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Modelling the Demand for Factors of Production in the Mechnical Engineering Industry of Northern Ireland, 1954-79

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I INTRODUCTION

There have been few attempts to model the demand for factors of production at a sub-national level, even though the main emphasis of regional policy in most countries has been an attempt to increase the demand for labour and capital through the availability of factor subsidies. Estimates of the parameters of production functions are more readily available with regard to the US and Japanese regional economies (e.g., Blackley, 1986; Uno, 1986; Sasaki, 1985; Lande, 1978), although many of these have used cross-sectional data (two particular exceptions are Alperovich, 1980; and Carlino, 1982), while Harris (1982a) has provided a comprehensive set of parameter results using time-series data for the regions of the UK (previous UK studies have been hampered through not having access to regional capital stock data, e.g., O'Donnell and Swales, 1979). Factor demand models have been estimated using Irish data (Higgins, 1981; Boyle and Sloane, 1982), while a recent paper by Hanseman (1986) estimated a quarterly factor demand model for Cincinnati (USA) using a Nadiri and Rosen (1969) type approach. This paper estimates an econometric model of the demand for the factors of production using data available for the Mechanical Engineering industry in Northern Ireland.[1] The primary aim is to demonstrate that it is possible to use Northern Ireland data and obtain meaningful results, and this should encourage a more extensive coverage of industries in the North in the near future.

Section II looks at different dynamic factor demand models and sets out the basic model that is estimated, while Section III discusses the data that are required. Section IV presents the method of estimating the model, the results obtained, and tests of the model's specification. Finally, the paper ends with a summary and conclusion.

II DYNAMIC FACTOR DEMAND MODELS

Most studies take as their starting point the assumption that there is an equilibrium (i.e., static) model underlying the industry's decision to produce output using factor inputs. An appropriate functional form for the production function is chosen, usually the translog function (since this offers the most flexible form if the number of factor inputs exceeds two), and the firm is assumed to minimise costs so that the theory of duality between production and costs can be invoked and the production function is replaced by a cost function.[2] Shephard's Lemma is used, when partially differentiating the translog cost function with respect to the log of input prices, to obtain the demand functions for inputs expressed in terms of shares of the inputs in total output. The model is then often subjected to estimation with no further consideration as to whether it is altogether likely that firms can instantaneously adjust factor inputs, following exogenous changes in input prices, i.e., the model is purely static. Dynamic misspecification will then be apparent since autocorrelation becomes a problem. Alternatively, the cost function can have added to it unrestricted dynamic responses[3]; for example, these may take the form of *short-run* increasing or decreasing returns to scale (e.g., Holly and Smith, 1986), so that while the model is dynamic, static (i.e., backward looking) expectations are assumed; or *external* costs of adjustment can enter explicitly into the cost minimisation model (Pindyck and Rotemberg, 1983), so that firms set out to minimise the expected sum of discounted costs. Alter-

1. By 1979, Mechanical Engineering accounted for 6.7 per cent of net output and 6.5 per cent of employment in manufacturing in Northern Ireland.

2. Actually, the translog cost function is not derived from any optimising procedure in the face of the translog production function (such as would be possible in the case of CES or Cobb-Douglas technology), but if both functions are considered to be approximations to the same underlying technology, the statement made in the text is not misleading.

3. As generated by the data rather than by incorporating internal costs of adjustment into the model.

natively, *internal* adjustment costs can be incorporated into the model; this is theoretically superior, but often leads to intractable models (unless static expectations are assumed or a suitable, and often more restrictive form of the cost function is used – see Mahmud, Robb and Scarth (1986) who use a generalised Leontief function).

This brief look at recent factor demands models indicates that the preferred approach centres on the use of translog (or similar) *cost* functions, with their associated flexibility when more than two factor inputs are included. The *dynamic* model to be estimated in this paper will be constrained to the two-factor input case because we only have 26 observations (using annual data). Therefore, using a CES production function will yield comparable results to the translog model (e.g., the elasticity of substitution between factors will take on almost identical values). Moreover, cost function models assume that the output market is perfectly competitive (or that the industry operates within a constrained maximisation/minimisation framework). This may be a justifiable assumption, given that Northern Ireland manufacturing industries are relatively small and are likely to be selling in large export markets (especially to mainland UK). However, early attempts at estimating a model suggested that the Mechanical Engineering industry in Northern Ireland exhibits increasing-returns-to-scale, and it is difficult to devise an *economic* rationale for why the industry should be satisfied with minimising costs instead of maximising profits. Moreover, we also wish to experiment by including an output demand equation in the model, and so a production (as opposed to a cost) function approach is preferred[4]. Finally, estimating models based upon a translog function typically involve using share equations where the value of inputs are expressed as a proportion of total costs (or revenues, if a production function is used). Homogeneity and symmetry properties must be imposed on factor prices, and when a dynamic factor demand model is estimated this leads to certain complications when estimating the model (see Holly and Smith, 1986), making the approach less attractive.

The underlying equilibrium model used in this paper is obtained from the joint assumptions of profit-maximisation behaviour and technology adequately described by a CES production function. Hence, we assume that the industry in aggregate maximises

4. Including an output demand function also makes the model more interesting for simulating the effects of changes in, for example, national demand for output; the effects of potential changes in exogenous regional factor prices are likely to be of less interest to policy-orientated economists.

$$\Pi = p^* - wL^* - cK^* \tag{1}$$

subject to

$$\log Y^* = \upsilon\sigma/(\sigma-1) \log [\alpha K^{*(\sigma-1)/\sigma} + \beta L^{*(\sigma-1)/\sigma}] + \lambda t \tag{2}$$

where

$$\log p^* = -\frac{A}{\epsilon} - (\eta/\epsilon) \log Q + (1/\epsilon) \log Y^* \tag{3}$$

giving the following optimal solutions (omitting time subscripts) for capital and labour services.

$$\log K^* = \sigma\log(\upsilon\alpha) - \sigma\log[c/(1+(1/\epsilon)p] + (\sigma+\frac{1-\sigma}{\sigma} \log Y^* + \lambda t \tag{4a}$$

$$\log L^* = \sigma\log(\upsilon\beta) - \sigma\log(w/(1+(1/\epsilon)p] + (\sigma+\frac{1-\sigma}{\sigma} \log Y^* + \lambda t \tag{4b}$$

where Y^* is output produced at price p^*; K^* is the flow of capital services (utilisation times capital stock); L^* is the flow of labour services (average hours times labour stock); c is the "user" cost of capital services; w is the average labour-cost per man-hour; λ is the neutral rate of technical progress; t is time; Q is exogenous demand for industry output (proxied by UK Mechanical Engineering industry output); α, β, σ and υ are the parameters of the production function representing capital efficiency, labour efficiency, the elasticity of substitution between factor services, and returns-to-scale respectively; ϵ and η represent price and income elasticities of demand for output respectively.

It is assumed that c_t and w_t are exogenous so that separate factor supply functions need not be introduced.

Turning to the dynamics of the model, we introduce adjustment costs by assuming that they take the form of lags between actual and desired dependent variables. Furthermore, we couple this partial adjustment model with the assumption that the firm's optimal position in any period depends upon the current values of the determining variables during that period. This use of static expectations has been widely criticised because it leads away from the dynamic processes we wish to explain, but too few years of data constrains us from introducing a more complicated model incorporating uncertainty.

Hence, the dynamic factor-demand component of the model which will be estimated is specified as

$$\begin{bmatrix} a_{11}(L) & a_{12}L \\ a_{21}L & a_{22}(L) \end{bmatrix} \begin{bmatrix} \log K_t \\ \log L_t \end{bmatrix} = \begin{bmatrix} b_{11}(L) & a_{12} \\ a_{21} & b_{22}(L) \end{bmatrix} \begin{bmatrix} \log K_t^* \\ \log L_t^* \end{bmatrix} + E_t \ldots \quad (5)$$

where $a(L) = 1 - a_1 L - \ldots - a_r L^r$

$b(L) = b_0 + b_1 L + \ldots + b_s L^s$

Note, the model is specified as a dynamic interrelated model, following the work of Nadiri and Rosen (1969, 1974) and Harris (1985), which allows a hierarchy of adjustment speeds when the determining variables change[5]. Factor demands that have lower adjustment costs respond faster and overshoot their long-run optimal targets.

Finally, the dynamic model in (5) is jointly estimated along with the CES production function (with actual value replacing optimal values – the industry is always "on" is production function), and the (inverted) industry demand function (3)[6].

III THE DATA

The data used in the model are mostly available from the annual Census of Production carried out in Northern Ireland since 1949.[7] From this we have information on employees broken down into manual and non-manual workers, and on capital broken down into vehicles, plant and machinery, and buildings. To combine these into aggregate capital and labour figures we construct Tornqvist index numbers, which are the discrete time counterpart to the Divisia index number. In discussing these index numbers, and the data in general, this section will consider first the labour side, then the capital services data and lastly price and net output data. Finally, we will define the Tornqvist index of aggregate factor input.

(A) *Employment, Labour Costs and Annual Hours Worked*

The average total number of workers employed each year is available for both manual and non-manual workers, along with their respective annual wage costs. However, although the hours worked by both groups of employees

5. Note, the cross-adjustment processes were initially restricted to a Koyck-lag because too few data were available to estimate a full unrestricted model.

6. This equation was estimated in dynamic form by introducing a Koyck-lagged adjustment process. Note also that Y* in (4) is replaced by (3), after a suitable rearrangement of the latter.

7. The Census covers all establishments employing more than 25 persons, and so we are excluding the small firm sector.

will be reflected in wage costs, the Census provides no information on total hours worked.

Hence it was necessary to use alternative sources, i.e., the average hours worked by male manual workers as recorded in the October Survey of earnings and hours[8], and the average total annual hours worked by all workers as recorded in the Survey of Labour Costs.[9] Comparing these when both sets of data were available it was found that the ratio of average total annual hours worked by all workers to hours worked by male manual workers (multiplied by 48) was consistently between 0.91-0.92. Hence, we used the average ratio to convert the October survey estimates into a series that could represent all workers for 1954-79. Therefore, we are assuming, in order to validate our approach for calculating hours worked, that the composition of the workforce (and hours worked by each component) was stable. Concerning the former, the average proportion of non-manual workers in the workforce was 22.6 per cent, with a variance of 3.85. The average proportion of females[10] was 12.59 per cent with a variance of 3.74. Hence, the composition of the workforce was generally stable. As to the hours worked by each group, we have no data, other than that for male manual workers.

These methods used in calculating the average total hours worked each year are imperfect, but the alternative was to assume labour services used up in producing output are proportional to the labour stock, and we think the bias from using this approach is greater than that which results from having overestimated hours worked.

Data on labour costs are constructed from adding on to wage costs (available separately for manual and non-manual workers) those other payments made by employers on behalf of the workforce, i.e., employers' National Insurance contributions, contributions to pension funds, the provision for redundancy, the cost of training, etc.[11] These additional payments are not recorded in the Census of Production, but rather can be obtained from the Survey of Labour Costs conducted by the Department of Employment in 1968, 1973, 1975 and 1978.[12] Total annual labour costs and total annual wages

8. See Department of Employment *Gazette*, various issues. Note, the survey has only been in operation since 1960; earlier figures were taken from Glass (1972).

9. More will be said on this data source when we consider labour costs.

10. Employment figures for males and females in Order 7 (1968 SIC) are available in the Northern Ireland Department of Manpower Services *Gazettes*. Furthermore, the female *manual* component in the manual labour force in Mechanical Engineering is fairly small (about 10.8% in 1975, for example), so any differences between male and female manual hours will not have a large effect on the estimates produced.

11. A comprehensive list is given in any of the Survey of Labour Costs (1968, 1973, 1975 and 1978). Note the Regional Employment Premium enters as a negative cost.

12. See, for example, the Department of Employment *Gazette*, December 1977.

and salaries payments per employee are used to calculate the "other" labour costs component, and this is then multiplied by the number of employees (taken from the Census of Production) to obtain the figures used in this study. Data for 1963 are available from the Census of Production on employers' National Insurance Contributions and contributions to pension funds, which were the major source of "other" labour costs in 1963. For 1949, the Census provides data on employers' National Insurance contributions. We interpolated, using an exponential time trend, for the missing years' figures.[13] Then "other" labour costs were divided between manual and non-manual workers on the basis of employment weights, because sufficient information was not available to calculate separate series for each group of workers.

Having obtained data on average total numbers employed, annual labour costs and annual total hours worked (for both manual and non-manual workers), the cost per employee per hour was calculated for each category. These figures were used, along with the employment figures, to construct aggregated Tornqvist index numbers of the quantity and price of labour services used each year. The exact method used to aggregate is discussed later.

(B) *Capital Stock, Capital Utilisation and the Cost of Capital Services*

The measure of capital *stock* needed must be an estimate of only those capital services available and hence it must usually be a *net* stock measure. We use the perpetual inventory approach to calculate capital stock and take *a priori* known service lives[14] of assets as an indication of the period over which depreciation effectively removes vintage capital from the stock. Furthermore, when dealing with plant and machinery estimates we adopt the Denison (1972, p. 102) approach of adding one-quarter net stock[15] to three-quarters gross stock, thus obtaining a reasonable depreciation pattern. The justification for these methods has been extensively discussed in Harris (1983), where further details are available.

Since investment data are available for vehicles, plant and machinery, and buildings we calculated three capital stock series. Investment data are available back to 1949 (in 1980 prices)[16], but not for earlier years. Hence, we calculated

13. That is, we interpolated "other" labour costs *per employee* by using an exponential time trend because these "other" costs have been steadily increasing as a proportion of all labour costs. We then multiplied the resultant figures by the number of employees.

14. That is, the official *UK* estimates used by the CSO – see Griffen (1976) – but updated in 1983 to allow service lives to decline over time. The methods used here to calculate capital stock by the perpetual inventory method are identical to those employed by the CSO, except for our use of the "Denison" modification.

15. Calculated using the straight-line method.

16. Note, a revised series was used, available from the Department of Economic Development, Belfast, because Census figures often underestimated investment when plants were not yet in production.

(from Northern Ireland's share of capital expenditure in 1949-53) the share of the 1948 UK net capital stock[17] in Mechanical Engineering attributable to Northern Ireland, and then assumed that this was equivalent to a single lump-sum investment undertaken in 1948. This is an overestimate because the 1948 UK capital stock comprises capital goods of various vintages. However, there would seem to be little alternative to the method used here, because we require the 1948 share of the capital stock to depreciate.

Turning to the cost of capital services, this is defined as

$$C = P_k \left(r + \delta - \dot{P}_k/P_k\right)(1 - g)(1 - Z)(1 - t_c)^{-1} \tag{6}$$

where P_k is the price of capital goods, δ is the rate of economic depreciation, r is the "opportunity cost" of funding investment, g is the rate of cash investment grant, Z is the present value of depreciation allowances[18], and t_c is the corporate profit tax rate. Separate series were calculated for vehicles, plant and machinery, and buildings using UK price indices and the grant and investment incentives structure applicable to Northern Ireland. To calculate r, we should use a forward looking measure that takes into account uncertainty, the taxation system and an imperfect capital market (see Harris, 1985, section II.d). However, lack of data means that we have in fact used an *ex post* measure of the rate of return on capital actually achieved. This is defined in Harris (1982b) and can be represented by

$$r_t = \frac{GVA_t - D_t - TLC_t}{UK_t} \tag{7}$$

where GVA is gross value added (defined to be equal to net output minus other non-industrial costs), D is depreciation (obtained when calculating K_t), TLC is total labour costs, and UK denotes capital services, all measured in current prices. Information on non-industrial costs was available as GVA minus Net Output from 1973-1979 when both sets of figures were available. For 1963 and 1968 data were available on payment for repairs, maintenance, insurance and rates and other relevant items which together cover the majority of items included in non-industrial costs. For intervening years it was necessary to extrapolate/interpolate using a regression equation that included both

17. Calculated using the straight-line method.

18. $$Z = t_c \left\{ i_v + i_n + \frac{d(1+i_n)}{r+d} \right\}$$

where i_v = rate of investment allowance; i_n = rate of initial allowance; d = rate of annual depreciation allowance on the reducing-balance method. See King (1974, p. 24, footnote 1).

net output and an exponential time trend (the latter because non-industrial costs have been rising as a proportion of net output).

A single series on r_t was calculated, and used in (6) for all three capital measures. Furthermore, we have excluded the $\dot{P}_k/P_k$ element in (6), which is to assume that the industry does not allow for the capital gains that are due to inflation.[19]

Using the cost of capital figures and the capital stock measures, Tornqvist indices were calculated for both the quantity and price of capital used each year.

Finally, on the capital side, we need a measure of capital utilisation. Since we have no data on "machine hours", two alternatives can be used. First, we could use the data available on fuel consumption (from the Census of Production) as a proxy (see Bosworth, 1979, and Heathfield, 1972). However, attempts to construct such a measure proved unreliable, perhaps because the dependence on the different ranges of fuels used by industry has changed dramatically over the period in question.[20] Therefore, capital utilisation was measured according to short-run fluctuations in the output-capital ratio from a peak-to-peak trend line (see Harris, 1983, section 2).

(C) *Output Price and Net Output*

Regional price indices are not available and so the UK index of wholesale prices for Mechanical Engineering was used. The net output figures were directly available from the annual Census, and these were deflated by the 1980-based wholesale price index to obtain a constant price series on the net quantity of output produced.

(D) *Tornqvist Price and Quantity Indices of Capital and Labour Services*

Diewert (1980) considered the problem of aggregation, and especially that of aggregating factor inputs. He argued that ". . . there was a strong economic rationale for using . . . a *superlative* index number formula (because) it corresponds to a *flexible* functional form, for the underlying production function"

19. Note this is consistent with (7), where the numerator excludes asset revaluation. We have excluded the latter because, first, *Northern Ireland* capital price indices are not available, but more importantly because inclusion of asset revaluation is only valid if industry believes inflation has increased the *market* value of the capital stock. Since second-hand markets for capital goods may not reflect the true economic value of the capital services embodied in equipment (grants are not available on second-hand items), it is unlikely producers revalue capital fully in line with inflation (if at all).

20. In 1960 and 1979 the relative dependence, measured in therms, was

	Coke and Coal	*Gas*	*Electricity*	*Other Liquid Fuels*
1960	54.5%	16.7%	14.3%	14.4%
1979	0.6%	16.3%	20.0%	63.2%

(Diewert, *op. cit.*, p. 486, original emphasis). Moreover, he argued in favour of a chain index (i.e., the base is changed to the previous period t - 1 rather than maintaining a fixed base when calculating the change in the aggregate from period to period). An index number with both properties is the Divisia index number, and its discrete approximation the Tornqvist index. This latter index is defined as:

$$\ln\left[\frac{p_i(T)}{p_i(T-1)}\right] = \sum_{\ell=1}^{M_i} \overline{V}_{i\ell}\ \left[\ln p_{i\ell}(T) - \ln p_{i\ell}(T-1)\right] \qquad (8)$$

where $\overline{V}_{i\ell} = \frac{1}{2}[V_{i\ell}(T) + V_{i\ell}(T-1)]$

and $$V_{i\ell} = \frac{p_i X_{i\ell}}{\sum_{\ell=1}^{M_i} p_{i\ell} X_{i\ell}} \quad (i = L, K)$$

and $p_{i\ell}$ is the price of the ℓ_{th} element of the M components that make up the aggregate factor input L (labour) or K (Capital); $X_{i\ell}$ is the corresponding quantity of this input, and so $V_{i\ell}$ is the share of the ℓ_{th} element in the total payments to the corresponding input aggregate. Hence, $\overline{V}_{i\ell}$ is the average share of total payments.

The Tornqvist quantity index number is the same as (8), with quantity $X_{i\ell}$ in place of $p_{i\ell}$.

IV RESULTS

The model set out in (5) was estimated, with the industry production and price functions, using the Multivariate regression algorithm available in version 3.5B of Time Series Processor (see Hall and Hall, 1980)[21]. This is a non-linear iteration programme[22] that has the advantage that cross-equation constraints are enforced, as well as the usual properties of the weighted-regression technique. The parsimonious version of the model estimated gave the following set of adjustment coefficients (Table 1)[23]:

21. Note, all variables in the model were normalised so that 1969=1. There were 26 observations available.

22. Hence, statistical inference is asymptotic. All t-values reported are obtained from a linearisation of the model after the final iteration that achieves convergence.

23. The final form of the adjustment lags in the model were obtained after testing various alternatives against the most general form possible, given a limited data set. The testing involved using the Likelihood Ratio statistic, which is outlined later in the text when discussing serial correlation.

Table 1: *Estimated Adjustment Parameter Coefficients of the Factor Demand Model*

$$\begin{bmatrix} \underset{(11.3)}{1-0.869L} & \\ \underset{(1.6)}{-0.258L}\underset{(3.0)}{+0.430L^2} & \underset{(4.5)}{1-0.477L}\underset{(3.2)}{-0.461l^2} \end{bmatrix} \begin{bmatrix} \log K_t \\ \log L_t \end{bmatrix} = \begin{bmatrix} \underset{(1.6)}{-0.218}\underset{(2.9)}{+0.387L} & \underset{(2.5)}{0.177}\underset{(2.4)}{-0.337L} \\ \underset{(0.8)}{0.074}\underset{(2.2)}{+0.218L} & \underset{(1.8)}{0.135}\underset{(3.6)}{-0.425L} \end{bmatrix} \begin{bmatrix} \log K^*_t \\ \log L^*_t \end{bmatrix}$$

(asymptotic t-values in parentheses)

The remainining estimated parameter coefficients are given in Table 2, along with the "goodness of fit" statistic R^2 for each equation.

Table 2: *Remaining Estimated Parameter Coefficients of the Interrelated Factor Demand Model*

Parameter	*Estimated Coefficient*	*Asymptotic t-value*
α	0.568	30.5
β	0.455	31.9
σ	0.413	7.3
υ	1.367	21.7
ϵ	-9.460	5.3
η	2.092	4.3
A	0.669	6.7
ϕ	0.959	47.9

$R_p^2=0.99$; $R_y^2=0.97$; $R_k^2=0.92$; $R_l^2=0.74$.

R^2 is the "goodness of fit" statistic for the price (p), output (y), capital services (k) and labour hours (l) equations respectively. $t_{0.95}$ (5 degrees of freedom) equals 2.0.

These results suggest that the Mechanical Engineering industry in Northern Ireland operates under a capital intensive technology (since α exceeds β)[24], with low substitution possibilities between capital and labour services (σ is equal to 0.455) and significantly increasing returns-to-scale ($\upsilon > 1$). Technical progress (specified to have occurred exponentially) did not have a significant effect (λ was insignificantly different from zero when included in the model) and so this variable was dropped. The price and income elasticities of demand for the industry's output suggests that demand is both sensitive to price changes and income elasticities.

24. Therefore, an increase in capital will lead to a more than proportionate increase in output than would an increase in labour. This also means that a capital intensive technology will see the demand for capital increase faster than the demand for labour if output is growing.

As to the factor demand equations, the own-adjustment coefficients indicate that capital services alter relatively quickly in disequilibrium. This suggests that capital *utilisation* has lower adjustment costs since most of the adjustment in capital services is via changes in utilisation. The own-adjustment coefficients in the labour hours equation show a more complicated adjustment process in response to excess demand for that factor (there are important lags still occurring after two years).

Turning to the cross adjustment coefficients, they show that *initially* capital services act as a dynamic short-run substitute for labour hours. That is, when there is a change in the optimal level of labour hours, capital services (and more specifically capital utilisation) adjust in the short run to compensate for actual labour hours not adjusting quickly enough. However, there appears to be a fairly strong complementary effect after one year, as indicated by the negative cross adjustment parameter in the capital services equation. The cross adjustment process in the labour hours equation is more complicated, although the general indication is that labour hours are initially a weak dynamic complement when there is excess demand for capital services, although over a lag of one-to-two years labour services act as a dynamic substitute.

Finally, for a non-linear model of the kind presented in this paper, it is important to test for misspecification. To test for serial correlation, we examine the residual autocorrelation functions for each of the four equations and compute the modified Box-Pierce statistic (see Ljung and Box, 1978) to test for randomness in the residuals. The results are given in Table 3, which suggest serial correlation is not a major problem. As a second test of serial correlation, we use a Breusch-Godfrey type test to consider the null hypothesis H_0 : S=0 in the model

$$Y_t = \beta X_t + S\hat{u}_{t-1} + e_t, \quad e_t \text{ NID } (0, \Sigma) \tag{9}$$

where $\hat{u}_{t-1}$ is the vector of lagged residuals obtained from estimating the model. That is, the lagged residuals from the model are included as additional regressors and a Likelihood Ratio statistic is computed to test the null hypothesis that S=0. The LR statistic is given as:

$$LR = 2 \log L (\psi_1) - 2 \log L (\psi_0) \tag{10}$$

with LR asymptotically distributed as $\chi^2(m)$ under the null hypothesis that S=0, where m is the number of restrictions needed to define the null hypothesis. The terms $L(\psi_0)$ and $L(\psi_1)$ denote the maximised likelihood functions obtained under the null and alternative hypotheses respectively. Since

$\chi^2_{0.95}(4)$ equals 9.49 and LR was calculated to be equal to 8.78, the null hypothesis was accepted.

Table 3: *Modified Box-Pierce Test of Random Residuals*

Equation	Q*(6)	Q*(12)
Price	10.72	18.47
Output	5.01	14.63
Capital	10.38	17.47
Labour	5.49	13.01

Note: Reject the null hypothesis of randomness if $Q^*(6) > \chi^2_{0.95}(6) = 12.59$ or $Q^*(12) > \chi^2_{0.95}(12) = 21.03$.

As a further test of misspecification, the model is considered for dynamic stability (this is an essential property since we assume that the underlying model is in equilibrium). This depends upon the roots of the determinant of the right-hand side in Table 1, which can be obtained from the determinantal equation:

$$1 - 1.346L - 0.047L^2 + 0.401L^3 = 0 \tag{11}$$

The eigenvalues of this equation are -2.077, 1.150 and 1.044. For the system to be stable the necessary and sufficient condition is that the moduli are all greater than one, and this is satisfied here.

Finally, the results of testing for the significance of the interrelatedness of the model are reported. Using the LR test described above, the null hypothesis that the cross adjustment parameters are zero was tested giving an LR statistic of 26.61, while $\chi^2_{0.95}(6)$ equals 12.59. Hence, the null hypothesis was rejected.

V SUMMARY AND CONCLUSION

This paper has shown that it is possible to get plausible and significant results from an econometric model of the demand for factor services, even though such models are not usually estimated at the regional level in the United Kingdom. As is obvious from the discussion of the data, the model's requirements are quite extensive, which may explain the paucity of work in this area.

The importance of having modelled the relationship between input and output in a dynamic context is apparent. It is possible to use the results to form the basis for an explanation of changes in the *relative* demand for inputs. Furthermore, such models play an essential role in *predicting* changes, especially if they are embedded in an econometric model of the regional economy. Many econometric models do not consider in any great detail the processes that determine the supply of regional industrial output. They often make use of an untested and, hence, possibly misspecified production function and/or adjustment process.

Finally, and not least, modelling the demand for factor services along with the production function, will allow inter(intra)-regional comparisons to be made, when coverage is extended to include other industries and regions. Perhaps then further insights can be gained as to why some regions perform less well than others, and whether supply-side conditions (technical efficiency, factor intensity, etc.) are important.

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