Integrating Distribution-Free Adaptive Kriging-based Reliability Analysis with SQP for Optimal Design Considering Uncertainties

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ABSTRACT: Reliability-Based Design Optimization (RBDO) are gradually viewed as a critical method in many engineering problems and practical applications. In this paper, we propose a new RBDO method based on the reliability process which we developed recently. The reliability process is termed as "distribution-free adaptive Kriging-based Reliability analysis" and it could give better efficiency both in function and Kriging evaluations. Hence, this research further incorporates this new reliability analysis method with SQP optimization technique, to form a double-loop RBDO for searching a structural optimal design under uncertainties. A case study is adopted to demonstrate the accuracy and efficiency of the proposed RBDO. The proposed RBDO method has been implemented using MATLAB/Optimization toolbox, and its results have been compared with the literature study using other methods such as EMP-based PSO.

1. INTRODUCTION

Recently, in many complex engineering design systems, the uncertainties are inevitable and the random variables in system start to be considered. Therefore, different types of reliability-based design optimization (RBDO) methods have gained more attention and been widely applied. It incorporates probabilistic analysis with an optimization technique to ensure the system reliability within an acceptable limit, which is not considered in the traditional deterministic design optimizations (DDO).

Different from DDO methods, which do not concern the variability and uncertainty of design variables or random variables, address only the performances but not the reliability and robustness, RBDO methods evaluate probabilistic constraints and take the variability and uncertainty of design variables and random variables into account by combining the reliability theory and optimization.

The main idea behind RBDO methods is to use reliability index obtained from previous systems or prototypes, if the value higher or lower then we sign, the design variables would be replaced and then use the new pairs of design variables as input parameters for next iteration. After several times iterations, the system would stop while it finds the optimal design variable that satisfied the reliability index as close we signed as possible.

2. RBDO PROBLEM FORMULATION

If an optimization task is formulated without uncertainty, that is, no random design variables and random design parameters are considered, which is termed as DDO and is expressed as Eq. (1):

$$\begin{gathered}\min f(d)\\ \textit{subject to } g(d) \le 0\\ d^L \le d \le d^U\end{gathered} \tag{1}$$

where $f(.)$ and $g(.)$ are the objective and constrained functions, d is the vector of design variables, d^L and d^U represent the lower and upper bounds of d, respectively.

A typical RBDO problem, which involves probabilistic constraints, is generally formulated as follows Eq. (2):

$$\begin{gathered} \min f\left(d, X, P\right) \\ \textit{subject to } prob.\left[g_i(d, X, P) \le 0\right] \le p_{fi} \\ d^L \le d \le d^U \\ d \in R^{NDV},\ X, P \in R^{NRV} \\ i = 1, \dots, NC \end{gathered} \tag{2}$$

where $prob.\left[g_i(d, X, P) \le 0\right]$ is the probability function which denotes the probability of satisfying the i^th constrained function $\left[g_i(d, X, P) \le 0\right]$, X is the vector of the random design parameters to represent the uncertainty, P is the vector of the random design variables, p_{fi} is the signed probability of satisfying the i^th constrained, and represents the probability of failure at the same time, NDV is the number of the design variables, NRV is the number of the random variables, NC represents the number of the probabilistic constraints, and i is the reliability requirement for the i^th constraints.

On the other hand, Reliability requirement can be equivalently expressed in terms of the generalized reliability index, which is defined as Eq. (3):

$$\beta(d) = -\Phi^{-1}(prob.\left[g_i(d, X, P) \le 0\right]) \tag{3}$$

Where Φ is the cumulative distribution function (CDF) of the random variables distributed by standard normal.

3. RBDO ALGORITHM

3.1. Reliability analysis

The reliability analysis method in this study is which we recently proposed termed as "Distribution-Free Adaptive Kriging using PSO enhanced DoE with Hollow-Hypersphere Space (DF-AK-PSO-HHs)". This method modified the original adaptive Kriging approach by using particle swarm optimization (PSO) to reduce the initial sample pool size and find the training points. Furthermore, proposing hollow-hypersphere space (HHs) to create a ring region is helpful to reduces PSO searching domain, and enhances the search efficiency as well. To validate the availability of the proposed methods, several problems are carried out and the results indicate that better efficiency can be proved both in function and Kriging evaluation, from Thedy and Liao (under review).

3.2. Optimization: sequential quadratic programming (SQP)

The optimization techniques would be used in the paper to address the problems is sequential quadratic programming (SQP). The SQP is very efficient and often chosen to collate in RBDO methods. It's a gradient-based iterative method for nonlinear optimization and commonly used on mathematical problems for which the objective function and the constraints are twice continuously differentiable.

3.3. Support vector machine (SVM)

PSO method is used in our reliability analysis method to find the training points, thus, the limit state functions found by different initial points may be slightly different. The uncertainty is existed, as a result, we may get different solution of failure probability from DF-AK-PSO-HHs methods. Though the variance of many times trails is relatively small than other methods such as MCS, it still cannot be ignored. In RBDO methods, the optimizer decides each step by the former outcomes of reliability index (or failure probability). If the reliability index jumps up and down at a fixed point, the optimizer may be confused and hard to converge to the extreme value. Thus, we come up the idea of combing machine learning to handle the uncertainty problem from reliability. The method used was support vector machine.

Support vector machine (SVM) is a supervised machine learning algorithm that uses a linear kernel function to learn the mapping from an input vector to a target vector, by finding the line that best represents the relationship between an input vector and a target vector in terms of minimizing the distance between them. The SVM algorithm is used for classification and regression problems. It can be used for both binary and multiclass classification problems, working by finding the optimal hyperplane that separates data into two classes, where each class has its own probability of being selected.

To use SVM in this RBDO process, in the beginning, the size of initial sample pools should be decided, which affect the effectiveness of SVM learning. The more points SVM learn initially, the more accurate the resulting surface will be to provide better predictions in the result. The positions of initial sample are often randomly determined. In this research, Latin Hypercube Sampling (LHS) is adopted to generate the positions of initial sample that are more scattered in the design domain.

After initial samples were designed, their reliability index can be found by DF-AK-PSO-HHs method as well. SVM could learn by sample pools and their corresponding reliability index and create the solution surface, the expected model should be able to throw back the only value when different designed value input, which solving the problem resulting from uncertainty.

In the following part, the results from two different methods would be compared and discussed.

3.4. *Numeric example*

To verify the proposed formulation for RBDO, the performance is demonstrated in this section for cantilevered beam problem about the allowable displacement. Objective function of this example is to find the minimum of cross-sectional area. Limit state function is considered that the tip displacement is required to be smaller than the allowable displacement (d_0 =2.2535 inches). The target reliability is set to 99.865% (i.e., β = 3). The problem can be expressed in the Eq. (4) and Figure 1:

$$Min.\ W = f(b,h) = b*h$$
$$\mathrm{G}_D = g(E,x,y,b,h) = d_0 - \frac{4L^3}{Ebh}\sqrt{(\frac{y}{h^2})^2 + (\frac{x}{b^2})^2} \quad (4)$$
$$s.t.\ \mathrm{Prob}(\mathrm{G}_D{<}0) \le \Phi(-\beta)$$

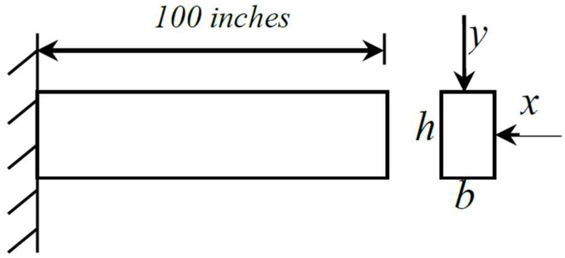

Figure 1: A cantilever beam under vertical and lateral bending.

In this case, the width (b) and height (h) of the beam are the design variables, with lower and upper bounds are 2 and 4 respectively, the modulus of elasticity (E) and the two loads (x, y) at the free end are the random variables with detailed statistics are provided in Table 1. The length of the cantilevered beam (L) is 100 inches.

Table 1: The statistics of the random variables used

Random variable	*x(lb)*	*y(lb)*	*r(psi)*
Distribution type	*normal*	*normal*	*normal*
μ	*500*	*1000*	*29E6*
σ	*100*	*100*	*1.45E6*

μ is the mean value and σ is the standard deviation

In the first part, the RBDO algorithm is the double-loop approach using SQP as the optimizer and using DF-AK-PSO-HHs for reliability analysis. From the limit state function, we can know it’s a nonlinear problem. According to literature study from Liao and Ivan (2013), the mean value of objective values from several trials is 9.2375, therefore, we use this value as the benchmark for comparison.

First of all, initial point should be determined. One question should be asked is which range of initial point can find out the local minimum in this RBDO method. The experiment is designed as following. 21 values of 2.0, 2.1, 2.2, …. ,3.9, 4.0 for each design variables were chosen, resulting in 441 initial points.

To ensure the procedure stability, we set a threshold that failure probability should larger than10^{-8}. The function of threshold is to prevent the DF-AK-PSO-HHs method spending too much time or to stop suddenly due to incalculably reliability index, which means the value is too small to be found. If the failure probability is smaller than this specified threshold, the model would stop and try the next pair of initial point to avoid taking redundant time to search, which is inefficient and memory-wasting even it may be able to run out the results eventually.

In the optimization part, different step size may affect the result and efficiency. To demonstrate this idea, 4 kinds of step size are used, which are 0.01, 0.05, 0.1 and 0.5. For each step size, three RBDO simulations are conducted. The results indicate that the range of initial points can find the local minimum are generally the same. This outcome is expected because the success of an RBDO simulation depends on whether reliability analysis is accomplished at the initial point. If the failure probability at the initial point is smaller than 10^{-8}, the reliability process will stop and the proposed RBDO will not able to find the optimum.

However, in terms of computational efficiency, there're clear gap between different steps size. Step sizes with 0.05 and 0.1 show better performances, which cost less time and fewer iterations. On the other hand, 0.01 and 0.5 may exceed the appropriate range of steps size, so more optimization iterations are needed. Thus, Table 2 only displays results of step sizes of 0.1 and 0.05. Hereby, using a suitable steps size is a critical decision in the case.

441 initial points are used to investigate the sensitivity of the proposed RBDO, as shown in Figure 2. The grid with green color represents that an RBDO task is completed with a promising result. The mean and standard deviation of all optimal values for each green grid and the comparison with literature study are expressed in Table 2. In order to compare the computational efficiency with the second methods, mean of iterations and mean of costing time are also recorded. It is found that only part of initial points can successfully find the optimum. The key is whether the reliably analysis is successfully conducted for a given initial point. The proposed RBDO still has room to be improved.

Table 2: Results of the proposed RBDO methods

	Liao and Ivan (2014)	*Current study*	
		Step size=0.1	*Step size=0.05*
Mean of obj. value	*9.2374*	***9.2221***	*9.2235*
Standard Dev.	*0.0356*	***0.0035***	*0.0073*
Max value	*9.3376*	*9.2440*	*9.2708*
Min value	*9.2216*	*9.2174*	*9.2168*
Mean of iterations	-	*66.7364*	*69.0310*
Mean of costing time(s)	-	*326.9317*	*326.4153*

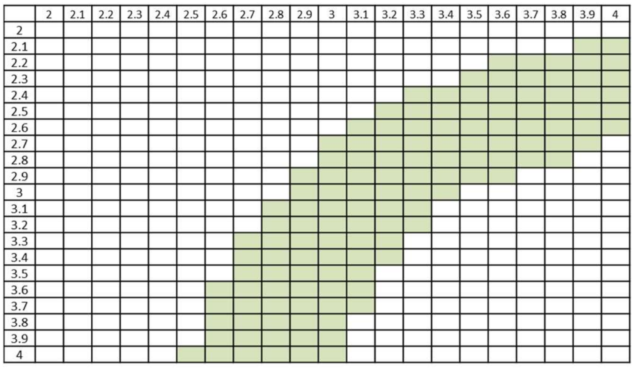

Figure 2: Region of initial point which be able to find the objective value

In the second part, we would like to know if it can bring out a better result through combining RBDO and SVM, and compare the computed efficiency as well.

First, the sample pools for SVM learning is needed. The same as the first method, the upper and lower bound of initial points are 2 to 4. The sizes of initial points in these analyses are 100, 200, 500 and 1000. Each point, which is a (b, h) pair, would be put in the DF-AK-PSO-HHs method to generate the corresponding reliability index. SVM would learn the pair and reliability index and establish the surface. To compare the result, Monte-Carlo Simulation (MCS) with sample size 10^6 is also used. 10 times simulations would be conducted with both RBDO combined SVM methods and MCS method using the same initial data to compare the results fairly.

According to Table 3, as initial sample pools increasing, the variance of objective value of SVM learning decrease, which shows an improvement on stability. However, this approach highly depends on the initial sample pools, and their corresponding reliability index from DF-AK-PSO-HHs method.

Table 3: Results of combing RBDO and SVM (average for 10 runs)

initial sample pools		*100*	*200*	*500*	*1000*
MCS	*b*	*2.6938*	*2.7194*	*2.7144*	*2.6985*
	h	*3.4220*	*3.3788*	*3.4012*	*3.4229*
	obj.	*9.2181*	*9.1885*	*9.2321*	*9.2365*
	var of obj.	*0.0002*	*0.0013*	*0.0010*	*0.0006*
	Func count	*32.6*	*35.5*	*57.9*	*53.3*
DF-AK-PSO-HHs	*b*	*2.7073*	*2.6991*	*2.7589*	*2.7164*
	h	*3.3850*	*3.3817*	*3.3483*	*3.3932*
	obj.	*9.1643*	*9.1274*	*9.2376*	*9.2171*
	var of obj.	*0.0166*	*0.0036*	***0.0001***	***0.0004***
	Func count	*78.4*	*69.7*	***45.6***	***37.3***

To compare the computational efficiency for the aforementioned two methods, number of limit state function evaluation for each approach is counted. "Function count" in Table 2 and Table 3 means the iteration numbers of design variables shifting from initial points to optimal points. In each shift, DF-AK-PSO-HHs method is used to compute the corresponding reliability index, and to determine the direction gradient. In average, the reliability index at each trial needs 32 runs using the proposed method.

In the first method with step size of 0.1, from Table 2, the average function count is 66.7, which means in the grid with green color in Fig. 2, from the initial points to the optimal points, 66.7 shifts are required in average. Omitting the area where is unable to find the local minimum, including 311 initial pairs, there're about $130 \times 66.7 \times 32$ = 277,472 times of limit state function evaluation in total, computed by 130 initial points resulting in convergent solution.

In the second method with initial sample pools of 1000, Omitting the useless points, there're about 550 points left to establish the feasible surface. 550×32 = 17,600 limit state function evaluation are needed for completing the surface. After completing the surface, the initial points would be easy and quick to converge to the optimal points, which is about 37.3 times of shifts. Because no more reliability analysis is needed, the time spending and the numbers of limit state function evaluation are both greatly reduced. In consequence, the second method provides a better performance in terms of computational efficiency.

4. CONCLUSIONS

RBDO method aims to optimize objective function under reliability constraints. In this study, a new RBDO algorithm is proposed by integrating DF-AK-PSO-HHs and SQP. The results show that the proposed RBDO methodology is feasible, and more stable and reliable than previous methods in limit region. It not only produces an accurate response but also reduces variance between each trial. However, the key is whether the reliably analysis is successfully conducted that depends on initial point. The proposed RBDO still has room to be improved. On the other hand, the approach of combining proposed RBDO method and SVM provide a better computational efficiency, but in order to establish the feasible surface, this method highly depends on the initial sample pools. It still needs some modifications to achieve our expectation on both stability and efficiency.

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