# Efficient Finite Element Reliability Analysis Employing Surrogate Model for Seismic Fragility Curve Derivation

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ABSTRACT: This paper proposes a new method for deriving seismic fragility curves. To improve the efficiency of seismic fragility analysis, reliability analysis is conducted in conjunction with finite element analysis, which is often termed the finite element reliability analysis (FERA). However, the computational cost of FERA can still be large, and the proposed method helps reduce it significantly by employing a sequentially updated surrogate model. It is noteworthy that, in the proposed method, surrogate model training does not require additional analysis because a surrogate model is constructed based only on the prior FERA results. The proposed method was tested through its application to a numerical example of a buried pipeline structure, and it was observed that the proposed method required approximately 60% of the computational cost of the conventional method, with a similar level of accuracy to the analysis results.

## 1. INTRODUCTION

As earthquake damage to structures has been considered serious, the seismic risk assessment of infrastructure has been receiving attention. Seismic fragility curves play an important role in predicting seismic losses and making decisions for earthquake mitigation. For example, seismic risk assessment tools such as HAZUS-MH (FEMA, 2013) and SYNER-G (Pitilakis et al., 2014) have introduced seismic fragility curves to evaluate post-earthquake losses in various seismic scenarios. Therefore, several researchers have attempted to derive seismic fragility curves for various structures accurately and efficiently.

A seismic fragility curve is often obtained by calculating the probability that a structure will suffer more than a pre-defined damage level with respect to various seismic intensity measures (e.g., peak ground acceleration (PGA) and spectral acceleration (SA)), considering various uncertainties affecting the structural response. Among the several types of methods developed for seismic fragility curve derivation, an analytical method has the advantage of not requiring empirical data and subjective judgment, and finite element analysis (FEA) is often coupled with reliability analysis in such an approach (Kwon, 2007). In particular, when the target structure has a high level of structural nonlinearity or complexity, reliability analysis needs to be performed in conjunction with sophisticated FEA, often termed the finite element reliability analysis

(FERA) (Lee and Moon, 2014). However, despite its accuracy, seismic fragility curve derivation using FERA can be computationally expensive because the damage probability of a structure needs to be calculated for various earthquake ground motions and intensities.

This study proposes a new method for efficiently deriving accurate seismic fragility curves. First, the proposed method employs the first-order reliability method (FORM) to reduce the cost of FERA. However, the computational cost of seismic curve derivation can still be expensive; therefore, the proposed method introduces a surrogate model to further reduce the cost. When structural damage probabilities are initially calculated for a few earthquake intensities, a surrogate model for the structural response is trained based on prior results. Then, a more optimal starting point for the subsequent FORM analysis can be obtained from the surrogate model. Moreover, as the seismic fragility analysis progresses, the surrogate model can be sequentially updated, which continuously increases the efficiency of the subsequent FORM analysis. It is also noteworthy that surrogate model training in the proposed method does not require additional FEA because a surrogate model is constructed based only on the prior FERA results.

## 2. PROPOSED METHOD

### *2.1. Finite element reliability analysis (FERA)*

Seismic vulnerability assessments should be made considering various uncertainty factors related to the target structure, which requires structural reliability analysis. In general, reliability analysis requires a mathematical definition of the limit state of interest, and the analysis may become straightforward when a limit state can be described as an algebraic function of random variables. In most cases, however, the seismic responses of a structure cannot be expressed explicitly; thus, a sophisticated finite element model needs to be introduced to FERA.

Several FERA tools have been developed and applied to various structures and vulnerability assessments. For example, Lee and Moon (2014) proposed FERUM-ZEUS for the seismic vulnerability evaluation of 3D building structures. Lee et al. (2008) developed FERUM-ABAQUS for component reliability analysis and system reliability analysis of an aircraft wing torque box, and Lee et al. (2016) proposed Python-based interface for FERUM and ABAQUS (PIFA) for flood fragility curve derivation. These FERA tools integrate two software packages with different expertise to take full advantage of their strengths in managing complex and diverse structural reliability problems. Figure 1 shows typical data flow in FERA between the two software packages.

Because the computational cost of FEA can be considerable, these FERA tools employ an analytical reliability analysis method, FORM, which can provide reliable results (e.g., failure probability) at a relatively low analysis cost. For this task, as shown in Figure 1, Finite Element Reliability Using MATLAB (FERUM), which is a reliability analysis software package developed by researchers at UC Berkeley (Haukaas et al. 2003), was introduced to perform reliability analysis employing FORM in FERA.

In this study, PIFA was introduced to derive an efficient seismic fragility curve. PIFA repeatedly calls ABAQUS®, which is a specialized software package for FEA, to obtain seismic responses of a structure during FORM analysis from FERUM. In the proposed method, a surrogate model is repeatedly constructed, based on the FEA results obtained during the prior FORM analyses.

### *2.2. Surrogate model construction*

In various engineering applications, surrogate modeling approaches have been adopted to replace or assist expensive physical models. Common techniques include support vector machine, neural networks, and Gaussian process regression (GPR) (Moustapha et al., 2018). For surrogate model construction, this study introduced GPR, which is a machine-learning-based non-parametric regression method (Rasmussen and Williams, 2006).

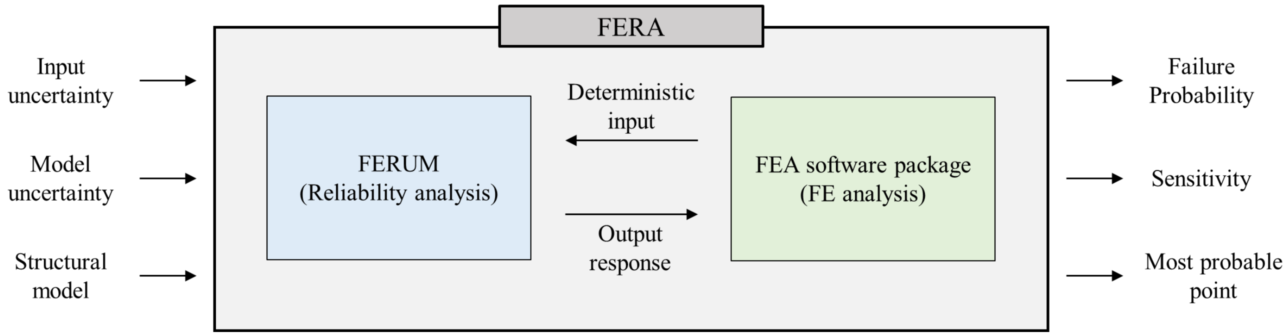


*Figure 1: Illustration of a typical data flow in FERA*

In GPR, the matrix of data points (**D**) consisting of the FEA results obtained from the prior FORM analyses and RVs are expressed as follows:

$$\mathbf{D} = \{(\mathbf{X},\ \mathbf{y})\} = \{(x_{ij},\ y_i) | i = 1,\ \ldots, N_{FE};\ j = 1,\ \ldots, N_{RV}\} \tag{1}$$

where **X** and **y** are the input matrix and output vector, respectively, $N_{FE}$ is the number of FEAs, $N_{RV}$ is the number of RVs, $x_{ij}$ is the input for the $i$th FEA and $j$th RV, and $y_i$ is the corresponding FE analysis result. Then, for the new input matrix of X*, $\widehat{f}_*(\cdot)$, can be determined through the multivariate normality assumption of GPR as follows:

$$\begin{pmatrix} \mathbf{y} \\ \hat{f}_*\left(\mathbf{X}_*, \hat{\boldsymbol{\theta}}\right) \end{pmatrix} = N\left(\mathbf{O}, \Sigma\right) = N\left(\mathbf{O}, \begin{bmatrix} \mathbf{K} + \sigma_{noise}^2 \mathbf{I} & \mathbf{K}_* \\ \mathbf{K}_*^{\mathrm{T}} & \mathbf{K}_{**} + \sigma_{noise}^2 \mathbf{I} \end{bmatrix}\right) \tag{2}$$

where $\hat{\boldsymbol{\theta}}$ is the vector of optimized hyperparameters, $N$ represents the multivariate Gaussian distribution, **O** is the zero matrix, **Σ** is the symmetric and positive semi-definite covariance matrix, **K** is the covariance matrix of the design points $x_{ij}$, **K*** is the covariance matrix between the given (**X**) and prediction (**X***) inputs, **K**** is the covariance matrix of the prediction input matrix **X***, and $\sigma^2$ is the variance of noise. In GPR, a kernel function is often used to construct the covariance matrix (i.e., **K**, **K***, and **K****) with respect to the Euclidean distance between two vectors **p** and **q**:

$$k(\mathbf{p},\ \mathbf{q}) = \sigma_{\mathbf{p}} \sigma_{\mathbf{q}} \rho_{\mathbf{pq}} \tag{3}$$

where **p** and **q** indicate the standard deviations of the two vectors, and $\rho_{\mathbf{pq}}$ is the correlation coefficient. This study introduced a square exponential kernel with two hyperparameters to construct the covariance matrix (Rasmussen and Williams, 2006).

To obtain an appropriate surrogate model, it is essential to optimize these hyperparameters, and maximum likelihood estimation is often employed in GPR. The probability of observing the given input and output data (i.e., **X** and **y**) with respect to Eq. 2 can be expressed as a conditional probability using a multivariate normal distribution as follows:

$$p\left(\mathbf{y} | \mathbf{X}\right) = \frac{1}{\left(2\pi\right)^{N_{FE}/2} \left|\Sigma\right|^{1/2}} \exp(-\frac{1}{2} \mathbf{y}^{\mathrm{T}} \Sigma^{-1} \mathbf{y}) \tag{4}$$

For computational convenience, the natural logarithm of conditional probability in Eq. 4 is taken as follows:

$$\mathcal{L}\left(\boldsymbol{\theta} | \mathbf{X}\right) = -\ln p\left(\mathbf{y} | \mathbf{X}\right) = \frac{1}{2} \mathbf{y}^{\mathrm{T}} (\mathbf{K} + \sigma_{noise}^2 \mathbf{I})^{-1} \mathbf{y} + \frac{1}{2} \ln\left|\mathbf{K} + \sigma_{noise}^2 \mathbf{I}\right| + \frac{N_{FE}}{2} \ln(2\pi) \tag{5}$$

where $\mathcal{L}(\cdot)$ is the negative log-likelihood function. Then, the optimized hyperparameters $\hat{\boldsymbol{\theta}}$ can be obtained through optimization minimizing $\mathcal{L}(\cdot)$:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \left(\mathcal{L}\right) \tag{6}$$

### *2.3. Surrogate-model-based FERA*

The computational cost of deriving seismic fragility curves can still be high even though FERA has introduced the FORM; Hence, in this study, a surrogate model constructed as described in Section 2.2 is employed. The proposed method calculates the probability of structural damage using FERA for the first few seismic intensities and learns the structural responses with various RV input values. The surrogate model can then be constructed, and a preliminary FORM analysis is conducted using the surrogate model for the next seismic intensity, which can provide an optimal starting point for FORM analysis. With the optimal point, the main FERA analysis is conducted. The flow chart of the proposed method is shown in Figure 2.

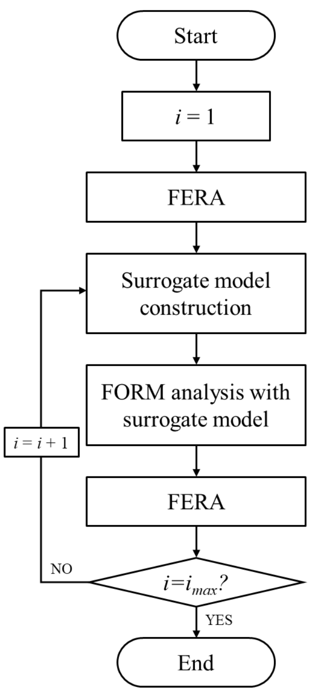


*Figure 2: Flow chart of surrogate-model-based FERA.*

In Figure 2, $i$ is the $i$th seismic intensity, and $i_{max}$ is the number of seismic intensities considered for seismic fragility curve derivation. In the proposed method, as the seismic fragility analysis progresses, the surrogate model can be updated sequentially, thereby, continuously enhancing the efficiency of the FORM-based FERA. Notably, surrogate model training in the proposed method does not require additional analysis because a surrogate model can be constructed based only on the outcomes of the previous FERA.

## 3. NUMERICAL EXAMPLE

### *3.1. FE model and input ground motions*

To test the proposed method, it was applied to API 5L X65, which is a widely-used buried gas pipe in Korea, as shown in Figure 3. In this example, the 3-D pipe-soil interaction element (PSI34) of ABAQUS was used to conduct soil-structure interaction analysis. The PSI34 element represents nonlinear constitutive behavior by modeling the interaction of a buried pipeline structure and the surrounding soil region (Audibert et al., 1984). A previous study (Yoon et al., 2019) calculated the seismic fragility of the structure, and seismic fragility curves for the same structure were derived in this study using the proposed method. Further details of the example problem can be found in the paper by Yoon et al. (2019).

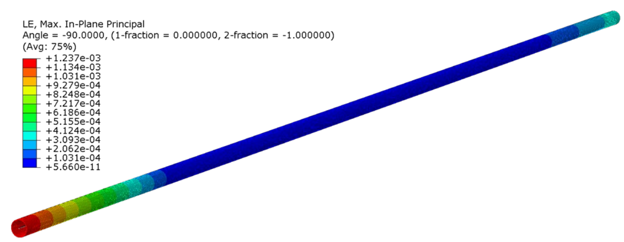


*Figure 3: Buried gas pipe model for ABAQUS.*

To consider the uncertainty of earthquake ground motions during seismic fragility curve derivation, in this example, seven input ground motions were selected, which are summarized in Table 1.

*Table 1: Selected input ground motions*

| | *Earthquake (year)* | *PGA (g)* | *Magnitude* |
|---|---|---|---|
| *EQ1* | *Gulf of Aqaba (1995)* | *0.109* | *7.2* |
| *EQ2* | *Borrego (1968)* | *0.130* | *6.63* |
| *EQ3* | *Chi-Chi (1999)* | *0.263* | *7.62* |
| *EQ4* | *Imperial Valley (1979)* | *0.351* | *6.53* |

| | | | |
|---|---|---|---|
| *EQ5* | *Kocaeli (1999)* | *0.187* | *7.51* |
| *EQ6* | *Landers (1992)* | *0.109* | *7.28* |
| *EQ7* | *Tabas (1978)* | *0.108* | *7.35* |

### *3.2. Random variables and limit states*

In addition to the uncertainty of the seismic ground motions, the uncertainty of the steel yield strength ($f_y$) and soil parameters of the internal friction angle ($\varphi$), cohesion ($c$), and unit weight ($\gamma$) were considered by assuming them as Gaussian RVs. The mean, coefficient of variation (COV), and correlation coefficient are presented in Table 2.

*Table 2: Statistical characteristics of RVs*

| *RVs* | *Mean* | *COV* | *Correlation Coefficient* | | | |
|---|---|---|---|---|---|---|
| $f_y$ | *467* MPa | *0.07* | *1.0* | *0* | *0* | *0* |
| $c$ | *5* kPa | *0.10* | *0* | *1.0* | *-0.7* | *1.0* |
| $\varphi$ | *38°* | *0.10* | *0* | *-0.7* | *1.0* | *0.3* |
| $\gamma$ | *18* kN/m$^3$ | *0.10* | *0* | *0* | *0.3* | *1.0* |

To calculate a fragility estimate, a limit state needs to be defined. Among the few studies where the damage states of a buried pipeline were defined as limit-state functions, the damage states proposed by Shinozuka et al. (1979) were applied in this study. Damage states were classified as minor, moderate, and major damages, which physically represent minor leakage, partial leakage, and complete break, respectively. Table 3 presents the corresponding structural responses. In the table, $\varepsilon_p$ denotes the maximum strain of the buried pipeline under earthquake loading, and $\varepsilon_y$ denotes the yield strain.

*Table 3: Damage states and the corresponding structural responses*

| *Damage state* | *Structural response* |
|---|---|
| *Minor* | $\varepsilon_p \leq 0.7\varepsilon_y$ |
| *Moderate* | $0.7\varepsilon_y \leq \varepsilon_p \leq \varepsilon_y$ |
| *Major* | $\varepsilon_y \leq \varepsilon_p$ |

### *3.3. Analysis results*

Seismic fragility curves were derived based on FORM-based FERA employing PIFA. However, for comparison purposes, the analysis was conducted with and without the surrogate model, which is hereafter referred to as the conventional and proposed methods, respectively.

For simplicity, in this study, fragility curves were derived for moderate damage states only. Figure 4 shows the seismic fragility curves obtained using the conventional and proposed methods, and it can be observed that the results are almost the same. However, the total number of FE analyses required for the conventional and proposed methods were 2,578 and 1,521, respectively, as shown in Table 4. This means that the computational cost of the proposed method was approximately 60% of that of the conventional method. This clearly shows that the proposed method can be used to derive accurate seismic fragility curves more efficiently.

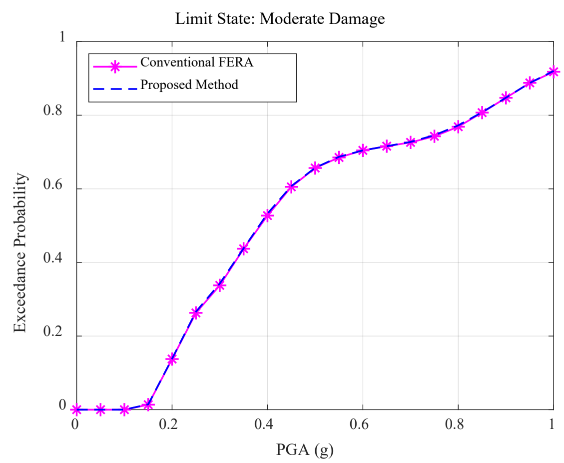


*Figure 4: Seismic fragility curves obtained from conventional and proposed methods*

*Table 4: Computational cost of conventional and proposed methods*

| | *Conventional method* | *Proposed method* |
|---|---|---|
| *EQ1* | *282* | *181* |
| *EQ2* | *181* | *137* |
| *EQ3* | *615* | *323* |
| *EQ4* | *331* | *186* |
| *EQ5* | *168* | *108* |
| *EQ6* | *425* | *275* |
| *EQ7* | *576* | *311* |
| *Total* | *2,578* | *1,521* |

## 4. CONCLUSION

In this study, an efficient method was newly proposed for seismic curve derivation based on repeated FORM-based FERA. By employing a sequentially-updated surrogate model, FORM analysis can proceed with optimal starting values of the random variables, significantly reducing the computational cost. In addition, because prior FERA analysis results are used, there is no additional cost to construct the surrogate model. The proposed method was applied to calculate seismic fragility estimates for a buried gas pipe, and the analysis cost was found to be approximately 60% of that of the conventional FERA method, with similar accuracy to the analysis results.

## 5. ACKNOWLEDGEMENT

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