Towards An Input-Output Decision Model For Ireland

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"If we try to apply this experience to the situation found in numerous development countries at the present time, it becomes clear that the existence of modern production facilities is a necessary but insufficient precondition for building up an efficient national economy Indeed, even if there is a labour force with the skills to operate such facilities, that is still no guarantee of economic success Only when these factors are accompanied by a consciously positive attitude of the people towards their daily work—without which any process of systematic economic activity is inconceivable—are all the conditions satisfied so that the concerted action of these three factors releases the forces which permit a rapid economic upswing"

—Ludwig Erhard, *The German Economic Review*, Vol 1, No 1, p 4, 1963

As will appear, the experiments, all based on Irish data, have been numerous, so I shall dispense as much as possible with preamble and define decision models as the models which decision model makers make, adapting the famous definition of economics To be a little more specific, decision models are designed to assist in making prudent economic policy decisions Their role is a comparatively minor one, incomparably less important than raising the head of steam amongst a polity the great majority of whom (though they may not always admit it) want to do tomorrow what they do today If the people concerned are a great people for talk you will hear much of "I'll tell you what's wrong with this country " It would be a mistake to assume that this indicates a deep dissatisfaction with the condition of the nation (whichever it be) It may merely mark an opening conversational gambit One finds invariably that the things that require to be done have to be done by someone other than the speaker

It has yet to be proved that plans (most countries now have a plan) formulated in advance are conducive to national well-being It may be asked why countries, which had attained an impressive rate of growth like 5-6% before any plan was conceived, require a plan Perhaps the answer is that the plan is needed to promote continued, orderly growth, in particular to ensure that the poorer classes improve their lot more than the better-to-do I have yet, however, to hear of a plan in which the poor are explicitly distinguished To give the planners their due they usually assume, with some justification, that if GNP increases the lot of the poor will improve still more. This is scarcely enough. In the writer's view income-distribution should be built into any plan worthy of the name. He shall not do so in this paper: so much the worse for it.

What a decision model does is to give some indication of what has to be done, starting now, to achieve desired ends in a given term of years, which ends are necessarily expressed in general terms. An aspired goal of $x\%$ a year may be enthusiastically adopted as well within the range of possibility. Realism begins to obtrude when it is pointed out that such a rate of increase requires that, on past experience, a rate of increase in savings (or a refrainment from consumption) greater than in the past, that certain industries must make plans to increase more than others, that wage and dividend distribution restraints must be exercised, starting now, if the desired ends are to be achieved. The more detailed the decision model the greater the reality, in the sense that the sacrifices entailed *now* will be brought home with the greater force. The ultimate detailed plan says to each individual in the nation "This means you!"

In the previous paragraph there was mention of "past experience". Sometimes it is argued in this and other countries that past experience, if this were not impressive, is irrelevant to what is going to happen in the future. There is a certain amount of justification for this view in countries at a very early stage of economic development: the less its validity the more the country in question has advanced along the path of economic development. Even in the less advanced countries the decision model approach to reality is not irrelevant since, if they cannot base the coefficients in their models on their own past experience, they can base these on the coefficients of other countries. It may even be valid to argue that "a new spirit is abroad" though planners must be on their guard against the assumption that the whole community is seized of the planners' enthusiasm for advance.

It is also sometimes argued that decision models imply a slavish adherence to past experience, that they postulate a measure of determinism, ignoring the random shocks of which we are so painfully aware, which experience has shown changes the whole shape of the economy. This is not so. Having set up our model we can try out a manifold infinity of assumptions as to the shape of things to come. In the future year of reference, 1970, 1975 or what you will, there will still be national income accounts, input-output tables and all the rest. We may, with some confidence, assign limits to the coefficients involved in our model to make them applicable to the future year of reference. If, for instance, the ratio of imports to GNP has remained almost constant during the past ten years at 40% we may assign future limits of perhaps not less than 35% and not more than 60%. We may well be sceptical about the public in a democracy accepting a saving ratio free or forced of 25% when, on the experience in recent years the percentage averaged about 10. We can use the experience of other countries more advanced than our own as a guide for ourselves in the future year of reference. Fortunately right policy and right decisions do not depend on our having a quantitively correct picture of the future year of reference. Using decision models with a built-in margin of imprecision we can determine in some detail what the best policy, having regard to all the hazards, would be

Every step we take, as individuals or as members of communities, implies some assumptions with regard to the future based on past experience Every investment a farm or a firm makes has this characteristic If I cross a road I take the greater care at mid-day than at midnight based on a largely subconscious recollection of past experience Only the future matters, but the future is, within limits, a function of the past At a less mystical level, there are in the past many persistent and demonstrable statistical regularities which it would be unwise to ignore in the future This is the underlying philosophy and the ultimate justification of the decision model approach The system breaks down only when the variations in the hypotheses are so large as to impart such a measure of imprecision to the results as to make the exercise useless for policy decisions The greater the attempted detail in decision model making the larger the imprecision in the results on the other hand, too broad classifications are the less useful for policy making as failing to bring to the attention of the public what is required of them A balance must be achieved between the two extremes

The paper is divided into five parts

1 A Macro-economic Approach,
2 An Input-Output (IO) Decision Model,
3 Sensitivity of the Model,
4 An Experiment in Linear Programming (LP) (applied to the Input-Output Model),
5 Summary and Conclusion

Part 1, based on the national income accounts, justifies its inclusion in an essentially IO approach in that the account system is an IO model with a single economic sector, that of the national economy as a whole This approach has the merit of simplicity of operation and produces results of absolute validity from very few hypothetical parameters In part 2 the model is formulated and applications to an Irish IO 9×9 sector table (kindly supplied by CSO) are developed In part 3 experiments are made of the effect on the answers of changing the coefficients in the Irish 1960 table are the deviations in the answers (e g sectoral outputs in 1970) of such a magnitude as to invalidate economic policy decisions taken now and modified as the years advance? The LP examination in part 4 addresses itself to the question of an optimal incremental export pattern for Ireland

Involved in this approach is the presentation of figures for some future year (1970 in this paper), usually mistermed "forecasts", i e something that is likely to happen, to stand or fall by ultimate comparison with actuality They have no such pretension Citation of 1970 or any other future year is merely accidental The method is designed to ensure that action taken *now* will be prudent in the light of all our recent past experience and realistic anticipations The IO approach ensures that all flows in the economy are taken into account and that these are in the proper relation to one another

The rest of the paper is purely methodological, comment being confined to argument to try to justify the hypotheses adopted as regards basic coefficients, the capital-output ratio, the import ratio and all the rest The paper is but raw material for economic analysis The methods expounded are designed for decision making but no decisions are taken here

It is the fate of inventors who become classic in their own lifetime that their inventions should be cited (and applied) more frequently than their names The author makes bold to dedicate (without permission but with affection) this paper to W W Leontief [1], the inventor of input-output

1 A MACRO-ECONOMIC APPROACH

At constant prices in year t (with $t=0$ in the base year) let

		Ireland 1960 ($t=0$) £m
	Y_t=net national product at market prices	631
	C_t=national consumption at market prices	572
	V'_t=net fixed capital formation	49
	V''_t=change in stock	11
	X_t=current exports	255
(1 1)	M'_t=current imports (positive or negative) in respect of profit, interest, etc , in consequence of investment from abroad N_t	0
	M''_t=other current imports	256
	N_t=net investment from abroad	1
	S_t=national saving	59

Except for M'_t these are the familiar macro-economic variables of the national income accounting system It seems desirable to isolate the import constituent M'_t, defined as above, since, at least in theory, a persistent import excess, i e a positive N_t, must be contemplated as a possibility during a period of rapidly increasing capital formation, and its effects considered A generally small, persistently positive, import excess has been a characteristic of the Irish economy except during the past few years For the theory expounded here, however, N_t and therefore M'_t may be positive, negative or zero

National consumption includes general government as well as household consumption The current values for Ireland in 1960 are displayed to indicate orders of magnitude (source *Economic Statistics 1963*, Tables 11, 12 (a)) We then have four national accounting identities as follows

	(i)	Product account	$Y_t=C_t+V'_t+V''_t+X_t-M'_t-M''_t$
(1 2)	(ii)	External account	$X_t-M'_t-M''_t+N_t=0$
	(iii)	Capital-Saving account	$V'_t+V''_t=S_t+N_t$
	(iv)	Consumption account	$C_t+S_t=Y_t$

These four accounts are articulated (or double-entry), for it will be noted that each of the nine entities specified at (1 1) occurs twice, once on the left side and once on the right side of the identities (1 2) In consequence, only three of the four identities are independent any one of the four can be derived from the remaining three In sum, there are three relations between the nine entities, six other relations are required to obtain a model from which each of the entities may be determined in any year t, given the values in base year $t=0$ It is assumed that during the growth period import and export price indexes (base year 0) are the same so that no entry for the trading gain is required in relations (1 2) It will be noted that N_t is positive when current imports exceed current exports and negative in the contrary case

These six *behaviouristic* equations (deterministic, as distinct from stochastic, in character) are found as follows First the *growth* equation

(1 3) $$Y_t=(1+r)^t Y_0,$$

where r is the annual growth rate of the economy A *consumption* equation is

(1 4) $$C_t=(1-s) Y_t,$$

so that, from (1 2) (iv), *saving* is given by

(1 5) $$S_t=s Y_t$$

The fixed capital relation is derived from the incremental capital-output ratio k whereby

(1 6) $$V'_t=k(Y_{t+1}-Y_t)$$

But, by definition of r, the growth ratio,

(1 7) $$Y_{t+1}-Y_t=r Y_t$$

Hence, from (1 6) and (1 7) we have the *fixed capital* equation

(1 8) $$V'_t=kr Y_t$$

As will appear later, this relationship and its implications are of fundamental importance for decision-making at the macro level As regards changes in stocks, let stock P_t at beginning of year t be

(1 9) $$P_t=p Y_t$$

Then since $V''_t=p(Y_{t+1}-Y_t)$ we have, from (1 7), the *stock* equation

(1 10) $$V''_t=pr Y_t$$

It will be convenient to take investment from abroad N_t next The required *external investment* equation, derived from (1 2) (iii), using (1 5), (1 8) and (1 10), is

$$(1\ 11)\qquad N_t=(kr+pr-s)\ Y_t$$

If n is the rate of interest per £ invested, total interest M'_t payable or receivable in year t in respect of foreign investment during the period of review is

$$(1\ 12)\qquad M'_t=n\sum_{t'=0}^{t-1} N_{t'}$$

Hence, from (1 3) and (1 11),

$$M'_t=n(kr+pr-s\ Y_0\sum_{t'=0}^{t-1}(1+r)^{t'}$$

$$=n(kr+pr-s)\ Y_0\{(1+r)^t-1\}/r$$

$$(1\ 13)\qquad =n(kr+pr-s)\ (Y_t-Y_0)/r,$$

the *external investment interest* equation

The *import* equation is

$$(1\ 14)\qquad M''_t=m_tY_t$$

The multiplier m_t is not a constant but an increasing function of time t of form to be discussed later

Finally the *export* equation is derived from (1 2) (i) but substitution from (1 4), (1 8), (1 10) and (1 14) It is unnecessary to write the complicated formula down since in practice (i e with actual figures) the value can be readily found

The model accordingly consists of a system of nine equations (of which three are the accounting identities (1 2), together with relations (1 3), (1 4), (1 8), (1 10), (1 11) and (1 14) above) to determine the nine entities specified at (1 1) The six behaviouristic equations involve five parameters r, s, k, p and n, together with m_t of determinate form in t—actually they would all appear in the export equation Each of the six equations expresses the relevant entity linearly in terms of Y_t and Y_0 or, if Y_t be regarded as given by (1 3), in terms of Y_0 alone Hence, given Y_0, the initial value of Y_t, and the parameters the values of all the entities specified at (1 1) are determinable in such a way that they must satisfy the accounting identities (1 2)

The epithet "behaviouristic" applied above to the six equations with parameters is intended to imply that the values attributed to the parameters determine how the economy is to behave in future of its own accord or as a result of conscious direction The analysis which follows is designed to indicate the kind of thinking which might go to determine reasonable values, or ranges of values, of the parameters, which might be deemed to apply to Ireland in the decade or so ahead

TABLE I

SOME MACRO-ECONOMIC DATA, IRELAND, 1947-1962

Year	At Constant (1953) Prices							At Current Prices		
	Gross national product	Gross fixed capital formation	Dwellings	Depreciation	Net national product at market prices	Net fixed capital formation	Imports as % of GNP	Net national product at market prices	Saving	Col 10 as percentage of col 9
1	2	3	4	5	6	7	8	9	10	11
	£ million						%	£ million		%
1947	440 2	38 0	5 1	14 7	425 5	23 3	41 7	319 2	2 9	0 9
1948	457 2	45 5	8 1	15 0	442 2	30 5	41 1	351 4	16 9	4 8
1949	482 6	60 6	14 5	16 4	466 2	44 2	38 7	377 3	35 1	9 3
1950	488 8	71 8	17 4	18 3	470 5	53 5	42 8	383 0	22 9	6 0
1951	497 4	81 7	19 5	19 1	478 3	62 6	44 1	402 8	8 0	2 0
1952	513 1	80 0	18 6	19 2	493 9	60 8	35 6	459 6	47 9	10 4
1953	525 6	80 7	16 3	21 8	503 8	58 9	39 2	503 8	58 6	11 6
1954	531 2	86 8	15 2	24 0	507 2	62 8	37 9	505 0	51 2	10 1
1955	541 1	90 2	15 8	24 9	516 2	65 3	41 1	526 3	40 5	7 7
1956	535 6	84 3	15 9	27 7	507 9	56 6	36 4	529 6	38 5	7 3
1957	540 6	70 6	10 5	28 4	512 2	42 2	34 4	548 8	50 0	9 1
1958	524 9	70 0	8 6	29 0	495 9	41 0	39 6	565 5	37 9	6 7
1959	548 9	72 5	9 0	31 6	517 3	40 9	41 3	600 1	60 5	10 1
1960	579 7	76 6	9 9	33 0	546 7	43 6	41 2	633 5	59 7	9 4
1961	607 7	88 0	10 1	34 2	573 5	53 8	44 7	677 4	68 0	10 0
1962	623	97	11	35 0	588	62	45 8	731	74	10 1

SOURCE National Income and Expenditure 1962 (revised data for years 1947-62)

NOTES Col 3 Current price data deflated by capital price index (CSO)
Col 4 Included in col 3
Col 5 Current price data deflated by capital price index (CSO)
Col 6 Col 2 less col 5
Col 7 Col 3 less col 5

The Capital-output ratio k In Table 1 some relevant macro-data are displayed covering the period 1947-1962 From the table, by reference to gross or net national product—the difference is depreciation—it will be apparent that 1947-1955 was a period of small but regular advance in the economy Accordingly it seems appropriate to estimate the average net capital-output ratio for this period This estimate is made by using the model

(1 15) $(Y_{t+1}—Y_t)—k'V'_t=u_t,\ t=1946, \quad , 1954,$

where u_t is a random variable and

(1 16) $k'=1/k,$

the inverse of the capital-output ratio By least squares procedure applied to (1 15) the estimate of k', namely $\hat{k}'$, is found as

(1 17) $\hat{k}'=\Sigma(Y_{t+1}—Y_t)V'_t/\Sigma V'^2_t$

Applying this formula to the data in columns 6 and 7 in Table 1 we find $\hat{k}'=0\ 1991$ which, for convenience, may be taken as 0 2, yielding a capital-output ratio for the period 1947-1955 of 5 By international standards this value is large There are three possible contributory causes for this

(a) under-utilisation of capacity,
(b) the sizeable proportion borne by dwellings—see column 4 of Table 1—with a high capital-output ratio in total gross fixed capital formation,
(c) the low output increment for agriculture during the period

Extrapolating the constant price value of output using the formula

(1 18) $\Delta y_t \equiv y_{t+1}—y_t=0\ 2V'_t$

and starting with the actual value of net national product for 1955, namely £518 million, the following "expected" values are found (in £ million) with the actual Y_t for comparison

Year	Δy_t	y_t	Y_t
1955	13	518	516
1956	11	531	508
1957	8	542	512
1958	8	550	496
1959	8	558	517
1960	9	566	547
1961	11	575	574
1962	12	586	588
1963	—	598	609

Comparison of the actual and "expected" development (i e of Y_t with y_t) of NNP from 1955 indicates that during the whole period 1956-1961 fixed capital was underutilised by reference to the probably not too exalted standard of 1947-1955 In 1962 the economy just reached the point it would have reached if the 1947-1955 trend had continued The figure inserted for Y_t for 1963 is based on C E V Leser's [3] estimate of gross national product It would appear that in 1963 real net national product exceeded the expected y_t, if by a small amount, but encouraging the hope that the economy has entered a phase of greater increase than in 1947-1955, and that 1958-1962 was not merely a phase of recovery * The definitive statistics for 1963 will be anxiously awaited Of course, there is the unprecedently healthy feature about the 1959-1962 phase that it was based largely on exports and the impetus induced in this sector once achieved may not slacken Furthermore, as will presently appear, the capital-output ratio of 5 is far too big for an economy aspiring to a large and sustained rate of growth, given the Irish propensity to save which is considered later

Admittedly, a capital-output ratio extending to the whole economy is not particularly significant for comparative purposes in time or between countries, in particular because sectors of the economy differ so much in capital (as distinct from labour) intensity This is particularly the case in Ireland where agriculture is an important sector (accounting for about a quarter of gross domestic product) and in agriculture one surmises that the role of fixed capital in promoting development is less important than in industry in the sense that current expenditure in the form of fertilisers, insecticides, medicines, etc , is likely to be more conducive to growth than is capital expenditure Apart from this point, in comparing capital-output ratios in Ireland and the United Kingdom it seems desirable to eliminate agriculture from the Irish computation, this is scarcely necessary in the case of the United Kingdom where agriculture accounts for only one-twenty-fifth of gross domestic product

Omitting agriculture from Irish capital formation and output, the *gross* capital-output ratio for the period 1953-1959, calculated by formula (1 17) is 6 8, practically identical with the United Kingdom figure similarly calculated for the same period of 6 7 It should be pointed out, however, that, for the purpose of this calculation, GDP for the years 1956-1959 was based on "expected" net national product y_t, not on actual Y_t since obviously there was under-utilisation of capital in this period The gross ratio has been used, V_t (V'_t in the formula) has been taken as gross fixed capital formation and Y_t as gross domestic product, both at constant market prices, not only because separate figures for depreciation in the agricultural sector have not been published for Ireland but also because some expert opinion favours the use of the gross concept in preference to the net on theoretical grounds for which there is much to be said It is satisfactory to observe that on this admittedly over-generalised test, the Irish non-agricultural economy does not make a bad showing, in utilising what capital there was, which is not to say that capital increments were adequate for a high rate of growth It may be worth observing that in advanced economies the gross ratio is about double the net ratio On the net basis, therefore, an incremental ratio of 3 for Ireland (including the agricultural sector) is not an unreasonable aspiration for the future

Stock ratio, p The stock ratio p, given by (1 9), at constant prices in each of the five years 1957-1961, was as follows (source CSO)

* Latest indications are that the £609 million shown for 1963 is an under-estimate

	p
1957	61 9
1958	63 1
1959	58 8
1960	59 7
1961	58 3
Average	60 4

The relative trendlessness in the figure will be noted, also its magnitude, due mainly to the contribution of livestock in which so much working capital is perennially locked up in Ireland A reduction in this stock ratio would be desirable for rapid economic progress It will have been noted that in the six behaviouristic equations the term kr is always accompanied by pr indicating that *pro tanto* a reduction in p has precisely the same effect as a reduction in the marginal net capital-output ratio k However, k is far more important, as an object of policy, than p it would be a major achievement to reduce p to 0 4, or by 0 2, whereas no sizeable advance in the economy can be contemplated unless net k is reduced from 5 to at least 4, or by unity For this paper p will be taken as 0 6

Saving ratio, s As saving fluctuates considerably from year to year (see columns 10 and 11 of Table 1) it will be convenient to divide the 15 years 1947-61 into three quinquennia and to use annual averages to the effect shown in Table 2

TABLE 2

SAVING AND INVESTMENT, 1947-1961

Period	Annual average (£ million at current market prices)				As percentage of national income, col —	
	Net national income at market prices	Saving	Net investment from abroad	Net capital formation at home	3	4
1	2	3	4	5	6	7
1947-51	366 7	17 2	30 2	47 4	4 7	8 2
1952-56	508 1	47 3	14 3	61 6	9 3	2 8
1957-61	605 1	55 2	0 1	55 3	9 1	0 0

Of course, columns 3 plus 4 equal column 5 In 1947-1951, a period of restocking, especially of consumer durables, the saving ratio was low (column 6) and foreign disinvestment high The proportion borne by investment from abroad in capital formation at home rapidly declined to practically zero (on average) in the latest quinquennium It should be emphasized that the figures in column 4 are net, i e they represent the balance of gross extern investment (direct and portfolio, including drawing down of Irish-owned assets abroad, reserves of extern-owned companies, subsidiaries and branches in Ireland invested in Ireland as well as direct investment of externs) over Irish investments abroad These two gross totals are not known separately Since foreign investment in the State during the period 1957-1961 was known to be relatively large, so also must have been Irish investments abroad It would be useful and revealing to have the gross figures separately analysed into their main constituents

From the present point of view, mainly of note is the fact that in each of the periods 1952-56 and 1957-61 the saving ratio equalled 9% In any speculations as to the magnitude of the ratio during the next decade or so it would appear unrealistic to assume a large departure from 9 or 10%, though with increasing income a higher ratio may be contemplated

The import ratio, m_t For any Irish model, m_t is possibly the most important element because of the relative magnitude of external trade in the national economy The simple average values of the ratio in the last three quinquennia were as follows

Period	Imports as % of GNP (constant prices)
1947-51	41 7
1952-56	38 0
1957-61	40 2

The absence of trend will be observed, more apparent perhaps from the single year percentages in column 8 of Table 1 One might be inclined to infer from these very stable figures that it would be proper to assume a 40% ratio in any Irish growth model This would not be correct for, as the following analysis shows, the stability of the ratio in Ireland has been very probably a reflection of the comparatively slow rate of growth of Irish GNP at constant prices

From the UN National Accounts Yearbook 1960 linear time trends were fitted to log (constant price GDP) and to log (ratio of constant price imports to GDP) during the seven-year period 1953-1959 for all the countries for which these data were available, for the purpose of determining exponential growth rates of both entities Ultimately three countries were omitted, namely Colombia, Ecuador and Iceland, where it was evident that there had been some interference with the "natural" trend of imports The results for the twenty-one countries are shown in Table 3 The coefficient of correlation between columns 2 and 3 is 0 67, highly significant with 21 pairs of observations The tendency for a hypothetical straight line of relationship to pass through the origin also appears to be a tenable hypothesis, i e that zero ratio of growth of GNP will be associated with zero rate of growth of the import ratio Ireland itself is an illustration In fact, the averages for the 21 countries are 3 6% for GNP and 4 1% for the import ratio Accordingly the simple hypothesis that a 1% rate of growth of GNP will be accompanied by a 1% growth in the ratio seems reasonable, if a little conservative as regards the growth in the ratio

It mıght be thought that as Ireland has already a comparatıvely hıgh ratıo of 40% ıts rate of growth for each 1% ın GNP mıght be less than that of countrıes wıth a lower percentage, ı e that there would be a tendency ın all countrıes of taılıng-off of the ratıo at some faırly hıgh percentage (60, 70, 80?) Such does not appear to be the case ın any very marked degree In fact, the correlatıon between the mean level of the ratıo and ıts rate of growth (ı e between columns 3 and 4 of Table 2) ıs

TABLE 3

PERCENTAGE ANNUAL AVERAGE RATES OF INCREASE IN GROSS NATIONAL PRODUCT AND IN THE IMPORT RATIO AT CONSTANT PRICES IN TWENTY-ONE COUNTRIES, 1953-1959

Country	Rate of ıncrease: Gross natıonal product	Rate of ıncrease: Import ratıo	Mean ımport ratıo
1	2	3	4
	%	%	%
Germany, FR	6 5	8 8	18 9
Chına (Taıwan)	6 3	11 0	23 8
Greece	6 3	9 4	22 7
Austrıa	6 2	8 5	16 0
Italy	5 3	1 9	14 2
Netherlands	4 3	3 2	51 5
France	4 2	0 8	13 1
Porto Rıco	4 2	4 0	69 3
Sweden	3 6	3 4	28 7
Canada	3 4	1 1	24 0
Portugal	3 4	3 0	23 3
Cyprus	3 1	7 2	62 8
Ceylon	2 9	3 8	40 0
Norway	2 9	1 9	44 7
Denmark	2 8	4 3	34 6
Belgıum	2 5	4 4	32 0
U S A	2 4	3 3	4 8
Unıted Kıngdom	2 1	2 1	21 7
Swıtzerland	2 0	1 8	28 5
Chıle	1 7	2 0	11 5
Ireland	—0 1	0 4	36 8

SOURCE Based on data from UN Yearbook of Natıonal Account Statıstıcs 1960
NOTE Countrıes arranged ın descendıng order of rates ın column 2 Rates ın columns 2 and 3 are exponentıal Means ın column 4 are geometrıc, ın consequence lower than the arıthmetıc averages for Ireland used ın the text for 1953-56 sımple arıthmetıc average ıs 38 5% compared wıth the geometrıc 36 8% shown ın the table

only —0 10 whıch ıs not statıstıcally sıgnıficantly dıfferent from zero, though the mınus sıgn wıll be noted The *possıbılıty* of a slıght tendency towards taılıng-off ıs allowed for ın depressıng the relatıonshıp to 1 1 It should be added that no account ıs taken of ıncreased lıberalızatıon of external trade ın future ın assessıng thıs relatıonshıp It ıs beyond doubt that the ratıo m_t would ıncrease more steeply ıf Ireland entered the Common Market

All parameters will accordingly be given fixed values in the experimental forecasts to be undertaken, except m_t which will increase percentagewise with Y_t For example, if the series starts with $m_0=0\ 40$ and a rise of 3% is postulated for the first and subsequent years in Y_t m will assume the successive values 0 40, 0 412, 0 424, , each 3% in excess of the preceding value *

The foregoing result was based on international (essentially cross-section) analysis The result is somewhat in conflict with C E V Leser's [2] analysis of Irish time data over the period 1947-1961 showing that a rise of 1% in Y_t is likely to be accompanied by a rise of about 0 4% in the ratio this author uses, namely $M/(M+Y)$, which is equivalent to a rise of about 0 6% in our ratio M/Y † As a third approach it may be pointed out that during the period of steep increase in Y from 1958-1962 (see Table 1) the ratio has increased from 40% to 46% and, on Leser's forecast for 1963 [3], the ratio has advanced further to 47% Admittedly it is extremely difficult to form any precise idea of what this crucial import ratio will be in the years ahead On the analysis indicated and with diminishing tariff barriers it seems likely to increase In these uncertain circumstances it would seem prudent to examine the effects on the result of a range of values between say 50-60%

The break-even case No large persistent deficit in the import excess, i e N_t, can be contemplated in future To show this, let the rate of increase be 4% ($r=04$), saving ratio 10% ($s=10$), capital-output ratio 5 ($=k$), year of reference 1970 ($t=10$) and let the interest rate on accumulated net foreign investment be 7% ($n=07$), then, from (1 3), $Y_{10}=£927$ million Then, applying (1 13), imports in 1970 in respect of outflow of interest would be $M'_{10}=£65$ million, and from (1 11), the current deficit $N_{10}=£115$ million, while exports $X_{10}=£499$ million A deficit equivalent to 23% of exports cannot surely be tolerated as a quasi-permanent phenomenon Having made the point, let us study the implications of a future break-even in the external account, i e that $N_t=0$ This will not, of course, ever be exactly the case in practice Even as a matter of policy it may be necessary to contemplate Ireland's developing an export excess (i e N_t negative) for our contribution to international social security All the inferences which follow on the assumption that N_t is exactly zero will not be affected if, in fact, N_t is a small fraction, positive or negative, of exports X_t

If $N_t=0$ it follows from (1 11) that

$$(1\ 19)\qquad s=r(k+p)$$

and, while emphasizing the desirability of decreasing the stock ratio p from its present level, we have agreed to give it the fixed value of 0 6 Then (1 19) becomes

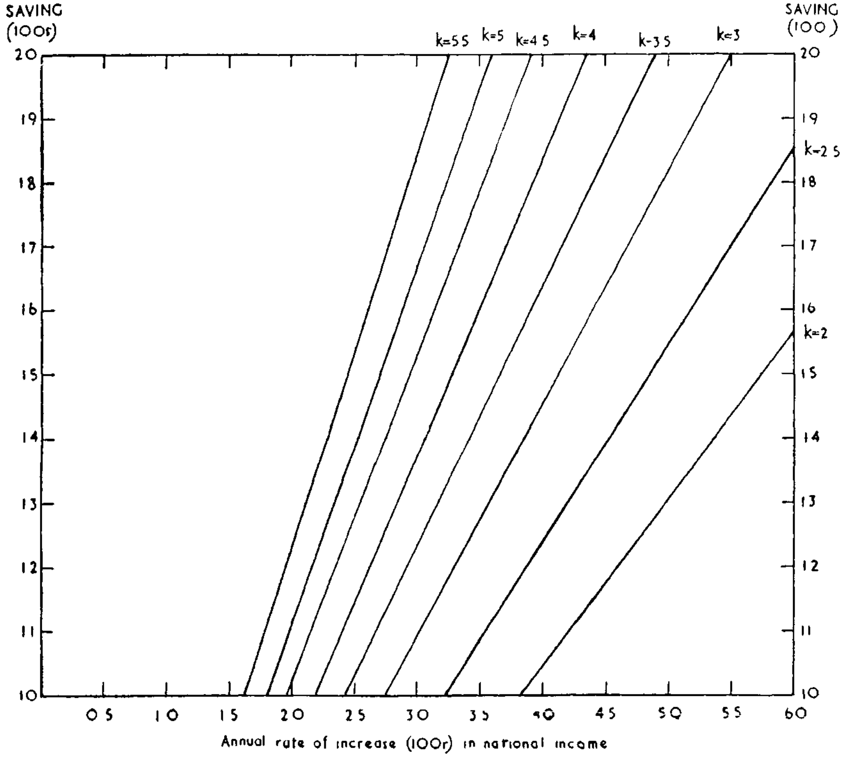
$$(1\ 20)\qquad s=r(k+0\ 6)$$

* Using (1 3) the proposed relationship is of the form $M''_t=M''_0 Y^2_t/Y^2_0$ This is not seriously proposed as a "law" It seems reasonable to assume that, at any rate, $M''_t=\text{constant}\times Y^{\alpha}_t$ where $\alpha>1$

† The difference in results is partly explained by the use of totals in current, instead of constant, prices in [2]

DIAGRAM 1

NOMOGRAPH OF INCREMENTAL CAPITAL-OUTPUT RATIO (k), RATE OF INCREASE ($100r$) IN NATIONAL INCOME, SAVING RATIO ($100s$)



Equation (1 20) is illustrated in the nomograph from which the implications of the relationship for the national economy can readily be appreciated From the base line it appears that with a saving rate of 10% an annual average rate of increase of the economy of nearly 4% is attainable with an incremental capital-output ratio k of 2, with $k=5\ 5$, the rate of increase is little over $1\frac{1}{2}$% Reading the chart vertically we can deduce, for example, that a 4% rate of increase implies a "reasonable" saving ratio of about $10\frac{1}{2}$% for $k=2$ and over 20% for $k=4\ 5$

The diagram shows the supreme importance for policy-making at all levels of the capital-output ratio National income is the sum of added value (employee compensation plus profit) of every enterprise in the nation and in considering new projects or extensions regard must be had to this ratio Private entrepreneurs are naturally interested only in the return on capital invested—broadly the ratio of profit to capital—but, in so far as public authorities are involved in the promotion, they must have regard to the other element in added value, namely employee compensation, as well It is only by conscious action by way of selection at the individual enterprise level that a reduction in the national capital-output ratio will be effected

To summarize this part of the paper, the macro-economic approach within the framework of the national accounts, which are integral, goes a certain distance in defining the problems, essentially political, facing a planning authority Can the capital-output ratio be reduced to a level consistent with a regular annual average increase of specified magnitude in the real national income having regard to the level of savings (voluntary or forced by way of taxation, etc) which the people will tolerate? Can markets be found for the large increase in exports required to pay for imports on the assumption that the rate of increase in foreign trade will almost certainly be greater than the increase in national income? Or, on the other hand, is there further scope for import substitution? So phrased, these problems are well known What the accounting approach does is to lend them some precision in measuring, subject to the behaviouristic hypotheses, the amounts involved There is no trouble, within the simple model proposed, in trying out a great number of different sets of hypotheses* and of selecting the optimal set, i e the set with the most desirable features It can at least be said that the target figures will be consistent Of course there is anything but certainty that the hypotheses (i e the parameters) of the initial plan will turn out to be more or less correct as years go on the evolution in the values of the parameters can be studied and the plan modified if necessary, in the light of these studies Obviously public authorities have considerable powers to influence the course of events in the desired direction, even if these powers are not absolute

The greater the extent to which citizens in their individual capacities are induced to feel involved for specific action in a particular plan the better the prospect of the objects of the plan being realised The foregoing macro-economic approach, useful enough to the planning authorities, does nothing to impart this kind of reality to the plan This will come about only when the plan is presented in reasonable detail Thus the different sectors of the economy will be brought to realise what is required of them The methodology for a detailed plan, again using Irish data for purposes of illustration, is described in the next part of the paper Unfortunately, the more detail one tries to impart to the model the greater the degree of uncertainty in the results In the third part of the paper an effort is made to cope with this element of uncertainty in the conviction that prudent policies can be adopted even if the prognostics in the form of figures are imprecise within even wide but more or less prescribable limits

Appendix to Part 1

The results of nine experiments with the macro-model are shown in the following table It may be verified that each trial satisfies the accounting identities (1 1) Of course, by special choice of the parameters s, k and r, the variables N and M' may be made as small as desired, both are very sensitive to what might appear small aberrations in the parameters The increases in imports M (and, in consequence, in exports, X) are pursuant to the analysis developed earlier A more conservative view with regard to the relationship between rise in imports and GDP is taken in part 2 the table is illustrative rather than realistic It is almost certain, however, that imports and exports will rise proportionately more than GDP (or GNP), as already remarked

* Some trials are given in an appendix to this part of the paper

VALUES OF MACRO-VARIABLES IN 1970 ON VARIOUS ASSUMPTIONS ABOUT THE PARAMETERS

Values in £m

Trial	Parameter		*Y*	*C*	*S*	*V*	*M′*	*M″*	*M*	*X*	*N*
	s	*k*	631	572	59	60	0	256	256	255	1
					Rate of growth 3%	*r= 03*					
1	05	3	848	806	42	92	29	462	491	441	50
2	10	3	848	763	85	92	4	462	466	459	7
3	15	4	848	721	117	117	—6	462	456	466	—10
4	15	5	848	721	127	142	9	462	471	456	15
					Rate of growth 4%	*r= 04*					
5	15	3	934	794	140	134	—3	561	558	564	—6
6	20	4	934	747	187	172	—8	561	553	568	—15
7	20	5	934	747	187	209	12	561	573	551	22
					Rate of growth 5%	*r= 05*					
8	20	3	1,028	822	206	185	—11	679	668	689	—21
9	20	4	1,028	822	206	236	17	679	696	666	30

NOTES

(i) The values at the column heads are those of 1960—see (1 1)
(ii) $M=M'+M''$, $V=V'+V''$

2 AN EXPERIMENTAL INPUT-OUTPUT DECISION MODEL

The object of this part of the paper is to develop a model, based on the Input-Output (IO) approach which can be used for forecasting purposes The model in type will be *decisional*, as distinct from the "onlooker" or purely prophetic Decision models are entirely hypothetical though naturally the hypotheses must be reasonable and as restricted as possible The model is primarily designed to show, in fairly considerable industrial detail, the economic pattern in some future year of reference on the assumption of different rates of increase in GNP However detailed, the pattern must be consistent in all its parts

The Curtailed Irish Table

Perhaps the best way to explain the model is, in the first instance, to display an IO table, namely that for Ireland in 1960, in which, for arithmetical convenience, the number of industrial groups has been reduced from the original 36 (for 1956) to 9 The data in the primary input section has also been recast very considerably—see Note to Table 4 *As a statistical presentation the figures in Table 4 are not to be taken too seriously* The data are designed for the purpose only of illustrating a method Nor is it suggested that, even if the figures were correct, results useful for decision-making would emerge for so dimensionally small a table as in 9 industrial groups The figures in Table 4 are, however, believed to be of the right order of magnitude

The task facing the analyst is to produce, on various hypothetical bases involving policy-decisions during the period from base to reference year, tables for the year of reference (which, for purposes of illustration here, will be taken as 1970, i e 10 ($=T$) years from the base year 1960, on the lines of the basic IO table

The table is compiled on the "sellers' price" principle Thus, on the first row, all the figures shown are valued at prices which farmers receive e g agriculture, etc , sells £77 million at farmers' prices to the food, etc , industries, £62 million to households, total gross agricultural output being £200 million Column 1 of the table shows the costs of agriculture thus agriculture purchases £3 million from metals, engineering, etc , at factory prices A result of this sellers' price treatment is that the cost of transport and distribution of all classes of goods (the principal constituent in line 9 of the table) is very large, for instance, the £132 million in the household expenditure column (line 9) includes about £70 million for transport and trade services, including the transport costs and trade margin for the products of agriculture, industry etc , as well as imports which in the table are valued c i f

The row for imports includes the value of all imports whether these are competitive with home industry or not In such treatment the practice here differs from that of the more common practice adopted by other countries of assigning competitive imports to the cells pertaining to home production While the author is rather doubtful of the competitiveness of most imports into Ireland—is Manitoba wheat competitive with Irish wheat in a normal year?—he is not concerned to make a major point of this issue It is more arithmetically convenient for illustrative purposes to use a single line for imports, while recognising that such treatment may impart an additional degree of imprecision to the interindustry coefficients The validity of the model to be described is not impaired by the present treatment of imports, i e the model described at (2 5) below could easily be elaborated to encompass both competitive and non-competitive imports

There is a considerable departure from the usual practice in the primary input section of Table 4 Thus row 1 of this section represents the disposable (i e after direct taxation) income of households (by way of employee compensation, dividends and non-corporate profits after tax) Row 2 contains all public authorities income including income from property and entrepreneurship as well as taxes Thus in the industrial part of the row are included direct taxes on employees, rates on business premises, import and excise duties on materials and products, etc However, the £26 million on the row in the household column is made up, for the greater part, of rates on dwelling-houses and import duties on consumer goods ready for use, i e this item is closely associated with imports valued c i f at £70 million and net rent included in the £132 million for services

Row 4 in the primary input section directs attention to a special difficulty in IO work In the industry part of the row the figures relate necessarily to companies, for the saving of non-corporate enterprises is, for the greater part, indistinguishable from saving of households and must be included therein, i e in the £34 million for households in Table 4 If, as seems likely in the future, the corporate proportionate share in the economy increases, then so will the coefficients pertaining to saving in the industrial sectors

TABLE 4

SUMMARY INPUT-OUTPUT TABLE FOR IRELAND 1960 (PRODUCERS' PRICES)

£ million

OUTPUT / INPUT	INTERINDUSTRY									FINAL DEMAND						
										Consumption				Exports		
	1 Agriculture, forestry, fishing	2 Food processing	3 Drink, tobacco	4 Textiles, apparel	5 Metals, engineering, vehicles	6 All other manufacturing, mining	7 Construction	8 Electricity, gas, waterworks	9 Services	Households	Government	Fixed capital	Stock changes	Visible	Invisible	
Non-Factor Input																
1 Agriculture, forestry, fishing	2.180	77.000	3.104	0.550	—	1.076	—	—	1.100	62.111	0.803	0.976	+1.695	49.000	0.750	200.345
2 Food processing	13.500	20.994	0.150	—	—	0.773	—	—	0.021	83.309	0.200	—	—1.027	38.200	3.000	159.120
3 Drink, tobacco	0.496	0.299	2.103	0.008	0.025	0.069	0.004	—	0.116	50.266	—	—	—0.030	7.000	11.527	71.883
4 Textiles, apparel	0.400	0.881	—	16.003	—	0.946	—	—	0.755	31.726	—	0.764	+1.259	10.500	5.600	68.834
5 Metals, engineering, vehicles	3.130	0.345	0.164	0.201	2.882	0.773	3.033	2.491	4.089	17.688	0.100	12.071	+1.513	6.400	1.500	56.380
6 All other manufacturing, mining	9.010	7.186	1.900	5.774	1.450	13.536	11.336	1.934	8.034	17.858	1.701	1.871	+1.302	15.500	1.500	99.892
7 Construction	0.150	1.033	0.049	0.355	—	0.271	1.898	0.278	3.104	5.492	12.957	40.322	—	—	—	65.909
8 Electricity, gas, waterworks	0.547	1.172	0.111	0.512	0.467	1.402	0.232	0.116	2.547	13.771	0.700	4.039	+0.029	0.059	—	25.704
9 Services	12.464	12.880	0.885	4.919	2.676	6.698	5.554	2.062	8.307	132.273	49.304	2.933	+2.914	15.505	29.078	288.452
Total home	41.877	121.790	8.466	28.322	7.500	25.544	22.057	6.881	28.073	414.494	65.765	62.976	+7.655	142.164	52.955	1,036.519
Imports	14.326	16.392	6.419	19.477	28.071	38.874	8.000	3.365	8.000	62.152	1.775	21.783	+4.400	2.804	0.540	236.378
Total Non-Factor Input	56.203	138.182	14.885	47.799	35.571	64.418	30.057	10.246	36.073	476.646	67.540	84.759	+12.055	144.968	53.495	1,272.897
Primary Input																
1 Disposable household income	130.149	15.702	5.977	16.190	12.483	25.016	31.527	7.301	183.909	—	—	—	—	—	43.479	471.733
2 Government income	15.010	3.016	43.421	1.726	3.868	4.703	3.045	3.557	40.961	25.889	—	—	—	—	12.698	157.894
3 Transfer payments	—7.317	—3.178	—	—	—	—2.955	—	—	—6.416	—65.174	85.040	—	—	—	—	—
4 Saving, etc.	—	2.098	0.600	1.119	1.958	3.310	0.280	1.600	11.235	34.372	5.314	—	—	—	0.828	62.714
5 Depreciation	6.300	1.900	1.800	1.100	1.000	3.000	0.800	2.900	15.300	—	—	—	—	—	—	34.100
6 Profits paid abroad (imports)	—	1.400	5.200	0.900	1.500	2.400	0.200	0.100	7.390	—	—	—	—	—	—	19.090
Total Primary Input	144.142	20.938	56.998	21.035	20.809	35.474	35.852	15.458	252.379	—4.913	90.354	—	—	—	57.005	745.531
Input = Output	200.345	159.120	71.883	68.834	56.380	99.892	65.909	25.704	288.452	471.733	157.894	84.759	+12.055	144.968	110.500	2,018.428

See Note on following page.

NOTE TO TABLE 4

CSO made available a preliminary version of a full 36 x 36 table for 1956 showing for each cell home production and imports separately This table at the request of the author was abridged by CSO as regards the interindustry section to a 9 x 9 table in the form shown above, with all the imports assigned to a single line of the table The Primary Input section of the table prepared by CSO was quite different in categorisation to that shown above The items estimated in the CSO table were (a) indirect taxes less subsidies, (b) wages salaries and employers contributions to social insurance, (c) profits and depreciation The categorisation of the Primary Input section of the table shown above is the author s responsibility For 1960 CSO supplied certain items for a 1960 table showing (1) all the categories of final demand (2) the interindustry row and column for agriculture forestry and fishing The missing interindustry cell entries i e all those in rows and columns 2-9 were estimated by the author using an iterative process due to R Stone and J A C Brown (5) starting with the known values for the year 1956 In the interests of simplicity certain relatively small entries mostly pertaining to Government in the south east zone of the table have been absorbed in other items In addition the division of the Export category of final demand between visible and invisible items is the responsibility of the author

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Row 6 in this section represents profits on externally-owned enterprises to the total of £19 million The obverse of this item, namely factor income from abroad of £56 million, appears in the two entries on rows 1 and 2 of the second last column as household and government income

National Accounting Identities

The object of the adjustments is to enable us to produce directly from Table 4 all the major national accounting identities It is an invariable feature of IO tables that the totals of corresponding rows and columns in the interindustry section should be identical, see, for example, that the figure of £200 million for agriculture at the end of row 1 agrees with the figure at the end of column 1 In addition, in Table 4 the row and column totals for primary input and final demand have been brought into close agreement Thus household and government income (£472 million and £158 million respectively) coincide with the column totals for expenditure and saving—with a negative entry for government transfers to households (including interest on the public debt) of £65 million Gross capital formation of £97 million (including stock changes of £12 million) is financed by saving £62 million, net investment from abroad £1 million (or a total of £63 million shown in the last column of Table 4) and depreciation £34 million Finally the external account imports of goods and non-factor services (£236 million) together with factor imports (£19 million) equal non-factor exports (£198 million, the sum of visible and invisible non-factor exports) factor exports (£56 million) and net investment from abroad (£1 million) or a total of £255 million the sum of the two figures shown at the foot of the export columns It has been judged expedient to distinguish invisible from visible exports to highlight the importance and the potential of the gross expenditure of visitors which accounts for almost all of the £53 million non-factor invisible export

Unitary Coefficients

Table 5 displays unitary coefficients derived from the data in Table 4 This differs from the more usual table of coefficients in that it covers not only the interindustry sector of the table but also the primary input and final demand parts for reasons which, it is hoped, will be evident from what follows in a word this procedure is designed to enable us to bring all the major national accounting entities into our model It will be noted, by comparison with the entries in Table 4, that subsidies, saving, foreign

TABLE 5

BASIC COEFFICIENT MATRIX FOR THE INPUT-OUTPUT TABLE 1960

INPUT \ OUTPUT	INTERINDUSTRY									FINAL DEMAND					
										Consumption				Exports	
	1 Agriculture forestry, fishing	2 Food processing	3 Drink, tobacco	4 Textiles apparel	5 Metals engineering, vehicles	6 All other manufacturing, mining	7 Construction	8 Electricity, gas, waterworks	9 Services	Households	Government	Fixed capital	Stock changes	Visible	Invisible
Non-factor Input															
1	01088	48390	04318	00799	—	01077	—	—	00381	12359	01189	01152		33801	01402
2	06738	13194	00209	—	—	00774	—	—	00007	16578	00296	—		26350	05608
3	00248	00188	02926	00012	00044	00069	00006	—	00040	10002	—	—		04829	21548
4	00200	00554	—	23248	—	00947	—	—	00262	06313	—	00901		07243	10468
5	01562	00217	00228	00292	05112	00774	04602	09691	01418	03520	00148	14242		04415	02804
6	04497	04516	02643	08388	02572	13551	17199	07524	02785	03554	02519	02207		10692	02804
7	00075	00649	00068	00516	—	00271	02880	01082	01076	01093	19184	47573	See text	—	—
8	00273	00737	00154	00744	00828	01404	00352	00451	00883	02740	01036	04765		00041	—
9	06221	08095	01231	07146	04746	06705	08427	08022	02880	26321	73000	03460		10695	54356
Total Home	20902	76540	11777	41145	13302	25572	33466	26770	09732	82480	97372	74300		98066	98990
Imports	07151	10301	08930	28296	49789	38916	12138	13091	02773	12368	02628	25700		01934	01010
Total Non-factor Input	28053	86841	20707	69441	63091	64488	45604	39861	12505	04948	1	1		1	1
Primary Input															
1	64962	09868	08315	23520	22140	25042	47834	28405	63758	—	—	—		—	—
2	07492	01895	60405	02507	06861	04708	04620	13838	14200	05152	—	—		—	—
3	—03652	—01997	—	—	—	—02958	—	—	—02224	—	—	—		—	—
4	—	01319	00835	01626	03473	03314	00425	06225	03895	—	—	—		—	—
5	03145	01194	02504	01598	01774	03003	01214	11282	05304	—	—	—		—	—
6	—	00880	07234	01308	02661	02403	00303	00389	02562	—	—	—		—	—
Total Primary Input	71947	13159	79293	30559	36909	35512	54396	60139	87495	05152	—	—		—	—
Input = Output	1	1	1	1	1	1	1	1	1	1	1	1		1	1

NOTE

Unitized version of Table 4 It will be seen that certain items in Table 4 are not included see text and Table 6

TABLE 6

ALGEBRAIC NOTATION OF THE MODEL

Output / Input	Interindustry 1	2		n	Final Demand: Households	Government	Fixed capital	Stock changes	Exports: Vis	Invis	Output = Input
Non-factor input											
1	a_{11}	a_{12}		a_{1n}	h_1	g_1	v'_1	$p_1(Y_1-Y_{10})/T$	x'_1	x''_1	Y_1
2	a_{21}	a_{22}		a_{2n}	h_2	g_2	v'_2	$p_2(Y_2-Y_{20})/T$	x'_2	x''_2	Y_2
n	a_{n1}	a_{n2}		a_{nn}	h_n	g_n	v'_n	$p_n(Y_n-Y_{n0})/T$	x'_n	x''_n	Y_n
Imports	b_1	b_2		b_n	h_m	g_m	v'_m	$p_m(M-M_0)/T$	x'_m	x''_m	M
Primary output											
1 Disposable household income	c_{11}	c_{12}		c_{1n}	—	—	—	—	—	(F_h)	
2 Government income	c_{21}	c_{22}		c_{2n}	h_g	—	—	—	—	(F_g)	
3 Transfer payments	c_{31}	c_{32}		c_{3n}	(D)	(B)	—	—	—	—	
4 Saving, etc	c_{41}	c_{42}		c_{4n}	(S_h)	(S_g)			—	(N)	
5 Depreciation	c_{51}	c_{52}		c_{5n}	—	—		—	—	—	
6 Profits paid abroad (imports)	c_{61}	c_{62}		c_{6n}	—	—	—	—	—	—	
Input=Output	Y_1	Y_2		Y_n	H	G	V'	V''	X'	X''	

NOTE See equation system (2 5)

investment and factor income receivable have been ignored These elements will be seen to be the strategic variables in the model

The notation to be used for setting down the equations and identities of the model is displayed in Table 6 Workers in this field have not yet succeeded in evolving a satisfactory algebraic notation for IO work and notationally Table 6 will probably be found to be no exception to this sorry experience The significance of the symbols may be clear from the illustrative Tables 4 and 5, noting that small letters denote unitary coefficients and capital letters values (in £ million) The number of industrial groups is n (=9 in the application) and T is the time period between base year and year of reference The entries in the stock changes column will be explained later Brackets ( ) around F, D, etc, indicate that the corresponding values are not deemed included in the values H, G, etc, at the foot of the column

The attention of the reader is directed to the fact that the algebraic symbols in this part 2 of the paper are somewhat different in scope from the same symbols in part 1

Final Demand Categories

It will be useful to consider briefly the various categories of final demand

Households The coefficients h_i cannot be accorded the kind of quasi-predetermination with which the interindustry coefficients a_{ij} are customarily endowed As is well-known these coefficients will depend on the *average* level of household expenditure, in accordance with Engel's Law They are functions of this average level If the economy is generally advancing at a given rate, say 4%, *total* household expenditure is likely to rise at about the same rate If we assume, as we shall, that the proportionate rise in population is the same as that of the labour force, then the values of the h_i will depend on the evolution of the labour force and, therefore, on labour productivity If the labour force increases at the same rate as GNP then productivity remains constant at its base year level and there is no logical reason for changing the h_i for the year of reference On the other hand, if total household consumption is to increase at the rate of r% and the labour force by f% then household expenditure on average will increase by $(r—f)$% approximately It is this $(r—f)$% or labour productivity which determines the value of the h_i By way of illustration C E V Leser has kindly supplied the following data (which, however, are to be regarded only as rough approximations at this stage) for the coefficients in 1970 on the assumption of a 3% a head a year growth in average consumption, based on income elasticity considerations The "actual" 1960 coefficients are shown for comparison The 3% increase is consistent with a rise of 4% in *total* consumption and 1% rise in population

The marked decline in the proportions for agriculture and food in 1970 is the familiar Engel's phenomenon When one considers that in the 10 years a rise in consumption of 3% a head a year is equivalent to a rise of 34% the changes are not very marked

The outcome of the application will depend on the view taken wthi regard to theo rpductivity increase $(r—f)$% It must be deemed advisable

TABLE 7

UNITARY PATTERNS OF HOUSEHOLD CONSUMPTION, IRELAND
1960 AND 1970

Home Production	1960	1970	
		A	B
1 Agriculture, forestry, fishing	12359	11085	10515
2 Food processing	16578	14869	14105
3 Drink, tobacco	10002	09317	08838
4 Textiles, apparel	06313	06531	06195
5 Metals, engineering, vehicles	03520	04735	04491
6 Other manufacturing, mining	03554	04166	03952
7 Construction	01093	01056	01002
8 Electricity, gas, waterworks	02740	02645	03772
9 Services	26321	27350	25944
Total above	82480	81754	78814
Imports	12368	13304	16034
Import duties	05152	04942	05152
	1	1	1

NOTES

1960 Table 5
1970 A C E V Leser, see text and Leser [4]
1970 B Upward adjustment of proportions for sector 8 and imports and import duties restored to 1960 level, other figures adjusted proportionately to 1970 A See text

to produce answers for all reasonable levels of productivity As will be pointed out in the concluding section of this paper, national planners using the present model will have a wide choice before them but will have ample opportunities of modifying the coefficients and therefore the original targets of the plan selected as the time-period of the plan advances

Government expenditure This is the strategic area over which public authorities have absolute control, in theory at any rate It is therefore an area in which it would be well to try many experiments with the model The government pattern as time evolves must be conditioned by actions in the private sector, for example if private saving is insufficient for the plan the government may have to create forced saving by taxation, or, if private investment in certain sectors is insufficient for the attainment of the prescribed targets, government may have to step in With a large IO table available, presented on the lines indicated here, the planning authorities could experiment with many alternative patterns with a view to determining the optimal course of action

Fixed capital formation In Tables 4, 5 and 6 this column classifies gross fixed capital formation by home industry of *production* of these goods and services, as distinct from industry of *use* At first sight it might appear desirable to evolve formulae for gross fixed capital formation (GFCF) by industry of use consistent with rises from Y_{i0} to Y_i between base and reference years in gross output of industry i, or, more generally, substitute for the single column for gross fixed capital formation 9 (or, in general, n)

columns showing industries of purchase or use From international experience during the post-war period it would in fact be easy to find the relation between rate of increase in each broad industrial sector and the rate of increase in GFCF, even in constant price terms Such an exercise would be rather different in concept from the more usual incremental net capital-output ratio in which the entities studied are net annual increases in added value and net fixed capital formation (or the net increment in physical capital) The main reason for the difference in approach is that the IO table deals essentially with *gross* entities though, of course, added value, industry by industry, is derivable from the primary input table as well as net capital formation as the difference between the GFCF column and the depreciation row In recent years, however, an increasing number of economists tend to favour the GFCF approach as distinct from the net and not only for the reason of the notorious statistical unreliability of depreciation statistics Such economists take the view that on the alleged mere replacement (i e depreciation), as distinct from a net increase of a physical capital good, there is likely to be an increment in productivity, because replacements are rarely identical with the goods they purport to replace and are more than likely to incorporate improvements If one be allowed to assume an arithmetical annual increase in the economy there would be little difficulty in evolving algebraic formulae based on international experience for GFCF, on the lines of the formulae below for stock changes

The writer is, however, rather sceptical about the value of such an exercise, though he remains open to conviction, and, should another view be taken, there would be no difficulty about changing the model in this, which is a mere detail He bases his scepticism on the showing of Table 8 pertaining to the whole economy of 20 countries during the period 1953-59

TABLE 8

INCREASE IN GNP AND RATIO OF GFCF IN TWENTY COUNTRIES, 1953-59

Country	Ann av increase GNP	Av ratio GFCF to GNP	Country	Ann av increase GNP	Av ratio GFCF to GNP
1	2	3	1	2	3
	%		*Continued*	%	
Germany (F R)	6 5	0 219	Portugal	3 4	0 155
China (Taiwan)	6 3	0 135	Cyprus	3 1	0 245
Greece	6 3	0 108	Ceylon	2 9	0 117
Austria	6 2	0 216	Norway	2 9	0 305
Italy	5 3	0 205	Denmark	2 8	0 166
Netherlands	4 3	0 235	Belgium	2 5	0 160
France	4 2	0 179	U S A	2 4	0 170
Porto Rico	4 2	0 192	U K	2 1	0 146
Sweden	3 6	0 208	Chile	1 7	0 105
Canada	3 4	0 246	Ireland	—0 1	0 147

SOURCE Based on data in UN Yearbook of National Accounts 1960

Countries are arranged in descending order of rate of increase in GNP It is true that there are certain regularities in the table and perhaps it is easy to account for some of the low ratios as well as the exceptionally high ones in regard to rates of increase For instance the low ratios in Greece and China may have been due to increased labour intensity and the high ratio in Norway to investment in shipping which is highly capital-intensive It will be noted that the U K and Irish ratios are practically identical Theoretically there can be no qualifications (for reasons of differential population changes or otherwise) as to the validity of formula (1 8) namely $V'=krY$, which it will be recalled (omitting the subscript time, t, as unnecessary), Y is net national product, V' net fixed capital formation, k the incremental capital-output rate and r the rate of increase At any level of the capital-output ratio the value of V'/Y should accordingly increase with r It was really with a view to examining whether such a relationship obtained in fact that the foregoing table was prepared

It would appear that, at this stage, the most sensible course would be to adopt experimental, but reasonable value or values of the ratio q given by

$$(2\ 1)\quad V'=qZ$$

where Z is the gross domestic product given by

$$(2\ 2)\quad Z=\sum_{j=1}^{n} c_j Y_j$$

with

$$c_j=\sum_{i=1}^{6} c_{ij}$$

Furthermore, the value of V' can be distributed proportionately amongs the industries using the formula (see Table 5)

$$(2\ 3)\quad V'_i=v'_iV'$$

For the applications to Ireland which follow q will be given different experimental values In each experiment the use of a single ratio q will allow some margin for manoeuvre we have already expressed our scepticism about too great a degree of specificity in models of this kind For example, if the demand for economic investment should increase, social investment (e g in dwellings) could be postponed, to keep total fixed capital investment within the planned aggregate in the year of reference

Stock changes For a growing economy allowance must be made for changes in stock, industry by industry It would appear reasonable, as certainly it is algebraically convenient, to try to express changes in terms of gross value of output of the industry, and of non-factor imports, the marginal figure of the IO table For the nine industrial groups and imports the relevant figures for 1960 are shown in Table 9

TABLE 9

THE STOCK RATIO FOR SECTORS AND FOR IMPORTS, IRELAND 1960

Sector and Imports	Stocks end of 1960	Gross output etc 1960	Stock ratio
	£m	£m	
1 Agriculture, forestry, fishing	204 0	200 345	1 0182
2 Food processing	14 0	159 120	0 0880
3 Drink, tobacco	8 6	71 883	0 1196
4 Textiles, apparel	11 7	68 834	0 1700
5 Metals, engineering, vehicles	7 8	56 380	0 1383
6 Other manufacturing, mining	12 5	99 892	0 1251
7 Construction	0 9	65 909	0 0137
8 Electricity, gas, waterworks	2 1	25 704	0 0817
9 Services	38 8	288 452	0 1345
Imports	59 5	236 378	0 2517
	359 9	1,272 897	—

NOTE

Principal sources CIP 1960, CD and Table 4 The segregation of import stocks involves an even larger element of estimation than is general in the rest of the paper

If the gross output of industrial group i be Y_i in the reference year and Y_{i0} in 1960, the base year, and if the stock ratio p_i be assumed to apply throughout, then, in the reference year, the increase in stock may be taken, as

$$(2\ 4)\quad V''_i=(Y_i-Y_{i0})p_i/T,\ i=1, 2, \quad , n,$$

with an analogous formula for stocks of imports—see (2 5) (iv) below Admittedly this formula is not very satisfactory in that it assumes an arithmetical rate of increase between base and reference years, whereas one would prefer the geometrical (or "compound interest") hypothesis The arithmetical formula has the immense advantage that thereby the equations in the IO model displayed below are maintained linear

Applying the formula to Irish data, with $T=10$, following are the actual formulae for stock increases in the reference year

TABLE 10

FORMULAE FOR INCREASES IN STOCK V'', IRELAND 1970

Sector	
1 Agriculture, forestry, fishing	0 10182 Y_1—20 4
2 Food processing	0 00880 Y_2— 1 4
3 Drink, tobacco	0 01196 Y_3— 0 9
4 Textiles, apparel	0 01700 Y_4— 1 2
5 Metals, engineering, vehicles	0 01383 Y_5— 0 8
6 Other manufacturing, mining	0 01251 Y_6— 1 2
7 Construction	0 00137 Y_7— 0 1
8 Electricity, gas, waterworks	0 00817 Y_8— 0 2
9 Services	0 01345 Y_9— 3 9
Imports	0 02517 M — 5 9

NOTE

Based on formula (2 4) applied to Table 9

Exports The coefficients x_i in this column of Table 3 are the least stable in the model There is no reason to suppose that proportions obtaining in 1960 will obtain in any future year of reference Clearly the future pattern depends on external demand Many alternative reasonable patterns may be postulated for exports, however, and the model will supply the whole consequential economic pattern The model, applied to the detailed IO table, will identify the exports which it is in the country's interest to promote It will be useful to set down a "reasonable" distribution for visible exports X' for 1970, i e based on British *import* expectations from all countries for that year—see Table 11

TABLE 11

VISIBLE EXPORT PROPORTIONS, IRELAND 1960 AND 1970

Sector	1960	1970
1 Agriculture, forestry, fishing	3380	2870
2 Food processing	2635	2237
3 Drink, tobacco	0483	0459
4 Textiles, apparel	0724	1026
5 Metals, engineering, vehicles	0441	0626
6 Other manufacturing, mining	1069	1515
7 Construction	—	—
8 Electricity, gas, waterworks	0004	0004
9 Services	1070	1069
Imports	0194	0194
	1	1

NOTE

1960 from Table 5 1970 based on projected % annual average increases in imports into U K (National Economic Development Council, "Growth of the U K Economy", page 53) as follows Food 1 7%, Drink and tobacco 2 8%, Manufactures 7 0%, Fuels 4 4%

Though used for illustrative purposes in the following experiments these export proportions for 1970 are very unsatisfactory To use the present model properly specific prognostics for visible exports must be made by experts for each sector, in fair sectoral detail

The Equations of the Model

With number of industrial groups n and period from base to reference year T the equations (by reference to the notation in Table 6) are as follows

(2 5)

(i) Interindustry $\Sigma_j a_{ij} Y_j + h_i H + g_i G + v'_i V' + p_i(Y_i - Y_{i0})/T + x'_i X' + x''_i X'' = Y_i$, $i = 1, 2, \quad , n$

(ii) Gross domestic product $Z = \Sigma_j c_j Y_j$

(iii) Gross fixed capital formation $V' = qZ$

			1960	1970	Increase %
1	Gross domestic product, Z		603 0	904 7	50
2	Government expenditure, G		67 5	114 7	70
3	Invisible exports (nearly all expenditure by visitors)	X'' (a)	53 5	107 0	100
		X'' (b)	53 5	133 8	150
4	Subsidies, etc , to households, D		65 2	91 3	40
5	Household income from abroad, F_h		43 5	50 0	15
6	Government income from abroad, F_g		12 7	15 0	18

These will be termed the *Basic Assumptions* (with alternative values at 3) To these may be added the all-important Basic Assumption 7, that for decision-making purposes the coefficient system as described in the text applies to reference year 1970

In this purely experimental paper there is no point in discussing at length the values selected for 1970 The major assumption is, of course, that of a 50% increase in head 1, gross domestic product, equivalent to a little over 4% per annum The 1970 values for heads 2-4 may be regarded as based roughly on the assumption for head 1 The 1970 values for heads 5 and 6 are an expression in figures of the opinion that in these figures, comparatively stable in recent years past, there will be a small increase Presumably if this model be adopted, trials will be made on other assumed values For head 3, regarded as having a considerable potential for expansion, alternative trials of 100% and 150% increases were made

As to the particular set of 6 variables selected, naturally choice leaned heavily towards the 3 variables, heads 2, 4 and 6, as those over which government has some control Other choices of predetermined variables could be made for instance, in a preliminary series, trials were made with the gross output of agriculture, etc , Y_1 and visible exports of agriculture, etc , X'_1 and food industries X'_2 as predetermined but the results were unsatisfactory as yielding bizarre values for some of the other variables For the present model to work properly it seems essential to keep all the n outputs Y_i endogenous This does not mean that one must lose control over them after a certain number of trials—one can be prodigal of trials on the computer—one can always obtain approximately the effect one wants, as will perhaps appear from what follows

Values must be assigned to the constants q and s in equations (2 5) (*iii*) and (*xii*) above The respective values were 0 14 and 0 07 in 1960 From the analysis in part 1 of this paper and, in particular, the showing of Table 8, it is obvious that these ratios must be raised considerably if the economy is to increase at a relatively steep gradient The following values are selected for investigation

q=0 17, 0 18, 0 19, 0 20 (4 values)
s=0 10, 0 12, 0 14 (3 values)

The trials were in four series A, B, C, D in increasing order of "reasonableness" by standards which may be apparent as the paper proceeds Each of the many trials emerged printed from the computer specifying (1) the assumptions for the particular trial, (2) the values of all the variables in 1970 (in £ million to one decimal place) together with percentage increases compared with 1960, (3) a set of five national income accounts showing for each item the 1960 and 1970 values together with the percentage increase, and (4) a table showing for each sector (a) output, (b) total exports and (c) visible exports, 1960 and 1970 values and percentage increase Most of these data are indicated in Table 13A for Trial D4 The items of (3) were so spaced that the computer sheets could be gummed without cutting to a master sheet with rubrics for photographing to produce all the copies required

The twelve trials in Series A were based on the coefficients shown in Table 5, i e those for 1960 with the following assumptions (a) the Basic Assumptions 1-4 with both values for head 3 (b), (b) two values for increase in agricultural output Y_1, namely 28 % and 40 %, (c) three values of the GFCF ratio q, namely 0 18, 0 19 and 0 20 The household saving ratio was fixed at 0 12 To permit agriculture to become predetermined Basic Assumption 5 was allowed to run free and head 6 was in fixed relation to head 5

By reference to net investment from abroad (i e the import excess N) only two of these trials could be regarded as satisfactory namely A1 and A2 for which the values of N in 1970 would be £2 million and £18 million respectively The basic assumptions for these trials were as follows

Trial	Agricultural output, Y_1 increase	Invisible exports, X'' increase	GFCF ratio, q	Import excess, N
	%	%		£m
A1	28	100	0 18	2 0
A2	28	100	0 19	18 1

Of course, the import excess for A2 may also be regarded as too great for Trial A12 it is impossibly large at £115 million even though the A12 assumptions provide for an increase of 40 % in agricultural output and 150 % increase in invisible exports The apparent anomaly is explained by the fact that in all these trials the level of the economy is deemed fixed at GDP equal to £905 million Some of the results can be rather unexpected, a fact which perhaps makes worthwhile the procedure outlined in this paper

Predetermining Y_1 and allowing heads 5 and 6 to become endogenous produced bizarre results in the 1970 Series A values of these entities which, as already remarked, have been very stable in recent years In fact in the twelve trials the 1970 value of household income from abroad F_h ranged from *minus* £121 million to *plus* £75 million, with corresponding nonsensical figures for government income from abroad F_g It was from this experience that the lesson was learned that no attempt should be made to give an exogenous status to any of the sectoral outputs Such procedure requires the release of a corresponding number of Basic Assumptions which are prone to appear in the solution with unacceptable values The status of these assumptions was maintained during subsequent series of

TABLE 12

INTERINDUSTRY AND IMPORT COEFFICIENTS FOR TRIAL D4

Sector	1	2	3	4	5	6	7	8	9
1	01113	46816	03600	00676	—	00689	—	—	00340
2	06895	12765	00175	—	—	00494	—	—	00006
3	00253	00182	02440	00010	00038	00044	00005	—	00036
4	00205	00536	—	19664	—	00604	—	—	00234
5	01598	00210	00189	00247	04459	00494	04229	08613	01265
6	04600	04369	02203	07095	02243	08657	15804	06687	02484
7	00077	00627	00067	00437	—	00173	02647	00961	00961
8	00438	01049	00219	01058	00882	01999	00501	00642	01257
9	06365	07832	01027	06044	04140	04284	07744	07130	02570
Total 1-9	21544	74386	09920	35231	11762	17438	30930	24033	09153
Imports	10174	12455	10787	34210	51329	47050	14674	15828	03352
Total	31718	86841	20707	69441	63091	64488	45604	39861	12505

trials Of course, there is nothing to prevent the experimenter from giving these six entities any values he pleases

One special inference of value which emerged from Series A is that an increase of 0 01 in the GFCF ratio q entailed an increase of £16 million in the import excess at the presumed 1970 level of the economy, for saving ratio (s=0 12) This fact shows the great importance of using capital efficiently—to avoid placing a strain on the balance of payments Series A also suffers from the defect that it implies a uniform rate of increase in all visible exports X'_i, e g that agricultural exports are to increase by the same percentage as industrial exports Such an assumption is quite unreal This set of trials was undertaken to try out the model It proved to be ideally adapted to the computer once programmed, each trial, fully printed, could be obtained in a few minutes Furthermore, it showed that the special feature of the model, namely the built-in system of national accounts (avoiding the tiresome iterative feature of other input-output models) was fully operational

It may not be necessary to give details of the evolution of the trials through the series B, C and D These culminated, through some 50 trials in all, in Trial D4, the outcome of which will presently be displayed Following is a summary of the lessons learnt and the modifications adopted in using the model (repeating some of the points already mentioned)

(i) The principal bell-wether is the import excess N, as a working rule the trial was rejected if this exceeded £10 million

(ii) Regard was had to increases in the output and exports of the different sectors by reference to recent past trend, was too much expected of agriculture? At one stage it was necessary to revise upward the coefficients for sector 8—electricity, gas, water—to induce an increase in output more in accordance with recent trend and expectations in the sector itself

(iii) Also having regard to recent trends it was decided to increase the unitary coefficients for non-factor input into agriculture from 0 28053 in 1960 (see Table 5) to 0 31718 This increase is an extrapolation of the trend in this proportion (at constant prices) in recent years The sector constituents were altered proportionately

(iv) Any experiments involving a prescription of the 1970 values for outputs or exports were failures These elements should always be endogenous in the present model Any desired effect can easily be obtained by modification of the coefficients

(v) The unitary coefficients adopted for household consumption are those of column B of Table 7 and the coefficients for visible exports are those for 1970 in Table 11

Table 13A
NATIONAL ACCOUNTS 1960 AND 1970 (TRIAL D4)

	1960 £m	1970 £m	Incr %		1960 £m	1970 £m	Incr %
ACCOUNT 1 GROSS DOMESTIC PRODUCT							
1 1 Disposable household income arising	428 2	638 4	49 1	Added value in sector of origin			
1 2 Government income arising	119 3	178 0	49 2	1 6 Agriculture forestry, fishing	144 7	193 6	34 3
1 3 *Less* subsidies, etc to sectors	—19 9	—28 8	44 5	1 7 Food processing	20 9	28 9	38 2
1 4 Saving and depreciation by sectors	56 3	87 9	66 2	1 8 Drink tobacco	57 0	81 2	42 5
1 5 Income paid abroad by sectors	19 1	29 1	52 2	1 9 Textiles apparel	21 0	33 8	60 9
				1 10 Metals engineering vehicles	20 8	36 3	74 4
				1 11 Other manufacturing mining	35 5	57 4	61 7
				1 12 Construction	35 8	56 7	58 3
				1 13 Electricity gas waterworks	15 5	32 6	110 1
				1 14 Services	252 4	384 2	52 2
Gross domestic product	603 0	904 7	50 0	Gross domestic product	603 0	904 6	50 0
ACCOUNT 2 HOUSEHOLDS							
2 1 Household expenditure (less 2 4)	476 6	661 9	38 9	2 5 Disposable household income arising	428 2	638 4	49 1
2 2 *Less* subsidies, etc to households	—65 2	—91 3	40 0	2 6 Household income from abroad	43 5	50 0	14 9
2 3 Household saving	34 4	83 5	142 8				
2 4 Indirect taxes (part)	25 9	34 3	32 3				
Total, households	471 7	688 4	45 9	Total households	471 7	688 4= V_p	45 9
ACCOUNT 3 GOVERNMENT							
3 1 Government expenditure	67 6	114 7	70 0	3 5 Government income arising	119 3	178 0	49 2
Subsidies etc to				3 6 Household indirect taxes (part)	25 9	34 3	32 2
3 2 Sectors	19 9	28 8	44 5	3 7 Government income from abroad	12 7	15 0	18 1
3 3 Households	65 2	91 3	40 0				
3 4 Government saving	5 3	—7 5	—242 0				
Total, government	157 9	227 3	43 9	Total government	157 9	227 3	43 9
ACCOUNT 4 SAVING-CAPITAL							
4 1 Gross fixed capital formation	84 8	153 8	81 4	Saving and depreciation			
4 2 Changes in stock	12 0	19 0	58 3	4 3 Sectors	56 3	87 9	56 2
				4 4 Households	34 4	83 5	142 8
				4 5 Government	5 3	—7 5	n m
				4 6 Net investment from abroad	0 8	8 9	n m
Total saving capital	96 8	172 8	78 5	Total saving-capital	96 8	172 8	7 5
ACCOUNT 5 EXTERN							
5 1 Non-factor exports	198 5	400 4	101 7	5 5 Non factor imports	236 4	445 2	88 3
Factor income from abroad				5 6 Income paid abroad by sectors	19 1	29 7	52 2
5 2 Households	43 5	50 0	14 9				
5 3 Government	12 7	15 0	18 1				
5 4 Net investment from abroad	0 8	8 9	n m				
Total, extern	255 5	474 3	85 6	Total extern	255 5	474 3	85 6

NOTES

(i) Accounts are not completely articulated, (ii) n m ="not meaningful" (iii) individual figures may not always add to totals shown because of rounding

TABLE 13B

OUTPUT AND DOMESTIC EXPORTS OF SECTORS (TRIAL D4)

Sector	Output			Exports, total			Exports visible (incl in Exports, total')		
	1960	1970	Incr	1960	1970	Incr	1960	1970	Incr
	£m	£m	%	£m	£m	%	£m	£m	%
1 Agriculture forestry fishing	200 3	283 5	41 5	49 8	85 7	72 3	49 0	84 2	71 9
2 Food processing	159 1	219 5	38 0	41 2	71 6	73 9	38 2	65 6	71 8
3 Drink tobacco	71 9	102 5	42 5	18 5	36 5	97 1	7 0	13 5	92 4
4 Textiles, apparel	68 8	110 6	60 7	16 1	41 3	156 6	10 5	30 1	186 8
5 Metals engineering, vehicles	56 4	98 3	74 3	7 9	21 4	170 5	6 4	18 4	187 0
6 All other manufacturing mining	99 9	161 7	61 8	17 0	47 5	179 1	15 5	44 5	186 8
7 Construction	65 9	104 2	58 1	0 0	0 0	—	0 0	0 0	—
8 Electricity gas, waterworks	25 7	54 2	110 7	0 1	0 1	19 2	0 1	0 1	19 2
9 Services	288 5	439 2	52 2	44 6	89 5	100 8	15 5	31 4	102 4
Total domestic exports				195 2	393 7	101 7	142 2	287 8	102 4

(vi) Pursuant to the analysis in part 1 the ratio of imports to GDP was increased from 39% in 1960 to 49% in 1970 The proportion to be borne by imports in the economy is nearly pure guesswork Here again it would be well to examine the effects of different assumptions as to its level With this model there is no trouble whatever All that requires to be done is to multiply the line of import coefficients across by a constant and readjust the columns to unity The computer will even do this and produce the modified answer in a few minutes

(vii) Compared with Table 5 for 1960 changes were made in the coefficients for government consumption and fixed capital formation to allow for (a) proportionately greater consumption for sector 8 —electricity, gas, waterworks—and (b) imports As always, other coefficients were reduced proportionately according to the 1960 coefficients, so that the sum for all sectors was unity It may be sufficient to furnish only revised coefficients in respect of these heads, deemed to apply in the year 1970

	Government consumption	Fixed capital formation
Electricity, gas, waterworks	01474	06781
Imports	03178	31068

We now give the full results for Trial D4, which the author regards as generally the most satisfactory as far as his investigations have gone with the present model It may be convenient first to set out in full the assumptions

Assumptions of Trial D4

(a) Basic Assumptions with 3—Invisible exports X''=£107 million
(b) Ratio of GFCF to GDP, i e $q=0\ 17$
(c) Ratio of household saving to household expenditure, i e $s=0\ 12$
(d) Inter-industry part of coefficient scheme as indicated in Table 12—see considerations outlined in (i)-(vii) above It will be noted that in this table the non-factor input totals were maintained at the levels of the 1960 Table 5 except in the case of agriculture—see (iii) above

The results for Trial D4 are shown in Tables 13A and 13B These are not exactly the solutions of equation system (2 5) but are easily reconcilable with them It may be well to specify the values of two important variables

	1960 £m	1970 £m	Incr %
Household expenditure, H(=2 1+2 4 of Table 13A)	502 5	696 2	38 5
Income paid abroad by sectors, P	19 1	29 1	52 2

To Tables 13A and 13B may be added Table 13C showing how GNP is derived from GDP

TABLE 13C

TRANSITION FROM GROSS DOMESTIC PRODUCT TO GROSS NATIONAL PRODUCT AT MARKET PRICES (TRIAL D4)

Category	1960	1970	Incr
	£m	£m	%
Gross Domestic Product at market prices for Input-Output purposes	603 1	904 7	50 0
Indirect taxes paid by final consumers	30 6	40 6	32 7
Adjustment for physical changes in stocks	—1 1	0	—
Gross factor income from abroad	53 0	61 3	15 7
Less factor income paid abroad	—19 1	—29 1	—52 2
Depreciation on government property	2 5	2 5	0
Gross National Product at market prices	669 0	980 0	45 5

As stated earlier, Trial D4, in the author's opinion, yields the most generally satisfactory results, assuming a GDP growth rate of 50%, of the many he has tried An import excess of *N* of £9 million in 1970 is no great matter with exports totalling £474 million and, with greatly increased household income, a household expenditure saving ratio *s* of 0 12 should be obtainable

It is no part of the author's function, however, to decide whether the plan outlined is feasible As also stated earlier, feasibility depends largely on the view taken as to the absolute size of exports in the two divisions visible and invisible (almost all visitors' expenditure) and of the sectoral pattern of visible exports

The full IO table for 1970 in the exact form of Table 4 for 1960 could readily be produced from the computer results shown above for Trial D4 This is not considered necessary as imputing a measure of detailed exactitude to this exercise to which it has no pretensions This remark serves as a reminder that the whole object of these calculations is to try (or, if one wishes, to pretend) to produce such an IO table for the reference year 1970 In the interest of brevity these tables must speak for themselves, without textual comment

3 SENSITIVITY

In this part 3 attention is directed to the effect on the answers of changes in the coefficients in the model

Interindustry

We first consider the interindustry coefficients, i e those of Table 12 except the last line The method adopted was to increase in succession each of the coefficients of magnitude exceeding 01, of which there are 35, by 20%, an arbitrary figure After each change the column total was restored to the Table 12 total values by proportionate adjustment to the other values (e g 21544 for column 1), all the other Trial D4 conditions being unchanged The computer executed these adjustments and then produced the 35 sets of answers Table 14 is based on these answers Attention is directed to changes in only the 9 sectoral outputs Y_i

TABLE 14
DEVIATIONS IN VALUE OF SECTOR OUTPUTS FOR SINGLE INTERINDUSTRY COEFFICIENT CHANGES OF 20%

Coefficient changed		Absolute deviation (£ million)		Col 4 as % total output 1970	Largest deviation	
Row	Col	Coeff	Result		as % of output	in sector
1	2	3	4	5	6	7
1	1	0 631	1 306	0 083	0 180	1
2	1	3 909	9 810	0 623	2 097	2
5	1	0 906	1 795	0 114	0 997	5
6	1	2 608	5 414	0 344	1 871	6
9	1	3 609	10 176	0 647	1 093	2
1	2	20 552	39 493	2 510	5 200	1
2	2	5 604	8 329	0 529	3 108	2
6	2	1 918	3 967	0 252	1 405	6
8	2	0 461	1 108	0 070	0 851	8
9	2	3 438	8 285	0 527	1 184	1
1	3	0 738	1 540	0 098	0 284	3
3	3	0 500	1 176	0 075	0 493	3
6	3	0 452	0 901	0 057	0 330	6
9	3	0 211	0 512	0 033	0 061	3
4	4	4 350	10 185	0 647	5 383	4
6	4	1 569	3 636	0 231	1 193	4
8	4	0 234	0 579	0 037	0 428	8
9	4	1 337	3 757	0 239	1 114	4
5	5	0 877	2 077	0 132	0 969	5
6	5	0 441	1 047	0 067	0 312	6
9	5	0 814	2 227	0 142	0 551	5
6	6	2 800	5 920	0 376	2 059	6
8	6	0 646	1 434	0 091	1 180	8
9	6	1 385	3 740	0 238	0 724	6
5	7	0 881	2 017	0 128	0 956	5
6	7	3 294	8 050	0 512	2 318	6
7	7	0 552	1 188	0 076	0 545	7
9	7	1 614	4 417	0 281	0 888	6
5	8	0 934	2 176	0 138	1 019	5
6	8	0 725	1 715	0 110	0 509	6
9	8	0 773	2 125	0 135	0 461	5
5	9	1 111	2 412	0 153	1 201	5
6	9	2 192	4 916	0 312	1 532	6
8	9	1 104	2 260	0 144	2 029	8
9	9	2 257	5 993	0 381	0 840	8

NOTES

The entries may be explained by reference to line 1

Cols 1-2 Changed coefficient is 01113 (see Table 12)

Col 3 Amount of change is £(01113×283 5/5)m =£0 631 m The £283 5 m is the 1970 output of sector 1 (see Table 13B)

Col 4 Sum of absolute values of changes in sectors 1-9, i e, $\Sigma \mid Y'_i - Y_i \mid$ where Y'_i is the changed value and Y_i is the Table 13B value

Col 5 Total output ΣY_i=£1,573 5 m

Cols 6-7 Highest deviation occurs in sector 1 where $Y'_1 - Y_1$=£0 510 m or 0 180% of Y_1=£285 3 m

It seemed worth illustrating columns 3 and 4 in a diagram which shows that the relation between the coefficient change and the aggregate deviation in sector values is nearly linear (the line passing through the origin as, of course, it should) except for the single aberrant result,* the reason for which is not understood Linearity is to be expected for small changes in the coefficients × output since algebraic expressions for the output deviations are almost linear in the coefficient change actually it is very easy to write down the expressions for $Y'_i—Y_i$ in terms of a small single coefficient change We prefer, however, to let the computer do the work

Each figure in column 5 represents the *average* percentage deviation resulting from the change indicated Except for the great coefficient change for the row 1 column 2 entry the average effect is negligible Even if the single coefficient change was doubled, i e to 40% (which would be large indeed) the effect on the answer would be unimportant The % deviation for the sector for which this figure is *largest*—columns 6-7—is another matter It is for the decision makers to decide whether these percentages are in some cases so large as to change policy determined from Trial D4, bearing in mind that these errors should be considered against the background of considerably larger percentage increases contemplated compared with 1960

Comparison of the sector numbers in columns 1 and 7 show that the sector of largest deviation tends to be associated with the row number, a result one would expect from the "strong diagonal" characteristic of the IO matrix Correspondence occurs in 25 out of 35 changes studied and is invariable for the largest changes (column 6) which are the only ones that matter

With a view to testing for non-linear (quadratic or higher) effect the results from *decreasing* the coefficients by 20% were produced for the 6 largest coefficients of the 35 used for this series of experiments The results, always as applied to the 9 sectors, may be summarised as follows

(i) For each sector for each coefficient change the deviation in sectoral value for decreasing 20% was nearly equal to the increasing 20% value with the sign changed, e g for the coefficient in the row-column 1-2 position the respective deviations were —£16 310 m and +£14 742 m The values would be equal in absolute value if the relationship were exactly linear

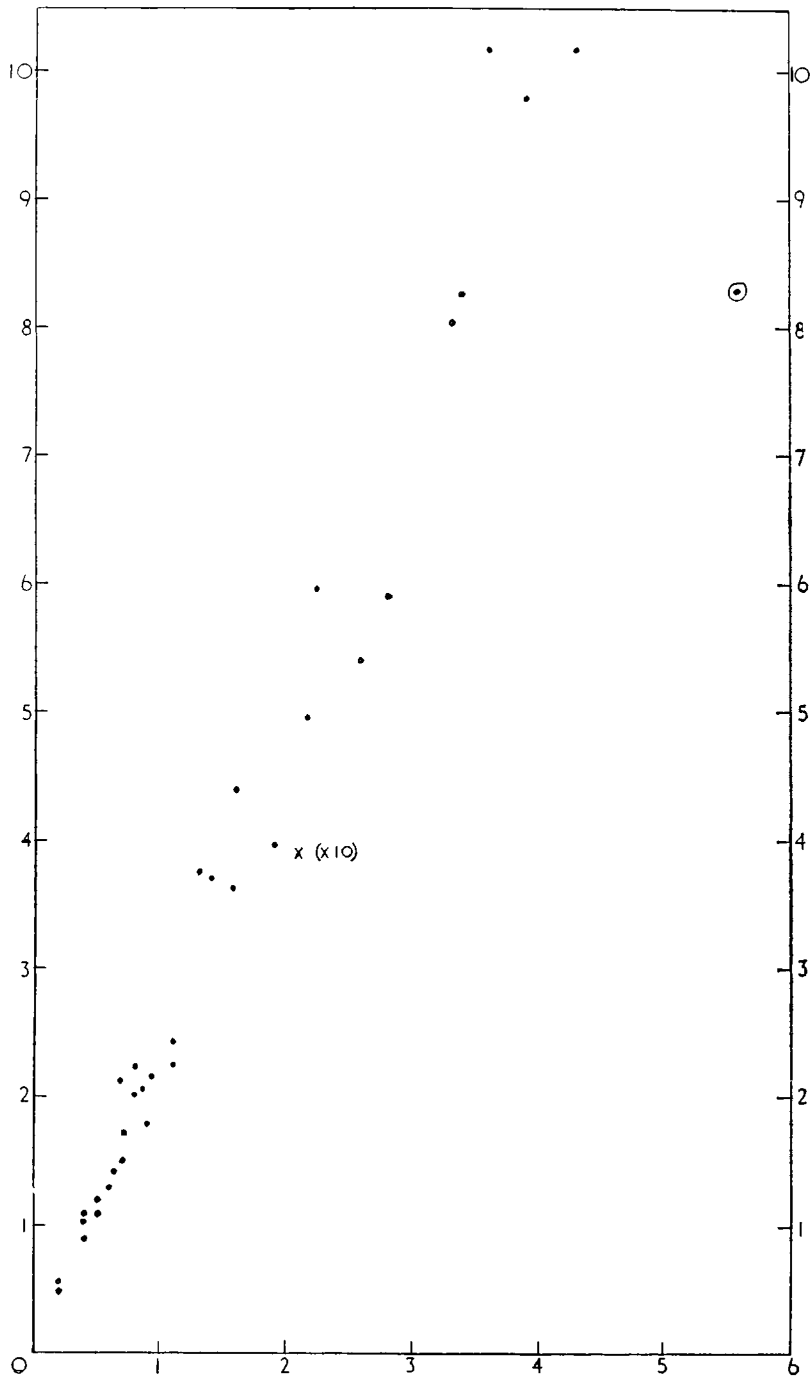
(ii) While relatively small, the non-linear effect was quite apparent In every case without exception (6×9=54 in all) it was found that $| Y'_i—Y_i | < | Y''_i—Y_i |$ where Y''_i is the solution in the decreasing 20% case

*That shown as ○ in the chart, on which it may be noted that the point x is graphed at one-tenth of the actual values—yet it is in line with the remaining 33 points

DIAGRAM 2

RELATIONSHIP BETWEEN ABSOLUTE DEVIATION IN (I) INTERINDUSTRY IO COEFFICIENTS (ABSISSA) AND (II) SECTOR OUTPUTS (ORDINATE)

Values in £m



(iii) Not only this, but for each change the statistic $2 \mid Y'_i - Y_i + Y''_i - Y_i \mid / (\mid Y_i' - Y_i \mid + \mid Y''_i - Y_i \mid)$ (e g in the case at (i) above $2 \times \mid 14\,742 - 16\,310 \mid / (14\,742 + 16\,310) = 0\,101$) is almost constant between sectors but different for each coefficient change There can be no doubt that this is an algebraic property, susceptible of proof

While much is to be learned from a study of single coefficient changes, clearly the investigation cannot be confined to this case only since it is not rigorous enough Table 15 summarises the result of a Monte Carlo experiment of 5 samples For each random sample, 10 coefficients of the largest 35 in Table 12 were selected for change by ±20% the signs also being randomized Thus the first sample consisted of row—decimal—column preceded by + indicating increase of 20% or — decrease —2 1, +9 1, +6 2, +1 3, +9 4, +5 5, —6 5, —5 8, —5 9, +6 9 After the recalculation of the selected coefficients the other coefficients were proportionately adjusted to add to the interindustry column totals of Table 12

Percentagewise, none of the deviations are alarming, being largest for sectors 2, 6 and 7 The largest percentage which appears is 3 4 in sample 4 for sector 2 To establish formally the interesting point that the averages shown in column 13 differ significantly in the different sections an analysis of variance was made of columns 8-12 to the following effect

	DF	*SS*	*MS*	*F*
Rows	8	22 77	2 846	9 78
Columns	4	0 40	0 100	0 34
Remainder	32	9 34	0 291	—
Total	44	32 51		

The difference between rows is overwhelmingly significant Sectors 2, 5 and 6 are relatively sensitive to coefficient aberration

This study of the effect on the solution of changes in the interindustry part of the IO table leaves one with the impression that, for decision making with a view to very large increases in the economy at some future date, variations of reasonable magnitude in the interindustry coefficients of the 1960 IO table should not materially affect the validity of the results in part 2 of this paper

Imports

Throughout this series of experiments on sensitivity the practice has been to examine one factor at a time, keeping all the values in the rest of the model constant, i e as they were in Trial D4 For this section the import row of coefficients for Trial D4 were reduced uniformly by 10% all the other column coefficients adjusted proportionately, so that the non-factor column totals were restored to their D4 values, e g 31718 for column 1—see Table 12 The effect on the solution will be seen in Table 16

Table 15

DEVIATIONS IN VALUE OF SECTOR OUTPUTS FOR INTERINDUSTRY COEFFICIENT CHANGES OF 20% IN RANDOM SAMPLES OF TEN

Sector	1970 output (D4) (£ million)	Absolute deviation (£ million), sample					Percentage deviation, sample					Average percentage deviation
		1	2	3	4	5	1	2	3	4	5	
1	2	3	4	5	6	7	8	9	10	11	12	13
1	283 5	3 346	0 273	0 618	0 844	4 030	1 180	0 096	0 217	0 298	1 422	0 643
2	219 5	5 151	5 189	5 932	7 504	4 906	2 346	2 364	2 703	3 419	2 235	2 613
3	102 5	0 330	0 457	0 595	0 168	0 235	0 322	0 446	0 580	0 164	0 229	0 348
4	110 6	1 210	0 278	0 271	1 636	0 518	1 094	0 251	0 245	1 479	0 468	0 707
5	98 3	1 286	2 056	0 785	0 290	2 132	1 308	2 092	0 799	0 295	2 169	1 333
6	161 7	3 514	2 292	3 965	3 518	1 246	2 173	1 417	2 452	2 176	0 771	1 798
7	104 2	0 186	0 568	0 703	0 141	0 358	0 180	0 545	0 675	0 135	0 344	0 376
8	54 2	0 364	0 399	0 605	0 816	0 072	0 672	0 736	1 116	1 506	0 133	0 833
9	439 2	3 590	2 126	2 186	3 587	1 811	0 817	0 484	0 498	0 817	0 412	0 606
Total or average	1,573 7	18 977	13 638	15 660	18 504	15 308	1 121	0 937	1 032	1 143	0 909	1 028

NOTES

Col 2	See Table 13B
Cols 8-12	Cols 3-7 as percentage of col 2
Col 13	Simple average of cols 8-12
Total or average	Cols 2-7 totals, cols 8-13 simple averages

TABLE 16

EFFECT ON TRIAL D4 SOLUTION OF A REDUCTION OF TEN PER CENT IN IMPORTS

Values in £m

Item	1970 value (D4)	Revised value	Change	
			Absolute	Percentage
1	2	3	4	5
Household expenditure, *H*	696 2	694 8	—1 4	—0 2
Non-factor imports, *M*	445 2	403 6	—41 6	—9 3
Income paid abroad, *P*	29 1	29 2	+0 2	+0 7
Visible exports, *X′*	293 4	255 3	—38 1	—13 0
Net investment from abroad, *N*	8 9	5 5	—3 3	n m
Sector				
1 Agriculture, forestry, fishing	283 5	271 4	—12 1	—4 3
2 Food processing	219 5	211 6	—7 9	—3 6
3 Drink, tobacco	102 5	102 1	—0 4	—0 4
4 Textiles, apparel	110 6	109 6	—0 9	—0 8
5 Metals, engineering, vehicles	98 3	100 7	+2 4	+2 4
6 All other manufacturing, mining	161 7	164 2	+2 5	+1 5
7 Construction	104 2	107 9	+3 7	+3 6
8 Electricity, gas, waterworks	54 2	56 8	+2 6	+4 8
9 Services	439 2	444 2	+5 0	+1 1

n m =not meaningful

In interpreting this table it will be borne in mind that for the results shown in column 3 GDP has been maintained unchanged at its D4 value of £904 7 million It is interesting to note that the model scales down visible exports almost *pro tanto* with imports, in fact, with a lessening in the import excess *N* from £8 9 to £5 5 million What the table exhibits is the phenomenon of import substitution household expenditure remains almost unchanged, the reduction in exports falls mainly on sectors 1 and 2 thereby reducing outputs Non-agricultural outputs increase through substitution for "former" imports

Household Expenditure

For this series of 9 trials each of the coefficients was increased by 10% in succession, as always with proportionate adjustment of the other coefficients in the column These 10% changes might be regarded as sizeable, since these, the Engel coefficients, are stable in time (Leser [4])

TABLE 17

EFFECT ON TRIAL D4 SOLUTION OF AN INCREASE OF TEN PER CENT IN HOUSEHOLD EXPENDITURE COEFFICIENTS

Values in £m

Sector	Increase in consumption of sector	Absolute sum sector output deviations	Increase in output of sector in Col 1	Percentage deviation	
				Col 3 of total output	Col 4 of output of Col
1	2	3	4	5	6
1	7 340	15 139	6 695	0 962	2 362
2	9 846	22 714	11 790	1 444	5 371
3	6 168	14 622	6 195	0 936	6 044
4	4 179	8 701	5 532	0 553	5 002
5	3 134	5 406	3 406	0 344	3 465
6	2 762	4 945	3 211	0 314	1 986
7	0 700	1 478	0 718	0 094	0 689
8	2 626	5 322	2 630	0 338	4 852
9	18 276	48 523	16 691	1 161	3 800

NOTES

Col 2 Product of total D4 consumption (£6 962 m) by 10% of consumption column coefficient

Col 3 Sum of col 4 entry and other 8 deviations with + sign In fact, these latter are usually negative

Col 5 Col 3 as percentage of £1,573 7 m , the sum of the 9 D4 outputs

Col 6 Thus 2 362% is £6 695 m (col 3) of £283 5 m , the D4 output of sector 1

Generally the results are as anticipated the figures in column 2 should really be multiplied by 2 for comparison with those in column 3 since the column 2 figure is offset exactly by negative changes in 8 coefficients, so, we find the column 3 figures nearly double those in column 2 in most cases Sector 9—Services is an exception the very large column 2 change results in an aggregate output deviation of well over twice the column 2 figure

The generally large percentages in column 6 shows that the model is sensitive to changes in the Engel coefficients Considerable attention should therefore be given to the deviation of these coefficients for 1970, if the model is to be used realistically, i e with a detailed schedule of sectors

The effect of changes in the household coefficients on the macro-variables total household expenditure, imports, visible exports and the import excess was negligible It paradoxically happens, however, that a repetition of this series of sensitivity tests, holding the import excess N constant and allowing GDP to be determined by the model, produced bizarre results The moral, already learned from Trials A, is in using the model do not depart from the Basic Assumptions, i e the particular set of variables which are assigned predetermined values

Exports

For each of the 7 exporting sectors the visible export coefficient was increased by 10% with proportionate adjustment of the remaining 6 sectors It may suffice to confine comment to the few aspects shown in Table 17

TABLE 18

SOME RESULTS OF CHANGING THE SINGLE VISIBLE EXPORT COEFFICIENTS BY 10%

Values in £m

Sector or coefficient changed	1970 output (D4)	Increase in output of own sector		Change in household expenditure	Col 5 per £ col 3
		Actual	Percentage		
1	2	3	4	5	6
1	283 5	6 415	2 3	+1 006	+0 16
2	219 5	7 533	3 4	+0 316	+0 04
3	102 5	1 352	1 3	—0 654	—0 48
4	110 6	3 970	3 6	—0 018	—0 00
5	98 3	1 998	2 0	—0 117	—0 06
6	161 7	5 149	3 2	—0 135	—0 03
9	439 2	2 791	0 6	—0 114	—0 04

NOTES

Col 3 E g the 6 415 represents the increase in the output of sector 1 in consequence of an increase of 10% in that sector's output coefficient

Col 5 The entries represent the changes in consequence of changes in coefficients as numbered in column 1

If the 10% change be regarded as too small the reader may multiply the percentages shown in column 4 by a suitable multiplier to decide whether policies determined on Trial D4 would stand if, in fact, the export coefficients were so changed Certainly the export coefficients are the most difficult to predict, as has already been pointed out

Though sector 2 output shows an increase of £7 5 million for its own coefficient increase, the same trial shows an increase of only £1 0 in sector 1 despite the fact that sector 2 draws so much of its input from sector 1 The explanation is that increased sales of sector 1 to sector 2 are effected by the reduced expoits of sector 1 to itself because of the reduction in its coefficient

Column 6 shows that £1 increase in sector 1 improves welfare (equated to household expenditure) more than a £ increase in any other sector This remark may serve as an introduction to the next part of the paper

4 A LINEAR PROGRAMMING EXPERIMENT

Problems of many types may be propounded for optimal solution in connection with the IO decision model presented in part 2 of this paper We might try to optimize household consumption, GNP, GDP, producttiviy, etc , subject to the constraints of the system It has been emphasised more than once that by far the most dubious set of coefficients used in the model are those for visible exports It seemed therefore that it might be useful to find the optimal pattern of these exports It is assumed that, if the showing of the model merits it, the national planners are prepared to consider a promotional campaign to stimulate those exports which the optimal solution favours, instead of leaving them to the free play of external demand It is certain that vigorous promotional effort will be required *in general* to stimulate exports if the 4% plan is to be realised It may suffice to state if Ireland is content merely to maintain its 1960 share of world markets (as measured by the 1970 UK import anticipations of NEDC—see Table 11), visible exports in 1970 would amount to £201 million compared with £293 million required by programme D4 The prognosis is, however, good, between 1960 and 1963 visible domestic exports rose in volume at the rate of 7 2% per annum, almost identical with the 7 3% required by D4—see Table 13B Since promotion will be necessary *in general* the optimum solution may serve as a guide in particularity of industrial sector

The problem proposed for solution was the following given an increment in total non-factor exports, i e $X'+X''$, of £1 million, how should this be distributed amongst sectoral exports so as to minimize GDP, i e Z? It was decided to treat invisible exports X'' as a single variable, though for the model D4 it is endowed with a unitary coefficient vector, like X' However, visible exports of the 7 sectors, X'_1, , X'_6, X'_9, were regarded as 7 variables, as well as X'_{10}=exports of imports, or re-exports

The Constraints

The preference function was, therefore, the variable Z which was included in the system, so that the equation defining it, namely (2 5) (ii) is one of the constraints of the system In fact, the constraints consist of the whole system of equations (2 5), except that in the 9 equations (2 5) (i), X'_i is substituted for x'_iX', so that the variable X' disappears from the system, as the LP problem is an incremental one, all the constants in the equations disappear also Exports of sectors 7 and 8, i e X'_7 and X'_8, negligible in 1960, were deemed zero This series of constraints numbered 19

It was deemed advisable to tie exports of services X'_9, consisting mainly of distribution costs on visible exports, rigorously to these exports using the proportionality of 1960 Thus an additional constraint is the equation

$$(4\ 1) \qquad X'_9=0\ 12 \sum_{i=1}^{6} X'_i$$

Finally there is the equation expressing that total exports equal £1 million It curiously happens, however, that, treating this constraint as an equation, an inacceptable solution emerged from the computer, for reasons which are not understood Instead it was decided to present the computer with a constraint in the form of an inequality

$$(4\ 2) \qquad X''+X'_9+X'_{10}+ \sum_{i=1}^{6} X'_i \lesseqgtr 1$$

There were accordingly 21 constraints in all, so that, in accordance with linear programming theory the optimal solution contains 21 non-zero variables One slack variable must be introduced in the inequality (4 2) to convert it into an equation and the 20 constraint equations each require an artificial variable, according to the method of solution used by the computer The number of variables in the LP system is accordingly 48, of which 27 are original and the remaining 21 either slack or artificial The optimal solution given in column 2 of Table 19

TABLE 19

OPTIMAL INCREMENTAL LINEAR PROGRAMMING SOLUTION FOR DISTRIBUTION OF £1 MILLION NON-FACTOR EXPORTS

Sector outputs	1970 value (D4)	LP incremental value	Col 3 as % col 4
1	2	3	4
	£m	£m	%
1 Agriculture, forestry, fishing, Y_1	283 5	1 361	0 48
2 Food processing, Y_2	219 5	0 335	0 15
3 Drink, tobacco, Y_3	102 5	0 131	0 13
4 Textiles, apparel, Y_4	110 6	0 118	0 11
5 Metals, engineering, vehicles, Y_5	98 3	0 163	0 17
6 All other manufacturing, mining, Y_6	161 7	0 227	0 14
7 Construction, Y_7	104 2	0 174	0 17
8 Electricity, gas, waterworks, Y_8	54 2	0 102	0 19
9 Services, Y_9	439 2	0 663	0 15
Exports, sector			
1 Agriculture, forestry, fishing, X'_1	84 2	0 893	1 06
9 Services, X'_9	31 4	0 107	0 34
Gross domestic product, Z	904 7	1 990	0 22
Gross fixed capital formation, V'	153 8	0 338	0 22
Change in stock, V''	19 0	0 181	0 95
Household expenditure, H	696 2	1 374	0 20
Household saving, S_h	83 5	0 165	0 20
Total subsidies, B	120 0	0 075	0 06
Government saving, S_g	—7 5	0 315	n m
Income paid abroad by sectors, P	29 1	0 042	0 14
Non-factor imports, M	445 2	0 835	0 19
Net investment from abroad, N	8 9	0 123	n m

n m =not meaningful

The optimal pattern of an increment of £1 million in non-factor exports would involve confining these exports to sectors 1 Agriculture, forestry, fishing and 9 Services There would be no invisible exports and no visible exports from the other sectors The resulting maximum value of GDP (i e Z) would be £1 990 million so that the incremental GDP-export multiplier is nearly 2 It is not surprising that the LP solution should so favour agricultural exports since this sector has a high content of added value as Table 5 has shown It will be noted that, despite our using the total incremental export constraint (4 2) in the form of an equality, exports actually total £1 million in the optimal solution (i e the sum of the increments for X'_1 and X'_9 in the table) This means that the slack variable in this constraint is zero

The reader may be interested to verify that the incremental values shown in Table 19 satisfy the equations of the model, e g $V'=0\ 17Z$, $S_h=0\ 12H$ etc In particular, the incremental extern account is

	£m		£m
Non-factor exports	1 000	Non-factor imports, M	0 835
Investment from abroad	—0 123	Income paid abroad, P	0 043
	0 877		0 878

The small difference in the sums is due to rounding

In absolute magnitude the percentages shown in column 4 are without significance since they depend on the £1 million incremental value chosen for exports if the figure used were £10 million the percentages would be exactly ten times as great as those shown The object of the column is comparative Of course, the percentages would be zero for the non-optimal basis variables not shown, since their incremental values are zero

Of greater practical importance than the optimal solution, useful only in establishing the optimal incremental value of GDP, is the determination of the effect on this value, if, in fact, the export increment of £1 million were differently distributed amongst sectors From LP theory it is known that when the variables in the optimal basis have been identified these can be eliminated from the expression for the preference function, using the equal number of equations, so that an expression can be derived for the preference function in terms of the non-optimal basis variables In the present application the computer produced the following expression for the preference function Z in its reduced form

(4 3) $$Z=1\ 990-0\ 4411X''-0\ 3611X'_3-0\ 8327X'_2-0\ 9486X'_4-1\ 1749X'_5-1\ 0833X'_6-1\ 9927X'_{10}-1\ 9834R$$

R is the slack variable which converts constraint (4 2) into an equality

The whole object of LP is to produce an expression for the preference function of form (4 3), i e one in which the constant term is followed by terms in the non-basis variables (necessarily non-negative) with non-positive signs (i e minus or zero) Then it is obvious that the constant term must yield the required optimal value for the preference function, since any positive values attributed to the variables in the reduced form expression must diminish the maximum value (i e the constant term, in the present case £1 990 million)

If all of the £1 million cannot be attributed to sectors 1 and 9, (4 3) shows how promotional policy should be directed Choice must be determined by the magnitude of the coefficients, in ascending order In order of merit, policy should favour sector 2—Food processing, then invisible exports X'' (mostly visitors' expenditure), industrial exports X'_3, X'_4 and X'_5 (always of given amount, say £100,000), having much larger coefficients with negative signs are not conducive to improving the added value of the economy in anything like the same degree as sector 1—Agriculture etc (which is in a class by itself), sector 2 and invisible exports

Suppose now that the £1 million increment, instead of being optimal, is distributed as in Trial D4, what would be the effect on the optimal solution? Since the increment is to be used to the full, the value of the slack variable R is zero The values of the other variables in (4 3) would be $X'_2=0\ 1638$, $X'_3=0\ 0337$, $X'_4=0\ 0752$, $X'_5=0\ 0460$, $X'_6=0\ 1111$, $X'_{10}=0\ 0140$, $X''=0\ 2672$ Using these values in (4 3), incremental GDP becomes £1 512 million instead of the optimal £1 990 million It would be for the planning authority to decide whether this reduction of the GDP export multiplier from 2 to $1\frac{1}{2}$ is worth attending to, having regard to the promotional effort involved, especially with agricultural exports, or whether exports should be allowed to follow extern market demand

Since the object of this exercise is not only to set up a decision model, but to acquire experience in its use, it must be recorded that many LP runs had to be made on the computer, varying the input, before the comparatively satisfactory solution recorded here emerged In this connection the author would like to pay an additional special tribute to the ingenuity and patience of Mr F M O'Carroll (of An Foras Taluntais)

5 Summary and Conclusion

The main conclusion from the macro-economic approach in part 1 is the importance of having regard to the incremental capital-output ratio in promoting economic development In parts 2-4 various aspects of an IO model showing sectoral detail, with built-in macro-economic relationships, are examined Whatever be thought of the two approaches the author claims for them the merit of being global, which property should inform any acceptable model

As stated in more general terms at the outset the main criticism always advanced against IO models of the type expounded here is that, as we do not know the values of the coefficients in the future year of reference, much dubiety must attach to results based on even the most painstaking prognosis of these entities Obviously there are grounds for such criticism and absolute reliance must not be placed on the results The analysis in part 3 of this paper has perhaps shown that, at least as regards the inter-industry coefficients, the effect of quite large aberrations in the coefficients on the results is less than one might have supposed The real question is from the decision making point of view, and policy decisions have to be made *now*, should regard be had to calculations of the type presented in this paper, subject to further work as indicated below? Are these calculations better than nothing in the decision making context? The author's answer is an unqualified "yes", to the question so posed In fact, the author cannot conceive of a plan without calculations on these lines being an integral part, if a very small part, of such a plan And the author knows of no better approach than the IO one Work summarized in this paper shows that, with a computer of suitable capacity and the experience gained, the model is easily and cheaply operational

To emphasize the point once more, this paper is experimental No pretence to reality can be claimed from the compression of the economy into a mere nine sectors, each very heterogenous in character The model is designed for application to a larger CSO model, for the base year 1960, of 36×36 dimensions, on the lines of that for 1956 which the Office kindly made available to ERI Furthermore, it is suggested that for each year in the development period 1960-1970 an IO table should be prepared, at current and constant prices With such tables the evolution of the coefficients and of the details of final demand can be studied, and the prognoses rendered more reliable It is certain that as the years advance changes must be made in any plan, if only because, amongst economic sectors, there will be laggards and leaders Remedial action, if it is to be prudent and timely, must be based not merely on the latest IO table but on the evolution of the coefficient values

As regards the model itself, i e as in the series of equations (2 5) provision should be made separately for competitive and non-competitive imports, as in the CSO 1956 table This would merely involve adding certain terms to the equation, with no change in principle

Much more investigational work could have been done even on the present model, if the author had the time and the resources Such matters as the evolution, industry by industry, of the rather crucial ratio of factor input in relation to total cost should have been studied if trends were discernible they should have been extrapolated to the year of reference For such study it would not be necessary to construct IO tables for past years It may be added that, by reference to the ratio net output gross output as published annually by CSO in connection with CIP there is little evidence of a trend in any sector except agriculture and, in the model, account has been taken of some lowering in this ratio for 1970 It may be at this point that the author should remark that though the year of reference is ostensibly 1970 he would be well content if the outcome would be something like the figures displayed in this paper, regarding them as applying on average to a series of years, say 1968-72 However, it is emphasized again that the figures are not designed as forecasts They are only intended as a guide to prudent action in the light of all the information available now and as it becomes available in the years ahead

References

[1] Leontief, W W — *The Structure of the American Economy 1919-39* (Oxford, Second Edition, 1951)

[2] Leser, C E V — "Imports and Economic Growth in Ireland", *ERI* Paper No 14 (1952)

[3] Leser, C E V — "The Irish Economy in 1962 and 1963", *ERI* Paper No 15 (1963)

[4] Leser, C E V — "Pattern of Personal Expenditure in Ireland", *SSISI* (1963)

[5] Stone, R and Brown, J A C — "A Long-term Growth Model for the British Economy", *Europe's Future in Figures* (North Holland Publishing Company, edited R C Geary, page 287 *seq* , 1962)

Abbreviations

CD	Census of Distribution
CIP	Census of Industrial Production
CSO	The Central Statistics Office, Dublin
ERI	The Economic Research Institute, Dublin
GFCF	Gross fixed capital formation
GDP	Gross domestic product at market prices
GNP	Gross national product at market prices
IO	Input-Output
LP	Linear Programming
NEDC	National Economic Development Council (UK)
SSISI	Statistical and Social Inquiry Society of Ireland

DISCUSSION

In proposing a vote of thanks, *Mr Oslizlok* thanked the Society not only for allowing him to speak to this excellent paper first but also for giving him an opportunity of expressing publicly the debt of gratitude which all those engaged in economic research owed to Dr Geary for his inspiration and leadership.

Dr Geary's latest work is without doubt a very valuable and a very timely addition to the analytical tools at the disposal of the economic planner. Of necessity, however, most of the paper is taken up by a presentation of a technique, which is not the easiest of subjects for comment: there are no right or wrong techniques and even a blunt instrument in the hands of a master of Dr Geary's calibre is bound to produce attractive results. This is not to suggest that the proposed input-output model is a blunt instrument, indeed (if one can pursue the metaphor) the instrument may well be too sharp for inexperienced hands.

Before proceeding any further I feel impelled to say that I do not share the view which Dr Geary expressed in an earlier work but which is of relevance in the discussion of tonight's paper, namely that "the only topic in economics worthy of serious consideration is economic programming at all levels". A "literary" economist, not to be included in the priesthood of the scientists, I agree however that economics is frequently concerned with matters of policy and I have few doubts about the advantages that can be derived from the proper use of a formal technical apparatus. It is doubtful whether a formal instrument like the input-output model can be used to solve economic problems but it is clear that it can be very useful in assisting in the search for a solution. Above all, the proposed method of projection on the basis of past trends may provide a safeguard against prognostications into the never-never land of unrealised trends, but is the 1947-62 experience regular or long enough to indicate in detail future potentialities?

The last query refers of course to the assumptions and not to the method—in a way this sums up my main reaction to Dr Geary's input-output model: as a method it is admirable, capable of amendment if applied to short-term prognostications where *marginal* changes appear more appropriate, a trickier problem arises in attempting to apply the method in the actual conditions of changing prices, my main concern, however, is that *formally* the model (as any other model) is always right. A few years ago, in the Netherlands (where the use of econometrics and economic models is much more advanced than in this country) an interesting discussion about models took place between Professor Witteveen of the Rotherdam School of Economics and Dr Holtrop of the Nederlandsche Bank which—to quote Dr Holtrop himself (International Monetary Fund Staff Papers, February 1957)—"proved that, within the limits of their own definitions and methods of approach, both authors *from a formal point of view* had been right in their *contradictory* conclusions". Elsewhere Dr Holtrop comments: "it is not difficult to imagine that the public showed itself rather flabbergasted by these contradictory opinions"[1] (Italics supplied)

The equations of a model will always give the right (formally that is) values for any change in any one of the variables That is the fun, but also the trouble, with algebraically streamlined economic models they are efficient, they ensure consistency and logical certainty, but any doubts about the underlying presupposed behaviouristic relationships which constitute the model are likely to be soon swept away (if entertained at all) by the algebraic permutations which follow I hasten to add that this cautionary remark is not addressed to Dr Geary who is too old a hand at this kind of work to fall into a trap which has, however, caught at one time or another most lesser economic modellers

I may perhaps be allowed to conclude these few arid thoughts with the only substantial point of difference with Dr Geary's approach In a decision model (which, as the title implies, the proposed input-output model purports to be) the presupposed rules of behaviour between the economic phenomena constituting the model must contain assumptions about causality and the direction of that causality To put it differently, only a model which includes as "causes" the variables which can be influenced by economic policy can, it seems to me, be properly described as a *decision* model I do not of course question the rather obvious fact that statistical observation very rarely reveals causal inter-connections, the point I am making is that, for this very reason, statistical observations by themselves—that is unaided by economic logic—must be considered poor material for decision models

Dr T K Whitaker I want to express my respect and admiration for the mastery of input-output and other statistical methods displayed by Dr Geary, for his enterprise as an innovator, and for his gift of rapid and lucid oral summarisation The Statistical Society is fortunate in being chosen by the author to sponsor the publication of this paper which will add greatly to its prestige

As Dr Geary mentioned, the paper is a synthesis of a series of confidential papers produced over the past year or so and made available to the Department of Finance I would like to say in public that the work done by Dr Geary, Dr Leser and their associates has made a most important contribution to the development of improved programming techniques in Ireland We hope in the Department of Finance to become more sophisticated—or rather more specific and efficient—in our programming as time goes on and the continuing co-operation of the Institute in devising helpful techniques will be most welcome

Dr Geary has himself emphasised that the future is only within limits a function of the past, it is not entirely its prisoner We do need and expect an improvement in the principal determinants of economic growth, including, in particular, a higher savings ratio and a lower capital-output ratio

Dr Geary is right to emphasise the particular importance of the capital-output ratio—the need for ensuring we get the utmost value from capital expenditure in terms of addition to net output This has implications both for the public and the private sector In the public sector it is necessary, however difficult it may be, to keep the kind of proportion between immediately and remotely-productive projects which will ensure the rate of economic progress on which social improvement fundamentally depends In the private sector, too, one would like to see the capital-output test becoming better known, so that there would be a very lively concern on the part of managements and workers to get the utmost value out of new investment Modernisation and adaptation will not be enough unless there is fully effective (involving in many cases continuous) use of new plant and machinery

Dr Geary deserves our best thanks for a most valuable paper

Dr C E V Leser This is obviously not a mere methodological study, it derives and gives important conclusions Of course, the inserted coefficients like consumption and export patterns as well as the mathematical model are to some extent arbitrary In the macro-economic model, ratios other than the import ratio may vary, for example, the savings ratio is not constant in the short run and need not be constant in the long run The increase in the import ratio is perhaps best seen, in the light of the input-output model, as a consequence of structural changes in the economy

There is some danger that the result of the linear programming experiment may be quoted out of context The most relevant linear programming problem is not the choice between £1 million of agricultural or £1 million of industrial exports, but between the use of scarce resources—labour or capital—towards increasing exports, the answer to such a problem may not favour agricultural exports to the same extent if at all

Replying to the discussion, *Dr Geary* said that he agreed with practically everything that had been said, except the too flattering comments on the paper itself, though he would be less than human if he were not gratified by them Mr Oslizlok was perfectly right in emphasising that no mathematical construct can supply all the answers This was the speaker's object in prefacing his paper by a quotation from Dr Erhard's article Furthermore, the opening and closing sections of the paper are full of protective colouring, against the suggestion of exact determination in planning, or still less, programming What the speaker hopes he has shown is the great flexibility of the input-output approach in its function of indicating, if not always what is feasible (unless "feasibility" be defined in a special way), at least what is impracticable It is the vast potential of the electronic computer (very imperfectly realised, even by econometricians) which has added a new dimension to the usefulness of input-output

The speaker would go even further than Mr Oslizlok in stating his conviction that (as is no doubt the case in Ireland) the planning authority should draw on the wisdom and experience of literary and political economists, sociologists and social psychologists, to say nothing of the great public who have to be persuaded to do certain things Economic development is very much a combined operation And, of course, Mr Oslizlok is right in emphasising the price aspect, which input-output decision-making can and should take into account

With regard to Dr Whitaker's comments, the Economic Research Institute esteemed it a privilege and an honour to be invited to co-operate with the Department of Finance in certain aspects of economic programming and we hope that this co-operation will continue, if only to enable us really to merit his too generous praise The speaker is glad to now feel able to place on record his indebtedness to the Department (in the person of Professor Louden Ryan) especially as regards the basic assumptions, and for helpful discussion at many stages He is pleased that Dr Whitaker agrees as to the importance of the capital-output ratio When regard is had to this parameter, the saving ratio and the external payments balance as economic weather gauges economic processes will take care of the rest

My colleague, Dr Leser, is right too If, instead of total exports, one constrained the factors one might come up with a different choice for optimal policy Linear programming helps with, but does not settle, the problem of choice As stated in the text there are many ways in which mathematical programming can be used in conjunction with linear programming, the intention of the short section was to call attention to this fact Dr Leser's observation enables the speaker to give a small puff to a monograph by the President and the speaker to be published shortly on "Elements of Linear Programming" in Griffin's well-known series *Mathematical Monographs and Course* in which labour and capital constraints are dealt with at some length