Coarse lattice results for glueballs and hybrids.

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A review of new results from lattice simulations of glueballs and heavy-quark hybrid mesons is presented.

1. INTRODUCTION

Glueballs and hybrid mesons are both colour-singlet bound states containing excitations of gluonic degrees of freedom and so are absent in the quark model, but expected in QCD. These gluonic excitations are a highly non-perturbative aspect of the theory and so making reliable predictions requires robust methods. Unambiguous experimental identification of these states has proven to be difficult. At present, the two lightest glueballs, the scalar and tensor, have candidate resonances; the $f_0(1500)$ [1] and $f_J(1710)$ (scalar) and the $\xi(2230)$ [2,3] (tensor). It now seems likely that the scalar glueball is mixed strongly with near-by conventional mesons. Recently, new data for the light-quark hybrid candidate, with exotic quantum numbers 1^{-+} have been reported [4], close to the latest lattice predictions [5,6] but, to date, no signal for a heavy-quark hybrid has been reported. More details on the current status of phenomenological interpretations of experimental resonances are presented in Ref. [7].

Clearly then, reliable lattice data for the properties of these bound states are helpful, both to interpret and predict experiment. Information beyond the masses of these states would seem to be required; the clearest example is the scalar glueball candidates, $f_0(1500)$ and $f_J(1710)$, which are both consistent with lattice mass predictions in the pure-gauge theory [8–10] of 1600 MeV with systematic errors of approximately 100 MeV.

Simulations of hybrids with light quarks were reported to the conference [5,6]. In discussing hybrids, this review will concentrate on developments in the study of the heavy-quark system, which is a natural starting point for theoretical studies of hybrids, since the Born-Oppenheimer approximation can be used to separate the quark and gluon degrees of freedom.

The presence of gluonic excitations in the rather heavy gluonic states makes them a challenging problem for lattice simulations. The vacuum fluctuations in the constituent gluon fields tend to be rather large and so the rapidly-decaying Euclidean correlators which must be measured in a Monte-Carlo calculation become dominated by statistical noise for rather small source-sink time separations and extracting the masses becomes difficult. Studying other properties is an even greater numerical problem. This year, simulations on improved, anisotropic lattices (with spatial lattice spacing, $a_s \ll a_t$, the temporal lattice spacing) were performed, with encouraging results. This technique shows promise as an efficient method for simulation of gluonic bound states as it tries to incorporate the cost advantages of coarse lattices without reducing the energy resolution too far.

In this review, new results for glueball simulations and the gluonic excitations of the static inter-quark potential from anisotropic lattices, as well as NRQCD studies on large ensembles of Wilson action configurations will be summarised. The direction of present and future work will be discussed.

2. ANISOTROPIC LATTICES

In any attempt to simulate a bound state efficiently on a coarse lattice, it is important to recognise the two scales in the problem: the state's size and its mass. Glueballs have been estimated to have a size of about 0.5 to 0.8 fm (although some studies suggest the scalar glueball might be significantly smaller), while their masses are more than 1.5 GeV. For the wave function to be distributed over three grid points, a spatial lattice spacing of about 0.2 fm is required, while setting the temporal lattice spacing by demanding $m_G a_t < 1$, which allows the correlator to be resolved over sufficient time steps from accessible statistics, needs $a_t < 0.1$ fm.

The construction of anisotropic lattice actions using mean-field improved perturbation theory is dealt with in Refs. [11,12] and (except where specifically noted) all anisotropic simulation results presented here are from this action. This action was designed to be $O(a_s^2)$ improved and to include terms that directly couple links only on adjacent time slices to ensure the free gluon propagator has no spurious modes. In all cases considered, the effective masses computed from correlators between identical source and sink operators converged to their plateaus from above, illustrating positivity of the transfer matrix for this action.

Using mean-link improved perturbation theory to determine the coefficients in the action has been shown to reduce the renormalisation of the bare anisotropy to the level of a few percent. The true anisotropy can be measured by computing the static potential along distinct lattice axes.

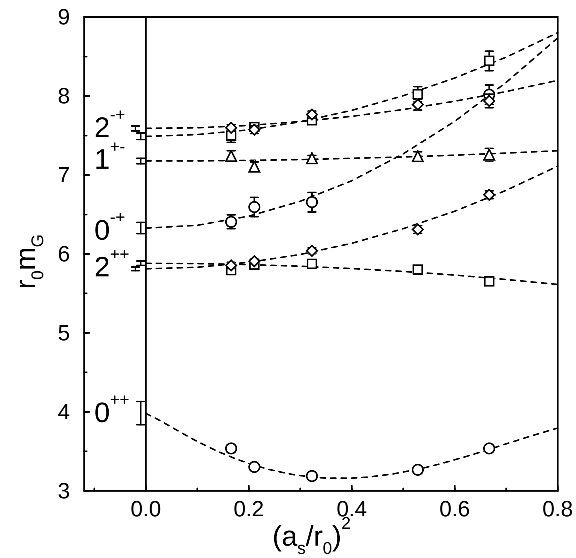

Figure 1. Scaling behaviour of the lightest five glueball states from 5:1 anisotropic lattice simulations. Data from 4 irreps are included, with symbols $\circ = A_1, \Box = E, \triangle = T_1, \diamond = T_2$.

3. GLUEBALLS

The scaling properties and continuum spectrum of glueballs has been studied extensively using anisotropic lattices [10,13]. The cut-off dependence of the lightest five glueball states, which have J^{PC} quantum numbers $0^{++}, 2^{++}, 0^{-+}, 1^{+-}$ and 2^{-+} is shown in Fig. 1. Data from five simulations at an input anisotropy of 5:1 are included on the plot and, for all but the scalar glueball, fits to the anticipated leading discretisation error for this range of lattice spacings,

$$\varphi(x) = r_0 m_G + c_1 (a_s/r_0)^4 \tag{1}$$

are included. For the scalar, a four parameter fit which models an $\alpha_s a_s^2$ along with the a_s^4 term is used.

The scalar glueball shows significant scaling violations even at 0.2 fm. Fig. 2 shows in more detail the scaling properties of the scalar for both the Wilson and anisotropic, improved action (as well as a new choice of anisotropic lattice action, which will be discussed later). At a lattice spacing, $a_s \approx 0.3$ fm, the dimensionless mass, $r_0 m(0^{++})$ is about $25 - 30\%$ lower than continuum estimates. The scalar glueball is believed to be a small state, and might never be well simulated on a coarse lattice with a Symanzik-improved action. The picture is complicated somewhat by the existence of a critical end-point in the space of fundamental-adjoint couplings [14] at which the correlation length in the scalar channel is seen to diverge. If this is the cause of the poor scaling of the lightest glueball, then it might be possible to construct actions whose renormalisation trajectory steers clear of the problematic fixed point neighbourhood. A class of actions of this form is under investigation with some promising initial findings. Included in Fig. 2 are three simulations performed with such an action. These points are much closer to the continuum predic-

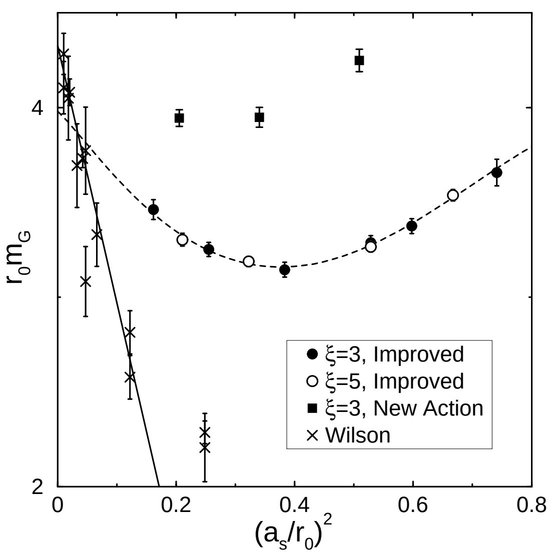

Figure 2. The scaling behaviour of the 0^{++} glueball for three different actions.

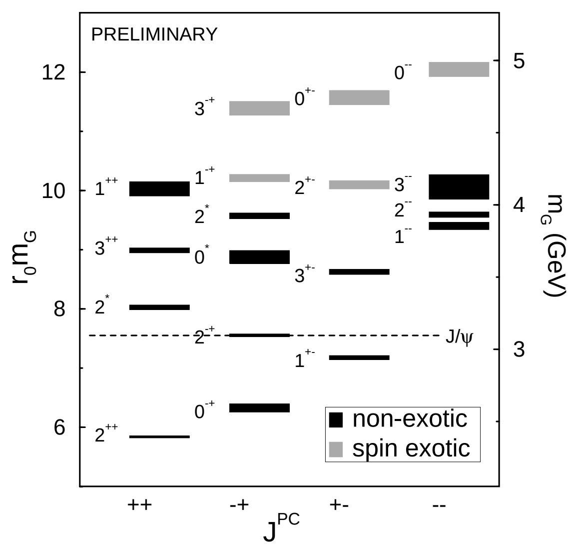

Figure 3. The continuum limit for the higher levels in the $SU(3)$ glueball spectrum . Data are from 5 simulations on 5:1 anisotropic lattices [13]

tions and show only a few percent scaling violations. Other glueballs are unaffected. For SU(2), a similar, but less severe dip is found [15] and the authors argue that the cut-off effects are reduced significantly with the use of the Landau gauge definition of the mean-link parameters, $u_\mu = \langle \frac{1}{3}\text{Re Tr } U_\mu \rangle_{\text{Landau}}$ rather than the plaquette definition used in *e.g.* Ref. [10]. The reduced severity of the dip here is still consistent with the fixed point idea as the point is further from the fundamental axis in SU(2) so should have a weaker influence. It is, however, important to note that these points are not necessarily mutually exclusive; the cause of the dip could be due in part to all of these effects. The discussion should be resolved with more reliable information on the size of the scalar, as well as simulations of SU(3) glueballs with the Landau-defined mean-link parameter and studies of new actions. It would also be interesting to see if the use of fixed-point techniques [16] can determine a better anisotropic action.

Fig. 3 shows the continuum limit of the spectrum of higher glueball states from five $\xi = 5$ anisotropic simulations. For all these states, the simple function of Eqn. 1 fits the data well and so was used in determining the continuum limits. No uncertainty from the scale is included in Fig. 3, however an estimate of this systematic effect is included in Table 1. The data show that that, for the pure-gauge theory, the pseudoscalar glueball is heavier than the tensor. This spectrum is still a preliminary result as ambiguities in the interpretation of the data remain: the assignment of a continuum spin label requires the resolution of degeneracies in the continuum limit across lattice irreps and the lattice operators creating more massive states can also overlap with both two light glueballs and torelon pairs (flux-tube-like excitations winding around the periodic boundaries of the finite lattice). The lightest spin-exotic state in this preliminary analysis is found to be very massive, with $J^{PC} = 2^{+-}$, $m_G = 4.1(2)$ GeV. These results are in agreement with the existing

Table 1
Continuum higher glueball masses in pure-gauge SU(3) (Preliminary). $r_0^{-1} = 410(20)$ MeV is used to set the scale.

J^{PC}	$r_0 m_G$	Mass (MeV)
2^{++}	5.84(2)	2390 ± 120
0^{-+}	6.33(7)	2590 ± 130
1^{+-}	7.18(4)	2940 ± 140
2^{-+}	7.49(4)	3070 ± 150
2^{*++}	8.02(4)	3290 ± 160
3^{+-}	8.63(5)	3540 ± 170
0^{*-+}	8.9(1)	3640 ± 180
3^{++}	9.0(1)	3690 ± 180
1^{--}	9.4(1)	3850 ± 190
lightest spin-exotic glueball		
2^{+-}	10.1(1)	4100 ± 200

Wilson action data [8] in this energy range.

In a lattice Hamiltonian formulation, the dynamics in the temporal direction are left continuous, and masses can be extracted by computing the energy eigenstates. The Symanzik improvement scheme can also be applied to choose a spatial discretisation with reduced contaminations from the finite cut-off. This idea is discussed in Ref. [17] and Hamiltonian methods have been applied to computations of the glueball spectrum [18].

Anisotropic lattice technology allows more properties of glueballs to be explored. A new calculation [19] to estimate the production rate of glueballs in the radiative decay of charmonium ($J/\psi \to \gamma G$) is a good example of this. Here, the matrix elements appropriate for creation of the lightest three glueball states (the scalar, tensor and pseudoscalar) of the form $\langle G|\ \mathrm{Tr}\ FF\ |0\rangle$ are under investigation.

The SESAM collaboration [20] has investigated the scalar and tensor glueballs on their large ensemble of unquenched (Wilson gauge and quark action) configurations. They find little change in the masses of these states as the sea quark mass runs down from the strange quark mass to near where the ρ meson becomes unstable ($m_\pi/m_\rho \approx 0.5$) but they do notice enhanced finite-volume dependence which they attribute to dynamical quark effects.

4. HEAVY-QUARK HYBRID MESONS

If heavy exotic mesons are to be seen as experimental resonances, the masses relative to threshold for decays into two heavy-light mesons are important in determining their widths. Constituent glue/quark models predict [21] that the dominant decay mode of the lightest exotic, 1^{-+} will be into a pair of mesons, where one of the mesons has $L = 1$ orbital angular momentum and so, in the model, if the hybrid is above this level it will probably be too broad to be seen. For the $b\bar{b}g$ hybrid, this channel has a threshold at 10.98 GeV (1.52 GeV above the Υ ground state). Nevertheless, the reliability of this model is questionable and if the hybrid is above the $2B$ meson threshold it may still be a broad state.

4.1. Excited gluonic potentials

The most straightforward way of studying heavy mesons is by use of the Born-Oppenheimer approximation. Since the fast-moving gluonic degrees of freedom have much shorter relaxation times than the slow-moving heavy quarks, the spectrum of this system can be approximated by solving Schrödinger's equation with the static potential as the interaction term. Hybrid mesons can be constructed by finding similar solutions with the inter-connecting parallel transport now transforming under some non-trivial representation of the symmetry group of the static-source pair. These representations include excitations with a non-zero component of spin along the $Q\bar{Q}$ axis, for example.

Anisotropic lattice simulations have been used to study excitations to the potential [22]. These modes are created on the lattice by connecting the static quark propagators (temporal Wilson lines) with linear combinations of path-ordered products of links that transform irreducibly and non-trivially under the "little group" of lattice rotations about the $Q\bar{Q}$ molecular axis, combined with two discrete symmetries (for details, see *e.g.* Ref. [22]). The standard ground state potential has quantum numbers Σ_g^+ and the lowest excitation is the Π_u, which carries one unit of angular momentum about the $Q\bar{Q}$ axis. In the static approximation, where the quark spins do not interact dynamically with the gluons, a hybrid meson of two quarks bound in this interaction potential can have a range of quantum numbers which include the exotic 0^{+-} and 1^{-+}. The new data are in agreement with existing Wilson action results [23] and a number of previously unresolved potentials have also been computed. A larger range of inter-quark separations, up to 2.5 fm, has been studied.

Using empirical fits to the Monte-Carlo data for the Σ_g^+ and Π_u potentials in the radial Schrödinger equation, the mass of the lightest b-quark hybrid is 1.3 GeV above the Υ(1S). The leading Born-Oppenheimer approximation neglects retardation effects and, above the two-B-meson energy where the lightest hybrid lies, the agreement with the experimental $b\bar{b}$ spectrum is poorer than below. The wave function of the potential model is found to be much larger than the small Υ meson. The peak in the radial probability density occurs at ≈ 0.6 fm for the wave function in the Π_u potential, compared with ≈ 0.2 fm for the ground state solution. This suggests that the lattice volume for hybrid studies needs to be larger than for conventional mesons. It also implies that a good choice of operator to create a ground state hybrid would be a highly extended object.

The rich spectrum of excited potentials has been investigated [24] within a diaelectric (bag-like) model of the QCD vacuum, giving results showing broad agreement with the Monte-Carlo data. In this model, the excitation energies are estimated by solving for the spectrum of confined gluon (and multi-gluon) states. This confinement is within a region where the physical vacuum has been expelled by the strong chromoelectric fields of the static-quark pair.

4.2. Hybrid mesons from NRQCD

Two groups [25,26] presented results to the conference from NRQCD simulations of $b\bar{b}g$ hybrid mesons which are included in Fig. 4. These simulations, both performed on $16^3 \times 48$ lattices at $\beta = 6.0$, find the 1^{-+} hybrid excitation to be 1.64(16) GeV and 1.35(25) GeV respectively above the ground state, suggesting the hybrid lies between the two thresholds noted in Sec 4. Both these groups used large, smeared operators. These results are also included in Fig. 4 and are consistent with the data from the Born-Oppenheimer approximation. In Ref. [26] simulations were performed with a higher-order-NRQCD action, and investigated the spin-dependent effects; preliminary data for the 0^{+-} exotic follow model predictions that it is heavier than the 1^{-+}.

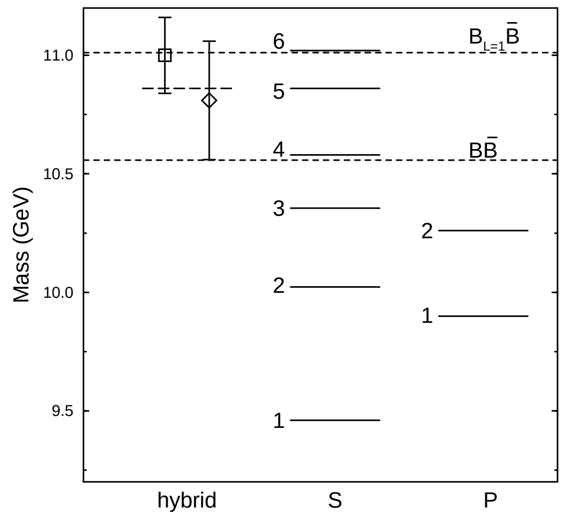

Figure 4. Quenched lattice predictions for the lightest exotic $b\bar{b}g$ (1^{-+}) hybrid. Solid lines are experimental data for S- and P-wave conventional mesons. The two thresholds are indicated with dashed lines. NRQCD data are from Refs. [25] (□) and [26] (◇) and the long-dashed line is the potential model prediction [22]

NRQCD simulations on anisotropic lattices are in progress and the results of some exploratory work were presented to the conference [22].

5. CONCLUSIONS

Recent developments in the use of anisotropic lattices have extended the computational advantages of coarse lattices to studies of glueballs and heavy-quark hybrids. An anisotropic lattice, with a fine temporal spacing, allows Euclidean correlators to be resolved clearly over a range of time slices, while preserving most of the economic advantages of coarse lattices by keeping the spatial lattice spacing large. The problems with the scalar glueball on a coarse lattice are still not fully resolved, however some first results from new actions suggest a solution might be found while retaining the convenience of the Symanzik improvement programme. Work on NRQCD and relativistic quark actions [27] on anisotropic lattices should extend these benefits to hybrid studies from the static approximation down to the light-quark sector.

The preliminary findings from the higher states in the glueball spectrum suggest a large number of glueball states at energies in the charmonium spectrum range. Details will be reported elsewhere. Other properties of glueballs, such as the matrix elements relevant for their creation in radiative charmonium decay, are now being studied on anisotropic lattices.

The first results from NRQCD actions of $b\bar{b}g$ hybrids have shown that these systems can be simulated reliably. While the data still have large uncertainties, including the prospect of large finite-volume corrections due to the highly extended nature of these objects, they suggest the heavy hybrid lies between the two thresholds (creation of 2 S-wave B mesons and creation of an S-wave and P-wave B meson pair).

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