

Enhancement of quality factor for TE whispering-gallery modes in microcylinder resonators

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Abstract: The enhancement of quality factor for TE whispering-gallery modes is analyzed for three-dimensional microcylinder resonators based on the destructive interference between vertical leakage modes. In the microcylinder resonator, the TE whispering-gallery modes can couple with vertical propagation modes, which results in vertical radiation loss and low quality factors. However, the vertical loss can be canceled by choosing appropriate thickness of the upper cladding layer or radius of the microcylinder. A mode quality factor increase by three orders of magnitude is predicted by finite-difference time-domain simulation. Furthermore, the condition of vertical leakage cancellation is analyzed.

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References and links

1. S. L. McCall, A. F. J. Levi, R. E. Slusher, S. J. Pearton, and R. A. Logan, "Whispering-Gallery Mode Microdisk Lasers," *Appl. Phys. Lett.* **60**(3), 289–291 (1992).
2. M. Arzberger, G. Bohm, M. C. Amann, and G. Abstreiter, "Continuous room-temperature operation of electrically pumped quantum-dot microcylinder lasers," *Appl. Phys. Lett.* **79**(12), 1766–1768 (2001).
3. Y. D. Yang, Y. Z. Huang, and Q. Chen, "High-Q TM whispering-gallery modes in three-dimensional microcylinders," *Phys. Rev. A* **75**(1), 013817 (2007).
4. Y. Z. Huang, and Y. D. Yang, "Mode coupling and vertical radiation loss for whispering-gallery modes in 3-D microcavities," *J. Lightwave Technol.* **26**(11), 1411–1416 (2008).
5. V. N. Astratov, S. Yang, S. Lam, B. D. Jones, D. Sanvitto, D. M. Whittaker, A. M. Fox, M. S. Skolnick, A. Tahraoui, P. W. Fry, and M. Hopkinson, "Whispering gallery resonances in semiconductor micropillars," *Appl. Phys. Lett.* **91**(7), 071115 (2007).
6. A. Taflov, and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time Domain Method, Third Edition*. (Boston: Artech House, 2005).
7. B. J. Li, and P. L. Liu, "Numerical analysis of the whispering gallery modes by the finite-difference time-domain method," *IEEE J. Quantum Electron.* **32**(9), 1583–1587 (1996).
8. F. L. Teixeira, and W. C. Chew, "Systematic derivation of anisotropic PML absorbing media in cylindrical and spherical coordinates," *IEEE Microw. Guid. Wave Lett.* **7**(11), 371–373 (1997).
9. W. H. Guo, W. J. Li, and Y. Z. Huang, "Computation of resonant frequencies and quality factors of cavities by FDTD technique and Pade approximation," *IEEE Microw. Wirel. Compon. Lett.* **11**(5), 223–225 (2001).
10. X. S. Luo, Y. Z. Huang, W. H. Guo, Q. Chen, M. Q. Wang, and L. J. Yu, "Investigation of mode characteristics for microdisk resonators by S-matrix and three-dimensional finite-difference time-domain technique," *J. Opt. Soc. Am. B* **23**(6), 1068–1073 (2006).
11. Q. Y. Lu, W. H. Guo, D. Byrne, and J. F. Donegan, "Compact 2D FDTD method combined with Padé approximation transform for leaky modes analysis," *J. Lightwave Technol.* (to be published).
12. B. E. Little, J. P. Laine, D. R. Lim, H. A. Haus, L. C. Kimerling, and S. T. Chu, "Pedestal antiresonant reflecting waveguides for robust coupling to microsphere resonators and for microphotonic circuits," *Opt. Lett.* **25**(1), 73–75 (2000).
13. D. Marcuse, *Light Transmission Optics, Second Edition*. (New York: Van Nostrand Reinhold Company, 1982).

1. Introduction

High quality (Q) factor microcavities have attracted great attention for physics research and engineering applications, such as cavity quantum electrodynamics, microlasers, light filters for optical communication, and biosensors. The whispering-gallery modes (WGMs) in the

semiconductor microdisk have the merit of high Q factors and ultrasmall mode volumes [1]. The WGMs in the microdisk are laterally trapped inside the cavity through total internal reflections at the cavity-air boundary, and vertically confined by a large refractive index difference. Semiconductor microcylinders that are vertically confined by semiconductor materials have better thermal conductivity and current injection efficiency than the microdisk on a pedestal [2]. We have found that the transverse magnetic (TM) WGMs almost do not have vertical radiation loss in the microcylinder resonator with vertical semiconductor waveguiding, even of the radius of 1 μm [3,4]. But the transverse electric (TE) WGMs in the microcylinder can couple with the vertical propagation modes when the radius is smaller than about 5 μm , which results in low Q factors for TE WGMs in small microcylinders [3,4]. Upper and lower distributed-Bragg reflectors (DBRs) are used to suppress the vertical radiation loss, and high Q WGMs are observed in the microcylinder with DBRs [5].

In this paper, we investigate the mode characteristics for TE WGMs in microcylinder resonators using the finite-difference time-domain (FDTD) technique [6]. The variations of mode Q factors with upper cladding layer thickness or radius of the microcylinder exhibit high Q factors for TE WGMs at conditions corresponding to the cancellation of the vertical leakage. The vertical leakage cancellation is caused by the destructive interference between up and down leakage waves as the up leakage wave reflects from the upper boundary. By calculating the propagation modes in the cladding layer and core layer, the leakage cancellation condition can be predicted analytically.

2. Numerical model

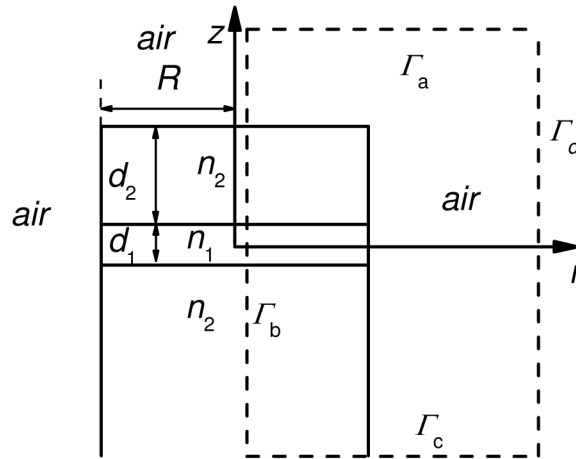


Fig. 1. Cross section of a microcylinder and the FDTD simulation region.

The FDTD method is widely used in modeling optical microcavities. Based on the circular symmetry, the 3D problem of a microcylinder can be transformed into a 2D one with the angular field dependence of $\exp(i\nu\phi)$, where ν is the angular mode number [7]. So the numerical simulation can be performed in the cross section of the microcylinder with an infinite lower cladding layer as shown in Fig. 1, with the FDTD calculating window bounded by Γ_a , Γ_b , Γ_c and Γ_d , where R , d_1 , and d_2 are the radius, the thicknesses of the core layer and the upper cladding layer, n_1 and n_2 are the refractive indices of the core and cladding layers, respectively. The perfect matched layer (PML) absorbing boundary conditions in circular cylindrical coordinates [8] are used on Γ_a , Γ_c and Γ_d , with the boundaries Γ_a , Γ_c and Γ_d placed 1, 5 and 2 μm away from the upper cladding layer, the core layer's lower and lateral boundaries, respectively. The spatial steps Δz and Δr are set to be 10 and 20 nm, respectively, and the time step Δt is chosen to satisfy the Courant condition. At the inner boundary Γ_b at $r = 4\Delta r$, the condition $\psi_m \propto r^m$ is used on the field components E_z and H_z based on the asymptotic behavior of the Bessel function [7]. In the simulation, an excitation source with a cosine

impulse modulated by a Gaussian function $P(x_0, y_0, t) = \exp[-(t-t_0)^2/t_w^2]\cos(2\pi ft)$ is added to one component of the electromagnetic fields at a point (x_0, y_0) inside the microcylinder, where t_0 and t_w are the times of the pulse center and the pulse half width, respectively, and f is the center frequency of the pulse. The time variation of a selected field component at a point inside the microcylinder is recorded as a FDTD output, then the Padé approximation [9] is used to transform the FDTD output from the time-domain to the frequency-domain, and finally the mode frequencies and Q factors are calculated from the obtained intensity spectrum.

3. Numerical results and discussion

We consider a microcylinder with the vertical refractive index distribution of $air/n_2/n_1/n_2 = 1/3.17/3.4/3.17$, $d_1 = 0.2 \mu\text{m}$, and $R = 1 \mu\text{m}$. The mode Q factor of the $TE_{9,1}$ mode versus the thickness of the upper cladding layer d_2 is calculated and plotted in Fig. 2. The dotted line at $Q = 550$ is the mode Q factor of $TE_{9,1}$ mode in the microcylinder with an infinite upper cladding layer. A strong oscillation of the Q factor is found with the increase of the thickness of upper cladding layer, and the peak values of 7.69×10^5 , 8.02×10^5 , and 5.70×10^5 are obtained at $d_2 = 0.84$, 1.42 , and $2.08 \mu\text{m}$, respectively. The peak values of the Q factor are close to the Q factor of 9.87×10^5 obtained by the s-matrix method [10], which neglects the vertical radiation loss. The results indicate that the vertical radiation loss almost vanishes for the $TE_{9,1}$ mode at $d_2 = 0.84$, 1.42 and $2.08 \mu\text{m}$. The vertical leakage cancellation phenomenon was also observed in semiconductor deep ridge waveguides [11], and a reflecting layer was added to suppress the leakage loss [12]. Using a long optical pulse with a narrow bandwidth to excite only one mode, the field distribution can be obtained with an impulse at $t_w = 10^4 \Delta t$, $t_0 = 3t_w$ and $f = 195 \text{ THz}$. Figures 3(a) and 3(b) depict the z -directional electric field E_z , and Figs. 3(c) and 3(d) depicts the z -directional magnetic field H_z , for $TE_{9,1}$ mode at $d_2 = 1.42$ and $1.50 \mu\text{m}$, respectively. The component E_z is confined well in the core and upper cladding layer as $d_2 = 1.42 \mu\text{m}$, but oscillates in the lower cladding layer as $d_2 = 1.50 \mu\text{m}$ corresponding to a vertical loss.

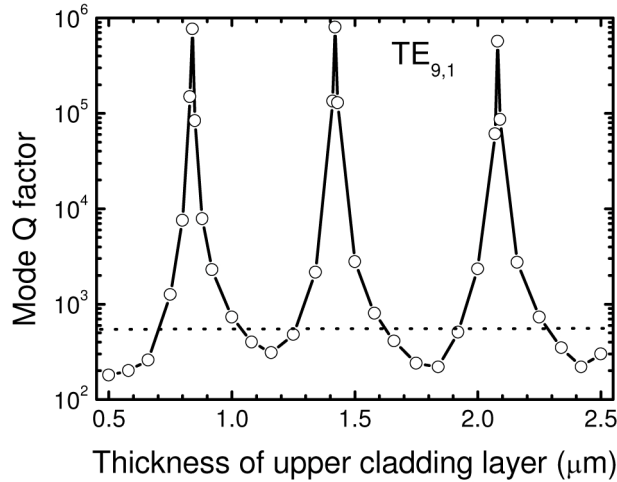


Fig. 2. Mode Q factor of $TE_{9,1}$ mode versus the thickness of upper cladding layer in the microcylinder with $R = 1$ obtained by FDTD simulation. The dashed line is the mode Q factor for the microcylinder resonator with an infinite upper cladding layer.

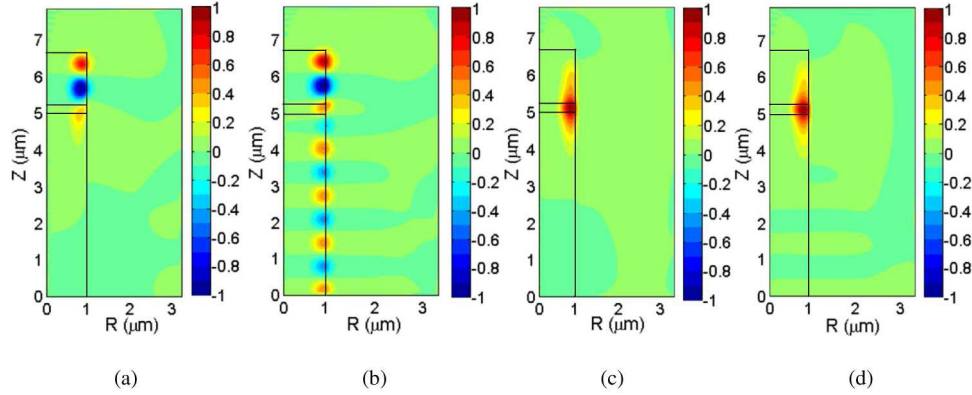


Fig. 3. Field distributions of E_z for $TE_{9,1}$ mode at the upper cladding layer thickness d_2 of (a) $1.42\mu\text{m}$ and (b) $1.50\mu\text{m}$, and H_z at d_2 of (c) $1.42\mu\text{m}$ and (d) $1.50\mu\text{m}$.

Because the Q factor of $TE_{9,1}$ mode in the microcylinder with an infinite upper cladding layer is only 550, we expect the enhancement of mode Q factor is caused by the reflection from the upper boundary of the upper cladding layer. In the microcylinder resonator, the TE WGM can couple to the HE vertical propagation mode of the upper and lower cladding layers when the mode wavelength of the TE WGM is smaller than the cut-off wavelength of the HE mode, which results in a vertical radiation loss [3]. The energy couples from the TE WGM to the HE modes in both the upper and lower cladding layers, and propagates in the z and the $-z$ directions as shown in Fig. 4.

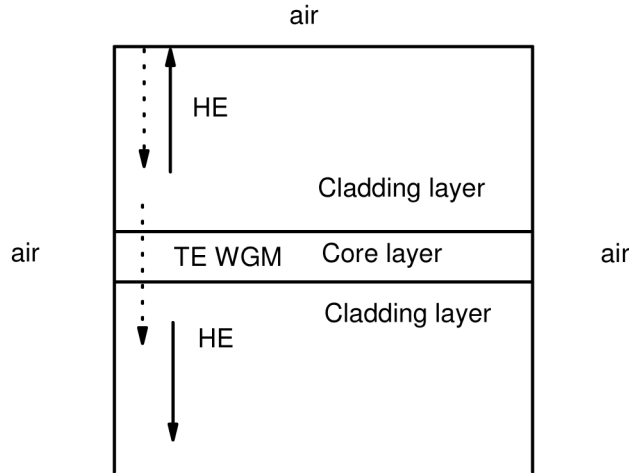


Fig. 4. The schematic diagram of the destructive interference for the vertical radiation waves.

The leaked HE mode in the upper cladding layer will be reflected by the upper boundary of the upper cladding layer, and then propagates through the upper cladding layer and the core layer to the lower cladding layer. Then two leaked HE modes will interfere with each other destructively or constructively, so as to decrease or increase the vertical radiation loss through the lower cladding layer. The propagation constant of the HE mode can be obtained by solving the eigen-equation of the circular waveguide [13] at a wavelength of $1.5315\mu\text{m}$, which is the WG mode wavelength of $TE_{9,1}$. Because E_z and H_z have a difference of π in the reflection phase shift, we focus on one component of the electromagnetic fields. If all the phase shifts are for the E_z component, the phase difference between the two HE modes in the lower cladding layer can be calculated by

$$\Phi = \theta_1 + 2\beta_2 d_2 + \beta_1 d_1 + \theta_2, \quad (1)$$

where β_2 and β_1 are the propagation constants of HE modes in the cladding layer and the core layer, θ_1 is the phase difference between the excited HE modes at the upper and lower boundaries of the core layer from the TE WGM, and θ_2 is the phase shift of the HE mode reflected on the upper boundary of upper cladding layer. In the microcylinder with an infinite upper cladding layer, the component E_z of the TE WGM is antisymmetric relative to $z = 0$ plane, which means $\theta_1 = \pi$. In fact, this is also a good approximation for the microcylinder with the thick upper cladding layer, and is suitable for our simulation when $d_2 > 0.5 \mu\text{m}$. Because the HE mode has the dominant component (E_z, H_r, H_ϕ) when the vertical propagation constant is small, we use the TM approximation to calculate the phase shift θ_2 at the incident angle of $\arccos(\beta_2/n_2 k_0)$ similar to a slab waveguide, where k_0 is the wavenumber in vacuum. Then the destructive interference condition can be obtained from $\Phi = (2m + 1)\pi$ as $d_2 = (0.64m + 0.17) \mu\text{m}$. When $m = 1, 2, 3$, it gives $d_2 = 0.81, 1.45$, and $2.09 \mu\text{m}$, which are in good agreement with the FDTD results in Fig. 1.

In the microcylinder resonator with the constant thickness $d_2 = 1.5 \mu\text{m}$, we calculate the mode Q factors of $\text{TE}_{v,1}$ modes versus the radius R and plot them in Fig. 5 as open squares. The Q factors of the same modes in the microcylinder resonator with infinite upper cladding layer are plotted as open circles. The mode wavelengths are chosen near $1.55 \mu\text{m}$ with the corresponding angular mode number v increases from 9 to 33 with a step of 1, as R increases from 1 to $3 \mu\text{m}$. Two peak values appear at $R = 1.08$ and $2.34 \mu\text{m}$, the corresponding angular mode numbers are 10 and 25, and the Q factors are 2.16×10^6 and 2.33×10^5 , respectively. The enhancement of Q factors is caused by the destructive interference similar to the microcylinder resonator with varying thickness of the upper cladding layer. The phase difference Φ in Eq. (1) is found to be 4.90π and 2.85π at $R = 1.08$ and $2.34 \mu\text{m}$, respectively, which correspond to $m = 2$ and 1. However, they are slightly smaller than $(2m + 1)\pi$ because some approximations are used in the calculation of the reflection phase shift.

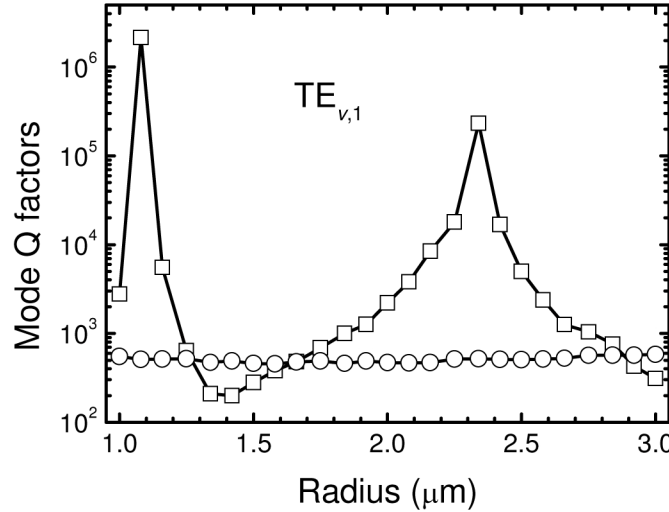


Fig. 5. The mode Q factors of $\text{TE}_{v,1}$ modes with the wavelengths near $1.55 \mu\text{m}$ versus the radius of microcylinder resonator obtained by FDTD simulation. The squares and circles are for the resonator with $d_2 = 1.5 \mu\text{m}$ and an infinite upper cladding layer.

For the first order radial mode $\text{TE}_{v,1}$, the vertical leakage can be totally canceled because only one propagation leaky mode exists, but high order radial modes will have different characteristics. For a microcylinder resonator with $R = 1.5 \mu\text{m}$, we calculate the mode Q factor of $\text{TE}_{11,2}$ mode versus the thickness of upper cladding layer d_2 and plot them in Fig. 6. Oscillation of the Q factor is also found with an increase of the upper cladding layer thickness,

two peak values of 3.70×10^3 and 2.37×10^3 are found at $d_2 = 1.06$ and $1.88 \mu\text{m}$, respectively. The magnitudes of the Q factors are two orders smaller than the value of 2.96×10^5 obtained by the s-matrix method which neglects the vertical radiation loss [10]. The result indicates that the vertical radiation loss does not vanish. The reason is that the $\text{TE}_{11,2}$ mode can couple to three propagation modes $\text{HE}_{11,2}$, $\text{HE}_{11,1}$, and $\text{EH}_{11,1}$. The destructive interference condition for the $\text{HE}_{11,2}$ can be obtained as $d_2 = 0.81m + 0.24$ at the mode wavelength $1.5609 \mu\text{m}$ of $\text{TE}_{11,2}$. When $m = 1$ and 2 , it gives $d_2 = 1.05$ and $1.86 \mu\text{m}$, which agree well with the FDTD results. However, the destructive interference cannot be realized at the same value of d_2 for the three propagation modes, and the vertical radiation loss does not vanish. Because the coupling between the modes with the same radial mode number is one order in magnitude larger than that between the modes with different radial mode numbers, the Q factor is mainly determined by the coupling with the $\text{HE}_{11,2}$ mode.

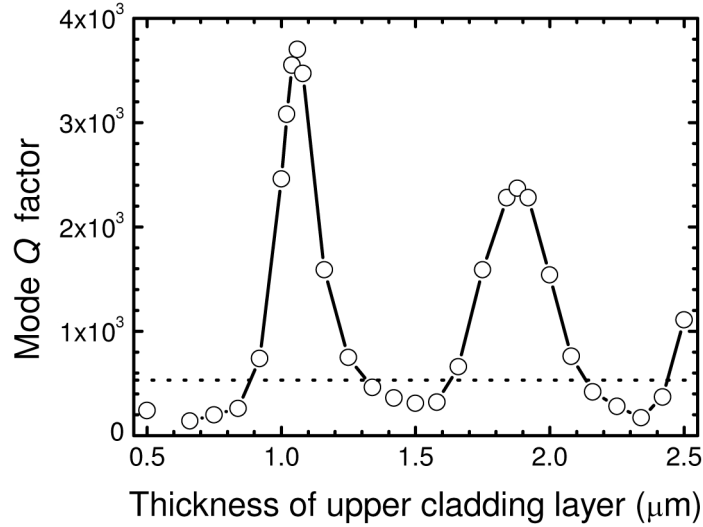


Fig. 6. The mode Q factor of $\text{TE}_{11,2}$ mode versus the thickness of upper cladding layer in the microcylinder with $R = 1.5 \mu\text{m}$ obtained by the FDTD simulation. The dashed line is the mode Q factor for the microcylinder resonator with an infinite upper cladding layer.

4. Summary

In conclusion, we have investigated the destructive interference between the leakage propagation modes in microcylinder resonators, which results in vertical leakage cancellation and Q factor enhancement. The results show that the destructive interference yields a high Q TE whispering-gallery mode in a microcylinder with a vertical semiconductor waveguiding when suitable thickness of the upper cladding layer or suitable radius of the microcylinder is chosen.

Acknowledgments

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