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The NARX neural network time series prediction technique and highly efficient reliability analysis method

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ABSTRACT: The reliability analysis theory represented by the stochastic simulation method requires repeated deterministic structural response analysis, which undoubtedly consumes a large amount of computational cost; meanwhile, the reliability obtained based on the stochastic simulation method has the characteristic of random convergence, so it is not worth advocating in practical engineering structures, which limits the application of the stochastic simulation method. In view of the above dilemma, this thesis proposes a strategy to train a nonlinear autoregressive with exogenous input model based on a neural network, and then measure the response characteristics of random samples in probability space, which greatly reduces the number of deterministic analyses and saves computational costs. Meanwhile, in order to circumvent the characteristics of random convergence of stochastic simulation methods, this study invokes the physical synthesis method based on probability density evolution theory to solve the system reliability. Finally, the accuracy and efficiency of the proposed method are systematically verified by the strong nonlinear structure. The analysis result show that the sample response characteristics predicted by the nonlinear autoregressive with exogenous input model are in high agreement with the real response. The proposed reliability analysis strategy has high computational efficiency without sacrificing much computational accuracy compared with the stochastic simulation method and the direct PDEM method.

With the increasing complexity of engineering structures, the challenge of being spared from structures destruction by increasingly severe disasters is inevitably faced, which raises the bar on demands of safe design and serviceability maintenance of modern engineering structures. Deterministic design approaches fail to meet realistic requirements due to these inevitable uncertainties associated with structural parameters and external excitation. Reliability assessment with the fundamental of probability theory and mathematical statistics expects to scientifically measure the consequence of structural response with due consideration of the randomness arising from both the external excitation and self-properties during the service period.

The core of structural reliability assessment is to address the potential for failure with randomness in scientific metrics. These methods can be categorized as two familiars depending on the differences in the stochastic system: component reliability and system reliability. There are various methods to deal with the static reliability of the structural components with a single failure mode, such as stochastic simulation methods Rubinstein and Kroese (2016), moment methods Der Kiureghian (2000) and so forth. Stochastic simulation methods via extensive deterministic analyses expect to access the statistical information of interest at the sample level, which predetermines the hindrance of practical application in actual structures due to the expensive computational cost. The moment methods try to find the nearest geometric distance from the centre of the normalized probability space to the failure boundary at the mean of iteration. The length is noted as the reliability index β, which is used to calculate the failure probability P_f. These methods are often limited by accuracy and efficiency and are difficult to apply to engineering practice.

To provide an efficient way out of the foregoing dilemmas, the probability density evolution method (PDEM) in virtue of the principle of probability conservation was put forwarded Li and Chen (2004). By integrating proper failure criteria into the PDEM, three types of efficient reliability analysis methods were successively presented, i.e. absorbing boundary condition Li and Chen (2006), equivalent extreme value events (EEVs) Chen and Li (2007) and physical synthesis method Li (2018), all of which are suitable for the safety assessment of dynamic or static stochastic systems.

On the other hand, machine learning theory and techniques for artificial intelligence have recently been explored for the responses estimation of stochastic systems Lucor and Karniadakis (2004). The gradual approximation of limit state surfaces via complex polynomial models is a common case of practice, called surrogate model or metamodel. The various reliability analysis methods aforementioned can then be embedded into the framework of machine learning to effectively perform reliability analysis, such as the response surface Rackwitz (1982), polynomial chaos expansion Crestaux et al. (2009), Kriging Krige (1951), neural networks Yegnanarayana (2009); Xiao et al. (2018a) and so forth. The introduction of metamodel into PDEM in the expectation of efficiency improvement without sacrificing accuracy is gaining traction in recent years, and several fruitful approaches have been proposed Tao and Li (2017); Jiang and Li (2017). However, some critical issues related to the PDEM-based reliability method have not been addressed. Firstly, most of these methods focus on the EEVs theory to analyze reliability with the signal failure path, which cannot be applied to time-varying reliability analysis of systems. In this regard, fewer expectations in guiding practical engineering design are met. Secondly, although these time-series prediction methods via Kriging Jiang and Li (2017) or support vector machine Tao and Li (2017) might work well for less complex structural reliability issues, the prediction accuracy is unacceptable when extended to the reliability analysis of real structures. Therefore, this study focuses on improving the efficiency of the PDEM-based reliability method in the analysis of the system or dynamic reliability problems for actual engineering structures. To emphasize the issue, we introduce the nonlinear autoregressive with exogenous inputs model (NARX) to replace the redundant response analysis with the well-trained nonlinear structure by a small-size training set to achieve efficiency improvement. In particular, the NARX has been demonstrated to be versatile and effective in this respect Li et al. (2021).

1. NARX NEURAL NETWORK MODEL

In recent years, as reliability issues have become increasingly complex with respect to the physical system due to the properties of strongly nonlinear and thousands of degrees of freedom, linear metamodels are inevitably prohibitive in satisfying the approximation of performance functions. The expansion of mathematical models with greater generalization capability to consider more features of stochastic systems is a feasible pathway. The nonlinear autoregressive with exogenous inputs (NARX) is a recurrent dynamic network, with feedback connections enclosing several layers of the network, which has an embedded memory that facilitates an excellent time-prediction architecture to propagate the information and back-propagation of the error. In this regard, the output $y(t)$ at the current moment t is not only related to the previous n_x input signal but is also influenced by the previous n_y step response sequence. Consequently, the definition of the NARX model is

$$y(t) = \mathscr{F}[\omega(t)] + \varepsilon_t \tag{1}$$

$$\begin{aligned}\omega(t) = [&y(t-\Delta t), y(t-2\Delta t), ..., y(t-n_y\Delta t);\\ &x(t-\Delta t), x(t-2\Delta t), ..., x(t-n_x\Delta t)]^\top\end{aligned} \tag{2}$$

where $\mathscr{F}[\cdot]$ is the structure of the autoregressive model reflecting the dynamic characteristics of the system; $\varepsilon_t \sim \mathscr{N}(0, \sigma_t)$ is the model residual supposed to be a normal independently distributed process with zero mean; $x(t)$ denotes the inputting time series, generally expressed as the excitation in systems analysis; correspondingly, $y(t)$ is the system response series, representing the output of the system, which is regressed on previous values of the output signal and previous values of an independent (exogenous) input signal; Δt is the time step chosen for NARX model; n_x and n_y are the number of time delay for external excitation and system output respectively.

2. PROPOSED METHOD

2.1. *Physical synthesis method for dynamic reliability analysis*

For the well-posed dynamic system, the interested physical quantity $Y(t)$, e.g., the inter-story drift ratio, should exit and uniquely depends on Θ Li and Chen (2009), that is $Y(t) = H(\Theta, t)$, where $H(\cdot)$ denotes the deterministic operator, and the velocity process of $Y(t)$ can be expressed as $\dot{Y}(t) = h(\Theta, t)$ with mathematical notation. Considering the probability-preserved augmented system $\{Y, \Theta\}$, where all the randomness is completely inscribed by Θ, the evolution principle of joint probability density of $\{Y, \Theta\}$ within the time bucket $[0, T]$ is derived as

$$\frac{\partial p_{U_p\Theta}(u_p, \theta, t)}{\partial t} + \dot{U}_p(\theta, t)\frac{\partial p_{U_p\Theta}(u_p, \theta, t)}{\partial u_p} = -\mathscr{H}\left[f\left(u_p, t\right)\right] p_{U_p\Theta}(u_p, \theta, t) \tag{3}$$

where

$$\mathscr{H}[f(u_p, t)] = \begin{cases} 1 & f(u_p, t) \in \Omega_f \\ 0 & f(u_p, t) \in \Omega_s \end{cases} \tag{4}$$

and $f[\cdot]$ denotes the general function, varies with different failure criteria. where $p_{Y\Theta}(y, \theta, t)$ denotes the joint PDF of $\{Y, \Theta\}$; and its initial condition satisfies $p_{Y\Theta}(y, \theta, t)|_{t=0} = \delta(y) p_\Theta(\theta)$ and $\delta(\cdot)$ is the Dirac's delta function, it is ∞ when $Y = y$, otherwise it is 0.

Noting any dynamic response of a particular representative point as $u_p(t)$ and severing the point as the observation window regarding the overall structural safe state, which can evade solving a series of partial differential equations concerning different failure criteria. Structural global reliability is approachable through solving the one-dimensional GDEE with the physical inspection of failure events. Introducing the sieve operator $\mathscr{H}$ to identify various failure events, the GDEE relating to probabilistic dissipation systems can be derived Li (2018)

$$\begin{cases} M(\Theta)\ddot{X}(t) + C(\Theta)\dot{X}(t) + G[X(t), \Theta] = \Gamma F(\Theta, t) \\ X = X_0, \dot{X} = \dot{X}_0 \\ \frac{\partial p_{U_p\Theta}(u_p, \theta, t)}{\partial t} + \dot{U}_p(\theta, t)\frac{\partial p_{U_p\Theta}(u_p, \theta, t)}{\partial u_p} = \\ -\mathscr{H}\left[f\left(u_p, t\right)\right] p_{U_p\Theta}(u_p, \theta, t) \\ p_{U_p\Theta}\left(u_p, \theta, t\right)\big|_{t=t_0} = \delta\left(u_p - u_{p_0}\right) p_\theta(\theta) \end{cases} \tag{5}$$

where $X = X_0, \dot{X} = \dot{X}_0$ denotes the initial condition for physical equation; $p_{U_p\Theta}\left(u_p, \theta, t\right)\big|_{t=t_0} = \delta\left(u_p - u_{p_0}\right) p_\theta(\theta)$ is the initial condition, u_{p_0} denotes the initial value of u_p.

Once the failure of the overall structure is identified through multiple failure criteria, the associated probability of representative point is dissipated, and the remaining probability of stochastic system represents the global structural reliability, i.e.,

$$R(t) = \int_{\Omega_{u_p}} p_{U_p}(u_p, t)\mathrm{d}u_p = \int_{\Omega_{u_p}} \int_{\Omega_\Theta} p_{U_p\Theta}(u_p, \theta, t)\mathrm{d}\theta\mathrm{d}u_p \tag{6}$$

where Ω_{u_p} denotes the distribution domain of u_p.

2.2. *NARX-based neural network model*

The NARX-based neural network (NARX-NN) model incorporates the concepts of an autoregressive exogenous time series model in that it includes previous outputs among model inputs. Consequently, its memory units store information in the input and hidden layers. Weights connect the neurons of the neighbouring layers, and the output of each neuron can be uniformly expressed as

$$s_i^{(l)} = f\left(\sum_{j=1}^{n} \omega_{ij}^{(l)} s_j^{(l-1)} + b_i^{(l)}\right) \tag{7}$$

where $s_i^{(l)}$ is the output of the ith neuron of the lth layer, $\omega_{ij}^{(l)}$ are the weights of the jth input, $b_i^{(l)}$ is the bias, n is the number of neurons of the $(l-1)$th layer, and the $f(\cdot)$ denotes the activation function. In this work, the Sigmoid function is used in the hidden layers, and the rectified linear unit function is used to efficiently prevent vanishing and exploding gradient problems in the outputting layer.

Furthermore, as shown in the study Xiao et al. (2018b); Ren et al. (2022), a backpropagation (BP) neural network with three layers (input layer, hidden layer and output layer) can approximate any continuous functions. Therefore, the BP neural network with three layers is applied in the present work.

2.3. *Optimization algorithm*

The best approximation to the original physical system can generally be trained effectively by setting a suitable number of samples to form the training set. The training process essentially minimizes the loss function by dynamically adjusting the neural network weights. The most commonly used loss functions are the mean square error loss function and the cross entropy loss function. The mean square error function is quoted as the loss function in this study, i.e.

$$E_d = \frac{1}{2}\sum_{i\in N_{\text{outs}}}(t_i - y_i)^2 \tag{8}$$

where E_d is the loss function, t_i is the ith label, and y_i is the ith prediction via the neural network model. To improve the training efficiency while comparing the effects of different algorithms on the target neural network, three types of commonly used optimization algorithms are adopted in this study: Levenberg (1944); Marquardt (1963), Scaled Conjugate Gradient algorithm Hestenes (2012); (Møller), Bayesian regularization algorithm MacKay (1992); Dan Foresee and Hagan (1997).

The E_d is the function of the connection weights ω, so the process of obtaining the optimal weights via the available training samples can be rewritten as

$$\omega^* = \text{argmin}_{\omega}\{E_d(\omega)\} \tag{9}$$

where ω is the weights matrix and ω^* denotes the optimal value. And the loss function can be expressed further

$$\begin{aligned} E_d = F(\omega) &= \frac{1}{2}\sum_{i\in N_{\text{outs}}}(t_i - y_i)^2 = \frac{1}{2}\sum_{i=1}^{m}(f_i(\omega_i))^2 \\ &= \frac{1}{2}\|\mathbf{f}(\omega)\|^2 = \frac{1}{2}\mathbf{f}(\omega)^{\top}\mathbf{f}(\omega) \end{aligned} \tag{10}$$

The optimal value ω^* can be approximated by iterations step-by-step from the initial value ω_0. Once the direction vector h and the step size α have been determined, the updated weights can be expressed as $\omega_{k+1} = \omega_k + \alpha_k h_k$. The strategy in which the direction vector h and step size α are determined for each iteration is different for different optimization algorithms.

With regard to the termination criterion, this study adopts an empirical approach in which the training process is terminated when any of the following criteria is satisfied:

(1). The maximum number of epochs (repetitions) is reached, herein, $N_{\max}^{\text{epoch}} = 3000$.

(2). The performance gradient falls below minimum gradient and $G_{\min}^{\text{gradient}} = 10^{-10}$.

(3). Validation performance has increased more than maximum failure times since the last time it decreased (when using validation) and the $N_{\max}^{\text{failure}} = 10$.

3. CASE STUDY

In this section, we testify the reinforced concrete (RC) plane frame structure to observe its efficiency and accuracy. For comparison, we also quote the probability density evolution method-based reliability analysis strategy without the help of a metamodel.

Herein, the finite element model of the corresponding RC structure is established in the finite element software ABAQUS Dassault Systèmes Simulia Cop (2020), of which the beam, as well as column components of the structure, are simulated with the fibre beam element.

For the strongly nonlinear stochastic systems that are used for testing and verification, the random sources were derived from external excitation. The stochastic ground motion physical model is utilized, where all the uncertainties from the source,

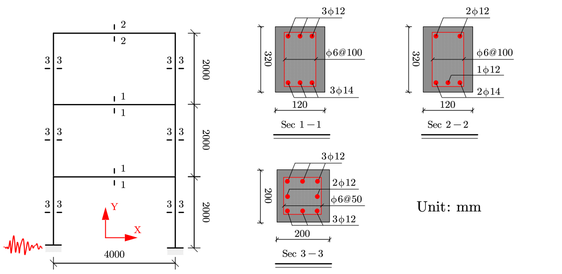


Figure 1: Elevation dimensions of reinforced concrete plane frame structure

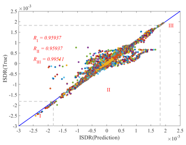
propagation path and local engineering site are incorporated Ding et al. (2018). Herein, the ground motion time history of the first group of the second site is generated via the narrow-band wave group superposition method Trifunac (1971). In addition, the inter-story drift ratio (ISDR) for the first floor of the reinforced concrete structure is selected as the concreted physical quantity.

3.1. Seismic reliability analysis of the plane frame structure

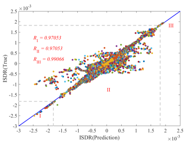
A three-story single-span RC plane frame is quoted to validate the proposed seismic reliability analysis method, shown in Fig.1. Meanwhile, the configuration details of reinforcement in the cross-section of the structural elements are also showcased. The acceleration peaks of random ground motion generated by the narrow band wave group superposition method are uniformly adjusted to 0.15 gravity relative to the frequency level, with unidirectional input along the span of the structure. The finite element model built in ABAQUS includes 147 elements and 299 nodes, using the explicit integration algorithm for the dynamic response analysis.

First, the random dynamic response of the plane frame structure is investigated via the PDEM. Herein, by minimizing the generalized-F discrepancy method Chen et al. (2016), 998 representative samples are selected to generate the corresponding ground motion. The reliability of the plane frame structure is 0.9899 via integrating the assigned probability for these samples in the safety domain, and the likelihood of failure is 1.01×10^{-2}. To verify the correctness of the failure probability, 10,000 deterministic analyses were performed based on the MCS, and the failure probability was obtained statistically as 1.01×10^{-2}. Unfortunately, the calculation cost is unaffordable since it took 3.71 days to conduct 998 deterministic structural response analyses and nearly four days for such a simple-form structure. The MCS method for large samples is even more overwhelming, taking a total of 32.38 days, which is an impressive amount of computation and confirms the efficiency barriers of the cruel MCS to calculate the actual engineering reliability.

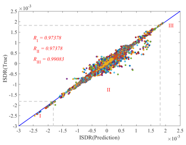
The NARX neural network is considered an alternative for the complex physical system, and 249 representative samples $\mathscr{X} = \{x_1, x_2, ..., x_{N_1}, N_1 = 249\}$ are generated by coarsening the partition of the probability space with the criterion of minimizing the generalized-F discrepancy. The physical response $\mathscr{Y} = \{y_1(t), y_2(t), ..., y_{N_1}(t), N_1 = 249\}$ of the quantity of interest is obtained by reapplying structural response analysis to these representative samples, which constitutes the training set $\mathscr{D}_{\mathrm{Tr}} = \{\mathscr{X}, \mathscr{Y}\}$. In order to obtain a neural network model with more generalization capability, three optimization methods are considered to optimize the weights. The trained NARX neural network model is used to predict responses of the 998 samples generated aforementioned with different optimization approaches and a uniform number of time delay. The results are shown in Fig.2. Here the point-in-time values of the ISDR response are divided into three regions according to the failure thresholds: $\mathrm{ISDR}(t_i) < -1/550$ (Region I), $-1/550 \leq \mathrm{ISDR}(t_i) \leq 1/550$ (Region II) and $1/550 < \mathrm{ISDR}(t_i)$ (Region III). For the double-sided reliability problem with the ISDR limit of 1/550, regions I and III are the failure domain, while region II is the safety domain. Therefore, the reliability will not be biased when the predicted response in the failure domain is accurate. In this sense, only the NARX neural network model trained with the Bayesian regularization method is capable of giving exact reliability results, which can be concluded later in the reliability calculation. Because whether it is for the Levenberg-Marquardt algorithm or the Scaled conjugate gradient algorithm, there are exaggerated responses by predicting the response within the safety threshold as exceeding



(a) Levenberg-Marquardt algorithm



(b) Scaled conjugate gradient algorithm



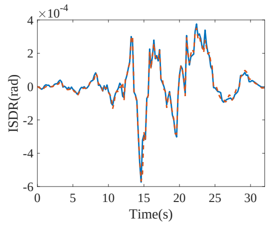
(c) Bayesian regularization algorithm

Figure 2: Comparison of real response and prediction for frame structure

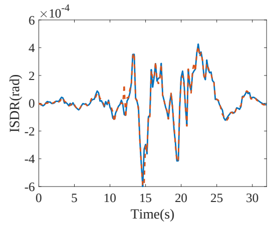
the failure threshold, which inevitably leads to an increase in the failure probability. Although the response prediction accuracy of all three algorithms is considerably high, over 97% for all three regions, accurate prediction for extreme areas is significant due to the specificity of the reliability problem. Of course, suitable algorithms often have a particular cost, about time cost.

A comparison of several typical predicted time histories versus the real one is given here considering the Bayesian regularization algorithm, shown in Fig.3. For a strongly nonlinear stochastic system, the NARX neural network model can capture the system's physical law and make accurate predictions. Tab.1 lists the comparison of reliability results of different methods. As the number of hidden layer neurons increases, the accuracy of the reliability analysis is significantly improved while the response prediction accuracy is improved. There is no doubt that it will increase the training time cost, but it is acceptable compared to the calculation burden of a large number of deterministic structural response analyses. It is seen that the number of neurons in the hidden layer in the interval $[180, 210]$ can achieve high accuracy reliability, and the prediction under this condition is close to the real-time series for each representative sample. In addition, the efficiency advantage is undeniable, as the proposed method takes about 19 hours to complete the reliability analysis, which is approximately 1/5 of the cost for direct PDEM and about 1/40 of the cruel MCS method.

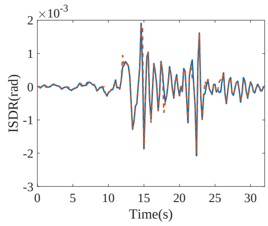
Another issue worth exploring is the number of hidden layer neurons here, which is rather empirical. Thus some usual formulas exist in the published literature. However, it may not apply to different problems, especially strongly nonlinear systems. To obtain a more usable reference, the target NARX neural network model is trained here based on different numbers of neurons in the hidden layer, which is later used as a metamodel for the stochastic system to make predictions. The regression relations are listed in Fig.4. When the small number of neurons in the hidden layer is adopted, it is difficult for the NARX neural network to fully capture the nonlinear system's properties, leading to lower prediction accuracy. However, the number of intermediate-layer neurons should not be increased blindly. Shown in the Figs.4h and 4i, the response prediction in the safety domain gradually worsens after the number of neurons exceeds 240, which means there may be an overfitting phenomenon. Although this has less effect on the reliability results, increasing the number of neurons is still prudent. In addition, another factor limiting the excessive increase in the number of neurons is the training cost, and an excessive number often means a high computational burden. In this study, 180 nodes are recommended as the number of intermediate layer neurons, and the Bayesian regularization method is selected to train the NARX neural network.



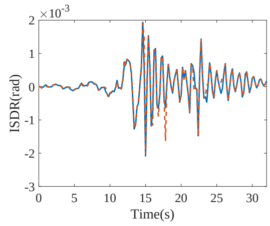
(a) Num 2



(b) Num 10



(c) Num 154

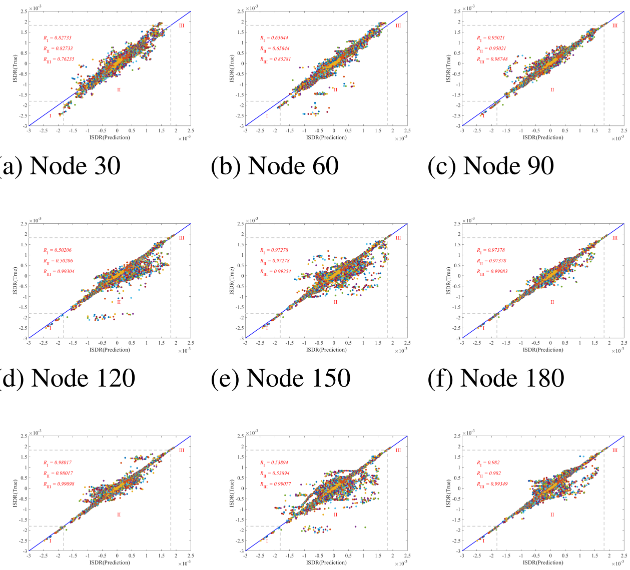


(d) Num 245

Figure 3: Predicted time series versus real one for the reinforced concrete frame structure

Table 1: The comparison between various reliability methods for plane frame structure

Items	NARX-PDEM (Num of Neurons for BR)							LM(180)	SCG(180)	PDEM(998)
	60	90	120	150	180	210	240			
$P_f(10^{-2})$	0.76	0.98	0.74	0.99	1.00	1.00	0.69	1.21	1.34	1.01
Error(%)	24.75	2.97	26.73	1.98	0.99	0.99	31.68	19.80	32.67	0.0
Time(h)	18.65	18.74	18.84	19.36	19.65	19.74	19.89	19.03	18.99	89.07
Reference(10^{-2})	1.01(Total time: 777.76 hours for MCS)									



(a) Node 30 (b) Node 60 (c) Node 90

(d) Node 120 (e) Node 150 (f) Node 180

(g) Node 210 (h) Node 240 (i) Node 270

Figure 4: Comparison of different number of neurons in the hidden layer for frame structure

4. Conclusion

This paper proposes an enhanced method to efficiently analyse the reliability of reinforced concrete structures with substantial non-linearity properties. The applicable nonlinear autoregressive with exogenous inputs model is embedded into the framework of system reliability analysis for the physical synthesis method. Only a few deterministic structural response analyses are performed by conducting the accurate time series prediction for these untried samples in the probability space. Three types of training algorithms are used for the training of the NARX neural network model. Subsequently, the prediction response of these encrypted samples by the well-trained NARX model are brought into the GDEE for reliability analysis, which can significantly improve the efficiency of PDEM-based reliability analysis without sacrificing the accuracy.

The efficiency of the proposed enhanced reliability analysis method is studied using the nonlinear stochastic system subjected to the strong stochastic ground motion. The analysis results show that the NARX neural network can ideally capture the strong nonlinear features of the stochastic system and make further predictions for these encrypted points by setting appropriate references of some hyperparameters.

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