An Application of the Samuelson-Stone Linear Expenditure System to Food Consumption in Ireland

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THE principal aim of this paper is to offer a structural analysis of the pattern of food consumption in Ireland over a period of twenty-one years. Such an analysis may also be useful as a basis for projections, although this aspect of the subject is only briefly examined here.

Formulation of the Model

The choice of model to be tested can, in the general m-good case, be formulated as follows:—

$$p_i q_i = b_i \mu + \sum_{J=1}^{m} B_{iJ} p_J \qquad (1)$$

Thus the expenditure per head of population on the ith good $(p_i q_i)$ is a function of total expenditure per head (μ)[1] and of all the prices (p_J) where $(J=1, 2, 3, \ldots m)$. In order to examine the properties of such a general linear system in greater detail a two-good example will be used. This can be written:

$$\begin{aligned} p_1 q_1 &= b_1\mu + B_{11}p_1 + B_{12}p_2 \\ p_2 q_2 &= b_2\mu + B_{21}p_1 + B_{22}p_2 \end{aligned} \qquad (2)$$

1. Total expenditure per head is taken here (as in much of the literature) to be synonymous with income per head.

or, in terms of quantities (q) only:

$$q_1 = \frac{b_1\mu}{p_1} + B_{11} + \frac{B_{12}p_2}{p_1}$$
$$q_2 = \frac{b_2\mu}{p_2} + \frac{B_{21}p_1}{p_2} + B_{22} \tag{3}$$

With the help of this example general properties of the model can be deduced.

It is well known that a useful and theoretical criterion of demand analysis is that the sum of the individual expenditures should add to total expenditure. In terms of equation system (2) this additivity criterion requires that

$$p_1q_1 + p_2q_2 = \mu \tag{4}$$

Substituting for p_1q_1 and p_2q_2 from (2) we get

$$(b_1+b_2) + p_1(B_{11}+B_{21}) + p_2(B_{12}+B_{22}) = \mu \tag{5}$$

Since μ, p_1 and $p_2 \neq 0$ then

$$b_1+b_2 = 1; \quad B_{11}+B_{21} = 0; \quad B_{12}+B_{22} = 0 \tag{6}$$

Thus the system will be additive if the b's add to unity and the price coefficients in each column add to zero, or, in general, if

$$\sum_{i=1}^{m} b_i = 1 \quad \text{and} \quad \sum_{i=1}^{m} B_{ij} = 0 \tag{7}$$

From these features of additivity other properties relating to the expenditure and price elasticities can be deduced:

Expenditure elasticities:

As the b's sum to unity, then the sum of the expenditure elasticities weighted by their "own" expenditure proportions will sum to unity. The expenditure elasticity of the ith good is $\frac{b_i\mu}{p_iq_i}$ and the expenditure proportion of the ith good is $\frac{p_iq_i}{\mu}$. In general, since $\sum_{i=1}^{m} b_i = 1$ then

$$\sum_{i=1}^{m} \frac{b_i\mu}{p_iq_i} \cdot \frac{p_iq_i}{\mu} = 1 \tag{8}$$

Price Elasticities:

As the price coefficients in each column sum to zero then the weighted sum of the price elasticities for each good with respect to one price plus the expenditure proportion of the good which has that price equals zero. In terms of the two-good example it is then required to show that:

$$\frac{\partial q_1}{\partial p_1} \cdot \frac{p_1}{q_1} \cdot \frac{p_1 q_1}{\mu} + \frac{\partial q_2}{\partial p_1} \cdot \frac{p_1}{q_2} \cdot \frac{p_2 q_2}{\mu} + \frac{p_1 q_1}{\mu} = 0 \qquad (9)$$

Differentiating equation (3) and substituting in equation (9)

$$-\frac{(b_1\mu + B_{12}p_2)}{\mu} + \frac{B_{21}}{p_2} \cdot \frac{p_1}{q_2} \cdot \frac{p_2 q_2}{\mu} + \frac{p_1 q_1}{\mu} = 0 \qquad (10)$$

Multiplying across by $\frac{\mu}{p_1}$:

$$-\frac{b_1\mu}{p_1} - \frac{B_{12}p_2}{p_1} + q_1 + B_{21} = 0 \qquad (11)$$

But from (3):

$$B_{11} = -\frac{b_1\mu}{p_1} - \frac{B_{12}p_2}{p_1} + q_1 \qquad (12)$$

$\therefore$ equation (11) becomes:

$$B_{11} + B_{21} = 0 \qquad (13)$$

Similarly it can be shown that if equation (9) were expressed in terms of elasticities with respect to the other price (p_2) and the last term were replaced by the expenditure proportion of the good which has this price (i.e., $\frac{p_2 q_2}{\mu}$) then

$$B_{22} + B_{12} = 0 \qquad (14)$$

In general, then, if $\sum_{i=1}^{m} B_{ij} = 0$ which it does because the model is additive, then:

$$\sum_{i=1}^{m} \frac{\partial q_i}{\partial p_j} \cdot \frac{p_j}{q_i} \cdot \frac{p_i q_i}{\mu} + \frac{p_j q_j}{\mu} = 0 \tag{15}$$

which is the required result.

Since the quantity consumed is being treated here as a particular linear function of total expenditure and of each price the equations are homogeneous of degree zero. Thus, if in the first equation of system (3), μ were increased by a given percentage and p_1 and p_2 were each increased by the same percentage the increases would cancel out leaving q_1 unchanged. This seems reasonable since a rational consumer would hardly be influenced one way or another if, for example, an increase in his income were offset by an equi-proportionate increase in all the prices with which he was faced. The major implication of this property of homogeneity is that for each commodity the sum of the expenditure elasticity and *all* price elasticities is zero. Thus, taking good 1 (q_1) from the two-commodity example it is required to show that:

$$\frac{b_1\mu}{p_1q_1} + \frac{\partial q_1}{\partial p_1} \cdot \frac{p_1}{q_1} + \frac{\partial q_1}{\partial p_2} \cdot \frac{p_2}{q_1} = 0 \tag{16}$$

Substituting for $\frac{\partial q_1}{\partial p_1}$ and $\frac{\partial q_1}{\partial p_2}$ from equations (3):

$$\frac{b_1\mu}{p_1q_1} - \frac{b_1\mu}{p_1q_1} - \frac{B_{12}p_2}{p_1q_1} + \frac{B_{12}p_2}{p_1q_1} = 0 \tag{17}$$

Similarly for good 2 (q_2).

Generalising then we have:

$$\frac{b_1\mu}{p_iq_i} + \sum_{i=1}^{m} \frac{\partial q_i}{\partial p_j} \cdot \frac{p_j}{q_i} = 0 \tag{18}$$

Thus, not only will an equi-proportionate change in total expenditure and each of the prices leave consumption unchanged but an equi-proportionate change in the expenditure elasticity and each of the price elasticities will also leave consumption unchanged.

In elementary (indifference curve) demand theory consistency of preferences is postulated for the rational consumer, i.e., the substitution effects are symmetrical. However, in adapting this theory to a market situation there is no guarantee that the consistency of preferences will remain (i.e., that the *community* indifference curves will not intersect). Whether or not the symmetry condition exactly reflects the facts of market behaviour (though it could not be very far out) there are good reasons for its adoption. Not least of these is the fact that it reduces the number of parameters to be estimated (from m^2-1 to $2m-1$) but, as will be seen later, it also allows the general model to be interpreted in a plausible manner. It can be shown that the model, as it stands, does not satisfy the symmetry condition. However, a more fruitful approach is to assume that this is the case and then to proceed to examine what restrictions are required to render the model symmetrical. The matrix of substitution elasticities is defined by Stone[2] in matrix notation as:

$$S = \hat{q}^{-1}(ai' + A\hat{q}^{-1})\mu \qquad (19)$$

where S = matrix of substitution elasticities
$\hat{q}$ = diagonal matrix of quantities
a = vector of quantity derivatives with respect to expenditure
i' = transposed unit vector
μ = vector of total expenditure
A = matrix of quantity derivatives with respect to price

In our two-good example this formula can be expanded as:

$$S = \begin{bmatrix} \frac{1}{q_1} & 0 \\ 0 & \frac{1}{q_2} \end{bmatrix} \left\{ \begin{bmatrix} \partial q_1/\partial \mu \\ \partial q_2/\partial \mu \end{bmatrix} [1 \quad 1] + \begin{bmatrix} \partial q_1/\partial p_1 & \partial q_1/\partial p_2 \\ \partial q_2/\partial p_1 & \partial q_2/\partial p_2 \end{bmatrix} \begin{bmatrix} \frac{1}{q_1} & 0 \\ 0 & \frac{1}{q_2} \end{bmatrix} \right\} \mu \qquad (20)$$

$$= \begin{bmatrix} \frac{1}{q_1}\cdot\frac{\partial q_1}{\partial \mu} + \frac{1}{q_1^2}\cdot\frac{\partial q_1}{\partial p_1} & \frac{1}{q_1}\cdot\frac{\partial q_1}{\partial \mu} + \frac{1}{q_1 q_2}\cdot\frac{\partial q_1}{\partial p_2} \\ \frac{1}{q_2}\cdot\frac{\partial q_2}{\partial \mu} + \frac{1}{q_1 q_2}\cdot\frac{\partial q_2}{\partial p_1} & \frac{1}{q_2}\cdot\frac{\partial q_2}{\partial \mu} + \frac{1}{q_2^2}\cdot\frac{\partial q_2}{\partial p_2} \end{bmatrix} \mu \qquad (21)$$

2. Stone J. R. N. Linear Expenditure Systems and Demand Analysis: an Application to the Pattern of British Demand *Economic Journal*, Vol. lxiv, September 1954.

The element s_{12} of matrix (21) is

$$s_{12} = \frac{\partial q_1}{\partial p_2} \cdot \frac{\mu}{q_1 q_2} + \frac{\partial q_1}{\partial \mu} \cdot \frac{\mu}{q_1} \tag{22}$$

which can be written:

$$s_{12} = \frac{\partial q_1}{\partial p_2} \cdot \frac{p_2}{q_1} \cdot \frac{\mu}{p_2 q_2} + \frac{\partial q_1}{\partial \mu} \cdot \frac{\mu}{q_1} \tag{23}$$

Thus the elasticity of substitution of good 1 for good 2 is equal to the elasticity of demand for good 1 with respect to the price of good 2 divided by the expenditure proportion of good 2 plus the expenditure elasticity of good 1. This is the familiar result that price elasticity consists of an income effect and a substitution effect. However, this says nothing about whether the substitution effects are symmetrical or not. For symmetry to hold, then, in our example, s_{12} must equal s_{21}, i.e., in the matrix (21) the off-diagonal elements must be equal. Therefore the following relation must hold:

$$\frac{\partial q_1}{\partial p_2} \cdot \frac{p_2}{q_1} \cdot \frac{\mu}{p_2 q_2} + \frac{\partial q_1}{\partial \mu} \cdot \frac{\mu}{q_1} = \frac{\partial q_2}{\partial p_1} \cdot \frac{p_1}{q_2} \cdot \frac{\mu}{p_1 q_1} + \frac{\partial q_2}{\partial \mu} \cdot \frac{\mu}{q_2} \tag{24}$$

This relation shows that the elasticity of demand for good 1 with respect to a change in the price of good 2, (income changing to compensate the price change) equals the elasticity of demand for good 2 as the price of good 1 changes. This relation was first developed by Slutsky[3] for the general case, by the use of determinants in 1915 and is consequently referred to as the Slutsky condition. Simplifying equation (24) and multiplying across by $\frac{q_1 q_2}{\mu}$ we get a form of the condition derived differently by Hicks[4]:

$$\frac{\partial q_1}{\partial p_2} + q_2 \frac{\partial q_1}{\partial \mu} = \frac{\partial q_2}{\partial p_1} + q_1 \frac{\partial q_2}{\partial \mu}, \tag{25}$$

The left hand side, for example, shows the effect on demand for good 1 of a change in the price of good 2 combined with such a change in income as would enable the consumer to buy the same quantities of all goods as before in spite of the change in the price of good 2. It is obvious that this change in income will be smaller the less important is q_2 in the consumer's budget. A similar interpretation applies to the right hand side. It is necessary, however, to discover what restrictions on the model are necessary for this equality to hold.

3. Slutsky E. Sulla teoria del bilancio del consomatore, *Giornale degli Economisti* July, 1915.
4. Hicks J. *Value and Capital*, Clarendon, Press 1939.

Substituting the differentials from equations (3) into equation (25) we get:

$$\frac{B_{12}}{p_1}+q_2\frac{b_1}{p_1} = \frac{B_{21}}{p_2}+q_1\frac{b_2}{p_2} \tag{26}$$

Multiplying across by p_1p_2:

$$B_{12}p_2+q_2b_1p_2 = B_{21}p_1+q_1b_2p_1 \tag{27}$$

Substituting the expressions for q_1 and q_2 from equations (3):

$$B_{12}p_2+b_1p_2\left(\frac{b_2\mu}{p_2}+\frac{B_{21}p_1}{p_2}+B_{22}\right) = B_{21}p_1+b_2p_1\left(\frac{b_1\mu}{p_1}+B_{11}+\frac{B_{12}p_2}{p_1}\right) \tag{28}$$

or

$$B_{12}p_2+b_1b_2\mu+b_1B_{21}p_1+b_1p_2B_{22} = B_{21}p_1+b_2b_1\mu+b_2p_1B_{11}+b_2B_{12}p_2 \tag{29}$$

which simplifies to:

$$B_{21}p_1(b_1-1)+b_1B_{22}p_2 = B_{12}p_2(b_2-1)+b_2B_{11}p_1 \tag{30}$$

This equality must hold if the symmetry condition is to be satisfied but there is no *a priori* reason why it should hold. It therefore remains to see what restrictions are necessary to make it hold. If the first and second terms on one side equalled the first and second terms on the other side respectively then the equality would hold but this would involve restricting the variables p_1 and p_2 as well as the parameters. Obviously the only way this equality can hold is if the two terms containing p_1 are equal and the two terms containing p_2 are equal, i.e. if:

$$b_1B_{22}p_2 = B_{12}p_2(b_2-1) \tag{31}$$

and

$$B_{21}p_1(b_1-1) = b_2B_{11}p_1 \tag{32}$$

Expressing the B's in terms of the b's, equation (31) gives:

$$B_{12} = b_1 \tag{33}$$

and

$$B_{22} = (b_2-1) \tag{34}$$

and equation (32) gives

$$B_{21} = b_2 \tag{35}$$

and

$$B_{11} = (b_1-1) \tag{36}$$

Thus the symmetry condition will hold if the restrictions on the B's shown in equations (33)–(36) are applied. These restrictions can be written more generally as follows:

$$B_{ii} = (b_i - 1) \tag{37}$$

and

$$B_{ij} = b_i \tag{38}$$

and compressed in one expression, using Kronecker's delta, as:

$$B_{ij} = (b_i - \delta_{ij}) \tag{39}$$

where $\delta_{ij} = 1$ if $i = j$; $= 0$ if $i \neq j$

Stone[5] includes a constant term (c) to make the restrictions fully general. Thus equation (39) may be written:

$$B_{ij} = (b_i - \delta_{ij})c_j \tag{40}$$

These most general restrictions on the B's do not violate the property $\sum_{i=1}^{m} B_{ij} = 0$ already described. Thus, for example, given $b_1 + b_2 = 1$ then

$$B_{11} + B_{21} = (b_1 - 1)c_1 + b_2 c_1$$

$$= -b_2 c_1 + b_2 c_1 = 0 \tag{41}$$

regardless of the value of c_1.

Similarly:

$$B_{22} + B_{12} = 0 \tag{42}$$

Consequently the properties of the expenditure and price elasticities (equations 8 and 15) still apply.

Also, of course, the homogeneous nature of equations (3) is not violated and the implication of homogeneity on the elasticities is unchanged since the terms containing the B's in equation (17) cancel out. Thus the symmetry condition can be accommodated without violating any of the other properties.

5. Stone, 1954, *op. cit.*

The restricted values of the B's in terms of the b's can now be substituted into the original system (2). Thus the first equation of this system becomes:

$$p_1q_1 = b_1\mu+(b_1-1)c_1p_1+b_1c_2p_2 \tag{43}$$

$$= -p_1c_1+b_1(\mu+p_1c_1+p_2c_2) \tag{44}$$

Following Stone[6] write $-k$ for c. Then

$$p_1q_1 = p_1k_1+b_1(\mu-p_1k_1-p_2k_2) \tag{45}$$

Similarly the second equation of system (2) becomes:

$$p_2q_2 = p_2k_2+b_2(\mu-p_2k_2-p_1k_1) \tag{46}$$

and the system may be generalised as:

$$p_iq_i = p_ik_i+b_i(\mu-\sum_{j=1}^{m} p_jk_j) \quad 47)$$

where k and b are parameters to be estimated.

This, then, is the standard form of the linear expenditure system. It postulates that expenditure on the i th good is divisible into two parts. The first part (p_ik_i) is committed expenditure on the i th good which consumers will maintain either from habit or due to necessity. The committed quantity (k_i) will not be affected by the level of total expenditure or by any of the prices. This might not be very realistic in the case of certain luxury items but due to fairly standard dietary habits especially in Ireland and constant biological needs, it would seem to be plausible for most food items. Since some proportion of total expenditure is "tied-up" in this way for each good it is clear that additional consumption on any good can only be made out of "residual" income and this is what the second term shows. The expression $\mu-\sum_{j=1}^{m} p_jk_j$ measures the income which is left over after *all* the committed purchases have been met. It is therefore termed supernumerary income and since the b's measure what proportion of it is devoted to each good, they can be interpreted as marginal propensities to consume out of supernumerary income and they must all sum to unity.

6. Stone, *ibid.*

The system as described here can also be interpreted in terms of the utility maximising behaviour of Paretian demand theory. The utility function: $u(q_1, q_2, \ldots, q_m)$ can be derived from the model as written in equation (1). This was done by Geary[7] in a note on previous work by Klein and Rubin[8] which laid more emphasis on constructing a theoretical cost-of-living index. Geary's method can be briefly illustrated by reference to a one-good example. Thus to find the utility function associated with the consumption of q_1 we begin with the first equation of system (3) and substitute for the symmetry-restricted B's. This gives:

$$q_1 = \frac{b_1\mu}{p_1} + \frac{(b_1 - 1)c_1p_1}{p_1} + \frac{b_1c_2p_2}{p_1} \tag{48}$$

$$\therefore\ q_1 + c_1 = \frac{b_1(\mu + c_1p_1 + c_2p_2)}{p_1} \tag{49}$$

Now since $\dfrac{\partial u}{\partial q_1} = \lambda p_1$, i.e., marginal utility is proportionate to price, then

$$p_1\partial q_1 = \lambda\partial u \tag{50}$$

But from (49)

$$p_1 = \frac{b_1(\mu + c_1p_1 + c_2p_2)}{q_1 + c_1} \tag{51}$$

$$\therefore\ \partial u = \frac{\lambda b_1(\mu + c_1p_1 + c_2p_2)\partial q_1}{q_1 + c_1} \tag{52}$$

Since compensating variation in income is implied by the model, utility maximisation is achieved by the consumer remaining on the same indifference curve, i.e., $\partial u = 0$. Integrating this function (52) we get, with $\lambda(\mu + c_1p_1 + c_2p_2) = 1$,

$$u = b_1 \log(q_1 + c_1) + C \tag{53}$$

where C is the constant of indefinite integration.

In terms of Stone's treatment the utility function (or preference field) for the multi-commodity case can be written

$$u = \sum_{i=1}^{m} b_i \log(q_i - k_i) \tag{54}$$

7. Geary, R., A Note on the Constant Utility Index of the Cost of Living, *Review of Economic Studies*, Vol. xviii 1950–51

8. Klein, L. R. and Rubin, H., A Constant Utility Index of the Cost of Living, *Review of Economic Studies*, Vol. xv 1948–49.

It seems intuitively obvious that utility should be some function of the excess of consumption over that part which is committed on the grounds that the latter part is in the nature of a basic minimum and hence does not enter into the consumer's efforts to maximise utility.

Equation (51), when generalised, becomes:

$$p_i q_i = p_i k_i + b_i\left(\mu - \sum_{j=1}^{m} p_j k_j\right) \tag{55}$$

which is the standard form of the model.

Thus the symmetry conditions also allow this utility function to be derived. It is clear that due to the hyperbolic form of this function the s_{ii} must all be negative, i.e., the marginal rate of substitution is diminishing. Also b_i must be positive and less than unity. If not the utility function (54) would be imaginary. All these features are derived from elementary demand theory and equation (55) shows that they are implied by the model.

This utility interpretation can be used to show how the same utility can be achieved in period t with income (μ_t) at prices p_{ti} as was achieved in period 0 with income μ_0 and prices p_{0i}.

Thus

$$\frac{\prod_{i=1}^{m} (q_{ti} - k_{ti})^{bi}}{\prod_{i=1}^{m} (q_{0i} - k_{0i})^{bi}} = 1 \tag{56}$$

where Π multiplies all expressions of the form which follow

or, introducing the price sets of both periods we have:

$$\prod_{i=1}^{m} \left(\frac{Z_t p_{0i}}{Z_0 p_{ti}}\right)^{b_i} = 1 \tag{57}$$

$$\text{where } Z = \left(\sum_{i=1}^{m} p_i q_i - \sum_{i=1}^{m} p_i k_i\right)$$

$$= \left(\mu - \sum_{i=1}^{m} p_i k_i\right)$$

i.e.,

$$Z_t = Z_0 \prod_{i=1}^{m} \left(\frac{p_{ti}}{p_{0i}}\right)^{b_i} \tag{58}$$

or

$$\mu_t = \sum_{=1}^{m} p_{ti} k_i + \left(\mu_0 - \sum_{i=1}^{m} p_{0i} k_i\right) \prod_{i=1}^{m} \left(\frac{p_{ti}}{p_{0i}}\right)^{b_i} \tag{59}$$

Thus the income (μ_t) which is required to keep utility constant in time (t) under price regime (p_{it}) is equal to the current cost of committed purchases plus the supernumerary income of the base period (0) raised by a geometric average of the price ratios. Thus a constant utility index of the cost of living is given by μ_t/μ_0 where μ_0 is given for some base year and μ_t is calculated by formula (59) for all other years. It can be shown that this utility function and the cost-of-living index based upon it is useful for making projections.

General Critique

Because the model conforms so well with demand theory there are circumstances, in practice, with which it cannot adequately cope. Strictly speaking complementary and inferior goods should be ruled out. This can be shown by examining the substitution effect, i.e., the compensated price effect. Thus, for example, substituting the restricted values of the B's in the left-hand side of equation (29) gives:

$$s_{12} = b_1 b_2[\mu - (p_1 k_1 + p_2 k_2)] \qquad (60)$$

Similarly it can be shown that:

$$s_{11} = (b_1 - 1) b_1 [\mu - (p_1 k_1 + p_2 k_2)] \qquad (61)$$

Therefore, in general,

$$s_{ij} = (b_i - \delta_{ij}) b_j (\mu - \sum_{j=1}^{m} p_j k_j) \qquad (62)$$

If $i = j$ then $\delta_{ij} = 1$ and s_{ij} must be negative since a change in the "own" price of a good will cause a change in the opposite direction in demand for that good. Since supernumerary income must be positive then the expression $(b_i - 1)b_i$ must be negative in which case b_i must be positive and less than unity. The fact that b_i must be positive rules out inferior goods. If $i \neq j$ then $\delta_{ij} = 0$ and s_{ij} must be positive which rules out complementary goods. In applying the model some negative b's do emerge suggesting that certain commodities are inferior. Such results do undermine the utility interpretation to a certain extent and while it would have been possible to eliminate the negative b's by aggregating the commodities into larger groups it was felt that these "recalcitrant" results were of interest in themselves.

This model might also be criticised on the grounds that the constant parameters are not realistic. For instance the committed expenditure might well increase slowly over time reflecting the way in which "luxuries" tend to become "necessities" as time elapses. Also, as has been noticed, there may be other factors which shift the function. Although the introduction of a trend term would destroy most of the theoretical properties, the parameters themselves could be rendered time dependent[9]; for example, by writing $b' + b''t$ for b and $k' + k''t$ for k where t = time. However, it was felt that even over a period as long as twenty years the parameters of Irish consumption habits probably have not changed very significantly for the traditional types of foodstuffs being analysed.

Finally the instantaneous response mechanism implied by the model may not be ideal. The model can be dynamised along Nerlovian lines but such an approach is considered to be more suitable for durable goods.[10] An alternative approach would be to use the dynamic version and then allow the equilibrium values of consumption which appear in the dynamic model to be determined by a static linear expenditure system.[11] It was felt, however, that such refinements would not be apposite here due to the perishability of foodstuffs and the regularity of their consumption.

Thus the model as formulated in equation (47) which is the standard form of the Samuelson-Stone linear expenditure system is used to analyse the consumption of foodstuffs in Ireland.

Application

The above model[12] was applied to the consumption of fifteen different foodstuffs and one category of other expenditure in Ireland over the twenty-one year period from 1947 to 1967.

Quantities of individual foodstuffs consumed per head per week were taken from the Irish Statistical Survey/National Income and Expenditure[13] up to 1958 and from the Irish Statistical Bulletin[14] thereafter. Annual retail prices were calculated from the quarterly data published in the June and December

9. *Programme for Growth* Vol. 1. Department of Applied Economics, University of Cambridge, Chapman and Hall, 1962.

10. Stone, R. and Rowe, D. A., The Market Demand for Durable Goods, *Econometrica*, Vol. 25, No. 3, 1957

11. Stone, R., and Croft-Murray, G., *Social Accounting and Economic Models*, Bowes and Bowes, London 1959.

12. The L.E.S. has recently been shown to be merely a special case of an even more general model which, for example, can accommodate complementary and inferior goods, See Brown, Murray and Heien: *The S-Branch Utility Tree: A Generalisation of the L.E.S.* Discussion Paper No. 18, State University of New York at Buffafo, May, 1968.

13. *Irish Statistical Survey/National Income and Expenditure*, Central Statistics Office, Dublin, 1948–1968.

14. *Irish Statistical Bulletin/Irish Trade Journal* and *Statistical Bulletin*, Central Statistics Office, Dublin, December, 1969.

Statistical Bulletins[15] the price of beef being taken as the unweighted average price of the two cuts; sirloin and shoulder. An attempt was made to estimate the expenditure proportions of all cuts with a view to getting a better (weighted) average price for beef but this made very little difference to the simpler method which had been used by Hart[16] and which is, perhaps, less arbitrary. Given prices, quantities and population[17] an annual series of expenditures for the total population was constructed for fourteen foodstuffs. The sum of these in each year was subtracted from the item "expenditure on food and non-alcoholic beverages"[18] giving a residual termed "other food". Finally, the total expenditure on all food was subtracted from the official figure for "total expenditure within the state", giving a second residual termed "other expenditure". Thus the sum of all the individual expenditures equalled the official totals and the two residual items "other food" and "other expenditure" bore the brunt of any inconsistencies. This calculation was repeated for the series at constant (1958) prices, the Consumer Price Index[19] being used to deflate the "hybrid" residual items which did not have a unique "own" price. Thus complete "additive" series for sixteen items over a period of twenty-one years were constructed at current and constant (1958) prices. The constant price series can be seen in conjunction with the values predicted by the model in Appendix A. In computing the model the data was converted to *per capita* terms by dividing by the population in each year.

The grouping of commodities was largely determined by the availability of data but it does seem to conform well with the criteria of "want independence" between groups and "price homogeneity" within each group.[20] The method of estimating the model is described, by reference to the two-good example, in Appendix B.

Results

The estimated parameters are set out in Table 1 in a way that illustrates the "mechanics" of the model. The first column shows the estimated quantities in natural units per head per week to which the consumers are committed. For

15. *Ibid.* for June and December, 1948–1969.

16. Hart, J., An Econometric Method of Forecasting the Demand for Food in Ireland in 1970, *Journal of the Statistical and Social Inquiry Society of Ireland*, 1966.

17. *Statistical Abstract of Ireland*, Central Statistics Office, Dublin, 1948–1968 for the Census Years 1951, 1956, 1961 and 1966. Estimates for intercensal years were obtained from the C.S.O., Griffith Barracks, Dublin.

18. *Irish Statistical Survey/National Income and Expenditure*, Central Statistics Office, Dublin, 1948–1968.

19. *Irish Statistical Bulletin*, December, *op. cit.*, 1964 and 1969.

20. Stone, R., Aitchison, J., and Brown, J. A. C., Some Estimation Problems in Demand Analysis. *The Incorporated Statistician*, Vol. 5, No. 4, April, 1955.

example, the average consumer maintains a committed consumption of 6·01 pints of fresh milk per week. He will actually consume more than this by devoting some of his supernumerary income to milk. Alternatively, in the case of bread, the actual consumption will be less than the committed part because, for this good, the marginal propensity to consume is negative. Column (2) shows committed consumption in expenditure terms for 1958. This year was chosen to simplify the exposition, since, being the base year, constant and current prices are the same. The third column shows the estimated b's. They are generaly quite small for the individual food items though those for pigmeat, mutton and fresh milk show that 4·6%, 3·1% and 3·0% of supernumerary income respectively, are spent on these goods. The existence of negative b's does undermine some of the theoretical properties of the model but they do appear reasonable, from a pragmatic point of view, for farmers' butter, bread, household flour and potatoes (the traditional inferior goods) though somewhat surprising for eggs and sugar. The interpretation of these parameters will be more fully discussed in terms of elasticities. As seen in column (3), however the b's do sum to unity.

Column (4) shows the estimated or predicted values for expenditure in 1958. The term, $b_i Z$, is formed by multiplying supernumerary income (Z) by each b. Since

$$Z = \sum_{i=1}^{16} p_i q_i - \sum_{i=1}^{16} p_i k_i$$

we form Z by adding up the actual individual expenditure and subtracting the sum of the estimated committed expenditures. We then form $p_i k_i + b_i Z$, i.e. the sum of the committed and uncommitted expenditure on the ith good which is, of course, the expenditure ($p_i q_i$). The sum of these predicted values (770.55) should approximate the actual figure of total expenditure which it does. Thus it can be seen from Table 1 that the predicted expenditure on fresh milk, for example, is made up as follows:

$$35{\cdot}28 \doteqdot 34{\cdot}28 + {\cdot}029653\ (770{\cdot}55 - 736{\cdot}82)$$

and for creamery butter as:

$$26{\cdot}15 \doteqdot 26{\cdot}03 + {\cdot}003499\ (770{\cdot}55 - 736{\cdot}82)$$

and similarly for other commodities.

TABLE 1: *Estimated Parameters of L.E.S.*

Item	col. (1) k_i oz/hd/wk or o.s.	col. (2) p_ik_i d/hd/wk 1958	col. (3) b_i	col. (4) $p_ik_i+b_iZ$ $(=p_iq_i)$ d/hd/wk 1958	col. (5) col. (4)/col. (2) $1+\frac{b_iZ}{p_ik_i}$ %
Fresh Milk	6·01 (*a*)	34·28	·029653	35·28	2·91
Creamery Butter	8·32	26·03	·003499	26·15	0·46
Farmers' Butter	3·51	9·65	—·022352	8·90	—7·77
Margarine	0·79	0·99	·008330	1·27	28·28
Cheese	0·59	1·29	·005771	1·48	14·73
Eggs	5·04 (*b*)	21·01	—·000627	20·99	—0·10
Beef	10·75	26·23	·002502	26·31	0·30
Mutton	4·76	11·91	·031050	12·96	8·82
Pigmeat	13·79	40·54	·045675	42·08	3·79
Poultry	3·41	6·94	·012832	7·37	6·20
Sugar	20·64	9·68	—·003080	9·58	—1·03
Bread	52·18	23·48	—·020575	22·79	—2·90
Household Flour	34·59	13·90	—·023119	13·12	—5·60
Potatoes	123·11	24·16	—·017468	23·57	—2·44
Other Food	—	20·56	·148561	25·57	24·37
Other Expenditure	—	466·17	·799348	493·13	5·76
Total	—	736·82	1·000000	770·55	—

(*a*) = pints; (*b*) = number; Z = Supernumerary Income; *d/hd/wk* = old pence per head per week.

It is obvious that for most commodities the differences between total expenditure on the good in question (col. 4) and the committed part (col. 2) are quite small, i.e. the proportion of expenditure which is committed is quite large. This is partly because supernumerary income (Z) in 1958 was still quite low due to a low income level but also because the *b*'s, with the exception of the last two items, are small fractions. In all cases where the *b*'s are positive the expenditure exceeds the committed part and where negative *b*'s occur the expenditures are less than the committed parts since these regression lines are falling. The extent to which expenditure differs from the committed part can be seen in column (5) which is obtained by dividing column (4) by column (2) forming the expression

$$1 + \frac{b_iZ}{p_ik_i}$$

The figures are given in percentage terms. Since Z is the same for all goods in any one year, the differences between goods on this basis are due to differences in the ratio $\frac{b_i}{p_i k_i}$. Thus the higher the b_i and the lower the $p_i k_i$ the more will expenditure on the *i*th good exceed the committed part. Margarine has a relatvely high b and a low pk with the result that expenditure on the good exceeds committed expenditure by 28·28%. The same is broadly true of other food, cheese and mutton which have figures of 24·37%, 14·73% and 8·82% respectively. Pigmeat is lower because, although it has a relatively high b, its committed proportion is also high. Alternatively, the more staple goods such as bread, potatoes, flour, sugar and eggs have high proportions of expenditure committed to their purchase while the propensity to consume additional quantities out of supernumerary income is negative. Although beef and creamery butter have positive b's they nevertheless fall, surprisingly, into this "staple" category. For a fuller understanding of consumer behaviour, however an examination of the various elasticities is required.

Expenditure Elasticities:

The expenditure elasticity of the *i*th good is given by:

$$\frac{b_i \mu}{p_i q_i}$$

The elasticities presented in Table 2, however, are expressed in quantity terms i.e., in calculating them the constant (1958) price values for μ and $p_i q_i$ were used. Table 2 shows these elasticities as averages over the period 1961–1964, for purposes of comparison with O.E.C.D. time-series elasticities, and for 1967. In addition this table shows what might be termed "supernumerary income elasticities" for 1967 given by the formula:

$$\frac{b_i Z}{p_i q_i}$$

which is also calculated at constant prices. Since

$$\frac{b_i Z}{p_i q_i} = 1 - \frac{p_i k_i}{p_i q_i}$$

TABLE 2: *Expenditure Elasticities of Quantity Demanded*

Item	O.E.C.D.	L.E.S.		
		$b_i\mu/p_iq_i$		b_iz/p_iq_i
	1961–64	*1961–64*	*1967*	*1967*
Fresh Milk	0·36	0·67	0·73	0·18
Creamery Butter	—	0·12	0·13	0·03
All Butter	—0·38	—0·51	—0·65	—0·16
Margarine	—	2·82	2·61	0·65
Cheese	2·11	2·37	1·84	0·46
Eggs	—0·49	—0·03	—0·03	—0·01
Beef	0·52	0·08	0·08	0·02
Mutton	1·24	1·48	1·62	0·40
Pigmeat	0·50	0·89	0·88	0·22
Poultry	1·06	1·41	1·07	0·27
Sugar	0·45	—0·30	—0·34	—0·08
Bread	—	—0·91	—1·10	—0·27
Bread and Household Flour	—0·8	—1·28	—1·60	—0·40
Potatoes	—0·4	—0·76	—0·89	—0·22
Other Food	—	2·86	2·32	0·58
Other Expenditure	—	1·21	1·19	0·30

these elasticities must all be less than unity since both p_ik_i and p_iq_i are positive. They are also, of course, directly correlated with the ordinary expenditure elasticities. Farmers' butter is included with creamery butter to give an overall elasticity for all butter, while flour is included with bread to give an overall elasticity for wheat products both figures being more compatible with the OECD[21] ones.

High positive elasticities of the two types described emerge for margarine, other food, cheese, mutton, other expenditure and poultry in that order. The figure for margarine is surprisingly high. However, both Hart[22] and Goreaux[23] have referred to the same phenomenon in terms of cross-section analysis. Margarine, in Ireland, of course, was never considered to be an inferior substitute for butter. The elasticity of demand for margarine with respect to the price of butter is, in fact, not significantly different from zero, suggesting that the relative cheapness of this good does not account for the growth in

21. OECD, *Country Studies*, Ireland, 1968.

22. Hart *op. cit.*

23. Goreaux L. M., Factors Influencing the Trend of Food Consumption, in *The State of Food and Agriculture*, F.A.O., 1960.

consumption. However, the growing use of margarine in cooking and baking, the development of better quality margarine and the promotion of this product in recent years on health and taste grounds all combine to provide a reasonable explanation for this high elasticity. The actual consumption of margarine rose dramatically from 0·84 ounces per head per week in 1948 to 2·5 in 1967. This trebling of consumption as the result of a much slower growth in income accounts for the high elasticity. Consumption in the initial year, 1947, was however, high (1·55 ounces) but this was clearly due to post-war abnormalities.

The high figure for other food is not surprising in view of the luxury-type nature of this category which includes such items as meals outside the home,[24] veal, lamb, cream, etc. The high elasticity for cheese corresponds quite well with that derived by the OECD. As in the case of margarine, the consumption of cheese has grown rapidly, from the low level of 0·61 ounces in 1948 to 1·4 ounces in 1967, although the smooth upward trend did not really begin until 1952. This clearly reflects changing tastes and there is some evidence to suggest that there is also a higher demand for better quality cheese. For example, the expenditure elasticity measured in current prices is 2·01 in 1967 which is higher than that at constant prices implying that there is a fairly strong quality effect. Also, of course, the availability of a wide range of cheeses which is probably related to the development of the supermarket, must also have had an effect.

In a cross-section study Leser[25] who also derived a high elasticity for mutton felt that there was "no obvious explanation for the special position of mutton as a luxury good". The OECD figure for mutton in Table 2 is also quite high. Consumption rose from 3·95 ounces per head per week in 1948 to 7·5 in 1967 but the upward trend has flattened out in more recent years. It is obvious that if the earlier observations were omitted from the analysis the elasticity would be substantially reduced. It is probable that in the earlier period the growing equality in the distribution of income brought the consumption of mutton within the compass of many more consumers thus raising the average consumption per head.

The consumption of poultry remained relatively stable until 1957 when it began an upward trend rising from 3·3 to 5·6 ounces per head per week in 1967. The elasticity is correspondingly high and is quite close to the OECD figure It is tempting to argue that the high elasticity is due, for example, to the substitution of poultry for beef, the consumption of which did not increase very much. However, this would be erroneous because such an occurrence would be

24. This item is included because the official figures for consumption are derived from production figures,

25. Leser, C.E.V., *Demand Relationships for Ireland,* The Economic Research Institute, Paper No. 4, 1962.

reflected by the cross-price elasticity and not by the expenditure elasticity. Nevertheless poultry used to be considered a luxury good when its relative price was high and there is good reason to believe that it is still regarded as a luxury item. Thus the expenditure elasticity of poultry may be high because of the high regard in which this food is still held quite apart from the fact that its price relative to beef has fallen due to the development of broiler production, etc. There is still something of the "Sunday-lunch" aura about poultry. Also the expenditure elasticity in current prices is significantly higher (1·37 in 1967) implying a shift to better quality fowl.

Fairly high elasticities also emerge for pigmeat and fresh milk in that order. It has already been noticed that the marginal propensity to consume pigmeat out of supernumerary income is the highest of all the individual foodstuffs yet its expenditure elasticity is lower because of the high proportion of expenditure which is committed. Both features can be rationalised in that a high commitment to the traditional bacon and rashers in Ireland is not surprising while a high marginal propensity to consume pork and ham might also be expected. The elasticity, however, is somewhat higher than that derived by the OECD This is probably because the longer time period used here includes more instances of sharp growth in consumption. Between 1948 and 1949 consumption rose from 11·79 to 14·50 ounces per head per week and between 1951 and 1955 from 12·57 to 15·50 ounces per head per week. Also of course the OECD study did not extract the effect of prices so there is no reason to expect strict correspondence between the elasticity estimates. The elasticity for fresh milk is also somewhat higher than the OECD estimate probably for the same reasons. The trend in milk consumption from 1952 onwards was very steady showing a decelerating increase from 5·46 to 7·0 pints per head per week in 1967.

Two commodities, creamery butter and beef, have surprisingly low elasticities In the case of butter Leser[26] derived a similar result from cross-section data. Thus, in Ireland, there is a situation where the elasticity for margarine is high while that for butter is low, a phenomenon which is reversed in the UK Part of the answer may be that butter is more or less complementary with inferior goods such as bread and potatoes while margarine is more widely used in baking and cooking. The elasticity for beef, however, is perhaps the most surprising of all even though the OECD figure is not very high. Since 1950 the consumption of beef has remained almost constant at a level of about 10 ounces per head per week. It is virtually independent of the growth in income or the passage of time. It will be seen later that the "own" price elasticity of demand for beef is also very low. Thus beef consumption seems to

26. Ibid.

be almost completely inelastic with respect to both income and price. One reason for the low expenditure elasticity might be that beef was not considered to be a luxury food in Ireland especially as there was no war-time shortage. In any event the higher elasticities for mutton, pigmeat and poultry do suggest a preference for these meats over that for beef. This preference may, in part, be due to the fact that some of these meats can be consumed on farms without process of sale while this does not apply to beef. Other single-equation models tested in relation to beef did not produce significantly higher expenditure elasticities.

The remaining commodities have negative expenditure elasticities. For example, if total expenditure or income increased by 10%, consumption of bread would fall by about 11% because the increase in income brings a range of other goods into the compass of the consumer who can buy them by further increasing his income as a result of reducing his purchase of bread. If bread already constitutes a large proportion of his expenditure then a small reduction will release a relatively large amount of income for purchasing the other goods. In the later years of this study bread did not form a very large proportion of total expenditure so that its consumption would have to be reduced by a fairly large amount if other goods were to be purchased. This reasoning probably accounts for the fairly high elasticities for bread, bread and flour and potatoes though obviously other secular forces such as the reduction of carbohydrate intake are also at work. Consumption of all these goods fell dramatically between 1947 and 1967; bread from 50·24 to 40·8 ounces, household flour from 48·48 to 20·8 ounces and potatoes from 148·6 to 97·6 ounces (per head per week).

The items, all butter, sugar and eggs have lower negative elasticities. The addition of farmers' butter to creamery butter renders this elasticity negative because the consumption of the former has almost fallen to zero. Over the whole period consumption of farmers' butter fell from 4·99 to 1·0 ounces. The figure for all butter for 1961–64 corresponds quite well with the OECD figure. The fact that sugar and eggs appear as inferior goods, though only marginally so, is fairly surprising though neither could really be termed luxury items. In the case of sugar it is probably the secular factors such as dietary habits and the minimisation of calorie intake which are responsible rather than income since the proportion of expenditure devoted to sugar is very small. After an initial increase up to 1953 the consumption of sugar fell slowly from 21·4 to 19·1 ounces per head per week. Although the OECD derived a negative elasticity for eggs as well, the one presented here is a good deal lower. The consumption of eggs has remained fairly stable at around 5·5 eggs per head per week from 1951 onwards suggesting that it is almost completely inelastic to the income changes that occurred over the period. Given the fact that eggs are fairly traditional for certain meals it is not surprising that this is so. Since the consumption of eggs may also be associated, to some extent, with Friday abstinence it is perhaps not surprising that the expenditure elasticity is marginally negative.

In general, there is a very wide dispersion in these elasticities suggesting that the consumption-income relationship in Ireland is very different for different goods. Leser[27] has found this to be also true of cross-section elasticities. However, as was already noted, the sum of the elasticities, however varied, when weighted, by their "own" expenditure proportions should equal unity and this is, in fact, the case. That is:

$$\sum_{i=1}^{16} \frac{b_i \mu}{p_i q_i} \cdot \frac{p_i q_i}{\mu} = 1 \qquad \text{since} \sum_{i=1}^{16} b_i = 1$$

and similarly when Z replaces μ.

Price Elasticities:

In this general simultaneous system there are 16 price variables for each good (1 "own" price and 15 "other" or "cross" prices). There are also 16 goods and a price change, as shall be seen, can be interpreted as compensated or uncompensated. Thus, as Frisch[28] has noticed, the number of price elasticities derivable from this type of model is very large. Actually the cross-price elasticities are almost all not significantly different from zero while the "own" price elasticities are. This suggests that the prices of other goods do not materially affect the consumption of the good in question and that most of the response is to the 'own" price of that good. Thus, only the "own" price elasticities will be shown here though passing reference may be made to a cross-price elasticity which appears interesting. The price-elasticity may be defined as being either compensated or uncompensated by income.

The uncompensated price-elasticity is the usual one where, for example, a fall in price leads to an increase in real income which is not offset by any exogenous income change. This elasticity can be expressed in terms of the parameters as follows (for the two-good case):

$$\frac{\partial q_1}{\partial p_1} \cdot \frac{p_1}{q_1} = \left[\frac{-b_1 \mu}{p_1^2} - \frac{B_{12} p_2}{p_1^2} \right] \frac{p_1}{q_1} \tag{63}$$

from equations (3)

27. Ibid.

28. Frisch, R., A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model with Many Sectors. *Econometrica*, April, 1959.

$$= \frac{-b_1(\mu - p_2k_2)}{p_1q_1} \tag{64}$$

(Restricting for symmetry)

$$= \frac{-b_1(Z + p_1k_1)}{p_1q_1} \tag{65}$$

Similarly it can be shown that[29]

$$\frac{\partial q_1}{\partial p_2} \cdot \frac{p_2}{q_1} = \frac{-b_1p_2k_2}{p_1q_1} \tag{66}$$

If, however, the price change is offset by an income change then the price elasticity measures the substitution effect and is given by:

$$s_{11} = \frac{\partial q_1}{\partial p_1} \cdot \frac{p_1}{q_1} + q_1 \cdot \frac{\partial q_1}{\partial \mu} \cdot \frac{p_1}{q_1} \tag{67}$$

$$= \left(\frac{-b_1\mu + b_1p_2k_2}{p_1q_1}\right) + \frac{b_1}{p_1} \cdot \frac{p_1}{q_1} \cdot q_1 \tag{68}$$

from equations (3) and restricting for symmetry.

Substituting for q_1 in the numerator of equation (68) from equations (3) we get:

$$s_{11} = \frac{-b_1(\mu - p_1k_1 - p_2k_2) + b_1b_1(\mu - p_1k_1 - p_2k_2)}{p_1q_1} \tag{69}$$

$$= \frac{-b_1Z(1 - b_1)}{p_1q_1} \tag{70}$$

29. This corresponds with the formula given in: Stone, R., *The Changing Pattern of Consumption*, Department of Applied Economics, Cambridge 1966, and in Goldberger, A. S., and Gamaletsos T., A Cross-Country Comparison of Expenditure Patterns, *European Economic Review*, Vol. 1, No. 3, Spring, 1970.

Similarly it can be shown that:

$$s_{12} = \frac{-b_1 b_2 Z}{p_1 q_1} \tag{71}$$

Thus, in general, the compensated elasticity is:

$$\frac{-b_i Z(\delta_{ij} - b_j)}{p_i q_i}$$

where $\delta_{ij} = 1$ if $i = j$; $= 0$ if $i \neq j$

Both measures of the "own" price elasticities are shown in Table 3 for 1967. The uncompensated elasticities are, in all cases, higher than the compensated ones. This is to be expected because if the price of one good rises and the consequent reduction in real income is not offset by an income change then the uncompensated response to the price change will be higher than if it were compensated. Since the compensation is such as to enable the consumer to buy the same quantities of all goods as before in spite of the change in the price of the good in question we might have expected the compensated price elasticities to be even lower. The fact that this is not the case suggests that the income compensation does not encourage the Irish consumer to revert to the *status quo* completely but rather to spend only part of the extra income on the dearer good while spending the remainder on other relatively cheaper goods. With the exception of the other expenditure category the differences between both types of elasticity are not very large but they are highest for other food, mutton, pigmeat, poultry and cheese, suggesting that the response to the prices of these goods is damped somewhat more by a compensating change in income than in the case of the other goods. If the formula for the uncompensated elasticity is divided by that for the compensated one the following expression results:

$$\frac{1 - b_i + (p_i q_i)/Z}{1 - b_i}$$

This explains the rather higher dampening of the price response for the above mentioned goods since the higher the expenditure as a proportion of supernumerary income and the higher the marginal propensity to consume the more will the uncompensated elasticity exceed the compensated elasticity. For example, if the price of mutton increased, the fall in income would have a strong effect on the consumer's reaction because the expenditure proportion of mutton is quite high while the marginal propensity to consume is also reasonably high and so, when the income effect is allowed to operate we would expect the price elasticity to be substantially higher than when the income effect is assumed to be exactly offset. Thus if the price of mutton rose by 10% consumption would drop by 5·4%. However, if income compensated the price increase the fall in consumption would be reduced to 4·3%.

It will be noticed that the elasticities in Table 3 are highly correlated with the expenditure elasticities for 1967 in Table 2. This, of course, is because of the homogeneity property of the model. It has already been noticed that because of this property the following relation from the two-good example holds:

$$\frac{b_1\mu}{p_1q_1} = -\frac{\partial q_1}{\partial p_1}\cdot\frac{p_1}{q_1}-\frac{\partial q_1}{\partial p_2}\cdot\frac{p_2}{q_1} \tag{72}$$

i.e., the expenditure elasticity equals minus the sum of the own and cross-price elasticities. Since the cross-price elasticities are all fairly low it is obvious that the "own" price elasticities will be correlated with the expenditure elasticities

In view of this we find that the highest negative elasticities appear for margarine, other food, cheese and mutton in that order. If the price of these goods rose by 10% consumption would fall by 5% and over. This is a fairly high measure of response. The elasticity of margarine with respect to the price of butter is only +0·003 implying that if butter price rose by 10% the increase in margarine would only be 0·03%. (This, in fact, was one of the higher cross-elasticities.) Poultry, pigmeat, milk and creamery butter have lower elasticities of —0·41, —0·34, —0·20, and —0·04 respectively while the figures for the remaining goods are all positive, ranging between +·01 for eggs to +·53 for bread and flour. These goods, of course, have been found to be inferior. Thus if the price of bread were to fall by 10% the increase in the consumer's income encourages him to further reduce his purchases of bread by 3·4% so as to purchase other goods that have come within his reach. This interpretation is not entirely plausible for eggs and sugar. However, both these positive elasticities are very small.

TABLE 3: *Own price elasticities in 1967: compensated and uncompensated*

Item	Compensated $-b_i Z(1-b_i)/p_i q_i$	Uncompensated $-b_i(Z+p_i k_i)/p_i q_i$
Fresh Milk	−·17	−·20
Creamery Butter	−·03	−·04
All Butter	+·19	+·21
Margarine	−·72	−·73
Cheese	−·50	−·59
Eggs	+·01	+·01
Beef	−·02	−·03
Mutton	−·43	−·54
Pigmeat	−·24	−·34
Poultry	−·34	−·41
Sugar	+·09	+·10
Bread	+·27	+·34
Bread and Household Flour	+·40	+·53
Potatoes	+·31	+·38
Other Food	−·57	−·84
Other Expenditure	−·06	−·85

Use of the Model for Projections

The model can readily be used for deriving projections. Although no actual projections are presented in this present paper it is worthwhile outlining briefly the methodology involved.

Since income and prices are the independent variables of the model, their future values must obviously be known before projections of food consumption can be made. Different annual average growth rates in supernumerary income can be tested. Since total committed expenditure will remain constant in the future the growth rates apply to supernumerary income only. For example, the Z in 1967 will become $Z(1\cdot 0r)^8$ in 1975 (say) where r = growth rate. It was noted earlier that utility was a function of real supernumerary income. Hence this treatment implies a growth in utility but it also assumes that no utility is derivable from committed consumption. With regard to prices, at least two methods can be used. The first merely assumes that *relative* prices will remain the same as in the base year 1967 while the second attempts to project *relative* prices in the following way:

1. A constant-utility index of the cost-of-living can be constructed along the lines suggested by Klein and Rubin.[30] The index is defined as:

$$\frac{\mu_t}{\mu_0}$$

where the subscripts *o* and *t* refer to the base year, 1958 (say), and all other years, respectively, and μ_t is given by the following equation:

$$\mu_t = \sum_{i=1}^{16} p_{ti}k_i + (\mu_0 - \sum_{i=1}^{16} p_{0i}k_i)\prod_{i=p}^{16}\left(\frac{p_{ti}}{p_{0i}}\right)^{b_i} \tag{73}$$

Equation (73) yields the income in any period (μ_t) which is required to give the same utility at prices p_t as income in 1958 (μ_0) gave at 1958 prices (p_0)

2. This index can then be used to deflate the actual prices of each good over the entire period, the deflated prices being:

$$p_{i_t}\left(\frac{\mu_0}{\mu_t}\right)$$

Thus, the set of prices so deflated, display, in any period, what might be termed the real price relatives.

3. These deflated price series can then be individually extrapolated to the projection year, 1975 (say). Using equation (73) we can then find the income in 1975 (say) which, at these projected prices, would give the same utility as the income in 1967 at 1967 prices. The 1975 price set is then deflated by the 1975 cost-of-living index so derived, i.e., by $\frac{\mu 1967}{\mu 1975}$. This procedure is used for all the projection years and it ensures that, if the growth in Z were zero, utility would remain constant.

The two sets of projections can then be derived from the equation:

$$q_i = k_i + \frac{b_i Z}{p_i} \tag{74}$$

30. Klein and Rubin *op. cit.*

where p_t is either the price projected in the manner described above or the constant 1967 price. Clearly if the projected price rises by more than r% per annum (i.e., the assumed growth in Z) then q_t will fall and vice versa. The *total* projected expenditure at constant prices will equal the total at projected prices. Thus, some relative prices will increase while others decrease, the net effect, weighted by the expenditure proportions, being zero. Consequently the real expenditure is the same for both price sets and this conforms with the constant-utility interpretation already described.

Concluding Comments

In this paper the overall approach was to impose a theory of consumer behaviour on the facts rather than to examine many different hypotheses. No apology is made for this. Demand theory is fairly well established especially in Western economies and if it had not fitted the facts in the Irish economy then the discrepancies themselves would have been of interest.

The model used conformed almost exactly with the theory of demand. It was homogeneous of degree zero, it satisfied the additivity criterion, it was capable of being (and was) restricted to satisfy the symmetry of substitution criterion, it conformed with the utility interpretation of demand theory and, finally, it was, of course, simultaneous, the main behavioural implication of which, was that the consumer was seen to respond to income and to the prices of *all* goods when deciding on the purchase of any particular good.

In short, the model used was an all-or-nothing one in the sense that if some of the results it produced appeal to common sense then the others (which may not) cannot be rejected out of hand. It is submitted that most of the results, in terms of parameters and elasticities, were eminently plausible while those which were somewhat surprising were capable of being rationalised. The expenditure (or income) elasticities were in some cases quite different to those derived elsewhere from *cross-sectional* studies but this was not unexpected in view of the conceptual differences between them.

Central Bank of Ireland.

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APPENDIX A

Actual and expected consumption at constant (1958) prices (£ million)

Year	Fresh Milk		Creamery Butter	
	Actual	Expected	Actual	Expected
1947	18.6730	20.4773	10.4740	16.6110
1948	19.3780	21.6407	13.3990	16.8019
1949	19.4950	21.9519	15.6280	16.8259
1950	20.3320	22.6744	17.4230	16.8676
1951	20.1650	22.3307	17.6570	16.7840
1952	19.9660	21.3985	17.9710	16.6278
1953	21.3680	21.8132	16.8630	16.6626
1954	22.5110	21.9406	17.0800	16.6378
1955	23.0810	22.6315	17.7570	16.6287
1956	22.9710	21.9680	17.7570	16.4392
1957	23.5800	21.3928	16.4030	16.3059
1958	23.3190	21.8709	15.9920	16.2094
1959	23.6200	22.2372	17.0040	16.2157
1960	24.1810	22.9834	15.7700	16.2314
1961	24.4310	23.3587	15.5220	16.2060
1962	24.5240	23.9247	15.7780	16.3369
1963	24.6970	24.5588	16.6630	16.5161
1964	24.8100	25.2287	17.1250	16.6831
1965	25.2610	25.5427	17.3960	16.7882
1966	25.3580	25.5741	16.4730	16.8655
1967	25.1310	26.0289	16.1680	17.0024

Year	Farmers' Butter		Margarine	
	Actual	Expected	Actual	Expected
1947	8.8650	7.4111	1.2540	0.2983
1948	8.3850	6.7259	0.6810	0.5323
1949	7.9990	6.4476	0.5910	0.5906
1950	7.4890	5.7883	0.6130	0.7668
1951	6.6900	5.9929	0.7270	0.6906
1952	6.5830	6.6156	0.6990	0.4928
1953	5.3560	6.2857	1.1280	0.5955
1954	5.3100	6.1408	1.1250	0.6436
1955	5.0260	5.4428	1.1930	0.8721
1956	5.0250	5.7690	1.1650	0.7339
1957	5.0020	6.1235	1.2540	0.5952
1958	4.7730	5.5131	1.3950	0.7896
1959	4.7610	5.1695	1.4720	0.9107
1960	4.5720	4.5121	1.6180	1.1715
1961	4.5490	4.1126	1.5310	1.3082
1962	4.5670	3.7144	1.5370	1.4356
1963	3.7450	3.3305	1.6280	1.6215
1964	3.0780	2.7520	1.7910	1.8170
1965	2.4030	2.5007	1.7170	1.8363
1966	2.0680	2.4243	1.8050	1.9039
1967	1.7320	2.0937	1.9710	2.0443

Actual and expected consumption at constant (1958) prices (£ million)

Year	Cheese		Eggs	
	Actual	Expected	Actual	Expected
1947	1.0470	0.5069	11.0490	13.6023
1948	0.8690	0.7194	11.1680	13.6348
1949	0.6220	0.7855	11.8850	13.6114
1950	0.8450	0.9364	13.1270	13.5423
1951	0.6750	0.8812	14.8590	13.5110
1952	0.5200	0.7103	14.0190	13.4893
1953	0.8710	0.7954	12.9040	13.4638
1954	0.8690	0.8289	14.3900	13.4238
1955	0.8310	0.9804	14.7170	13.3168
1956	0.8560	0.8844	14.7030	13.2204
1957	0.8210	0.7877	14.3790	13.1710
1958	0.8430	0.9187	14.4730	13.0097
1959	1.0820	1.0077	14.4380	12.9680
1960	1.0770	1.1556	13.5980	12.8834
1961	1.2060	1.2226	13.2750	12.8103
1962	1.3460	1.3288	13.3250	12.8454
1963	1.4920	1.4528	12.9060	12.9338
1964	1.4990	1.5822	12.7030	12.9623
1965	1.6420	1.6430	13.0140	13.0124
1966	1.6480	1.6763	12.5440	13.0526
1967	1.9340	1.7784	11.8220	13.1042

Year	Beef		Mutton	
	Actual	Expected	Actual	Expected
1947	26.2790	16.7883	7.1920	6.1021
1948	23.0880	16.9483	6.4070	7.0911
1949	18.1090	16.9602	5.2460	7.4462
1950	15.9590	16.9733	4.9860	8.2518
1951	19.0400	16.8987	4.9080	7.9421
1952	16.8160	16.7658	6.4670	7.0294
1953	15.3190	16.7892	6.4390	7.4763
1954	14.1850	16.7609	6.7600	7.6464
1955	15.0210	16.7150	7.8880	8.4191
1956	15.3960	16.5456	8.8030	7.9068
1957	15.5260	16.4301	8.9320	7.3636
1958	14.4740	16.3087	9.3600	8.0334
1959	14.6540	16.2994	10.7340	8.4697
1960	15.0010	16.2905	11.1060	9.4492
1961	15.5400	16.2575	11.0220	9.9683
1962	16.1960	16.3606	11.6780	10.5449
1963	17.3690	16.5204	11.9150	11.2063
1964	16.8380	16.6109	11.6590	11.7848
1965	16.2980	16.6599	11.2360	11.7283
1966	17.1250	16.7402	11.4350	12.0254
1967	18.2910	16.8716	11.8100	12.6645

Actual and expected consumption at constant (1958) prices (£ million)

	Pigmeat		*Poultry*	
Year	*Actual*	*Expected*	*Actual*	*Expected*
1947	22.6280	23.9174	5.1950	3.8513
1948	22.4780	25.3861	4.8960	4.2588
1949	27.6100	25.8836	4.8770	4.3945
1950	26.0920	26.9505	5.3540	4.7127
1951	23.7740	26.4396	5.0560	4.5874
1952	24.6170	25.2039	5.0810	4.1734
1953	26.5570	25.7678	4.4790	4.3821
1954	28.1210	25.9547	4.2680	4.4518
1955	28.9200	27.0756	4.4610	4.7955
1956	27.3100	26.1063	4.9120	4.5587
1957	27.5670	25.2586	4.1870	4.3109
1958	25.8970	26.0861	4.0880	4.5732
1959	27.4660	26.6633	4.1770	4.8696
1960	26.4080	27.8655	4.5220	5.2521
1961	27.7250	28.5124	4.4830	5.5843
1962	28.3720	29.5653	4.8280	5.8450
1963	29.1170	30.6414	5.8780	6.1878
1964	31.6250	31.8812	6.0540	6.2795
1965	35.0490	32.3076	6.6660	6.4461
1966	34.0800	32.7482	7.6770	6.5192
1967	32.0340	33.6581	7.3970	6.8990

	Sugar		*Bread*	
Year	*Actual*	*Expected*	*Actual*	*Expected*
1947	3.5930	6.4102	14.6090	16.8378
1948	4.5720	6.3620	16.2520	15.9444
1949	4.3130	6.3044	16.3210	15.5049
1950	5.6120	6.1636	16.2100	14.4875
1951	6.0010	6.1847	16.7630	14.7713
1952	6.3520	6.2862	15.7960	15.7114
1953	6.4330	6.2165	15.1340	15.1871
1954	5.8980	6.1806	15.1820	14.9668
1955	5.7120	6.0507	14.9900	13.9882
1956	6.5240	6.0526	14.5960	14.3728
1957	6.0550	6.0764	13.5390	14.8015
1958	5.9010	5.9346	13.5250	14.1221
1959	5.8430	5.8751	12.9110	13.8177
1960	5.8580	5.7536	12.4040	13.1731
1961	6.1480	5.6664	12.3440	12.7349
1962	5.3600	5.6560	12.3910	12.5079
1963	5.8060	5.6428	12.4790	12.2205
1964	5.1920	5.6363	12.0880	11.9300
1965	5.7110	5.6304	12.1340	11.8239
1966	5.4070	5.6336	11.7300	11.7604
1967	5.6430	5.6113	11.5640	11.7228

Actual and expected consumption at constant (1958) prices (£ million)

Year	Household Flour		Potatoes	
	Actual	Expected	Actual	Expected
1947	12.5870	10.7775	18.8480	16.7490
1948	8.5480	9.9165	18.7360	16.1215
1949	9.5790	9.3830	18.0570	15.8570
1950	8.7920	8.1226	18.0030	15.1940
1951	8.5230	8.5007	17.7340	15.3123
1952	9.2840	9.7339	17.5090	16.0556
1953	8.8550	9.6818	14.5440	15.6164
1954	8.3830	8.8355	14.7090	15.4084
1955	8.4090	7.6653	14.5650	14.7421
1956	8.2620	8.2309	14.0360	14.9140
1957	7.6590	8.8035	13.4820	15.2267
1958	6.7750	8.1295	12.4780	14.6101
1959	6.7580	7.7681	13.2140	14.2423
1960	6.7250	7.0757	12.9580	12.9961
1961	6.3010	6.7213	12.6870	13.0906
1962	6.3250	6.3781	12.7350	12.8088
1963	6.3690	5.9956	12.6330	12.1815
1964	6.0000	5.5839	12.4980	11.9817
1965	5.2180	5.3998	12.3470	12.6462
1966	5.2380	5.2829	12.2000	12.1195
1967	5.2650	5.0542	12.0610	11.5334

Year	Total Food		Other Expenditure	
	Actual	Expected	Actual	Expected
1947	168.0360	168.0051	261.1640	259.1698
1948	167.3600	170.1690	289.5400	285.4162
1949	166.0950	174.0078	304.1050	294.7807
1950	169.4960	179.0387	324.5040	316.3177
1951	167.7630	175.2483	318.0370	307.1558
1952	169.2300	170.7026	285.1700	283.0283
1953	170.0650	172.5621	296.3350	294.2728
1954	175.7640	173.1394	295.6180	298.8445
1955	185.2290	177.2770	313.7710	321.1743
1956	174.9190	171.8993	301.8810	304.2528
1957	173.6410	169.1391	283.4590	289.5878
1958	173.5610	171.9502	304.0470	305.6691
1959	177.9600	175.3307	314.1400	316.7726
1960	183.9470	180.3113	336.4530	339.4413
1961	183.4360	183.3193	349.6640	349.8466
1962	188.9820	186.8648	360.2180	362.3203
1963	191.0040	193.0593	380.2960	378.6931
1964	198.5100	197.9310	393.9900	394.4470
1965	200.1900	202.1469	401.9100	400.3404
1966	203.4730	201.7699	399.9270	401.3149
1967	202.2860	205.3524	414.2140	411.7991

APPENDIX B

Method of Estimating the Model

The system to be estimated was as follows:

$$p_{1t}q_{1t} = b_1\mu_t + (b_1-1)c_1p_{1t} + b_1c_2p_{2t} + \ldots b_1c_{16}p_{16t} + u_{1t}$$

$$p_{2t}q_{2t} = b_2\mu_t + b_2c_1p_{1t} + (b_2-1)c_2p_{2t} + \ldots b_2c_{16}p_{16t} + u_{2t} \qquad (1)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$p_{16t}q_{16t} = b_{16}\mu_t + b_{16}c_1p_{1t} + b_{16}c_2p_{2t} + \ldots (b_{16}-1)c_{16}p_{16t} + u_{16t}$$

where $t = 1, 2, \ldots 21$; u = disturbances

Clearly the parameters cannot be estimated by taking the equations one at a time since c_j ($j = 1, 2, \ldots 16$) appears in each equation and would not, except coincidentally, have the same estimated values in all the equations. The iterative procedure outlined by Stone[1] can be divided into three stages the first of which begins the process and the remaining two form a cycle of operations. For ease of exposition the method will be outlined in terms of the two-good example.

1. In the first stage provisional estimates of the b's are formed by assuming all the c's are zero and fitting the resulting "naive" form:

$$p_{1t}q_{1t} = b_1\mu_t + v_{1t}$$
$$p_{2t}q_{2t} = b_2\mu_t + v_{2t} \qquad (2)$$

using least squares estimators with uncorrected sums of squares. This method ensures that $b_1+b_2 = 1$[2]. The vectors of disturbances (v_1 and v_2) contain the variation in the expenditure which is due to price.

1. Stone 1954, *op. cit.*

2. In fact as long as $p_{1_t}q_{1_t} + p_{2_t}q_{2_t} = \mu_t$ the b's will sum to unity. The formula for estimating b_1 is

$$b_1^* = p_{1t}q_{1t}\mu_t/\mu_t^2 \text{ and } b_2 \text{ is } b_2^* = p_{2t}q_{2t}\mu_t/\mu_t^2$$

$$\therefore b_1^* + b_2^* = \frac{\mu_t(p_{1_t}q_{1_t} + p_{2_t}q_{2_t})}{\mu_t^2} = 1$$

2. In this stage an attempt is made to extract from v_{1t} and v_{2t} the components of c_{1t} and c_{2t} left out of the "naive" model. Thus, substituting the provisional estimates, b_1^* and b_2^* (say) we can write:

$$
\begin{aligned}
v_{1t} &\equiv p_{1t}q_{1t} - b_1^*\mu_t = (b_1^* - 1)c_1p_{1t} + b_1^*c_2p_{2t} + u_{1t} \\
v_{2t} &\equiv p_{2t}q_{2t} - b_2^*\mu_t = b_2^*c_1p_{1t} + (b_2^* - 1)c_2p_{2t} + u_{2t}
\end{aligned} \tag{3}
$$

The estimates, c_1^* and c_2^*, are derived by minimising the sum of squares of the disturbances in these equations over all time periods.

3. With these estimates, c_1^* and c_2^*, we can now derive new estimates for b_1 and b_2, b_1^{**} and b_2^{**} (say), by estimating the following equations:

$$
\begin{aligned}
p_{1t}q_{1t} + p_{1t}c_1^* &= b_1(\mu_t + c_1^*p_{1t} + c_2^*p_{2t}) + e_{1t} \\
p_{2t}q_{2t} + p_{2t}c_2^* &= b_2(\mu_t + c_1^*p_{1t} + c_2^*p_{2t}) + e_{2t}
\end{aligned} \tag{4}
$$

With b_1^{**} and b_2^{**} it is now possible to re-estimate c_1 and c_2 by returning to equations (3) in the second stage.

Thus, the first stage gives initial estimates of b which enable estimates of c to be derived in the second stage which, in turn, enable new estimates of b to be derived in the third stage. It is then possible to re-estimate c and b by proceeding from the second to the third stage and back again. This process is continued until stable estimates of b and c are eventually reached. In the present case, convergence occurred after fifty-three iterations. There are, of course, alternative estimation procedures such as those described by Barten[3] but it was felt that the method described above was quite adequate.

3. Barten, A. P., "Maximum Likelihood Estimation of a Complete System of Demand Equations, *European Economic Review*, Vol. 1, Fall, 1969.