

CLOSED-LOOP CONTROL OF A HOLLOW CYLINDER APPARATUS

B.C. O'Kelly

Department of Civil, Structural and Environmental
Engineering, University of Dublin, Trinity College,
Ireland; Ph + 353 1 6082387, Fax + 353 1 6773072,
email: bokelly@tcd.ie

P.J. Naughton

School of Engineering
Institute of Technology, Sligo, Ireland;
Ph + 353 71 9155222, Fax + 353 71 9155390,
email: naughton.patrick@itsligo.ie

ABSTRACT

The instrumentation and closed-loop control of a new hollow cylinder apparatus (HCA) that facilitated accurate generalized stress path measurements on soils over the pseudo elastic to failure strain range are described. Axial and torsional loads were applied across the length of the hollow cylindrical test specimen that was subjected to lateral confining pressures using the HCA. The control algorithms that were used to continually adjust the stresses induced in the specimen in order to follow a prescribed stress path are presented. The development of the control algorithms, including analysis of the stress path measurements and the methods used to achieve the necessary precision of measurement and control, are described. Numerical corrections were implemented in the control algorithms to account for the effects of restraint and penetration of the rubber membranes that were used to enclose the specimen walls. Proving test results that demonstrated the capability of the new apparatus to follow complex stress paths and regulate the stress components induced in the specimen to within 1 kPa of the targeted values are presented.

INTRODUCTION

The system of loads and confining pressures that are applied to the test specimen in the hollow cylinder apparatus (HCA) must be carefully controlled and the dimensions of the specimen must be revised regularly in order to accurately follow specified stress paths in generalized stress space. The apparatus must be automatically closed-loop controlled to achieve the necessary precision of measurement of the specimen deformational response and control of the loads and confining pressures.

This paper presents the control algorithms developed to target specific stress paths in generalized stress space using a new HCA. The methods used to axially load and torque the specimen, apply the confining pressures, update the specimen dimensions, compute the mean stress components and correct for the effects of the rubber membranes that were used to enclose the specimen walls in real time during a test are

described. The results of proving tests that were designed to show the capabilities of the new apparatus are also presented.

PRINCIPLE OF HOLLOW CYLINDER TESTING

The HCA subjects a test specimen to four surface tractions: an outer cell confining pressure p_o , an inner bore confining pressure p_i , an axial load W , and a torque T , Figure 1a. The axial w , radial (u_i , u_o), and twist θ , deformations of the specimen are shown in Figure 1b. Figure 1c presents the stress components (σ_z , σ_r , σ_θ , $\tau_{z\theta}$), and Figure 1d the principal stresses (σ_1 , σ_2 , σ_3) induced in an element of the specimen wall. The application of the torque causes rotation α_σ of the σ_1 - σ_3 stress axes.

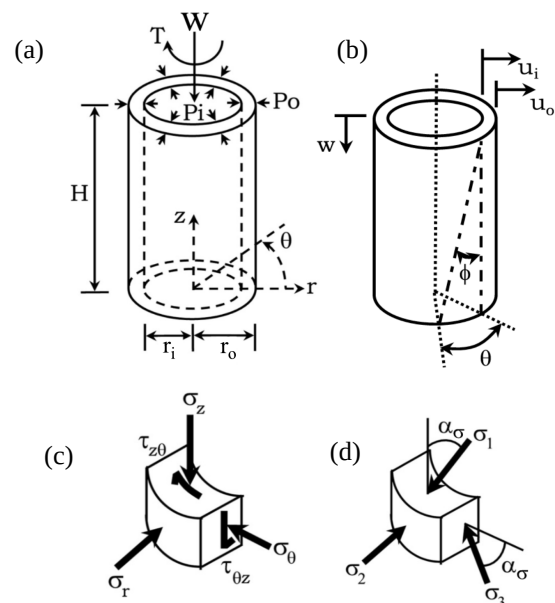


Figure 1. (a) Loads and confining pressures; (b) Deformational response; (c) Mean stress components and (d) Mean principal stresses acting on an element of the specimen wall.

OVERVIEW OF UCD HCA

The new HCA, developed at University College Dublin (UCD), Ireland [1,2,3], comprised the hollow cylindrical test specimen (35.5 mm inner radius, 50.0 mm outer radius and 200 mm long) contained in a pressure cell that was mounted on a mobile table, Figure 2. Zero-backlash, axial and torsional loading mechanics were secured to a reaction frame located beneath the pressure cell. The ends of the hollow cylindrical specimen were in contact with annular platens inside a 340 mm diameter by 600 mm long acrylic cylinder. The outer cell chamber, the specimen and its inner bore cavity were independently sealed. Digital pressure-volume controllers [4] independently controlled the hydrostatic confining pressures that were applied to the inner and outer walls of the specimen via rubber membranes, and the specimen back-pressure that was applied via the bottom loading platen. Axial and torsional loads were induced across the length of the specimen by restraining the top end of the specimen while axial and twist boundary displacements were applied to its lower end via a 25.0-mm diameter, loading piston. Ribbed, annular sintered bronze discs that were fastened to the loading and reaction platens ensured the full transfer of the applied torque across the length of the specimen.

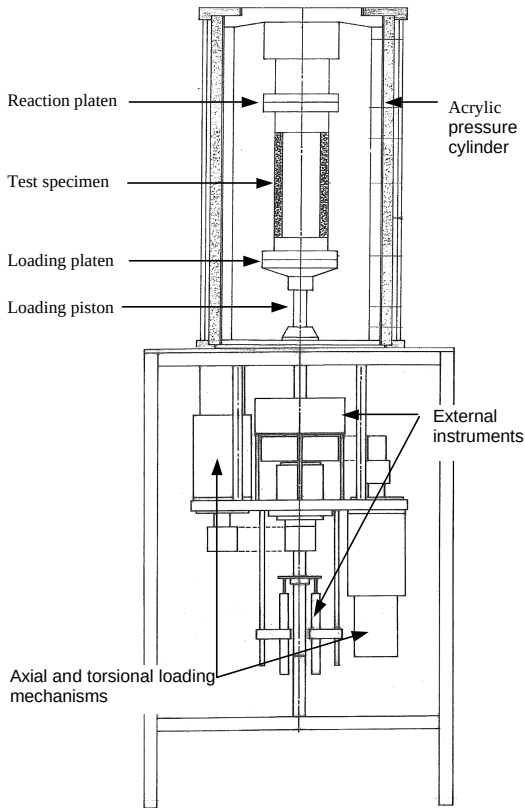


Figure 2. UCD HCA.

Screw and spline ball bearings that were fitted about the lower end of the loading piston actuated its smooth vertical, rotary or spiral movement. Separate drive units that comprised a motor and precision gearbox were coupled to the screw and spline bearings using pre-tensioned block and tackle. The axial and torsional working loads of the mechanisms were 19.3 kN

and 103 N.m, which facilitated generalized stress path testing to failure of all geomaterials. Slack in the axial and rotary directions was limited to 1 μ m and 15 arc-seconds when the motion of the loading piston was reversed.

Instrumentation that recorded the axial load, torque, confining pressures and the deformational response of the specimen were located both inside and outside the pressure cell. The axial and torsional loads induced across the length of the specimen were recorded using a combined thrust-torque transducer that was integrated into the top reaction platen. Internal instrumentation in contact or close proximity to the specimen walls recorded the actual deformational response over a central gauge length of the specimen. Two double-axis inclinometers recorded the axial and twist deformations over the mid third of the specimen length. A single-axis inclinometer also recorded the twist deformation. Two proximity transducers recorded the radial displacements at mid height of the inner and outer specimen walls.

The axial and twist deformations were also determined from the displacement and rotation of the loading piston that were recorded using two linear voltage displacement transducers (LVDT) and a rotary encoder that were located beneath the pressure cell. The radial displacements of the inner and outer specimen walls were also determined from the changes in the volumes of the specimen and its inner bore cavity that were recorded by the pressure-volume controllers and a volume change apparatus.

The operation range and calibrated resolution of the different instruments are presented in Table 1. Calibration issues were discussed by [3]. The accuracy of the instrumentation, expressed as a percentage of full scale output, was better than 0.5 %. The stress and strain components were resolved to 0.25 kPa and 0.5×10^{-3} % strain.

Table 1. Range and calibrated resolution of instrumentation.

Instruments		Range	Resolution
Pressure-volume controllers (x3)	Pressure	0 – 2 MPa	30 Pa
	Volume	0 – 200 ml	1 ml
Stepper motors (x2)	Axial	∞	9×10^{-4} deg
	Rotary	∞	3.3×10^{-4} deg
Thrust-torque transducer (x1)	Axial	0 – 4 kN	0.1 N
	Torque	0 – 50 N.m	0.00002 N.m
Inclinometers	Axial (x2)	0 – 20 mm	0.0025 mm
	Twist (x3)	0 to $\pm 30^\circ$	0.003°
Proximity transducers (x2)		0 – 6 mm	1×10^{-5} mm
Hollow shaft encoder (x1)		∞	0.018°
LVDTs (x2)		0 – 50 mm	0.01 mm
Volume change apparatus (x1)		0 – 200 ml	1 ml

RELATIONSHIP BETWEEN SURFACE TRACTIONS AND CONTROL PARAMETERS

The stress components (σ_z , σ_r , σ_θ , $\tau_{z\theta}$), were computed from the axial load, torque and confining pressures applied to the test specimen by considering the specimen as a thick-walled hollow cylinder with an isotropic linear-elastic constitutive response. The axial, radial and circumferential normal stresses were derived from equilibrium considerations alone. The circumferential shear stress $\tau_{z\theta}$, was derived assuming a linear distribution of the shear stress across the thickness of the

specimen wall [5]. The mean values of the stress components were computed by averaging each of the stress components over the specimen volume. The surface tractions were related to the mean effective stress components using Eqs. 1, 2 and 3.

$$\frac{p_o}{p_i} = \left(\frac{r_o^2 - r_i^2}{4r_i^2 \ln\left(\frac{r_o}{r_i}\right)} + \frac{1}{2} \right) s_r' \pm \left(\frac{1}{2} - \frac{r_o^2 - r_i^2}{4r_i^2 \ln\left(\frac{r_o}{r_i}\right)} \right) s_q' \quad (1)$$

$$W = p(r_o^2 - r_i^2) \left(\frac{s_r' + s_q'}{2} + s_z' \right) \quad (2)$$

$$T = \frac{3p(r_o^4 - r_i^4)(r_o^2 - r_i^2)}{4(r_o^3 - r_i^3)} t_{qz} \quad (3)$$

where r_o and r_i are the actual outer and inner specimen radii.

DATA ACQUISITION AND CONTROL HARDWARE

The instrumentation was connected to a computer for closed-loop control, Figure 3. The three pressure-volume controllers interfaced via a general purpose interface bus (GPIB) and the two units that actuated the loading piston via a Compumotor® peripheral component interconnect (PCI) card. Two data acquisition (DAQ) systems logged the deformational response of the specimen. A 24-bit, eight channel system was dedicated to the internal instruments (DAQ-1). A 16-bit, 24-channel system (DAQ-2) serviced the external instruments. Noise reduction was achieved by utilizing the built in noise filters on the DAQ systems and by carefully screening the signal cables.

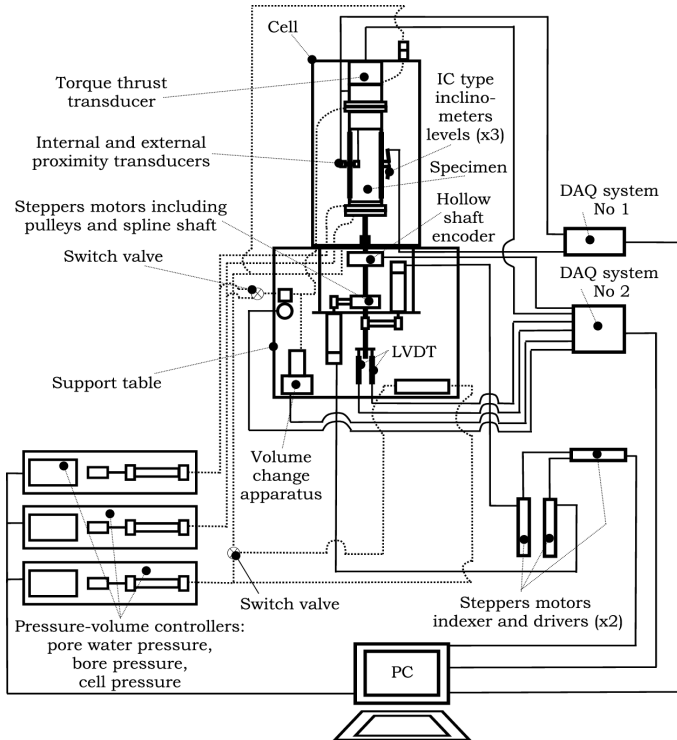


Figure 3. Data acquisition and control of UCD HCA.

DEVELOPMENT OF CONTROL ALGORITHMS

The control algorithms communicated with the instrumentation, adjusted the surface tractions and computed the actual dimensions of the specimen in real time in order to determine the actual states of stress and strain in the specimen. The algorithms for drained and undrained stress path testing followed the same general operating principles (Figure 4). The specified stress path that was read from a data file. The specimen was subjected to incremental changes of the surface tractions. The surface tractions that were to be targeted were computed from the mean effective stress components that were in turn computed from the principal effective stresses corresponding to transitional stress points located along the specified stress path. The magnitudes of the targeted surface tractions were regularly revised based on the actual dimensions of the specimen that were computed from its recorded deformational response. The deformational and pore water pressure responses of the specimen were allowed to equilibrate at each transitional stress point. A constant specimen back pressure was applied for drained loading conditions whereas the pore water pressure was recorded for undrained loading conditions.

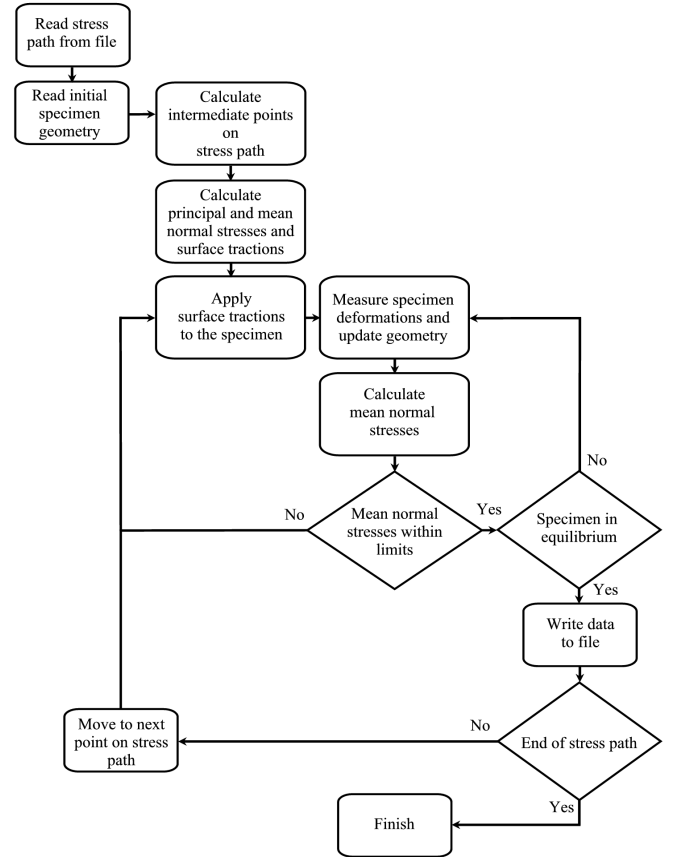


Figure 4. Flow chart of control algorithms.

The stress path was specified in terms of the following control parameters for drained test conditions: the mean effective confining stress p' , the effective major to minor principal stress ratio R' , the intermediate principal stress parameter b , and the rotation α_σ , of the major-minor principal

stress axes. Under drained test conditions, the stress path was expressed in terms of p' , the major to minor principal stress difference t , the b parameter and α_σ . It was possible to cause failure of the specimen using any combination of p' , b , α_σ and either R' or t . Both b and α_σ could be altered during the undrained stage of a test since these parameters are independent of the pore water pressure response.

The principal change points on the stress path were computed in terms of the control parameters. Further transitional stress points located along the stress path were determined by limiting the maximum change in any of the control parameters, Table 2. The values listed in Table 2 could be reset by the user subject to the new values being less than the maximum difference specified for the different control parameters.

The magnitudes of the surface tractions corresponding to the targeted stress points along the stress path were computed, applied to the specimen and regularly adjusted until the difference between the actual and targeted values fell within specified limits, Table 3. These limits could also be reset by the user at any stage during the test. The default limits where such that the mean effective stress components were within 0.25 kPa of the targeted values assuming that the inner and outer specimen radii were 35.5 and 50.0 mm, respectively, and $p_o = p_i$. Also included in Table 3 are the limits set by [6].

Table 2. Max difference between target stress points.

Control parameters	p'	R'	t	b	α_σ
Max difference	5 kPa	0.1	5 kPa	0.1	5°

Table 3. Default limits for application of W , T , p_o and p_i .

Parameter	W (kN)	T (kN.m)	p_o (kPa)	p_i (kPa)
Default limits for UCD HCA	0.0001	0.00004	0.25	0.25
Limits used by [6]	0.02	0.001	0.15	0.15

The deformation of the specimen was allowed to equilibrate for each point targeted along the stress path. Equilibrium was deemed to have been achieved for drained test conditions when the rates of change of the strain components fell within the limits given in Table 4. The excess pore water pressures were also allowed to stabilize for undrained test conditions and the specimen was deemed to have equilibrated when the rates of change of both the strain components and the pore water pressure were sufficiently low, Table 4. The limits set by [6] and [7] are also listed in Table 4. The control algorithm looped and the surface tractions were recomputed, based on the actual specimen dimensions, and reapplied until the specimen had equilibrated. The control algorithm required the difference between the actual and targeted values of the mean effective stress components to be less than 1 kPa. The specimen was assumed to be approaching the onset of failure if it had not equilibrated within a specified number of loops through the control algorithm in which case the equilibrium check was bypassed and the surface tractions were incremented in order to follow the specified stress path as closely as possible.

APPLICATION OF SURFACE TRACTIONS

The axial and torsional loads developed across the length of the specimen were controlled by the two stepper-motors that actuated the displacement and rotation of the loading piston and platen. The stepper motors were regulated in a feedback loop by the thrust-torque transducer.

Significant interaction occurred in applying the axial load and torque to the specimen due to the manner in which the loading piston was displaced and rotated by the screw-spline shaft. This interaction was overcome by only allowing the control algorithm to incremental the axial load or the torque at any instant. Two step changes were also specified in the control algorithms depending on the difference between the actual and targeted values, Table 5. These step changes were set to reduce the impact that a change in the magnitude of one parameter would have on that of another.

Table 4. Rates of strain and pore water pressure to assess equilibration of specimen.

Rates	Units	Current study	[6]	[7]
Axial, radial and circumferential normal strains	strain /min	5×10^{-5}	2×10^{-5}	Less than 5 % change in pervious one min
Circumferential shear strain		1×10^{-5}	5×10^{-6}	
Pore water pressure	kPa/min	0.5	0.3	

Table 5. Step rates for applying axial load and torque.

Difference between actual and target load	Stepper motor step change as fraction of one full revolution
≤ 0.02 kN	0.000005
> 0.02 kN	0.00004
≤ 0.001 kN.m	0.0000005
> 0.001 kN.m	0.00001

The magnitude of the circumferential normal stress was very sensitive to the difference in the actual and targeted values of the outer cell and inner bore confining pressures. Analysis indicated that these confining pressures had to be within 0.25 kPa of the targeted values in order to achieve the 1 kPa tolerance required on the circumferential normal stress. The accuracy of the standard pressure-volume controllers was limited to 1 kPa of the target confining pressure, which was inadequate. A novel system was developed in which the pressure-volume controllers were switched from pressure to volume control modes so that the volumes of the outer cell or inner bore chambers could be increased or decreased to facilitates more precise adjustments of the confining pressures if the target pressure could not be achieved with the necessary precision. Confining pressures could be targeted in a reasonable time by 20 mm³ volume changes, which corresponded to 40 steps of the motors that actuated the controllers under enhanced precision mode.

CALCULATION OF STRESS COMPONENTS

The mean stress components were computed from the applied surface tractions and the actual dimensions of the specimen were measured in real time. The point in the specimen where the stresses were computed for the purposes of the analysis and the restraint and penetration of the rubber membranes that enclosed the specimen walls also had to be considered.

The values of the mean stress components were calculated relative to the mid height of the specimen for the purposes of the analysis, which required a correction for half of the specimen weight in calculating the actual and targeted values of the mean axial normal stress σ_z . The zero pressure readings were set on the pressure-volume controllers at the start of the test and corresponded to the mid height of the specimen.

Corrections were applied for membrane penetration effects [8, 9] using the method reported by [10]. The amount of membrane penetration during the initial isotropic loading stage of the test was computed from the global specimen deformations recorded using the external instruments during increments and decrements of the confining pressures. Membrane penetration effects were automatically corrected by the control algorithms during subsequent stages of the test based on analysis of the amount of membrane penetration during the initial loading stage.

The walls of the hollow cylindrical specimen were enclosed by separate inner and outer rubber membranes, and since the cross-sectional area of the specimen was relatively small, membrane restraint effects could have produced fairly significant errors in the recorded values of the axial load and torque. Membrane restraint effects were also automatically corrected by the control algorithm using the method developed by [11].

PROVING TESTS

Two, identically prepared, saturated test specimens of medium-to-dense Leighton Buzzard sand [3] were both isotropically consolidated to a mean effective confining stress of 200 kPa. Different stress paths were then followed to arrive at the same end point in generalized stress space. The stress paths (Tests A and B, Figure 5) involved anisotropic consolidation of the specimens which was expressed in terms of R' and α_σ . The actual mean stress components deviated by less than 2.5 % from the targeted values and the surface tractions were smoothly controlled, Figure 6.

The control of the rotation α_σ of the σ_1 – σ_3 stress axes was studied using a plot of circumferential shear stress versus the normal stress difference $(\sigma'_z - \sigma'_\theta)/2$. Constant values of R' should appear as quadrants centered on the origin and constant values of α_σ would be represented by lines radiating from the origin. The rotation of the major principal stress increment $\alpha_{\Delta\sigma}$ was computed from the stress changes that occurred between two points closely spaced along the stress path. The capability of the new apparatus to follow specified stress paths was assessed by examining the $\alpha_{\Delta\sigma}$ rotations that occurred between Stress points i and ii, Test A, and Stress points iii and iv, Test B. Tests A and B followed smooth circular paths during α_σ rotation at constant R' (Figure 7a) and produced identical $\alpha_{\Delta\sigma}$

rotation (Figure 7b) validating the excellent stress path control that can be achieved using the UCD HCA.

The ability of the UCD HCA to capture the stress-strain responses of the specimens during reversals of the stress paths was also studied. The turning points in the stress and strain data plotted in Figure 8 for the circumferential shear direction matched each other indicating that the instant response of the axial and torque loading mechanisms.

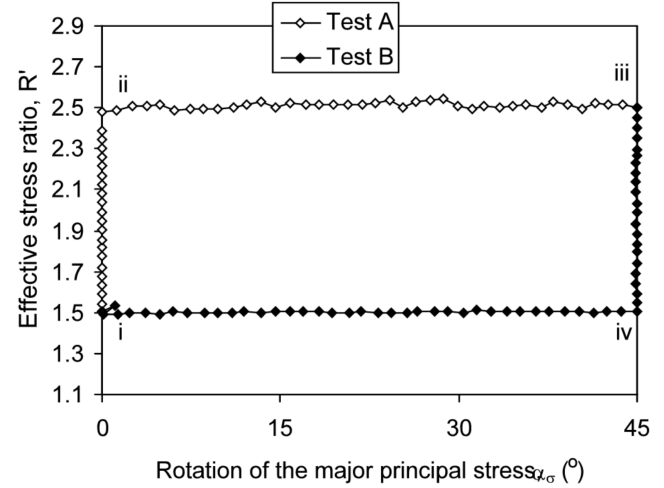


Figure 5. Actual and target stress paths.

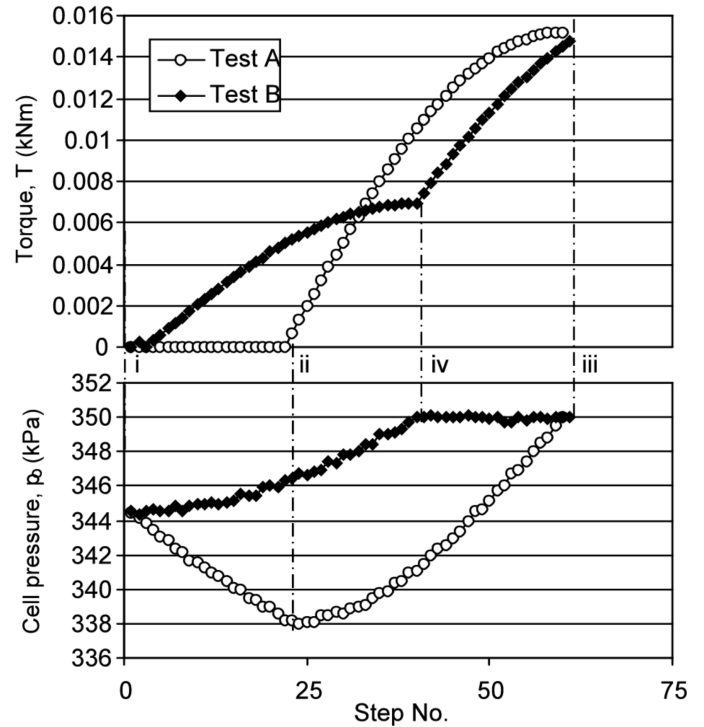


Figure 6. Control of torque and cell confining pressure applied during Tests A and B.

CONCLUSIONS

The instrumentation and algorithms used to closed-loop control the surface tractions applied to hollow cylindrical test specimens by a new hollow cylinder apparatus were described. The precision of the instrumentation and the tolerances between

the actual and targeted stress points achieved in implementing the control algorithms allowed accurate generalized stress path measurements on sand over the pseudo elastic to failure strain range.

The mean stress components (σ_z , σ_r , σ_θ , $\tau_{z\theta}$), were computed relative to the mid height of the specimen and targeted within 1 kPa of the specified values. Corrections were applied to the mean stress components by the control algorithms for the effects of restraint and penetration of the rubber membranes that were used to enclose the specimen.

Test results that demonstrated the capability of the apparatus to follow specific stress paths in generalized space were presented. Excellent compliance was achieved in the axial and torque loading mechanisms resulting in no time lag in the system during reversals of the stress paths.

REFERENCES

- [1] O'Kelly, B.C. & Naughton, P.J., 2003, "Development of the University College Dublin hollow cylinder apparatus", *Proceedings 56th Canadian Geotechnical Conference*, Winnipeg, 29 September–01 October, CD Rom.
- [2] O'Kelly, B.C. & Naughton, P.J., 2003, "Development of a new hollow cylinder apparatus for generalised stress path testing", *Ground Engineering*, **36**, pp. 26–28.
- [3] O'Kelly, B.C. & Naughton, P.J., 2005, "Development of a new hollow cylinder apparatus for stress path measurements over a wide strain range", *Geotechnical Testing Journal*, **28**, 4.
- [4] GDS Instruments Ltd., 1998, "GDS 2MPa/200cc advanced pressure/volume controller", *Technical literature*.
- [5] Naughton, P.J. & O'Kelly, B.C., 2005, "Stress non-uniformity in hollow cylindrical test specimens", *Proceeding Joint ASME/ASCE/SES Conference on Mechanics and Materials*, June 1-3, 2005, Baton Rouge, Louisiana, USA.
- [6] Shibuya, S., 1988, "A servo system for hollow cylinder testing of soils", *Geotechnical Testing Journal*, **11**, pp. 109–118.
- [7] Wijewickreme, D., Uthayakumar, M. & Vaid, Y.P., 1994, "Automatic stress path control system for hollow cylinder torsional testing of soils", *Transportation Research Record* 1432, pp. 1–8.
- [8] Naughton, P.J. & O'Kelly, B.C., 2003, "The anisotropy of Leighton Buzzard sand under general stress conditions", *Proceedings Third International Symposium on Deformation Characteristics of Geomaterials*, ISLyon, September 22–24, Lyon, pp. 285–292.
- [9] O'Kelly, B.C. & Naughton, P.J., 2005, "Engineering properties of wet-pluviated hollow cylindrical specimens", Paper submitted to *Geotechnical Testing Journal*.
- [10] Sivathayalan, S. & Vaid, Y.P., 1998, "Truly undrained response of granular soils with no membrane penetration effects", *Canadian Geotechnical Journal*, **35**, pp. 730–739.
- [11] Tatsuoka, F., Sonoda, M., Hara, K., Fukushima, S. & Pradhan, T.B.S., 1986, "Failure and strength deformation of sand in torsional shear", *Soils and Foundations*, **26**, pp. 79–97.

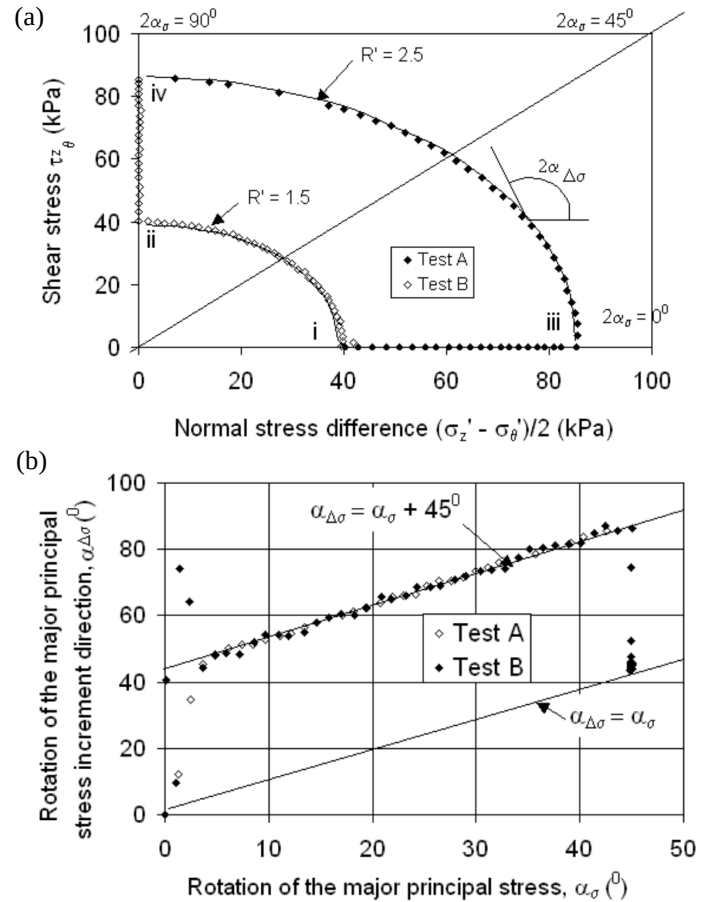


Figure 7. (a) $\tau_{z\theta}$ versus normal stress difference and (b) $\alpha_{\Delta\sigma}$ rotations for Tests A and B.

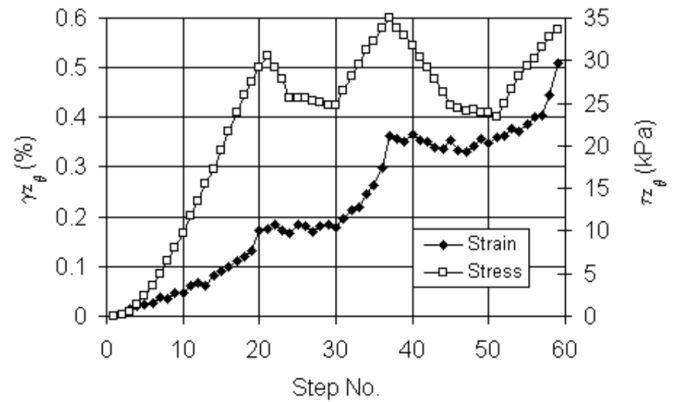


Figure 8. Variations of circumferential shear stress and strain during stress reversal.