

Article

Structural Reliability of Tall Buildings Under Wind Loads with Tuned Mass Damper Fluid Inerters

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Abstract: This study investigates the reliability of tall buildings subjected to dynamic across-wind loading, focusing on the Tuned Mass Damper Fluid Inerter (TMDFI). While existing literature emphasises the effectiveness of TMDFI in mitigating seismic hazards, research on its reliability regarding wind hazards remains limited. A wind-sensitive benchmark 76-storey building is modeled to compare the performance of the TMDFI against a traditional tuned mass damper (TMD) and an uncontrolled structure. A Monte Carlo Simulation (MCS) approach comprising 31,500 simulations is employed to assess reliability under uncertain damping ratios and varying turbulence intensities at reference wind speeds of 20 to 40 m/s. Key performance metrics, including peak acceleration and root mean squared (RMS) displacement responses, are derived through spectral analysis in the frequency domain. Results indicate that the TMDFI offers superior reliability, allowing an additional 6–7 m/s in reference velocity before reaching significant failure at the ISO limit state. Peak acceleration and RMS displacement are reduced by up to 64% to the uncontrolled structure. The TMDFI consistently outperforms both the TMD and uncontrolled configurations across all turbulent cases and wind velocities examined.

Keywords: inerter; tuned mass damper; vibration control; tall building



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1. Introduction

Tall buildings face unique design challenges due to their increased sensitivity to dynamic wind loading. As building height increases, structural stiffness typically decreases, resulting in lower natural frequencies that can align with the energy-rich, low-frequency components of turbulent wind spectra [1]. Simultaneously, the wind speed, and hence aerodynamic load, increases with height due to the wind shear effect, amplifying the dynamic excitation of upper floors. This combination can lead to significant across-wind responses, vortex shedding, and resonance phenomena, particularly in flexible, slender structures commonly found in modern cities.

Wind-induced vibrations in tall buildings can affect structural performance, compromise occupant comfort, and disrupt the operation of sensitive equipment on upper floors. As architectural and urban trends continue to favor taller, more flexible structures, these challenges become more pronounced. In such systems, damping plays a critical role in controlling dynamic response, often serving as the primary mechanism for mitigating wind-induced motion. Accurate estimation and effective utilization of damping are therefore essential to ensure safety, serviceability, and occupant well-being under wind loading. However, despite decades of advancement in structural dynamics, the accurate estimation of damping in tall buildings remains an unresolved challenge. As noted by

Kareem and Gurley [2] in the classical text on the subject, reliable damping values are often only obtainable post construction, and this uncertainty continues to necessitate probabilistic approaches in response prediction and vibration control design. Determining the precise level of structural damping in a structure is challenging, often involving the installation and design of complex monitoring systems and algorithms [3–5]. In many tall, flexible buildings, the inherent structural damping is often insufficient to effectively mitigate structural vibrations [6]. If inherent structural damping is insufficient to control wind-induced vibrations, engineers must design and implement auxiliary damping devices, such as tuned mass dampers or fluid viscous dampers, to supplement energy dissipation and achieve acceptable performance levels [7]. Experimental studies conducted in the past few years have shown that installing dampers can increase the first-mode damping ratio by more than 500% [8]. This substantial enhancement in damping underscores the critical role of dampers in improving the dynamic performance of tall buildings.

Damping devices used for vibration control in tall buildings are typically classified as passive, semi-active, active, or hybrid systems. Passive dampers, such as tuned mass dampers (TMDs), operate without external power and rely on mechanical properties to dissipate energy. The passive TMD is the most commonly used auxiliary damper and is installed in many of the world's tallest buildings [9,10], including in super-tall buildings [11] and in tall modular structures [12]. Passive dampers are simple, reliable, and low-cost. They are activated by the motion of the structure and require no power source to operate. In recent studies, passive TMDs have been shown to perform well in tall buildings under wind loads [13,14]. Other passive dampers such as tuned liquid dampers (TLDs) [15,16] and tuned liquid column dampers (TLCDs) [17] have also been proposed in recent years. Semi-active systems adjust their properties in real time based on structural response but do not add energy to the system [9]. Semi-active TMDs have been used for wind-induced vibration control of tall buildings [18], and magnetorheological (MR) [19,20] dampers have been utilized for this purpose, too. Active control systems, such as active mass dampers (AMDs), apply external energy through actuators to counteract structural motion, enabling precise control but requiring continuous power and complex control algorithms [21]. Active tuned mass dampers (ATMDs) have been used to mitigate wind induced vibrations in multistorey buildings [22]. Recently, an ATMD system was proposed and installed in a 600 m tall skyscraper to suppress wind-induced building vibrations [23]. AMDs have also been used for vibration control of tall flexible structures subjected to wind loads [24]. Hybrid systems combine elements of active and passive control, offering a balance between energy efficiency and adaptability; for example, a hybrid mass damper may use a passive TMD supported by active feedback control to enhance performance across a wider range of excitation conditions [25]. These types of damping systems have been used in recent studies to suppress wind-induced vibration of tall buildings such as hybrid TLCDs [26] and hybrid TMDs [27].

The performance of damping systems in tall buildings depends on their type, mass, and stroke length. Passive dampers are simple and reliable but require large mass and stroke, offering modest vibration suppression. Semi-active and active systems provide better control and adaptability, with increased stroke lengths for enhanced performance, but they introduce complexity, power reliance, and higher costs. Hybrid systems balance these traits, though they still require some external input and tend to be complex to design and maintain [25].

Recently, inerter-based systems have been widely investigated and implemented to mitigate wind-induced motions in tall buildings [28]. Inerter-based dampers amplify inertial effects without added damper mass, achieving superior vibration suppression compared to traditional dampers across a wide frequency range. With less mass required

and reduced stroke lengths, they offer efficient, effective vibration control, making them a practical evolution in structural control for tall buildings. Initially proposed by Smith [29], the ideal inerter is a linear mass-less two-terminal mechanical element resisting the relative acceleration at its terminals through an inertance coefficient, b , measured in units of mass.

The inerter behaves as an inertial weightless element whose gain depends on the relative acceleration observed by its terminals and its inertance. A TMD coupled with an inerter has been used in several studies for vibration control of tall flexible structures such as buildings, wind turbine blades, and towers [30–33]. Recently, the Tuned Mass Damper Fluid Inerter (TMDFI) has seen interest in the literature for both the seismic control of structures [34] and for the attenuation of wind-induced vibrations [35,36] in tall flexible structures. The fluid inerter device consists of a piston, encased in a cylinder containing fluid. When the piston moves through the cylinder, it forces fluid through a helical coil channel which generates a resistive force due to the inertia of the moving fluid. Due to the presence of the helical channel, a portion of this resistive force is proportional to the acceleration of the piston and therefore the relative acceleration of the chosen inerter terminals [37]. A diagram of the idealised fluid inerter device is shown in Figure 1, with the key fluid inerter design parameters given in Table 1.

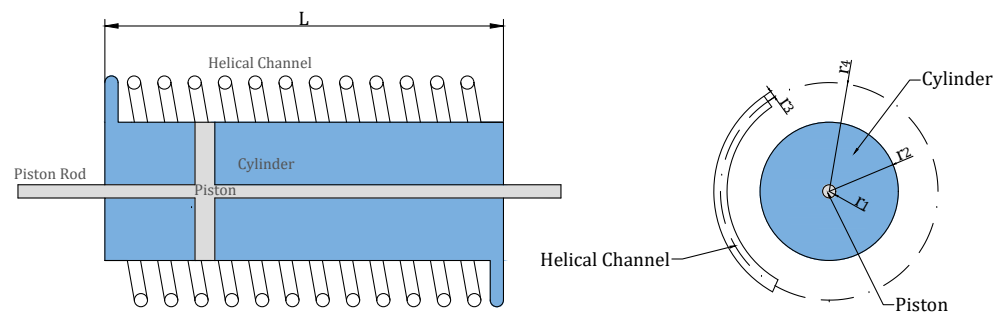


Figure 1. Ideal fluid inerter, adapted from [34].

Table 1. Physical Properties of the Helical Fluid Inerter.

Symbol	Units	Meaning
r_1	m	Radius of the piston
r_2	m	Inner radius of cylinder
r_3	m	Inner radius of helical channel
r_4	m	Radius of helix
h	m	Pitch of helix
n_t	-	Number of turns of helix
L	m	Inner length of cylinder
$l = n_t \sqrt{h^2 + (2\pi)r_4^2}$	m	Length of helical channel
R_b	m	Bend radius of helical channel
$A_c = \pi(r_2^2 - r_1^2)$	m ²	Cross-sectional area of cylinder
$A_h = \pi r_3^2$	m ²	Cross-sectional area of helical channel
$m_{hel} \approx \rho_f l A_2$	kg	Mass of liquid in helical channel
ν_f	cSt	Kinematic viscosity of fluid
ρ_f	kg/m ³	Mass density of fluid
$\mu_f = \rho_f \nu_f \times 10^{-6}$	Pa.s	Dynamic viscosity of fluid

Considering the geometric properties of the fluid inerter, the inertance b required to determine the inertance force may be approximated as

$$b \approx \left[\frac{m_{hel}}{1 + (h/2\pi r_4)^2} \left(\frac{A_c}{A_h} \right)^2 \right] \quad (1)$$

Recently, notable applications of reliability theory in inerter-based dampers have been proposed. Much of the literature has investigated the reliability of these devices under seismic actions. Giaralis and Taflanidis [38] and Ruiz et al. [39] both employed Monte Carlo Simulations, finding that TMDIs offer enhanced structural control and outperform TMDs in reducing damage. Petrini et al. [40] and Zhou et al. [41] used Uncertain Parameter Variation methods, showing that TMDIs can maintain or improve occupant comfort and structural reliability even under uncertain parameters, particularly when the inertance is sufficiently large. Most recently, Peng and Sun [42] used the probability density evolution method, confirming that optimally designed TMDIs provide superior control to TMDs at the same mass ratio, especially under stochastic excitation conditions.

Aims of This Study

Having presented the literature on the dynamics of tall buildings, the use of damping devices, and their reliability under various loading conditions, we now aim to investigate the effectiveness of the Tuned Mass Damper Fluid Inerter (TMDFI) in improving structural reliability under dynamic across-wind loading. While much of the existing research has focused on its application for mitigating seismic hazards, this study seeks to fill the gap in understanding its performance in wind-sensitive environments. By comparing the TMDFI to both a traditional tuned mass damper (TMD) and an uncontrolled structure, we aim to assess its potential to enhance structural performance and reliability under uncertain damping properties and varying wind conditions.

2. Methodology

This paper uses the 76-storey building proposed in [43] as a benchmark model. The structure has a height of 306.1 m and a total mass of 153,000 metric tons. The building has a square plan area with a depth (D) and breath (B) of 42 m. The structure is slender with an aspect ratio of 7.3, rendering it particularly susceptible to wind effects. Both the plan and elevation of this benchmark structure are shown in Figure 2.

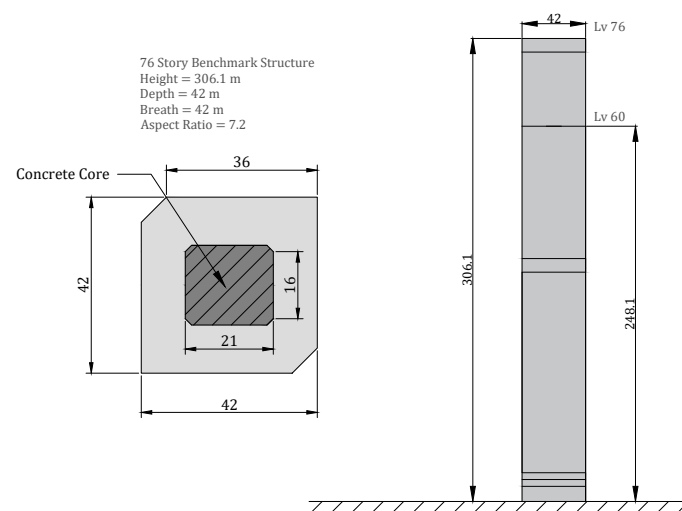


Figure 2. Elevation and plan view of the benchmark structure, adapted from [43].

The 76-storey building is modeled as a vertical cantilever using Bernoulli–Euler beam theory. A finite element model of the structure is constructed by considering the portion of the building between two adjacent floors as a classical beam element of uniform thickness. This results in a model with 76 degrees of freedom (DOF) (translational). The first three calculated natural frequencies of the 76-DOF model are 0.16, 0.765, and 1.992 Hz.

The input across-wind action to the 76-DOF model is computed using the across-wind model developed in [44]. This model captures the vortex shedding frequency within the overall across-wind model. For structures square in plan such as this, the vortex shedding frequency emerges as an obvious peak on the across-wind power spectrum, with the overall spectrum dominated by this effect, therefore having the appearance of a single peak. The model proposes that the diagonal terms of the across-wind power spectrum matrix are given as

$$\mathbf{S}_{FF}[k, k](\omega) = \frac{\sigma^2}{\omega} \left\{ A \frac{H(C_1)\bar{\omega}^2}{(1 - \bar{\omega}^2)^2 + C_1\bar{\omega}^2} + (1 - A) \frac{C_2^{0.5}\bar{\omega}^2}{1.56[(1 - \bar{\omega}^2)^2 + C_2\bar{\omega}^2]} \right\} \quad (2)$$

which specifies the power spectral density (PSD) of the k th floor. The parameters involved in Equation (2), as outlined in [44], are

$$\begin{aligned} H(C_1) &= 0.179C_1 + 0.65\sqrt{C_1} \\ C_1 &= [0.47(D/B)^{2.8} - 0.52(D/B)^{1.4} + 0.25]/(H/\sqrt{S}) \\ C_2 &= 2 \\ A &= (H/\sqrt{S})[-0.118(D/B)^2 + 0.358(D/B) - 0.214] + [0.066(D/B)^2 - 0.26(D/B) + 0.894] \\ \sigma &= \frac{1}{2}\rho[U(z)T_I]^2\bar{C}_L B\Delta z \\ \bar{C}_L &= 0.045(D/B)^3 - 0.335(D/B)^2 + 0.868(D/B) - 0.174 \\ S &= B \times D \\ \bar{\omega} &= \frac{\omega}{\omega_s} \\ \omega_s &= \frac{2\pi S_t U(z)}{B} \\ S_t &= 0.002(D/B)^2 - 0.023(D/B) + 0.105 \end{aligned} \quad (3)$$

where $\bar{C}_L$ is the mean of the RMS lift coefficient of the rectangular building, C_1 and C_2 are experimentally determined parameters corresponding to the spectrum bandwidth, σ is the root mean square (RMS) of the across-wind force, z is the height of the k^{th} floor from ground level, ω_s is the vortex shedding frequency in rad/s, S_t is the Strouhal number, S is the cross-sectional area of the building, ρ is the air density taken as 1.225 kg/m³, and Δz is the tributary height of the k^{th} floor. $U(z)$ is the mean wind speed at height z above the ground level given as

$$U(z) = V_{ref} \left(\frac{z}{H_{ref}} \right)^\alpha \quad (4)$$

where V_{ref} represents the reference velocity (commonly referred to as the design wind speed), H_{ref} is the reference height at which this reference speed is taken. For this study, H_{ref} is assumed to be 810 m, representing the characteristic height of the atmospheric boundary layer [40]. α represents the dimensionless shear exponent that depends on the terrain's surface roughness and is assumed to be 0.35, corresponding to a congested urban area [45]. To effectively model the off-diagonal terms of the across-wind power spectra, Ref. [44] proposes a vertical coherence function r_{ij} . This function is obtained from wind

tunnel testing and appears in the model to correct for the vertical decay in the co-spectra of across-wind forces between two floors as the distance between the floors increases. The coherence function used is given as

$$r_{ij}(z_k, z_j) = \exp \left[- \left(\frac{|z_k - z_j|}{B\delta} \right)^2 \right] \quad (5)$$

where z_k and z_j are the heights of the floors considered and δ is the experimentally determined coefficient determining the rate of decay, taken as 5.56. This yields an overall across-wind force power spectral density function of

$$\mathbf{S}_{FF}[k, j](\omega) = r_{ij}(z_k, z_j) \sqrt{S_{FF}[k, k](\omega) S_{FF}[j, j](\omega)} \quad (6)$$

Although the model above is deterministic in nature and is rooted in empirical data, for the purpose of a reliability assessment, both the reference velocity and the turbulence intensity are varied. The reference velocity is varied from 20 m/s to 40 m/s at a constant 1 m/s interval. The range of wind velocities considered is selected with aims of encapsulating extreme wind loading conditions across differing regions with varying levels of wind exposure. The turbulence intensity is varied by considering three cases: a low-turbulence case, a medium-turbulence case, and a high-turbulence case. This discretization of turbulence intensity is taken to maintain the computational tractability of the model. The turbulence intensity values selected for this study are presented Table 2.

Table 2. Chosen Turbulence Intensity Values.

Turbulence Intensity Case	Value Assigned
Low	6%
Medium	9%
High	12%

The turbulence intensity approach utilized in this study represents a simplification of the complex process below the atmospheric boundary. The three selected turbulence intensity values are taken as constant along the height of the buildings. The turbulence intensity plays a key role in shaping the power spectral density (PSD) of wind loads, particularly in across-wind direction. In the current study, turbulence intensity influences the variance of fluctuating wind speeds at each height and therefore directly affects the magnitude of the aerodynamic force spectra.

Specifically, the PSD of wind-induced lift forces is proportional to the square of the local turbulence intensity, as seen in the formulation adapted from Gu and Quan [46], where the spectral density at each height is computed as

$$\sigma^2(z) \propto [U(z) \cdot \text{TI}]^2$$

This relationship implies that higher turbulence intensity leads to larger fluctuating force components and thus increases the energy content across the frequency domain. This relationship is evident in the across-wind power spectrum generated, shown in Figure 3. A direct comparison of the across wind power spectrum at 40 m/s in the three selected turbulence cases is given in Figure 4.

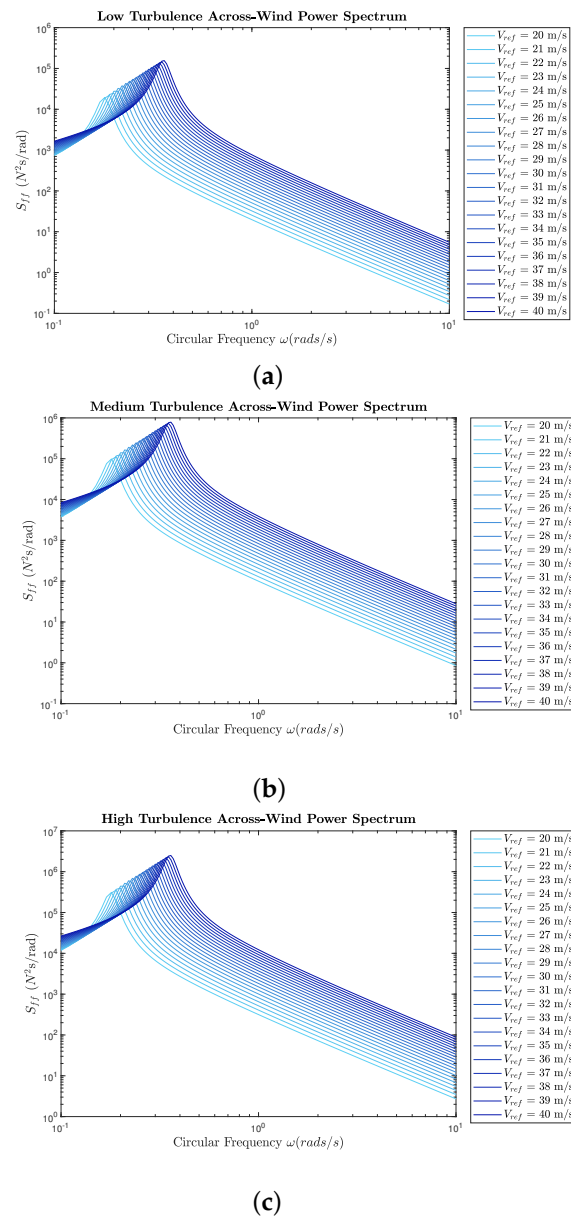


Figure 3. Across-wind power spectra. (a) Low turbulence intensity. (b) Medium turbulence intensity. (c) High turbulence intensity.

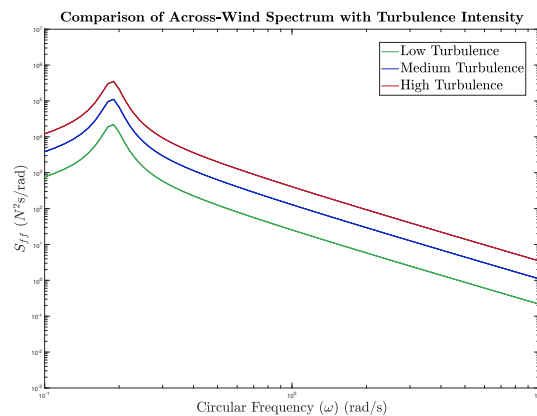


Figure 4. Comparison of across-wind power spectrum at $V_{ref} = 40$ m/s.

2.1. Response Model

To model the structure's response, the spectral method of modal analysis is utilized, with analysis of the structure completed in the frequency domain. The displacement response spectrum can be obtained directly from the power spectrum of the wind forces by solving

$$S_{qq}(\omega) = H^*(\omega)S_{ff}(\omega)H^T(\omega) \quad (7)$$

The transfer function matrix $H(\omega)$ used in the frequency-domain spectral analysis is computed directly in physical coordinates using the full coupled system matrices. For each frequency ω , the transfer matrix is calculated as

$$H(\omega) = \left(-\omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K} \right)^{-1} \quad (8)$$

This represents a full system (non-modal) transfer function. $\mathbf{M}$, $\mathbf{K}$, $\mathbf{C}$ denote the mass, stiffness, and damping matrices, respectively. The acceleration response spectrum is obtainable from the displacement response spectrum through the following relationship:

$$S_{aa}(\omega) = \omega^4 [S_{qq}(\omega)] \quad (9)$$

The root mean squared (RMS) structural responses are available through the computation of the following quantities:

$$\sigma_a = \sqrt{\int_{\omega_i}^{\omega_n} S_{aa}(\omega) d\omega} \quad (10)$$

$$\sigma_d = \sqrt{\int_{\omega_i}^{\omega_n} S_{qq}(\omega) d\omega} \quad (11)$$

where σ_a and σ_d are vectors ($\in \mathbb{R}^{76 \times 1}$) that contain the floor-wise RMS acceleration and displacements, respectively. The peak acceleration ($\ddot{q}_{max}$) response is then estimated using a standard gust factor approach [47] such that

$$\ddot{q}_{max} = g_f \sigma_a \quad (12)$$

where the gust factor g_f is given as

$$g_f = \sqrt{2 \ln(T_s) f_s} + \frac{0.6}{\sqrt{2 \ln(T_s) f_s}} \quad (13)$$

where f_s is the fundamental natural frequency of the benchmark structure. T_s represents the sampling time and is assumed to be 600 s. This spectral response model is used for all the structural arrangements considered. With each structural arrangement modeled through modification of the mass, stiffness, and damping matrices, these modifications are discussed below.

2.2. Uncontrolled Case

The uncontrolled case represents the benchmark structure's response without any control device installed. The mass and stiffness matrix for the uncontrolled structure are notated as M_{76} and K_{76} , both ($\in \mathbb{R}^{76 \times 76}$). The damping matrix C_{76} is developed using the Rayleigh damping method. The method is based on the identification of two variables a_0 and a_1 that are used in conjunction with the mass and stiffness matrices to develop the damping matrix. To this end, the damping matrix is a linear combination of the mass and stiffness matrices such that

$$C = a_0[M] + a_1[K] \quad (14)$$

where the dimensionless coefficients a_0 and a_1 are determined based on the desired damping ratios through the following matrix operation:

$$\frac{1}{2} \begin{bmatrix} 1/\omega_i & \omega_i \\ 1/\omega_j & \omega_j \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \zeta_i \\ \zeta_j \end{bmatrix} \quad (15)$$

where ω_i and ω_j are the natural circular frequencies of the i th and j th modes in rad/s. ζ_i and ζ_j are the desired damping ratios (given in %) between the i th and j th modes.

The selection of damping ratios in this study follows the probabilistic framework introduced by Bashor et al. [48], which provides empirically derived damping ratios (ζ) for tall buildings subjected to wind loading. This framework accounts for inherent variability in structural damping and offers a realistic representation of damping effects in serviceability conditions.

Specifically, Bashor et al. [48] proposes a lognormal distribution of damping ratios (with a mean of 1 and a covariance of 0.3) based on full-scale measurements of tall buildings. The stochasticity in the model is introduced through these empirically derived (ζ) values.

2.3. Control with a Tuned Mass Damper

The properties of the tuned mass damper (TMD) are modeled by appending an additional row and column to the uncontrolled system matrices, resulting in a 77 DOF structural model. The TMD is assumed to be located at the top floor of the structure and as such all modifications to the system matrices occur in the last two rows and columns. The TMD is tuned to the building's fundamental natural frequency using the formulae proposed in [49] and given below.

$$\omega_{r,TMD} = \frac{1}{1 + \mu} \quad (16)$$

$$\zeta_d = \sqrt{\frac{3\mu}{8(1 + \mu)}} \quad (17)$$

where $\omega_{r,TMD}$ and ζ_d are the tuning ratio and damping ratio of the TMD, respectively. μ denotes the ratio between the mass of the damper and the fundamental modal mass of the building, and it is taken as 2.5%. These parameters are then used to generate the combined TMD system matrices, denoted as M_{TMD} , K_{TMD} , and C_{TMD} , all ($\in \mathbb{R}^{77 \times 77}$), through the following matrix operations:

$$M_{TMD}[1 : 76, 1 : 76] = M_{76} \quad (18)$$

$$M_{TMD}[76 : 77, 76 : 77] = M_{76}[76 : 77; 76 : 77] + \begin{bmatrix} 0 & 0 \\ 0 & m_f \mu \end{bmatrix} \quad (19)$$

where m_f is the fundamental modal mass of the building. The notation $[i : j, i : j]$ refers to the indices for assignment in the resultant matrices, where the comma delineates the row and column indices and the $:$ operation notes the span in vector space of the assignment.

$$K_{TMD}[1 : 76, 1 : 76] = K_{76} \quad (20)$$

$$K_{TMD}[76 : 77, 76 : 77] = K_{76}[76 : 77; 76 : 77] + \begin{bmatrix} (2\pi f_s \omega_{r,TMD})^2 m_f \mu & -(2\pi f_s \omega_{r,TMD})^2 m_f \mu \\ -(2\pi f_s \omega_{r,TMD})^2 m_f \mu & (2\pi f_s \omega_{r,TMD})^2 m_f \mu \end{bmatrix} \quad (21)$$

$$C_{TMD}[1 : 76, 1 : 76] = C_{76} \quad (22)$$

$$C_{TMD}[76 : 77, 76 : 77] = C_{76}[76 : 77; 76 : 77] + \begin{bmatrix} 4\pi\zeta_d\omega_r TMD f_s m_d & -4\pi\zeta_d\omega_r TMD f_s m_d \\ -4\pi\zeta_d\omega_r TMD f_s m_d & 4\pi\zeta_d\omega_r TMD f_s m_d \end{bmatrix} \quad (23)$$

2.4. Control with a Tuned Mass Damper Fluid Inerter

Modifying the uncontrolled system matrices to include a Tuned Mass Damper Fluid Inerter (TMDFI) is not as straightforward as the inclusion of a TMD. First, the optimal TMDFI parameters must be selected, and then the inertance force must be linearized to allow for analysis in the frequency domain.

For a view of the mechanical diagram of the proposed TMDFI system, refer to Figure 5 below.

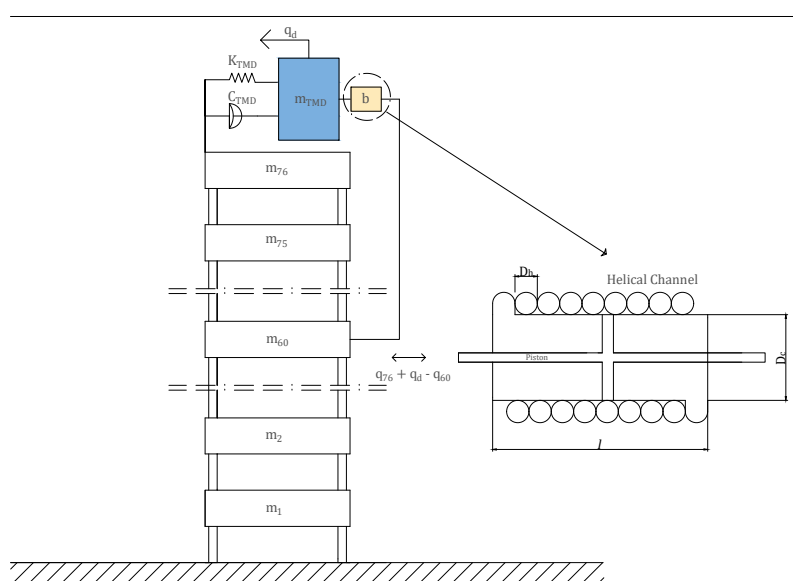


Figure 5. Mechanical diagram of TMDFI system, adapted from [35].

The TMDFI parameters employed in this study are provided in Table 3; these are optimal TMDFI parameters. The inertance force depends on the selected TMDFI parameters, as shown in Table 3. Optimal tuning of the TMDFI is achieved by adjusting the tuning ratio and diameter ratio of the fluid inerter element. In this study, an optimization procedure is employed to minimize the variance in the structure's modal displacements under white noise excitation (S_0), following the approach in [35]. The procedure uses the variance of the modal displacements as the cost function $\Pi(r, \omega_r)$, with the optimization problem taking the general form of

$$\begin{aligned} \text{minimize } \Pi(r, \omega_r) &= \sigma^2 = \frac{1}{m_0} \int_{-\infty}^{\infty} |\Lambda_{11}(\omega)|^2 S_0 d\omega \\ \text{subject to } &I \leq x \leq u \end{aligned} \quad (24)$$

where Λ_{11} is the first component of the transfer matrix $H(\omega)$ and $x = [r, \omega_r]$. Using an analytical technique as per [50], the integral in Equation (24) can be obtained as

$$\Pi(r, \omega_r) = \pi S_0 \frac{(r^2 + 1)^2 + \omega_r^4 \gamma^2 + \omega_r^2 \chi}{2\mu\omega_i^3 \zeta_{d,eq} \omega_r \epsilon^2} \quad (25)$$

where $\gamma = 2\mu\phi^2 - \mu\phi + 1$, $\chi = \left\{ \mu\phi^2 r^4 + \mu(r^2 + 1)^2(1 - 2\phi) - 2(r^2 + 1) + 4\zeta_{d,eq}^2(\mu\phi^2 + 1) \right\}$ and $\epsilon = (2\mu\phi^3\omega_r^2 - 3\mu\phi^2\omega_r^2 + \mu\phi\omega_r^2 + \phi\omega_r^2 + \phi - \omega_r^2)$. $\zeta_{d,eq}$ represents the equivalent damping ratio for the TMDFI system and is defined as

$$\zeta_{d,eq} = \frac{0.21097 \sqrt[3]{\mu S_0(1+r^2)} r^{10/3}}{D_c^{2/3} \omega_b \omega_r \sqrt[3]{(1+r)(\mu\phi^2 r^2 + r^2 + 1)}} \quad (26)$$

where $r = \frac{D_c}{D_h}$, ω_b is the natural frequency of the building in rad/s, ω_d is the natural frequency of the damper in rad/s, the tuning ratio ω_r is defined as the ratio between ω_d and ω_b , and ϕ is the normalized displacement ratio between the top of the building and the height of the inerter hook from ground level determined as 0.7.

Table 3. TMDFI parameters used.

Parameter	Value	Units
Tuning ratio (ω_r)	Variable	-
Diameter ratio (r)	Constant	-
Fluid dynamic viscosity (μ_f)	0.009	Pa.s
Fluid density (ρ_f)	997	kg/m ³
Tuned mass weight (m_d)	1.88×10^3	kg
TMD spring stiffness (k_d)	3.04×10^6	N/m
Fluid inerter channel length (l)	2	m
Fluid inerter channel diameter (D_c)	1.09	m
Fluid inerter helix diameter (D_h)	0.05	m
Fluid inerter bent radius of helix (R)	0.57	m
TMDFI hook floor (F_l)	60th	Floor no.
Normalized displacement ratio (ϕ)	0.7	-
Cross-sectional area of cylinder (A_c)	0.93	m ²
Cross-sectional area of helical channel (A_h)	1.96×10^{-3}	m ²

Considering Equation (24), the constraint variables, I and u are vectors ($\in \mathbb{R}^{2 \times 1}$), which are arbitrarily taken as $I = (1, 1)$ and $u = (38, 100)$. This gives a solution to the optimization problem of $[(r, \omega_r) = (38, 22.4)]$ for the benchmark problem considered. For further information on the optimization procedure and the arbitrary selection of I and u , readers should refer to [35].

Although this study is primarily concerned with building acceleration, the above optimization approach (minimization of the frequency-weighted modal displacement response of the augmented system) is sought as, follows:

- Displacement and acceleration responses are strongly correlated in lightly damped structures like the 76-story benchmark building, especially near the fundamental mode, which dominates the response.
- Acceleration is proportional to the second derivative of displacement; hence, improvements in displacement generally lead to improvements in acceleration response.
- Optimization based directly on acceleration spectra is also tested but found to be less stable numerically due to higher sensitivity to high-frequency components and noise in the power spectral density functions.

Once the optimal TMDFI parameters are identified, the inertance force is linearized to account for deviations from the ideal inerter device. These deviations arise from non-linear control forces generated by the TMDFI. A statistical linearization technique is used

to linearize the non-linear damping force. The linearized damping coefficient ($c_{ch,lin}$) is given as

$$c_{ch,lin} = c_{hc} \sqrt{\frac{8}{\pi}} \sigma_{\dot{q}_{\bar{d}}} \quad (27)$$

$$c_{hc} = 0.03426 \left(\frac{2\rho A_c}{\sqrt{D_h r}} \right) \left(\frac{A_c}{A_h} \right)^2 \quad (28)$$

where $\sigma_{\dot{q}_{\bar{d}}}$ is the variance of the relative velocity between the inerter terminals. c_{hc} represents the non-linear damping inerter damping coefficient. In Equations (27) and (28), the standard deviation of the relative displacements at DOF $\dot{q}_{\bar{d}}$ is unknown and depends on the damping coefficient $c_{ch,lin}$. It is observed in Equation (27) that a one-step solution is not possible, and therefore an iterative approach is utilized. By assuming an initial value of $c_{ch,lin}$, an approximation of $\dot{q}_{\bar{d}}$ is determined. This approximated value is then used to generate a better estimate of $c_{ch,lin}$ in the next iteration. This iterative approach is taken until the values of $\dot{q}_{\bar{d}}$ and $c_{ch,lin}$ converge. The model assumes that convergence has occurred when the absolute difference between the values is less than 1×10^{-5} .

With both the optimal TMDFI parameters identified and the non-linear damping force linearized, the TMDFI system matrices can be constructed. Similar to the TMD, the combined system matrices are notated as M_{TMDFI} , K_{TMDFI} , and C_{TMDFI} , all $\in \mathbb{R}^{77 \times 77}$, generated based on the following matrix operations:

$$\begin{aligned} M_{TMDFI}[1 : 76, 1 : 76] &= M_{76} \\ M_{TMDFI}[F_L, F_L] &= M_{76}[F_L, F_L] - m_f \mu \\ M_{TMDFI}[F_L, 77] &= M_{76}[F_L, 77] - m_f \mu \\ M_{TMDFI}[77, F_L] &= M_{76}[F_L, F_L] - m_f \mu \\ M_{TMDFI}[77, 77] &= (m_f \mu + m_f \mu (r)^2) \end{aligned} \quad (29)$$

where F_L denotes the index of the floor number to which the inerter hook is attached.

$$\begin{aligned} C_{TMDFI}[1 : 76, 1 : 76] &= C_{76} \\ C_{TMDFI}[76 : 77, 76 : 77] &= C_{76}[77, 77] + \left[c_{ch,lin} \sqrt{\frac{8}{\pi}} \sigma_{\dot{q}_{\bar{d}}} \right] \end{aligned} \quad (30)$$

$$\begin{aligned} K_{TMDFI}[1 : 76, 1 : 76] &= K_{76} \\ K_{TMDFI}[F_L, F_L] &= K_{76}[F_L, F_L] + (2\pi f_s \omega_r)^2 m_f \mu \\ K_{TMDFI}[F_L, 77] &= K_{76}[F_L, 77] - (2\pi f_s \omega_r)^2 m_f \mu \\ K_{TMDFI}[77, F_L] &= K_{76}[F_L, F_L] - (2\pi f_s \omega_r)^2 m_f \mu \\ K_{TMDFI}[77, 77] &= (2\pi f_s \omega_r)^2 m_f \mu \end{aligned} \quad (31)$$

* $c_{ch,lin}$ and $\sigma_{\dot{q}_{\bar{d}}}$ are dependent on the input.

2.5. Monte Carlo Simulations

Numerical simulations are conducted for both uncontrolled and controlled models. Reliability assessment under each turbulence case, as presented in Table 2, is performed using a one-step Monte Carlo Simulation (MCS). This method estimates the probability of failure for each structural arrangement at a given reference wind velocity, based on the wind loading and response models described earlier. Stochastic damping ratios (ζ_i, ζ_j) are generated for each MCS sample, and the output response is evaluated against an acceleration limit state. The limit state, defined by ISO 10137:2007 [51], represents the

maximum allowable peak acceleration for an office building with a 1-year return period. For the benchmark structure with a fundamental frequency of 0.16 Hz, this limit is 0.1376 m/s^2 . In this paper, we examine limit states at 80%, 90%, 100%, 110%, and 120% of the prescribed ISO peak acceleration limit value, as outlined in Table 4.

In this study, 21 reference wind velocities are considered, uniformly distributed between 20 and 40 m/s. For each simulation, random samples of (ζ_i, ζ_j) are generated. This is used to generate a realization of the uncertain damping matrix using the Rayleigh damping method. This realized damping matrix is then used to calculate the RMS acceleration, RMS displacement, and peak acceleration for the three structural arrangements. The calculated peak acceleration is then checked for exceedance under the five expanded limit states; see Table 4. The required number of Monte Carlo samples is validated by assessing the convergence of peak acceleration at a reference wind speed of 30 m/s and a turbulence intensity of 0.06. A total of 10,000 samples are evaluated, with the mean and standard error of the mean calculated at intervals from 50 to 10,000 samples. Results show minimal variation beyond 500 samples, so 500 samples are used for each wind speed. With 21 reference wind velocities, this results in 10,500 simulations per turbulence intensity, totaling 31,500 simulations across all cases. A flowchart of the model outlined above is given in Figure 6.

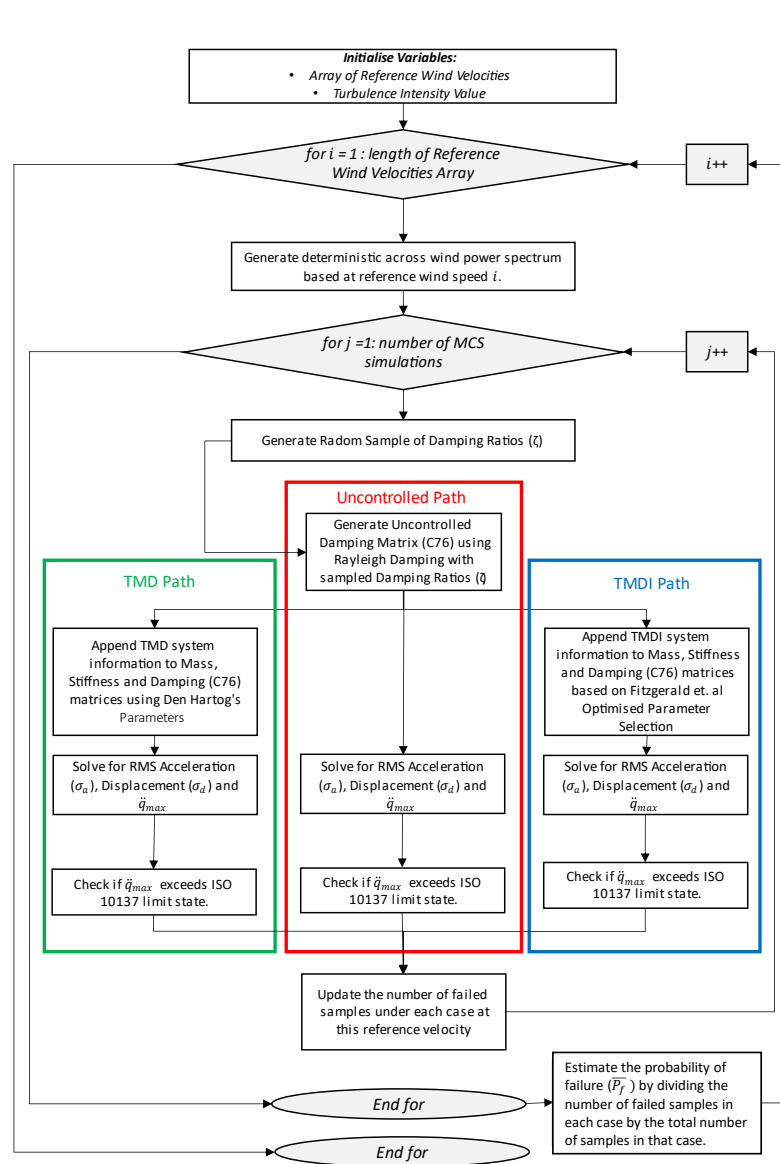


Figure 6. Flow chart of model utilized [33,49].

Table 4. Expanded Limit State Values.

Limit State	Label	Value (m/s ²)
80% of ISO limit	A1	0.1101
90% of ISO limit	A2	0.1238
100% of ISO limit	A3	0.1376
110% of ISO limit	A4	0.1513
120% of ISO limit	A5	0.1651

3. Numerical Results

3.1. Acceleration Limit State and Fragility Curves

Fragility curves are used to graphically depict the progression of the probability of failure, $\bar{P}_f$, across the studied wind reference velocities and turbulence intensity levels. Each curve plots the probability of exceedance over the ISO acceleration limit against the reference velocity, resulting in nine fragility plots—one for each turbulence case (Figure 7). These curves show a complex relationship between $\bar{P}_f$ and the reference velocity, observed as inflection points across the five expanded limit states. These inflections arise due to the discretization of 21 sampled reference velocities, with Piecewise Cubic Hermite interpolation used to generate smooth curves. This method preserves zero-probability values at consecutive intervals and ensures that $\bar{P}_f$ does not exceed a value of one, offering a smoother alternative to linear interpolation. The inflection points are primarily due to the initial 1 m/s discretization. While the spectral method of analysis is computationally efficient, there is a trade-off between increasing the Monte Carlo Simulation (MCS) resolution at each reference velocity and increasing the number of velocity samples, both of which extend computation time.

In Figure 7, it is observed that the reference velocity required to exceed the five limit states decreases (in all cases) with increasing turbulence intensity. This is the expected result as it is observed that the total power of the across-wind power spectrum increases with turbulence intensity. The fragility curves shift to the left with increasing turbulence intensity. To assign magnitudes to this shift, the reference velocity at which >95% of the MCS samples exceed the A3 (ISO 10137 [51]) limit state are compared, with this comparison tabulated in Table 5.

Table 5. Reference Wind Velocity Required for 95% of Samples to Fail at the A3 Limit State.

Turbulence Case	Control Case	V_{ref} for 95% Failure Rate at A3 Limit (m/s)
Low	Uncontrolled	35
	TMD	38
	TMDFI	-
Medium	Uncontrolled	26
	TMD	28
	TMDFI	32
High	Uncontrolled	20
	TMD	23
	TMDFI	27

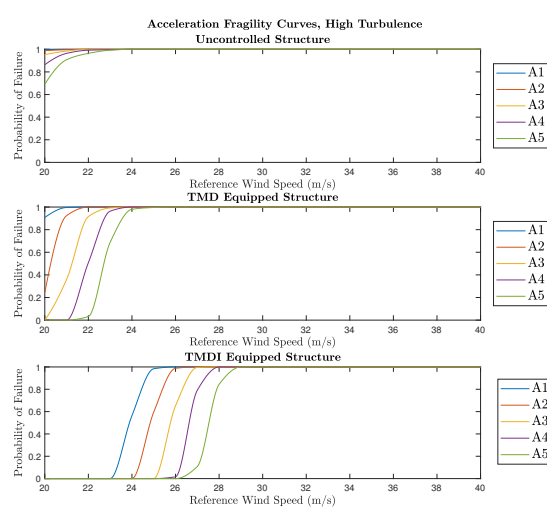
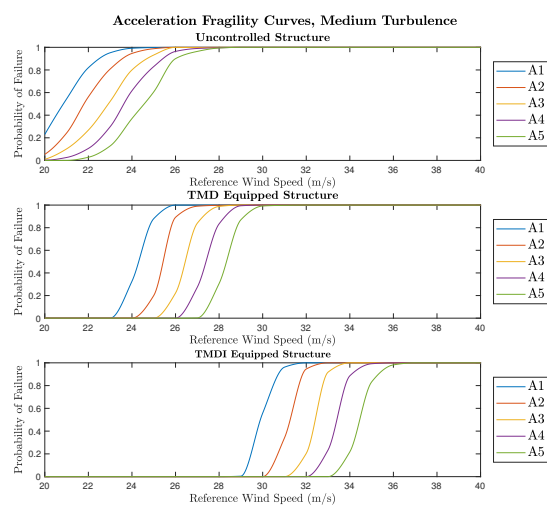
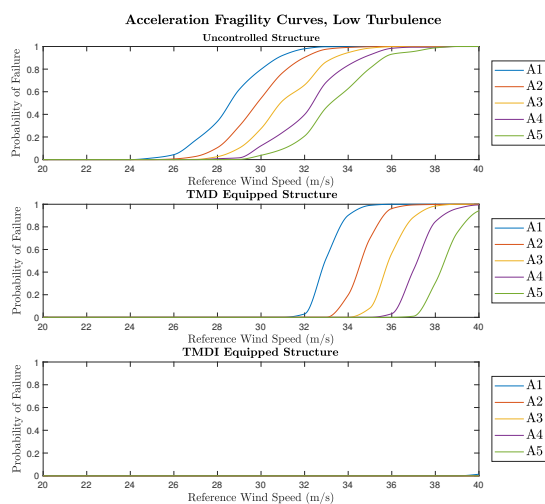


Figure 7. Acceleration fragility curves. (a) Low turbulence acceleration fragility curves. (b) Medium turbulence acceleration fragility curves. (c) High turbulence acceleration fragility curves.

Additionally, it is evident that the TMDFI increases the reference velocity required for a 95% failure of the A3 limit state in all turbulence cases. Notably, this increase gives the TMDFI controlled structure similar performance to the uncontrolled structure but at reference velocities of 6–7 m/s higher. The TMDFI also offered 3–4 m/s of additional reference velocity before significant failure compared to the TMD-controlled structure. This represents an improvement in reliability for the TMDFI compared to the two other structural arrangements.

3.2. Peak Acceleration Response

To understand the improvement in reliability at the A3 limit state, the peak top-floor acceleration at each reference velocity is analyzed. Figure 8 shows the mean and standard error of peak acceleration for each structural arrangement under low, medium, and high turbulence. The mean peak accelerations from the MCS samples for the TMDFI and TMD are compared to the uncontrolled structure, with the percentage improvement in mean acceleration for each control scheme presented at each reference velocity.

Figure 8 shows that the TMDFI consistently outperforms the TMD in reducing peak acceleration across all turbulence cases. However, the percentage of improvement decreases with increasing wind speed for both control systems. This decline is not uniform, with the TMDFI experiencing a more pronounced drop in performance, particularly at higher turbulence intensities. The magnitude of this degradation is detailed in Table 6, where the TMDFI shows greater sensitivity to increased turbulence.

Aggregating the results across all reference velocities, the TMDFI shows a 60.8% improvement over the uncontrolled structure and 28.9% over the TMD.

Table 6. Comparison of Percentage Improvement in Peak Acceleration for Each Structural Arrangement.

Turbulent Case	Control Case	% Improvement at 20 m/s	% Improvement at 40 m/s	% Degradation in Performance
Low	TMD vs. Uncontrolled	31.89%	31.53%	−0.36%
	TMDFI vs. Uncontrolled	64.38%	61.73%	−2.65%
	TMDFI vs. TMD	32.49%	30.20%	−2.29%
Medium	TMD vs. Uncontrolled	32.15%	31.79%	−0.36%
	TMDFI vs. Uncontrolled	64.64%	57.56%	−7.08%
	TMDFI vs. TMD	32.49%	25.77%	−6.72%
High	TMD vs. Uncontrolled	32.33%	31.47%	−0.86%
	TMDFI vs. Uncontrolled	62.74%	53.84%	−8.90%
	TMDFI vs. TMD	30.41%	22.37%	−8.04%

3.3. RMS Displacement Response

The RMS displacement response values were recorded to evaluate the performance differences between the acceleration and displacement responses of the control devices. Similar to the peak acceleration, Figure 9 presents the mean and standard error of the mean for the RMS displacement response across low-, medium-, and high-turbulence cases.

The plots reveal a notable relative improvement in the RMS displacement response for both the TMD and TMDFI across the sampled reference wind velocities. Specifically, the relative RMS displacement of the TMDFI improves with increasing reference velocity in all turbulence cases. These improvements are detailed in Table 7, with comparisons made at 20 m/s and 40 m/s, representing the minimum and maximum mean RMS displacement values recorded. The TMDFI shows a significant enhancement in RMS displacement, outperforming the TMD by 8.4% to 20.4% at 20 m/s and by 42.1% to 49.3% at 40 m/s.

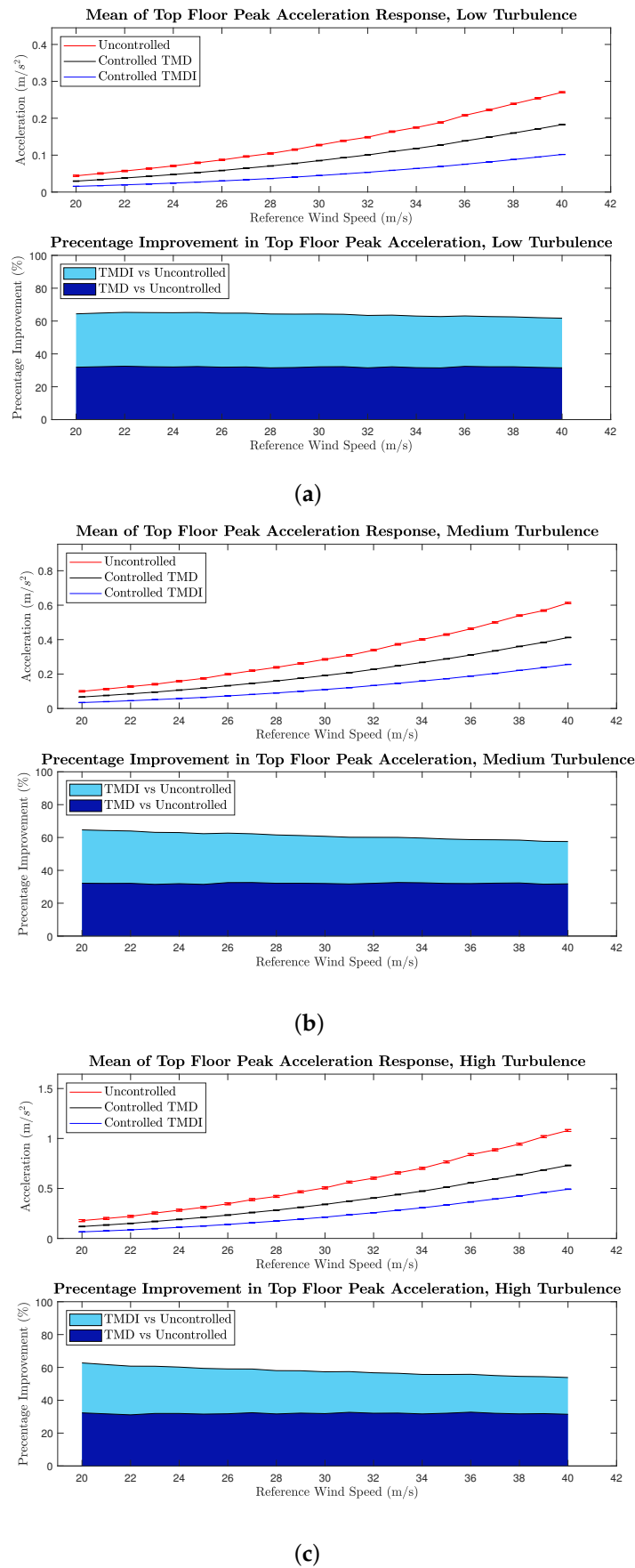


Figure 8. Magnitude of acceleration response. (a) Low turbulence. (b) Medium turbulence. (c) High turbulence.

Table 7. Comparison of Percentage Improvement in the Mean RMS Displacement Response.

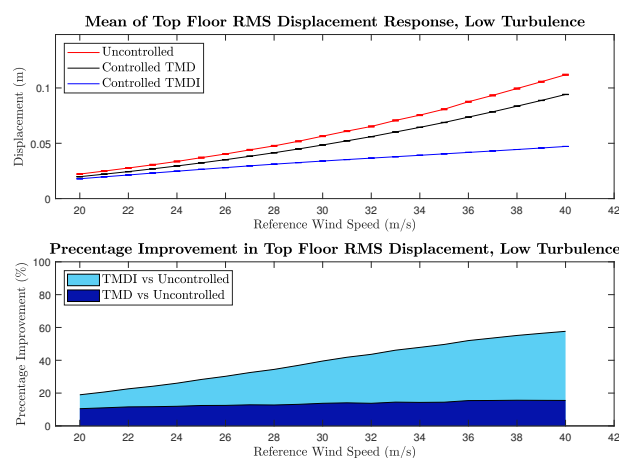
Turbulent Case	Control Case	% Improvement at 20 m/s	% Improvement at 40 m/s	% Improvement in Performance
Low	TMD vs. Uncontrolled	10.48%	15.53%	5.05%
	TMDFI vs. Uncontrolled	18.96%	57.64%	38.68%
	TMDFI vs. TMD	8.48%	42.11%	33.63%
Medium	TMD vs. Uncontrolled	10.67%	15.76%	5.09%
	TMDFI vs. Uncontrolled	24.98%	63.10%	38.12%
	TMDFI vs. TMD	14.31%	47.34%	33.03%
High	TMD vs. Uncontrolled	10.79%	15.46%	4.67%
	TMDFI vs. Uncontrolled	31.24%	64.85%	33.61%
	TMDFI vs. TMD	20.45%	49.39%	28.94%

3.4. Contrasts Between the Displacement and Acceleration Response

This improvement in the displacement response contrasts with the degradation in performance of 2.1% to 8.2% when considering the TMDFI against the TMD in the peak acceleration response (Table 6). This disparity, with degradation in the peak acceleration whilst improvement in the RMS displacement response, can largely be attributed to two factors.

3.4.1. Optimization Procedure Used

This study optimizes for minimized modal displacements, resulting in TMDFI parameters that emphasize displacement response over acceleration. The observed 33% to 38% improvement in TMDFI displacement response as wind speed increases from 20 m/s to 40 m/s is not solely attributable to this bias. Rather, it likely arises from increased power in the across-wind spectrum, which amplifies modal displacements and relative floor accelerations, thereby enhancing TMDFI performance through greater inertance force. At lower speeds, the across-wind spectrum generates insufficient motion for high inertance forces, resulting in comparable performance between TMD and TMDFI. As wind velocity and turbulence intensify, higher terminal floor accelerations improve TMDFI effectiveness via increased inertance. Consequently, the acceleration between floors, driven by enhanced across-wind power, correlates with the improved TMDFI displacement response. However, this relationship does not fully account for the observed divergence between improved RMS displacement and the decline in peak acceleration response with increasing velocity.



(a)

Figure 9. Cont.

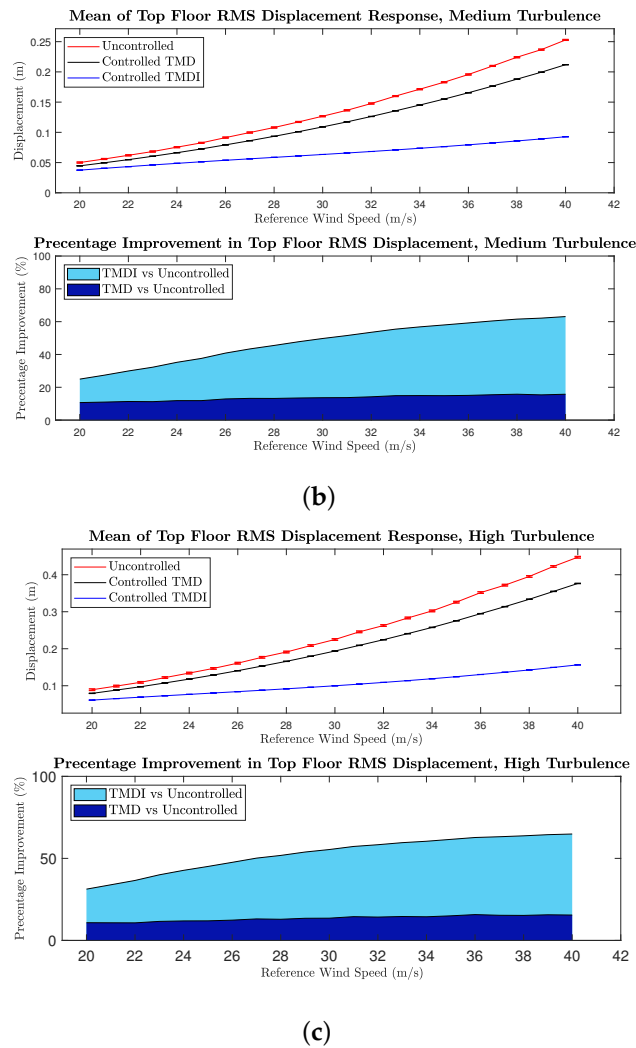


Figure 9. Magnitude of RMS displacement response. (a) Low turbulence. (b) Medium turbulence. (c) High turbulence.

3.4.2. Differences in the Response Spectra

Figure 10 provides a direct comparison of these spectra, displaying the maximum values from all 10,500 samples in the low-turbulence case at each circular frequency (ω) within the studied frequency band. The first three natural circular frequencies are denoted as $Fq.1$, $Fq.2$, and $Fq.3$. Notably, the area under these curves is directly related to the RMS and peak responses.

Figure 10 illustrates that the TMDFI significantly reduces the displacement response, particularly near the fundamental natural frequency. It also shows that higher-order natural frequencies ($Fq.2$ and $Fq.3$) have a greater impact on the overall acceleration response than on the displacement response, evidenced by a steeper decline in the displacement spectra beyond the fundamental frequency. To assess the effect of increasing reference velocity, response spectra for both control arrangements were generated using deterministic damping parameters ($\zeta_i, \zeta_j = 1\%$) in the low-turbulence case, as depicted in Figure 11. Three key observations emerge: (1) the influence of the 2nd and 3rd modes increases with reference speed in both response spectra; (2) TMDFI and TMD most effectively reduce response spectra near the fundamental frequency; and (3) the effect of higher natural frequencies is more pronounced in the acceleration response. These findings elucidate the improved RMS displacement response alongside the degradation in peak acceleration response with rising reference velocity. While both control devices diminish response

around the fundamental frequency, increased responses at higher frequencies lead to overall acceleration performance degradation. Conversely, the reduced influence of the 2nd and 3rd frequencies facilitates an overall improvement in displacement as reductions at the fundamental frequency prevail. These trends are consistent across medium- and high-turbulence cases, with deterministic response plots presented in Figure 12.

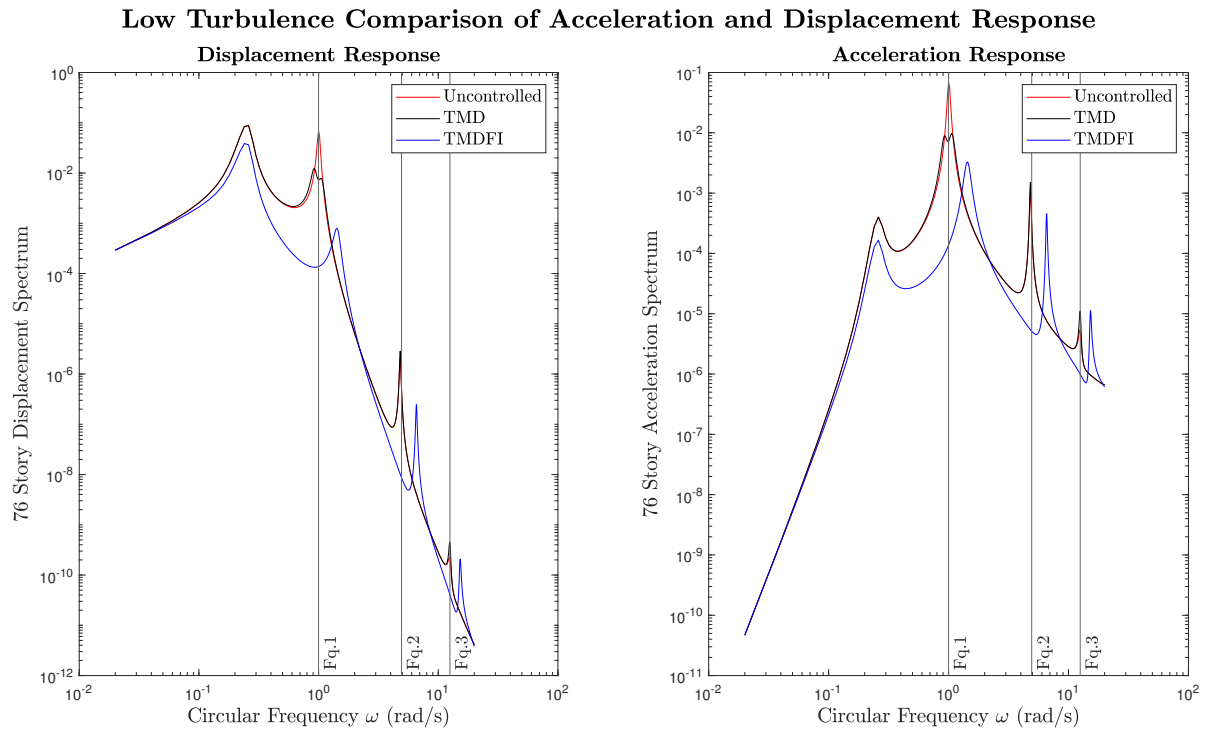


Figure 10. Comparison of the maximum response in displacement and acceleration spectra.

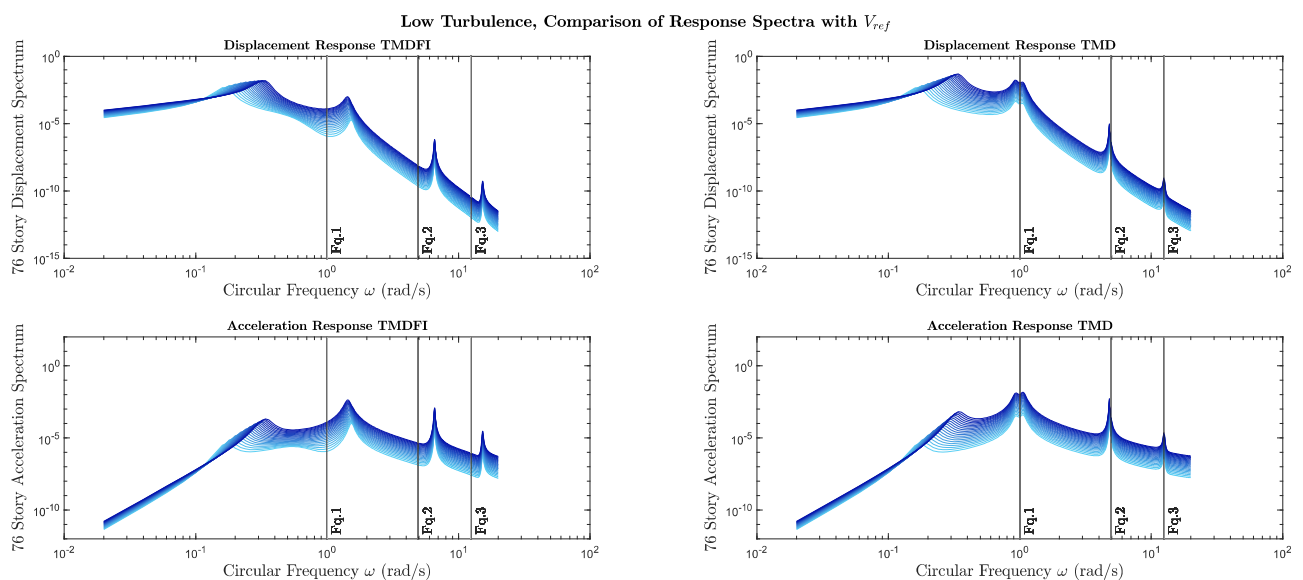


Figure 11. Low-turbulence response spectra with mean damping parameters ($\zeta_i, \zeta_j = 1\%$). Darker colors represent increasing V_{ref} cases.

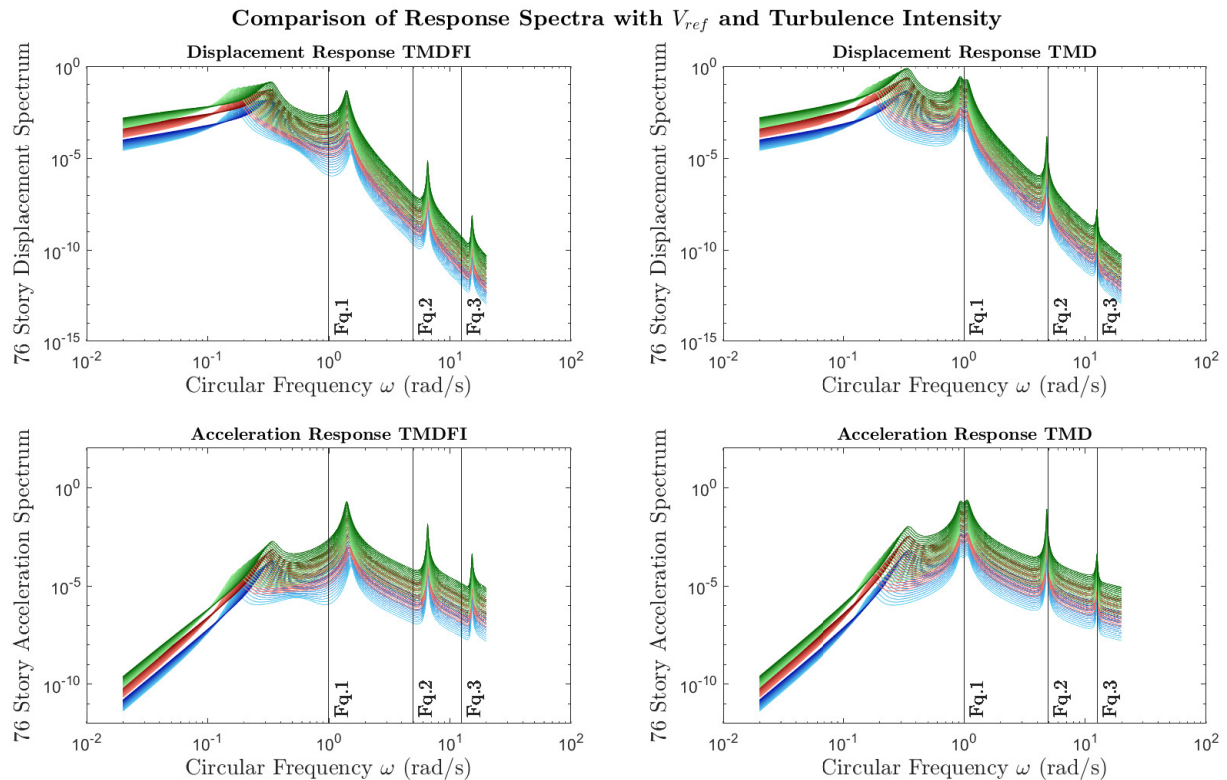


Figure 12. Low-turbulence response spectra with mean damping parameters ($\zeta_i, \zeta_j = 1\%$). Blue = low turbulence, red = medium turbulence, green = high turbulence. Darker colors represent increasing V_{ref} cases.

3.5. System Uncertainty

We established a deterministic explanation for the disparity between displacement and acceleration responses and now investigate the influence of uncertain damping ratios. To evaluate this, we analyze the frequency band to identify conditions that produce the largest response variations. For each structural arrangement, we record the maximum and minimum response values at each frequency (ω) across all reference velocities and turbulence cases. For each sampled pair of damping ratios, the response is updated if it exceeds the previous maximum or falls below the previous minimum. This process, applied to all 10,500 samples per turbulence case, yields bounded maximum and minimum response plots.

As illustrated in Figure 13 for medium turbulence intensity, uncertain damping parameters introduce variability primarily in two regions of the response spectra: the background response zone (frequencies below the fundamental frequency) and around the natural frequencies, especially in the uncontrolled case. The controlled configurations (TMD and TMDI) show minimal variation (approximately 1×10^{-2}) at resonant frequencies, indicating effective response stabilization. Conversely, damping uncertainties in the uncontrolled case significantly influence whether the resonant response exceeds the background response, as evidenced by greater variability and a higher standard error of the mean.

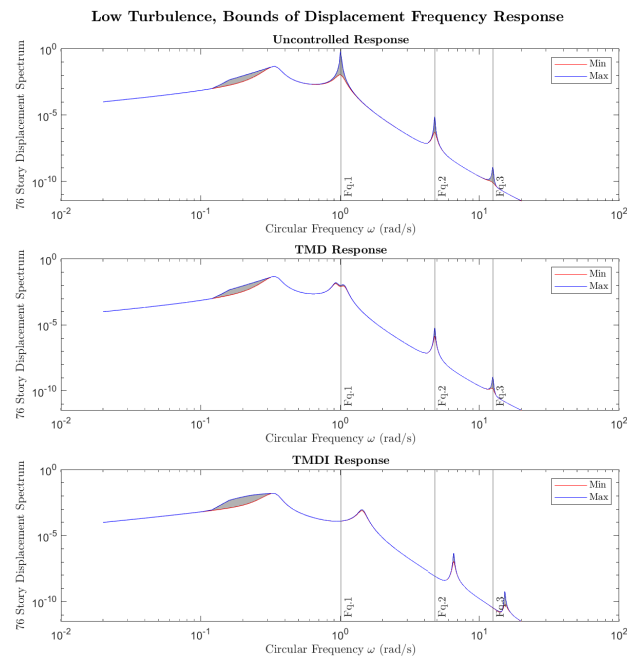
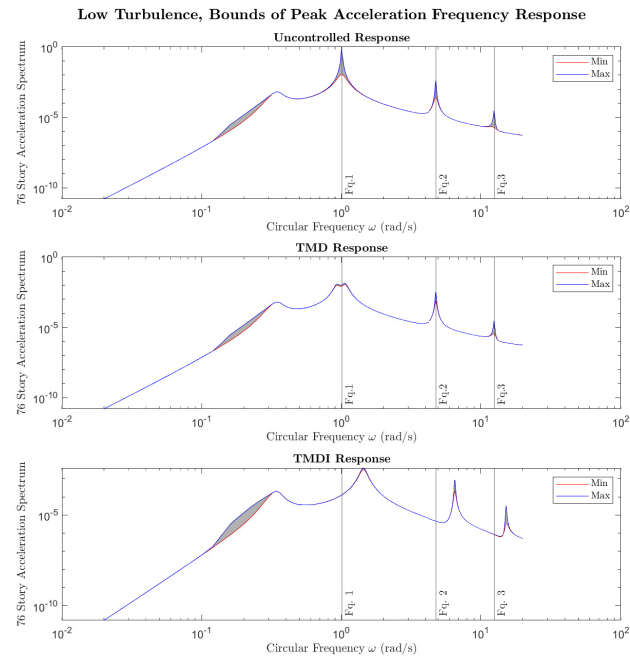


Figure 13. Minimum and maximum of acceleration and displacement response, medium turbulence. (a) Low turbulence. (b) Medium turbulence.

3.6. Influence of Turbulence Intensity

Increasing turbulence intensity leads to larger fluctuating force components and thus increases the energy content across the frequency domain. In contrast, increasing wind velocity primarily affects the mean structural response by scaling the steady aerodynamic forces while also shifting the frequency content of the PSD. Our parametric study, which considers a range of wind velocities (20–40 m/s) and three turbulence intensity levels

(low, medium, and high), demonstrates that while wind velocity dictates the overall load magnitude, turbulence intensity has a dominant effect on peak structural responses, particularly for accelerations.

3.7. Limitations of Study

This study employs a frequency domain analysis to efficiently evaluate structural responses under wind excitation. This approach significantly enhances computational efficiency and enables a broad parametric investigation. However, it inherently provides response statistics—such as peak acceleration and displacement—derived from spectral integration rather than instantaneous time-domain responses. While this is a standard and widely accepted method in wind engineering, it does mean that peak accelerations are estimated using a gust factor approach, and displacement comparisons are based on root mean square values rather than direct time histories.

In addition, the Monte Carlo Simulation (MCS) framework used to evaluate probabilistic response characteristics adopts a uniform sample density across all reference wind velocities. This ensures consistency in the analysis but may be less efficient in cases where the probability of exceedance is very low. In such cases, a more adaptive sampling strategy could provide more refined probability estimates, albeit at a higher computational cost. Given the range of turbulence intensities and damping uncertainties considered, we do not expect this to significantly impact the overall findings, but it is an avenue for future refinement.

The current study represents an analytical framework to evaluate the reliability of a TMDI system. Further investigation and experimental work are required to assess the applicability of the proposed framework with regard to real scenarios.

3.8. Applicability to the Analysis and Reliability Evaluation of Tall Buildings

The method proposed in this study offers an attractive approach for designers of tall buildings in real scenarios. The benefits to engineers and designers are given below.

- The method outlined in this work can be readily implemented to a structural analysis process, and it requires only a structures geometric information and reduced mass and stiffness matrices. With this information, the designer can quickly assess the relative performance of the inclusion of a TMD or TMDI against an uncontrolled option.
- The proposed method can be readily modified to include different wind load models.
- The use of fragility curves allows for the reliability analysis to be communicated graphically.

4. Conclusions

This study evaluated the reliability and performance of the Tuned Mass Damper Fluid Inerter (TMDFI) using a benchmark structure comparing it with a traditional tuned mass damper (TMD) and an uncontrolled case. A deterministic across-wind model assessed performance over wind velocities (20–40 m/s) and turbulence intensities, incorporating lognormally distributed damping ratios ($\mu = 1\%$, $\text{Cov} = 0.3$) via a one-step Monte Carlo Simulation (31,500 samples, 80 h CPU time).

Peak acceleration and RMS displacement were calculated for each damping realization. Fragility curves showed the TMDFI significantly outperformed both the uncontrolled structure and TMD, with the uncontrolled structure failing 95% of the time between 20 and 35 m/s, while the TMDFI only failed beyond 27 m/s. Increased turbulence reduced reliability in all cases, highlighting its critical effect.

The TMDFI reduced peak top-floor acceleration by 60.8%, compared to 28.9% for the TMD, maintaining superior performance despite some degradation at higher velocities and

turbulence. In terms of RMS displacement, the TMDFI improved by 18.9% to 64.8% over the uncontrolled case, showing robust performance under varying conditions.

The study underscored the impact of uncertain damping ratios on structural response and demonstrated the TMDFI's effectiveness in mitigating both acceleration and displacement, confirming its potential for enhancing structural reliability of tall buildings under dynamic wind loading.

The methodology proposed in this study offers an attractive method that can be readily implemented to the analysis of tall buildings in real scenarios.

Future work on this topic should focus on experimental validation of the proposed TMDFI system, and indeed other inerter-based dampers, to complement the current simulation-based findings. In particular, full-scale or real-time hybrid testing is envisioned to assess the dynamic performance of the damper under realistic wind loading conditions. Such studies will provide critical insight into device behavior, implementation challenges, and system robustness. In parallel, deployment of the TMDFI in a real structural setting, supported by structural health monitoring (SHM), would offer valuable data to evaluate long-term performance and inform future design practice. These experimental efforts are essential to advance the practical adoption of inerter-based damping systems in tall buildings.

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Conflicts of Interest: Author C  l  n McEvoy was employed by the company Arup. The remaining author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Nomenclature

A_c	Cross-sectional area of the cylinder	m^2
A_h	Cross-sectional area of the helical channel	m^2
a_0	Rayleigh damping coefficient	-
a_1	Rayleigh damping coefficient	-
B	Width of the structure	m
b	Inertance	kg
C	Damping	Ns/m
$\overline{C_L}$	Root mean squared lift coefficient	-
C_{TMD}	Damping matrix (tuned mass damper)	Ns/m
C_{TMDFI}	Damping matrix (tuned mass damper fluid inerter)	Ns/m
C_1	Across-wind spectral bandwidth coefficient	-
C_2	Across-wind spectral bandwidth coefficient	-
C_{76}	Damping matrix (uncontrolled structure)	Ns/m
c_{hc}	Tuned mass damper fluid inerter damping coefficient	-
$c_{ch,lin}$	Linearized tuned mass damper fluid inerter damping coefficient	-
D	Depth of structure	m
D_c	Fluid inerter channel diameter	m
D_h	Fluid inerter helix diameter	m

F_I	Inertance force	N
F_L	TMDFI hook floor	-
f_s	Vortex shedding frequency	Hz
g_f	Gust factor	-
H	Height of building	m
H_{ref}	Height of atmospheric boundary	m
I_u	Turbulence intensity	-
K	Stiffness	N/m
K_{TMD}	Stiffness matrix (tuned mass damper)	N/m
K_{TMDFI}	Stiffness matrix (tuned mass damper fluid inerter)	N/m
K_{76}	Stiffness matrix (uncontrolled structure)	N/m
L	Characteristic length (Strouhal)	m
M	Mass	kg
$\overline{M}$	Mean	-
M_{TMD}	Mass matrix (tuned mass damper)	kg
M_{TMDFI}	Mass matrix (tuned mass damper fluid inerter)	kg
M_{76}	Mass Matrix (uncontrolled structure)	kg
k_d	Damper spring stiffness	N/m
m_d	Tuned mass weight	kg
m_{hel}	Mass of fluid in helical channel	kg
m_f	Fundamental modal mass	kg
N	Number of simulations	-
N_H	Number of failures in N simulations	-
n_t	Number of turns in the helix	-
P_f	Probability of failure	-
$\overline{P}_f$	Estimate of the probability of failure	-
$\overline{q}_d$	Relative acceleration between inerter terminals	m/s ²
$\ddot{q}_{max}$	Peak acceleration	m/s ²
R	Fluid inerter bent radius of helix	m
Re	Reynolds number	-
r_{ij}	Coherence function of across-wind spectra	-
r_1	Radius of the piston	m
r_2	Inner radius of the cylinder	m
r_3	Inner radius of the helical channel	m
r_4	Radius of the helix	m
S	Plan area	m ²
S_{FF}	Across-wind power spectrum	N ² s/rad
S_{aa}	Acceleration spectrum	(m/s ²) ² /Hz
S_{qq}	Displacement spectrum	(m) ² /Hz
S_0	Intensity of white noise	-
S_t	Strouhal number	-
T_I	Turbulence intensity	-
T_s	Sampling time	s
$\ddot{u}$	Absolute acceleration of inerter terminals	m/s ²
V	Wind velocity (generic)	m/s
V_{ref}	Reference wind velocity	m/s
z	Height above ground level	m
α	Terrain power law coefficient	-
β	Reliability index	-
$\forall_{11}$	First component of transfer matrix	-
δ	Coherence function decay coefficient	-
ζ_d	Tuned mass damper damping ratio	-

ζ_i	Damping ratio of mode i	-
ζ_j	Damping ratio of mode j	-
$\zeta_{d,eq}$	Tuned mass damper fluid inerter equivalent damping ratio	-
l	Length of helical channel	m
μ_f	Dynamic viscosity of the fluid at reference temperature	Pa s
ρ	Density of air	kg/m ³
ρ_f	Mass density of the fluid at reference temperature	kg/m ³
σ_a	Root mean squared acceleration response	m/s ²
σ_d	Root mean squared displacement response	m
σ_m	Variance of the mean	-
$\overline{\dot{q}_d}$	Variance of the relative terminal accelerations	m/s ²
ν_f	Kinematic viscosity of the fluid at reference temperature	cSt
φ	Normalized displacement ratio	-
ω	Circular frequency in the studied bandwidth	rad/s
ω_b	Fundamental natural frequency of the building	rad/s
ω_d	Natural frequency of the tuned mass damper fluid inerter	rad/s
ω_i	Natural circular frequency of mode i	rad/s
ω_j	Natural circular frequency of mode j	rad/s
ω_r	Tuning ratio of tuned mass damper fluid inerter	-
$\omega_{r,TMD}$	Tuning ratio of tuned mass damper	-
ω_s	Vortex shedding frequency	rad/s

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