%%%%%%%%% %%%%%%%%%% Study of Buonacy effect %%%%%%%%%%
Volume_Bouy=Vol_Bouy(bred7,Volume);

```

MATLAB CODES

```

%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Neck Study %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
neck=0;
neck=Neck(bred7);
neck_mm=2*neck*scale;
neck_m=neck_mm/1000;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Find the pressure inside the bubble %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Bubble_Pressure=Find_Pressure(Pressure,single_time);
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% FIND THE CONTACT ANGLE %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Contact_Angle_Deg=Find_the_contact_angle(bred7);
Contact_Angle_Rad=pi*Contact_Angle_Deg/180;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Interface of the bubble %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Interface=bred7*scale;
Foot_Interface=abs(Interface(:,1)-Interface(end,1));
Foot_Interface(:,2)=Interface(:,2);
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Calculating the Radius, Area, Volume, Pressure %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Centroid_Level_mm=Centroid_Level*scale;
Volume_mm=Volume*scale^3;
Bubble_Height_mm=Bubble_Height*scale;
Bubble_Height_m=Bubble_Height_mm/1000;
Tip_Pressure = Base_Pressure - Bubble_Height_m*9.81*liquid_density;
Volume_Bouy_mm=Volume_Bouy*scale^3;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Contact area and perimeter %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Contact_Perimeter_m=pi*Hole_Diameter_m;
if neck==0;
    Contact_Area_m=pi*(Hole_Diameter_m^2)/4;
else
    Contact_Area_1_m=pi*(neck_m^2)/4;
    Contact_Area_2_m=pi*(Hole_Diameter_m^2-neck_m^2)/4;
end
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Calculating Forces acting on the bubble %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% BUOYANCY %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Buoy_Force_Newton=Volume_Bouy_mm*...
(liquid_density_mill-air_density_mill )*9.81;
Buoy_Force_microNewton=Buoy_Force_Newton*1e6;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% CAPILLARY %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Capillary_Force_Newton =Contact_Perimeter_m*surface_tension...
*sin(Contact_Angle_Rad);
Capillary_Force_microNewton=Capillary_Force_Newton*1e6;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%CONTACT PRESSURE %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Upward_Force_Newton=Bubble_Pressure*Contact_Area_m;
if neck==0;

```


MATLAB CODES

```

Downward_Force_Newton=Tip_Pressure*Contact_Area_m;
else
    Downward_Force_Newton=Tip_Pressure*...
    Contact_Area_1_m+Base_Pressure*Contact_Area_2_m;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Contact_Pressure_Force_Newton=Upward_Force_Newton-Downward_Force_Newton;
Contact_Pressure_Force_microNewton=Contact_Pressure_Force_Newton*1e6;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Dynamic %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Net_Force_Newton=Capillary_Force_Newton-...
(Contact_Pressure_Force_Newton+Buoy_Force_Newton);
Net_Force_microNewton=Net_Force_Newton*1e6;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Creation of 'Total' Array Components %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
T=time;
V=Volume_mm;
Z=Centroid_Level_mm;
V_B=Volume_Bouy_mm;
B_H=Bubble_Height_mm;
C_A=Contact_Angle_Deg;
B_P=Bubble_Pressure;
B_F=Buoy_Force_microNewton;
C_F=Capillary_Force_microNewton;
C_P=Contact_Pressure_Force_microNewton;
N_F=Net_Force_microNewton;
B_Foot= Bubble_Foot_mm;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Creating Total Matrix %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
total(k,1) = T;
total(k,2) = V;
total(k,3) = Z;
total(k,4) = V_B;
total(k,5) = B_H;
total(k,6) = 180-C_A;
total(k,7) = B_P;
total(k,8) = B_F;
total(k,9) = C_F;
total(k,10) = C_P;
total(k,11) = N_F;
total(k,12) = B_Foot;
total(k,13) = Contact_Area_m;

```

```

total(k,16) = neck_mm;
time= time+abs(t)/FrameRate;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Uncertainty analysis %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Error(k,1)=total(k,1);
Error(k,2)=3*total(k,2)*Delta_scale/scale;
Error(k,3)=total(k,3)*Delta_scale/scale;
Error(k,4)=3*total(k,4)*Delta_scale/scale;
Error(k,5)=total(k,5)*Delta_scale/scale;
% Error(k,12)=total(k,12)*Delta_scale/scale;
Error(k,12)=0;
Error(k,6)= 5;
CA_sin=[sin(Contact_Angle_Rad);
        sin(Contact_Angle_Rad+pi*Error(k,6)/180);
        sin(Contact_Angle_Rad-pi*Error(k,6)/180)];
Err_sin=abs(max(CA_sin)-min(CA_sin))/2;
Error(k,7)= 4;
Error(k,8)= 9.81*density_difference_mill*Error(k,4)*1e6;
Error(k,9)=pi*surface_tension_mill*(total(k,12)*Err_sin+...
        Error(k,12)*sin(Contact_Angle_Rad)+...
        Error(k,12)*Err_sin)*1e6;
Error(k,13)=0;
Diff_P=Bubble_Pressure-Tip_Pressure;
Delt_Diff_P=Delta_Base_Pressure+Error(k,7);
Error(k,10)=(Diff_P*Error(k,13)+Delt_Diff_P*total(k,13)...
        +Delt_Diff_P*Error(k,13))*1e6;
% Error(k,10)= Contact_Area_m*(Delta_Base_Pressure+Error(k,7))*1e6;
Error(k,11)=Error(k,8)+Error(k,10)+Error(k,9);
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(1)
plot(Foot_Interface(:,2),Foot_Interface(:,1),'-r*')
grid on
hold on
axis equal
end
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Experimental Data %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%% Definition of the principal experimental parameter %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
T
        = Frame_Data(13);
        %[Celsius]

```


MATLAB CODES

```

Hole_Diameter_m      = Hole_Diameter/1000;           %[m]
liquid_density        = 998;                         % Water      %[Kg/m^3]
= 1472.6-2.88*T;      % HFE-7000                    %[Kg/m^3]
air_density           = 1.225;                       %[Kg/m^3]
density_difference    = liquid_density-air_density;%    [Kg/m^3]
surface_tension       = 0.073;                       % Water      %[N/m]
= 0,0124;             % HFE-7000                    %[N/m]
Water_level          = 0.03;                         %[m]
Base_Level           = 0.009;                        %[m]
Water_column         = Water_level-Base_Level;        %[m]
Base_Pressure        = Water_column*liquid_density*9.81; %[Pa]
liquid_density_mill   = liquid_density*1e-9;          %[Kg/mm^3]
air_density_mill      = air_density*1e-9;             %[Kg/mm^3]
density_difference_mill = liquid_density_mill...
                                -air_density_mill; %[Kg/mm^3]
surface_tension_mill  = surface_tension*1e-3;         %[N/mm]
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Uncertainty Analysis %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
delta_H_mm           = 0.5;
delta_H_m            = delta_H_mm/1000;
delta_WL             = delta_H_m;
Delta_Base_Pressure   = 9.81*liquid_density*delta_WL;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Set the scale of the bubble %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
BubblescalingHSV     = rgb2hsv(Bubblescaling);
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Separating Channels %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
BubblescalingH       = BubblescalingHSV(:, :, 3);
figure('Units','centimeters','Position',[5 5 20 20])
imshow(BubblescalingH);
if q==1
title('Select the internal points','FontSize',10,'fontname','times')
else
    title('Select the external points','FontSize',10,'fontname','times')
end
axis equal;
axis tight;
axis ij;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Selects the 2 calibration points %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

MATLAB CODES

```

[X2,Y2]=ginput(2);
close;
DistancePix      = X2(2)-X2(1);
%% %%%%%%%%%% Calculate the Scale %%%%%%%%%%
scale_matrix(q)  = ActualDist/DistancePix;
%% %%%%%%%%%%

%% %%%%%%%%%%
%% %%%%%%%%%% Making the bubble symmetric by performing the %%%%%%%%%%
%% %%%%%%%%%% average between the two sides of the bubble %%%%%%%%%%
%% %%%%%%%%%%
function boundary=make_bubble_symmetric(bound)
bound_sort=sortrows(bound,1);
bound_aux=bound_sort;
[c1 c2]=size(bound_aux);
k=1;
for z=1:max(bound_aux(:,1))-min(bound_aux(:,1))+1
j=1;
for i=1:c1
    if bound_aux(i,1)==max(bound_aux(:,1))&&bound_aux(i,1)~=0
        A(j,1:2)=bound_aux(i,:);
        j=j+1;
        bound_aux(i,:)=[0 0];
    end
end
A=sortrows(A,2);
B(1,:)=A(1,:);
B(2,:)=A(end,:);
B(:,2)=B(:,2)-(B(1,2)+B(2,2))/2;
boundary(k,:)=B(2,:);
clear A
clear B
k=k+1;
end
boundary(end+1,:)=[boundary(end,1) 0];
%% %%%%%%%%%%

%% %%%%%%%%%%
%% %%%%%%%%%% Centre of Gravity and Bubble Volume Calculation %%%%%%%%%%
%% %%%%%%%%%%
function Xg_Vol=COG_Vol(Interface)
clear B_Mat Xg Vol

```



```

[c1 c2]=size(Interface);
for i=2:1:c1
    r=Interface(i-1,2);
    R=Interface(i,2);
    H=Interface(i,1)-Interface(i-1,1);
    v=pi*H*(R^2+r*R+r^2)/3;
    xg=Interface(i,1)-H*(R^2+2*r*R+3*r^2)/(4*(R^2+r*R+r^2));
    B_Mat(i-1,1)=xg;
    B_Mat(i-1,2)=xg*v;
    B_Mat(i-1,3)=v;
end
Xg=sum(B_Mat(:,2))/sum(B_Mat(:,3));
Vol=sum(B_Mat(:,3));
Xg_Vol(1)=Xg;
Xg_Vol(2)=Vol;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%% Calculation of the volume affected %%%%%%%%%%%
%% %%%%%%%%%%% by the buoyancy effect %%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function Buoy_Vol=Vol_Bouy(bred7,Volume)
[c1 c2]=size(bred7);
L=max(bred7(:,2));
i1=find(bred7(:,2)==L);
l=min(bred7(i1:end,2));
i2=find(bred7(:,2)==l);
%% %%%%%%%%%%% First part of Bred8 %%%%%%%%%%%
k=2;
while (bred7(k,2)<l);
    xB = bred7(k,1);
    yB = bred7(k,2);
    bred8(k,:) = [xB yB];
    k=k+1;
end
%% %%%%%%%%%%% Interpolating Middle part for bred8 %%%%%%%%%%%
xB = bred7(k,1);
xA = bred7(k-1,1);
yB = bred7(k,2);
yA = bred7(k-1,2);
yC=1;
xC=xA+(xB-xA)*(yC-yA)/(yB-yA);

```

```

bred8(k,:) = [xC yC];
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Completing bred8%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

k=k+1;
for p=i2:1:c1
bred8(k,:)=bred7(p,:);
k=k+1;
end
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

No_Buoy_Vol=Just_Volume(bred8);
Buoy_Vol=Volume-No_Buoy_Vol;
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%% Calculation of the bubble internal pressure %%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function Bubble_Pressure=Find_Pressure(FiltP,single_time)

[e1 e2]=size(FiltP);
for i=2:1:e1
    if FiltP(i,1)==single_time
        Bubble_Pressure=FiltP(i,2);
    elseif FiltP(i-1,1)<single_time & FiltP(i,1)>single_time
        xc=single_time;
        xa=FiltP(i-1,1);
        xb=FiltP(i,1);
        ya=FiltP(i-1,2);
        yb=FiltP(i,2);
        Bubble_Pressure=ya+(yb-ya)*(xc-xa)/(xb-xa);
    end
end
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% %%%%%%%%%%% Calculation of the contact angle on the bubble base %%%%%%%%%%%
%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function Contact_Angle=Find_the_contact_angle(interface)

bred23=interface;
bred23(1:end-5,:)=[];
passo=4;
Ps=complete_the_matrix(bred23,passo);

```



```

[e1 e2]=size(Ps);
for i=1:1:e1
bred23(end+1,1)=Ps(i,2);
bred23(end,2)=Ps(i,1);
end
bred_poli=bred23;
%% %%%%%%%%%%%%%%% Contact Angle %%%%%%%%%%%%%%%
[e1 e2]=size(bred_poli);
z=e1-passo;
zz=passo;
dynamic_find_circ;
Contact_Angle=tang;
%% %%%%%%%%%%%%%%%

```

APPENDIX B

TRANSAT CODES

```
!! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!! !!!!!!!!!!!!!!! Definition of the simulation conditions !!!!!!!!!!!!!!!
!! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

&PROJECT_NAME
PROJECTNAME = "orthomesh"
/
&execution
autonumthreads = .false.
/
&FLOW_CONDITIONS
xgravity = -9.81
ygravity = 0.0
zgravity = 0.0
hydrostatic = .true.
STEADY = .false.
AXISYM = .true.
compressible = .false.
ldebug = 0
/
&INITIAL_CONDITIONS
RADIUS = 0.0
ADD_GRAVITY_HEAD = .false.
DENSITY_REF_PHASE = "gas"
```

TRANSAT CODES

```
REINIT_DOMAIN = .true.
impose_solenoidal = .false.
/
&PHASE1
RHOG = 1.225
VISCg = 1.79e-6
LAMBG = 0.026
CPG = 1005.0
C_DIFFG = 1e-3
/
&PHASE2
RHOL = 998.2
VISCL = 1.003e-3
LAMBL = 1.025
CPL = 4187.0
C_DIFFL = 1e-3
/
&EQUATION_TO_BE_SOLVED
solve_pressure      = .true.
solve_uvelocity     = .true.
solve_vvelocity     = .true.
solve_levelset      = .true.
/
&CONTROL_PARAMETERS
stime = 0.0
NTIMET = 40000
ctimet = 1500
NTIMPl = 1000
NSAVE = 1000
restart_store = 10000
DTIME = 1.0e-4
EPSEXP = 1.0e-2
EPSPEXP = 1.0e-2
EPSIMP = 1.0e-2
EPSPIMP = 1.0e-2
MAXIT = 50
NSWP = 4
NCONVE = 1
IMON = 2
JMON = 2
KMON = 2
```


TRANSAT CODES

```
/
&LEVEL_SET_INPUT
CORRECT_MASS=localglobal
NUM_REINIT_STEPS=25
WEBER = 0.073
EQ_CONTACT_ANGLE = 67.0
STWIDTH=2
INTERFACE_WIDTH=1
ACTIVATE_TRIPLE_LINE = .false.
WRITE_DYN_CONTACT_ANGLE = .true.
reinit_method = "weno"
/
&TIME_SCHEME
NTSCH = 1
ADAPTTIME=.true.
CFLMAX=4.0
CFLMIN=2.0
STNMAX=1.0
STNMIN=0.8
DIFMAX=20.0
DIFMIN=10.0
!
solve_implicit = .true.
autorelaxation = .true.
nitnsimpl = 2
/
&CONVECTION_SCHEME
scheme_density      = 4
scheme_uvelocity    = 4
scheme_vvelocity    = 4
scheme_levelset     = 2
/
&ImplicitSolvers
pressuresolver = "gmres"
defaultsolver = "sip"
preconditioner = "sip"
itersip = 3
check_sip_coefficients = .false.
/
&PHASE_CHANGE_INPUT
correlation = "fixed"
```

TRANSAT CODES

```
mdot   = 0.0
latent = 1.0
tsat   = 1.0
tsc    = 1.0
tsh    = 1.0
Lliqmdot = 1.0
Uliqmdot = 1.0
Lgasmdot = 1.0
Ugasmdot = 1.0
/
&INPUT_OUTPUT_OPERATIONS
restart_method = "norestart"
RASCII = .false.
WASCII = .false.
GASCII = .true.
NSUFFIX = 7
/
&VISUALIZATION
L3DPLOT = .false.
!
L2DPLOT = .true
write_kplane = .true.
write_jplane = .false.
write_iplane = .false.
IPLANE = 10
JPLANE = 2
KPLANE = 2
!
DATA_FORMAT = "paraview"
!
WRTPRES=.true.
WRTUVEL=.true.
WRTVVEL=.true.
WRTLSET=.true.
wrtLSembed=.true.
wrtmarkLSembed=.false.
wrtHembed=.false.
wrtmach=.false.
wrtmarkerGL=.false.
wrtDeltaEmbed=.false.
wrtmdot=.false.
```


TRANSAT CODES

```
wrtkdirac=.true.
wrtwalldist=.false.
wrtcurvature=.true.
wrtkblk=.true.
/
&POSTPROCESSING
ACTIVATE_NONDIM_NUMBERS = .true.
KEPSILON_PROCESS = .false.
max_values = .false.
integral_surface_forces = .false.
inertia_force = .false.
interfacial_area = .false.
Bubble_rise = .false.
Phase_choice = "gas"
frequency_level_set = .false.
activate_timesignals = .false.
detect_drops_bubbles = .false.
/
&REF_STATE
REFPRES = 1.013e+5
REFTEMP = 300
refl = 0.003
refu = 0.05
/
&INFLOW_CONDITIONS
IWRVIS = 0
NWRMAX = 6010
THETA = 0.0
UINOSC = 0.0
VINOSC = 0.0
WINOSC = 0.0
/
&BC_INPUT
NEXIT = 0
masscond = 0
forcemode = "instant"
lsexit = 0
/
&AVERAGING
IAVER = 0
JAVER = 0
```

TRANSAT CODES

```
KAVER = 0
LOAVTI=.false.
NUAVTI=1
NSTAAV=10000
/
&embedded_interface
activate_embedded_interface = .false.
rhoembed = 1.225
cpembed = 1.0
lambdaembed = 1005.0
LSformat = "ascii"
hembedinside = .false.
/
&K_EPSILON_INPUT
mol_visc_factor = 20.0
turb_intensity = 0.1
sigma_k = 1.0
sigma_eps = 1.32
Ce1 = 1.44
Ce2 = 1.92
Ce3 = 2.9556
Cmu = 0.09
itkemax = 2
LowRemodel = .false.
Twolayermodel = .false.
LReChoice = 0
TLChoice = 0
OneEqnmodel = .false.
ldiss_factor = 0.01
/
&TAUW_TIME_AV
LOTAUW=.true.
NUTAUW=1
/
&TIME_LIMIT
TLIMIT = 518400000.0
/
!! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

!! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!! !!!!!!!!!!!!!!! Creation of the Cartesian grid !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
```


TRANSAT CODES

```
!! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

&GLOBAL
NBLOCKS=2
PLOT3D_OUTFILE="orthomesh.grda"
PLOT3D_FORMAT="ascii"
OUTPUT_PRECISION="double"
TECPLOT_OUTFILE="orthomesh.grd"
WRITE_TECLOT=.false.
GRID_DIM="3D"
MERGE_GRID=.true.
SCALE_FACTOR=0.0001
/

&DOMAIN_LENGTH
XBEG(1)=00, XEND(1)=100, YBEG(1)=00, YEND(1)=08, ZBEG(1)=00, ZEND(1)=01
XBEG(2)=00, XEND(2)=100, YBEG(2)=08, YEND(2)=65, ZBEG(2)=00, ZEND(2)=01
/

&GRID
XGRID(1)=149, YGRID(1)=8, ZGRID(1)=2
XGRID(2)=149, YGRID(2)=93, ZGRID(2)=2
/

&BLANKING
exclude_blank = .false.
/

&TransATBCS
WRITE_TRANSAT_BCS=.true.
transat_bcfile = "transat.bc"
default_bnd_type = "sym"
/

!! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

!! !!!!!!!!!!!!! Boundary Condition Script for the grid !!!!!!!!!!!!!!!!!!!!!!!
!! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

&BOUNDARY_CONDITIONS
NRBDA = 10
!-----
! NBOUND = 1 in BLK1
```

```

!-----
cbnd( 1,1) = "west "
cbnd( 1,2) = "inflow"
cbnd( 1,3) = "nofor"
!
bnd_inflow_type(1) = 0
inflowprofile(1) = "laminar"
vbnd(1,1) = 0.015719  ![m/s]
vbnd(1,2) = 0.0
vbnd(1,3) = 0.0
!
nbnd( 1,1) =      2
nbnd( 1,2) =      2
nbnd( 1,3) =      2
nbnd( 1,4) =      8
nbnd( 1,5) =      2
nbnd( 1,6) =      2
!-----
! NBOUND =      2 in  BLK1
!-----
cbnd( 2,1) = "east "
cbnd( 2,2) = "optflow"
cbnd( 2,3) = "nofor"
pexit(2)    = 107    ![Pa]
!
nbnd( 2,1) =    149
nbnd( 2,2) =    149
nbnd( 2,3) =      2
nbnd( 2,4) =      8
nbnd( 2,5) =      2
nbnd( 2,6) =      2
!-----
! NBOUND =      3 in  BLK1
!-----
cbnd( 3,1) = "south"
cbnd( 3,2) = "sym  "
cbnd( 3,3) = "nofor"
!
nbnd( 3,1) =      2
nbnd( 3,2) =    149
nbnd( 3,3) =      2

```


TRANSAT CODES

```
nbnd( 3,4) =    2
nbnd( 3,5) =    2
nbnd( 3,6) =    2
!-----
! NBOUND =    4 in  BLK1
!-----
cbnd( 4,1) = "botto"
cbnd( 4,2) = "sym  "
cbnd( 4,3) = "nofor"
!
nbnd( 4,1) =    2
nbnd( 4,2) =   149
nbnd( 4,3) =    2
nbnd( 4,4) =    8
nbnd( 4,5) =    2
nbnd( 4,6) =    2
!-----
! NBOUND =    5 in  BLK1
!-----
cbnd( 5,1) = "top  "
cbnd( 5,2) = "sym  "
cbnd( 5,3) = "nofor"
!
nbnd( 5,1) =    2
nbnd( 5,2) =   149
nbnd( 5,3) =    2
nbnd( 5,4) =    8
nbnd( 5,5) =    2
nbnd( 5,6) =    2
!-----
! NBOUND =    6 in  BLK2
!-----
cbnd( 6,1) = "west "
cbnd( 6,2) = "wall"
cbnd( 6,3) = "nofor"
!
film_thickness(6) = 3.5e-5
!
nbnd( 6,1) =    2
nbnd( 6,2) =    2
nbnd( 6,3) =    9
```

TRANSAT CODES

```
nbnd( 6,4) = 100
nbnd( 6,5) = 2
nbnd( 6,6) = 2
!-----
! NBOUND = 7 in BLK2
!-----
cbnd( 7,1) = "east "
cbnd( 7,2) = "optflow"
cbnd( 7,3) = "nofor"
pexit(7) = 107 ! [Pa]
!
nbnd( 7,1) = 149
nbnd( 7,2) = 149
nbnd( 7,3) = 9
nbnd( 7,4) = 100
nbnd( 7,5) = 2
nbnd( 7,6) = 2
!-----
! NBOUND = 8 in BLK2
!-----
cbnd( 8,1) = "north"
cbnd( 8,2) = "wall"
cbnd( 8,3) = "nofor"
!
nbnd( 8,1) = 2
nbnd( 8,2) = 149
nbnd( 8,3) = 100
nbnd( 8,4) = 100
nbnd( 8,5) = 2
nbnd( 8,6) = 2
!-----
! NBOUND = 9 in BLK2
!-----
cbnd( 9,1) = "botto"
cbnd( 9,2) = "sym "
cbnd( 9,3) = "nofor"
!
nbnd( 9,1) = 2
nbnd( 9,2) = 149
nbnd( 9,3) = 9
nbnd( 9,4) = 100
```


TRANSAT CODES

```
nbnd( 9,5) = 2
nbnd( 9,6) = 2
!-----
! NBOUND = 10 in BLK2
!-----
cbnd( 10,1) = "top "
cbnd( 10,2) = "sym "
cbnd( 10,3) = "nofor"
!
nbnd( 10,1) = 2
nbnd( 10,2) = 149
nbnd( 10,3) = 9
nbnd( 10,4) = 100
nbnd( 10,5) = 2
nbnd( 10,6) = 2
/
!! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

!! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!! !!!!!!!!!!!!!!! Initial condition definition !!!!!!!!!!!!!!!!!!!!!!!
!! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

SUBROUTINE initialconditions

  Implicit none

  include "precision.h"
  include "get_set_interfaces.h"

  Integer:: i, j, k, ii
!-----!
! TransAT: Do not edit above this line !
!-----!

  Integer :: nij,k,ni,nij
  Real(iprec):: d1,rad
  Real(iprec), Dimension(2):: c1, pc
  Real(iprec), Dimension(:), Pointer :: p,u,v
  Real(iprec), Dimension(:), Pointer :: x,y,phi

  Real(iprec), External :: sphere

  nij,k = get_integer('nij,k')
```

TRANSAT CODES

```
ni  = get_integer('ni')
nij = get_integer('nij')

call set_pointer(p,'pressure')
call set_pointer(u,'uvelocity')
call set_pointer(v,'vvelocity')

call set_pointer(x,'cellcenterX')
call set_pointer(y,'cellcenterY')

call set_pointer(phi,'levelset')

Do ii=1,nijk
  p(ii) = 0.0
  u(ii) = 1.0e-6
  v(ii) = 0.0
Enddo

rad = 0.0008
c1(1) = 0.0d0
c1(2) = 0.0d0
write(*,*)' rad : ', rad
write(*,*)' xcen: ', c1(1)

Do ii = 1,nijk
  pc(1) = x(ii)
  pc(2) = y(ii)
  d1 = sphere(pc,c1,rad,"gas")
  phi(ii) = d1
!   if (x(ii) < 1.0e-3) then
!     phi(ii) = -1.0e-3
!   endif
Enddo
END SUBROUTINE initialconditions
!-----
! USAGE: Every variable is stored in a 1-Dimensional array.
!       The transformation from 3D to 1D is as follows
!       If i, j, k are indices for the x, y, and z directions and
!       ni,nj,nk are the maximum values, then:
!       ii = i + (j-1)*ni + (k-1)*nij, where nij = ni*nj, nijk = nij*ni
```


TRANSAT CODES

```
! Use routine 'set_pointer(your_pointer,keyword) to set a
! pointer to an array. The pointer has to be declared as
! Real(iprec), Dimension(:), Pointer :: your_pointer
!
! Pointers keywords:
! - cellcenterx
! - cellcentery
! - cellcenterz
! - cellcornerx
! - cellcornery
! - cellcornerz
! - uvelocity
! - vvelocity
! - wvelocity
! - pressure
! - temperature
! - concentration
! - levelset
! - vof
!
! The Namelist in the input file called '&INITIAL_CONDITIONS'
! can be used to specify input that can be used in this routine
! eg: radius of a bubble or drop
!
! There are three function to get a copy of global variables:

! get_integer(keyword)
!     -> keywords:  - ni
!                   - nj
!                   - nk
!                   - nij
!                   - nijk
!                   - scheme_density
!                   - scheme_Uvelocity
!                   - scheme_Vvelocity
!                   - scheme_Wvelocity
!                   - scheme_temperature
! get_real(keyword)
!     -> keywords:  - dtime
!                   - radius
! get_logical(keyword)
```

TRANSAT CODES

```

!      -> keywords:  - solve_pressure
!
!                  - solve_Uvelocity
!
!                  - solve_Vvelocity
!
!                  - solve_Wvelocity
!
!                  - solve_temperature
!
!                  - solve_concentration
!
!                  - solve_levelset
!
!                  - solve_vof
!
!                  - activate_embedded_interface
!-----

!-----!
! To create a distance function for circles and spheres  !
! NOTE: for circles one coordinate can be sent in as zero !
!-----!

FUNCTION sphere(pc,cc,radius,phase)
    include "precision.h"
    Real(iprec):: sphere
    Real(iprec), Dimension(2), Intent(in):: pc, cc
    Real(iprec), Intent(in):: radius
    Character(LEN=*), Intent(in):: phase

    Integer:: iliquid

    iliquid = -1
    If (phase == "liquid") iliquid = 1

    sphere = real(iliquid,iprec)*(radius - SQRT((pc(1)-cc(1))**2 &
                                                +(pc(2)-cc(2))**2))

END FUNCTION sphere
!! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!!!!!!! User Processing !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

Subroutine user_processing() Bind(C)
    Implicit none

    include "precision.h"
    include "get_set_interfaces.h"

```


TRANSAT CODES

!!!!!!! Variables Declaration !!!!!!!!!!!!!!!

```
Integer:: ii,i,j,k
integer:: ni, nij, nj, iteration
real(iprec), dimension(:), pointer:: phi,x,y,p
real(iprec):: timenow,X1,X2,Y1,Y2,Xp,Yp
Character (len=20) :: temp
```

!!

```
timenow = get_real("ctime")
iteration = get_integer("timestep")
ni      = get_integer("ni")
nj      = get_integer("nj")
nij     = get_integer("nij")
```

!!!!!!!!!!!!!! Get to Phi !!!!!!!!!!!!!!!

```
call set_pointer(phi,"levelset")
```

!!!!!!!!!!!!!! Get to Cell Centers !!!!!!!!!!!!!!!

```
call set_pointer(x,"cellcenterx")
call set_pointer(y,"cellcentery")
```

!!!!!! Get the value of the pressure !!!!!!!!!!!!!!!

```
call set_pointer(p,"pressure")
```

!!

!!!! Writing the interface :Start !!!!!!!

!!

```
if (mod(iteration,10)==0) then
  write(temp,*) iteration
  temp = adjustl(temp)
  Open(10,file='Interface/Int_'//trim(temp)//'.txt')
  write(10,100)timenow,p(10)
  10 format(E11.4)
```

TRANSAT CODES

!!!!!!!!! Along Horizontal direction !!!!!!!!!!!!!

```
do j=1,nj
  do i=2,ni
    ii=i+(j-1)*ni
    if ((phi(ii)*phi(ii-1).LT.0)) then
      X1=x(ii-1)
      X2=x(ii)
      Y1=phi(ii-1)
      Y2=phi(ii)
      Xp=(X2-X1)*(0-Y1)/(Y2-Y1)+X1
      Yp=y(ii)
      write (10,100) Xp,Yp
    endif
  enddo
enddo
```

!!!!!!!!! Along Vertical direction !!!!!!!!!!!!!

```
do i=1,ni
  do j=2,nj
    ii=i+(j-1)*ni
    if ((phi(ii)*phi(ii-ni).LT.0)) then
      Y1=y(ii-ni)
      Y2=y(ii)
      X1=phi(ii-ni)
      X2=phi(ii)
      yp=(Y2-Y1)*(0-X1)/(X2-X1)+Y1
      xp=x(ii)
      write (10,100) Xp,Yp
      100 format(E11.4,' ',E11.4)
    endif
  enddo
enddo
close(10)
```

!!

!!!! Writing the interface : End !!!!!!!!!!!!!

!!

```
end if
END SUBROUTINE user_processing
```