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Notes and Comments

Efficiency and Size: Comments and Extensions*

DOUGLAS TODD
London

A recent contribution to this Review[1] considered a number of measures of business efficiency and applied these to a sample of firms in the Irish Plastics Industry. The purpose of this note is to extend the analysis somewhat further and to draw attention to some features of the efficiency measures chosen.

It must be recognised at the outset that making inter-firm efficiency comparisons based on summary measures is a dubious exercise. Quite apart from the comments of R. J. Ball and cited by M. Ahmed, it has been stressed that environmental factors are of great importance. Hall and Winsten (1959) place emphasis on these, which lead one to doubt whether conventional accounting information can enable one to make inferences about resource allocation within firms. Indeed, Amey (1969) argues that the concept of efficiency in business enterprises is wholly meaningful only in the context of an individual firm viewed at different moments of time and with reference to a budget plan. Nevertheless, in practice one rarely has anything other than accounting information and often imperfect information at that. On a pragmatic basis it remains legitimate for one to ask whether one firm or industry is more or less efficient than another. Certainly, the question is important for policy and in this as well as other respects the general approach adopted by Ahmed is both relevant and of interest.

One of the summary measures considered by Ahmed is an index of social efficiency, defined as

$$\bar{Z}=(wL+qK)/V$$

Where $\bar{Z}$=index of social efficiency, w=money wage rate, L=labour input, K=value of capital, V=gross value added, q=social cost of capital.

*The author is a Senior Economic Adviser in H M Treasury. It is understood that the comments in this paper are to be attributed in no way to any Government Department. I am grateful to Peter Richardson for programming and computational assistance and advice; the usual disclaimer applies.

1. M. Ahmed, "Size and Efficiency: the Case of the Irish Plastics Industry," *The Economic and Social Review*, Vol. 8, No. 1.

With respect to one of Ahmed's experiments, the actual market wage rate is taken to equal the "social efficiency wage", whilst the average rate of return on capital for the Irish plastics industry is used to represent the opportunity cost of capital. For any firm, this total factor productivity index can then be written as

$$TPF=(wL+qK)/(wL+rK),\ q=15.5 \text{ per cent} \tag{1}$$

or

$$TFP=1-\gamma(r-q), \text{ where } \gamma=(K/V) \tag{2}$$

In this form, the efficiency index is almost identical to that suggested by Jack Downie some years ago now in his classic work "The Competitive Process"[2]. It is also the form used by several other researchers in this field (e.g., Dunning and Rowan, 1968). A practical deficiency of this efficiency measure can be seen in the following way. The expected value of (2) is:—

$$E(TFP)=1-E(\gamma r)+qE(\gamma) \tag{3}$$

Since q is *defined* as $E(r)$ it is clear that the value of $E(TFP)$ depends on the sign of cov (γr). If there is statistical independence between the observed rate of return on capital and the capital output ratio over the sample of firms, the mean value of the index must equal unity. $(E(\gamma r)$ can then be written as $E(\gamma)\ E(r)=qE(\gamma).)$

One would expect capital intensity, which in this context is the ratio (K/V), to rise with size of firm and Ahmed's sample tends to confirm this. The problem, therefore, can be turned into one concerned with the relationship between the rate of return and size of business. There is no necessary reason why the two *should* be positively correlated; indeed reasons associated with the goals of the firm and the existence of managerial discretion might suggest that the reverse is more likely. Analytically, the result will depend on the relationship between size, the cost/output ratio and the ratio (K/V),[3]. The covariance term $E(\gamma r)$ could therefore take any given value for a number of reasons and the resulting index TFP, when defined in this way, will be uniquely dependent on this.

A related point concerns the correlation likely or otherwise, between the index *TFP* and the rate of return r. If (2) is rewritten as

$$\begin{aligned} TFP&=(1+\gamma q)-\gamma r \\ &= a+\gamma r+\mu,\ \gamma<0 \end{aligned} \tag{4}$$

We have the form of a simple two variable linear regression relationship with μ being the disturbance term and $a=(1+\gamma q)$. It is clear from (2) and (3)

2. Downie, (1958) in fact, defines his Index in terms of sales and not value added.

3. Numerous studies have examined the relationship between profitability and size of firm. An analytical view of the relationship is presented clearly in Joseph Steindl (1947).

that if (4) is applied to a cross section of firms, a perfect correlation between *TFP* and r can occur only if all firms in the sample have identical capital/output ratios. In other words, if this condition exists, the direct implication is that *TFP* and r can be used interchangeably as efficiency indicators.[4]

The index depends very heavily on not only the value for the social cost of capital q, but also on the way this variable is defined. In the case considered, $q=E(r)$ leads to certain difficulties of interpretation. The argument can be extended to the case where a social wage is selected which is different to the actual market wage w. In this case the index becomes a weighted factor productivity measure with weights equal to the social valuations of relative factor returns; that is

$$TFP=w^*(L/V)+q(K/V)$$

Where w^* = the social or shadow wage.

Ahmed considers this case also but, clearly, any results are highly sensitive to the choice of w^* and q. For example, a value of q which is taken to be the same as the mean rate of return for the industry is not consistent with a value of w^* which is equated to the minimum legal wage. Some value closer to the long survival rate of return for firms in the industry is preferable as Ahmed recognises.[5] However, the actual value for the rate of return on capital r, which appears in the denominator is influenced by numerous factors. As a consequence of this, one is never sure about whether or not the index *TFP* is picking up factors associated with market imperfections, local monopoly factors or what. In other words, the caution advised by Hall and Winsten is clearly of relevance when applying this kind of index across firms; even if they are drawn from the same industry. These remarks remain relevant in what follows.

An alternative approach to the measurement of productive efficiency and one not considered explicitly by Ahmed, is to estimate a technical efficiency frontier from the sample of firms. This method first suggested in an important paper by M. J. Farrell (1957), is simple and has a number of attractive properties. The precise details of the approach and its technical and analytical features are set out in a number of papers (Aigner and Chu, 1968), (Timmer, 1970), (Todd, 1971). It is assumed that firms are operating in a competitive environment and producing homogeneous output. Very briefly, the object is to construct an envelope from a set of observations on factor inputs which is *technically* efficient in the sense that no firm in the sample can lie outside

4. Dunning and Rowan (1968) examine the relationship between *TFP* and r but do not seem to recognise the implication of a high correlation between these two variables. A discussion of this point may be found in Todd (1971).

5. Ahmed op cit, p. 13.

(closer to the origin) this convex set of points. By means of such a construction it becomes possible to compare any observed firm in the sample with a hypothetical firm *assumed* to be located on the frontier and on a ray projected from the origin. Figure 1 Below illustrates this method of interpretation.

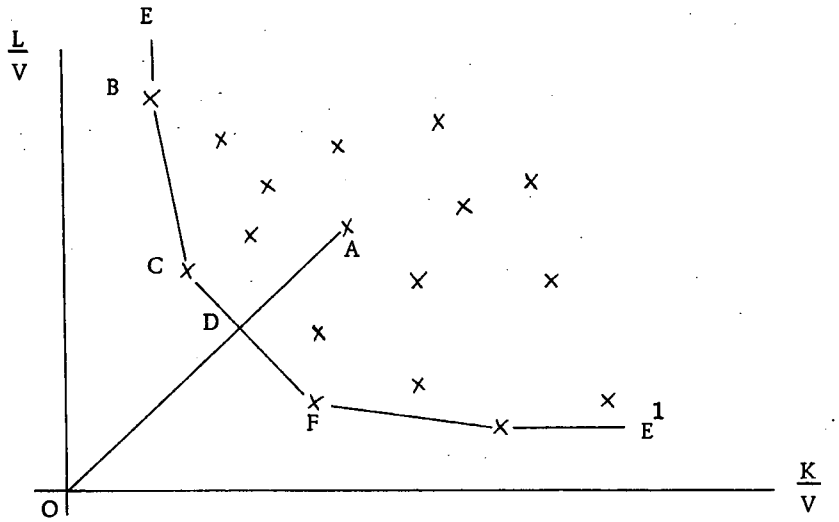

Fig. 1 *The Technical Efficiency Frontier.*

Each of the points represents a firm located in the (L/V), (K/V) plane. The frontier E-E^I is made up of facets from firms BC and F etc. which have equal technical efficiency. The frontier constructed in this way resembles the textbook isoquant.[6] Firm A, for example, is not technically efficient in that it does not lie on the frontier E-E^I. Its measured technical efficiency is given by the ratio $OD/OA<1$. One can see, therefore, that firm A is being compared with the hypothetical firm D which is located on the efficient frontier and producing a unit of net output (V) whilst employing the same ratio of labour to capital as firm A.[7] A feature of this approach which has some appeal is its close correspondence to the analysis of X-inefficiency in the firm. For example, one could regard the ratio OD/OA as a measure of X-inefficiency in the case of firm A. For completeness, such an interpretation would demand

6. It also has a number of characteristics associated with the "coefficient of resource utilisation"; see Debreu (1951).

7. It is important to recognise that the above description assumes constant return to scale for all firms. An extension of the analysis to include the case of increasing returns is provided in Farrell and Fieldhouse (1962).

an additional dimension to Figure 1 which might represent a managerial or organisational input in the formal production function of the firms.

The description of the industrial structure in the plastics industry given in the paper suggests that reasonably competitive conditions prevail. However, it is suggested also that significant product differences may exist.[8] In this respect the weaknesses inherent in the *TFP* index must carry over automatically to the analysis set out below. Using the sample of inter-firm data most generously provided by Ahmed in Table 1 of his paper, technically efficient frontiers have been estimated for the three size groups of firm in the sample and for the whole sample of firms also.[9]

There is a problem about how one should define labour input. It can be argued that the money wage is a better measure of labour input than is physical numbers employed because the former ought to reflect more close differences in the quality characteristics of labour between firms. Ideally we would like to have an index of man-hours for the labour input and an index of the flow of capital services.[10] However, it is of interest to see how Ahmed's same data set can be used in the way described and efficient frontiers have been computed using both numbers employed and the wage bill (W); each normalised for net output.

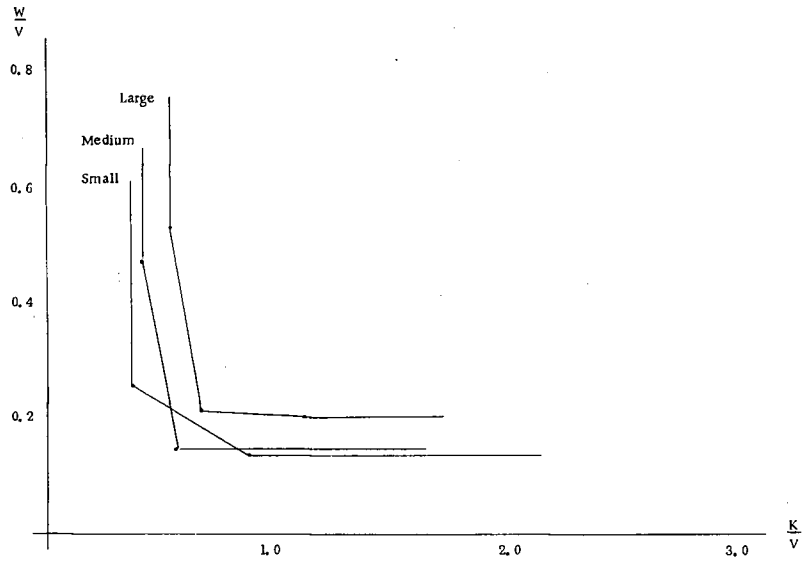

Fig. 2 *Technical Efficiency Frontiers (W/V, KV).*

8. Ahmed op cit, p. 6.

9. The precise method of constructing the envelope is given in Farrell (1957).

10. These remarks apply to the measurement of total factor productivity also. For an example of efficient frontiers constructed using the above definitions see P. Meller (1975).

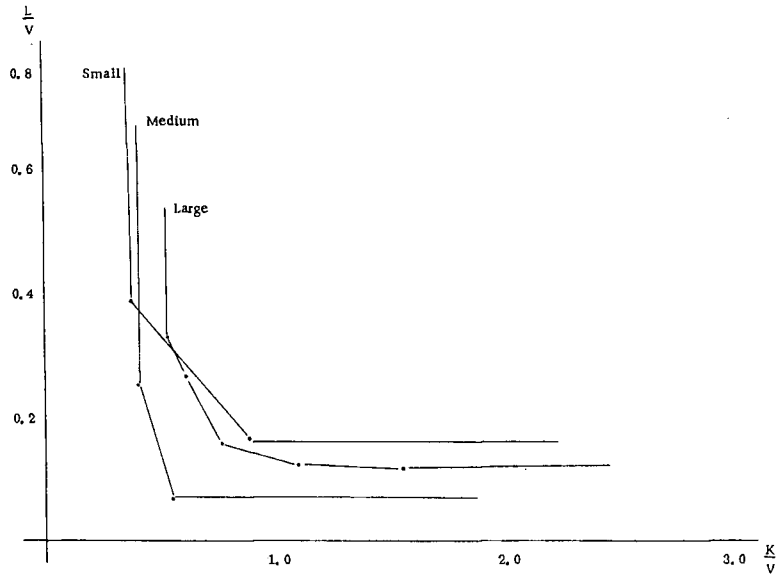

Fig. 3 *Technical Efficiency Frontiers (L/V, KV).*

Figures 2 and 3 show these efficiency frontiers under the two alternative assumptions about labour input. In both cases the frontiers intersect but inspection suggests that the medium sized group of firms are capable of being technically more efficient than the large firms. If the money value labour input assumption is preferred, one might also infer that the small firms are likely to be more efficient potentially than other size groups. Hence, these results lend some additional explicit support to Ahmed in his conclusion than in this particular industry, the large sized firms are likely to be the least efficient in resource use of labour and capital.

However, a difficulty with the Farrell approach is that the estimated frontiers use marginal data only from the available sample of firms. It follows that the location of the frontier "revealed" from this sample is extremely sensitive to any outlying observations. [11] Further, since the technique does not embody the usual principles of statistical inference it is not immediately obvious as to how one can evaluate the efficiency (in statistical terms) of the envelope generated by the observations. Whilst mean efficiencies for each size group can be calculated, such measures are relevant to the particular frontier of the given size class only; they do not carry over to calculations relating to some other technical frontier.

11. For a fuller discussion on these matters see Timmer (1970).

A very rough comparative indication is to estimate the technical efficiency frontier for the whole sample and compare means and variances of the three size classes with respect to this reference point. Secondly, the same statistics can be calculated for the "own" frontier of each size class. Again, it must be emphasised that this procedure depends heavily on any outlying points which might determine the exact location of the frontier.

Table 1A: *Overall efficiency frontier*

	Mean technical efficiency index	*Standard deviation*
Small	0·521	0·277
Medium	0·541	0·246
Large	0·538	0·218

Table 1B: *"Own" efficiency frontier*

	Mean technical efficiency index	*Standard deviation*
Small	0·547	0·293
Medium	0·593	0·271
Large	0·733	0·290

Tables 1A and 1B set out the relevant data for the case where the labour input is measured by the wage bill per unit of value added. With respect to the overall frontier there are no differences between mean technical efficiency which are statistically significant. There is very slight evidence, however, that the variance decreases with size of business unit and this would be consistent with results from the analysis of profitability and size of firm. In terms of the own efficiency frontier comparisons, mean efficiency with respect to "own" frontiers increases with size; variances are more or less constant.

Given the various weaknesses in the approaches suggested, together with the relatively small sample of experience in this particular industry, the results remain fairly consistent. Small and medium sized establishments seem to be capable of achieving greater technical efficiency; at least in terms of the static framework considered. In no case does the overall industry frontier have a large firm located on it.

Finally, it is of interest to see how the efficiency indices using the Farrell method compare with what is probably the most popular performance indicator, namely, the rate of return. Each of the firms in the sample were ranked in terms of the two alternative technical efficiency indices (ϵ_W and

ϵ_L) and the rate of return on capital (defined as gross profits divided by total assets). Spearman's rank correlation coefficient is given below for the two cases.

$\epsilon_{w.r}$	$\epsilon_{L.r.}$
$\rho = 0.953$	$\rho = 0.869$

These suggest that as a measure of static efficiency or aggregate performance in this particular industry, the simple rate of return might serve most purposes. It does not, of course, tell one anything explicit about relative factor usage.

One can speculate about these results within Jack Downie's conceptual framework. The suggestion would be that since the average size of firm is small in this industry with the smaller firms being capable of achieving greater technical efficiency, the "innovating mechanism" is a dominating factor. If this were not so, the "transfer mechanism" operating over time would lead to a concentration of resources in larger business units. Presumably, the small firms survive because they are successful in product innovation; so successful in fact that they offset those forces which might lead to industrial concentration.

It must be emphasised again that in the essentially pragmatic approaches such as those used in this note and Ahmed's paper, one is departing from strict theoretical requirements in a number of important respects. For many reasons, official and much other accounting information is not produced solely for the purposes of meeting the demands of economic researchers. Nevertheless, one often is faced with the task of seeing what information about an economic process a given data set is capable of yielding. The above can be regarded as an extension of one such an attempt.

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