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Bayesian empirical vulnerability modelling for hierarchical levels of damage

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ABSTRACT: A fully simulation-based Bayesian workflow for inference and model class selection is presented to perform ensemble modelling of the tsunami fragility curves for mutually exclusive and collectively exhaustive (MECE) damage states and the related uncertainties for a given class of buildings. The workflow employs adaptive Markov Chain Monte Carlo Simulation (MCMC), based on likelihood estimation using point-wise intensity values. Among the set of viable models considered, Bayesian model class selection is used to identify the simplest model that fits the data best. The proposed workflow is demonstrated for empirical fragility assessment based on observed damage data to 1-storey masonry residential and timber residential buildings due to the 2009 South Pacific Tsunami in the American Samoa and Samoa Islands.

1. INTRODUCTION

Tsunami empirical fragility curves are models derived based on observed pairs of damage (*D*) and intensity data (*IM*) for buildings and infrastructures usually collected, acquired, and even partially simulated in the aftermath of disastrous events. (Koshimura et al. 2009a, Reese et al. 2011; Charvet et al. 2017). Empirical fragility modelling is greatly affected by how the damage and intensity parameters are defined. Mutually exclusive and collectively exhaustive (MECE) damage states are quite common in the literature as discrete physical damage states. In most probabilistic risk formulations, this condition is necessary for leading to the mean rate of exceeding loss (Behrens et al. 2021). As the measure of *IM*, tsunami empirical fragility curves usually employ the tsunami flow depth. However, other measures like current velocity (De Risi et al. 2017a), debris impact, and scour are more appropriate *IM*s for higher damage states (see Charvet et al. 2015). To this end, De Risi et al. (2017a) developed bivariate tsunami fragilities, which account for the interaction between the two *IM*s, namely, tsunami flow depth and current velocity.

Fragility curves for MECE damage states are distinguished by their nicely "laminar" shape; in other words, the curves should not intersect. When fitting empirical fragility curves to observed damage data, this condition is not satisfied automatically. For example, fragility curves are usually fitted for individual damage states separately and they are filtered afterwards to remove the crossing fragility curves (e.g., Miano et al. 2020). De Risi (2017b) used multinomial probability distribution to model the probability of being in any of MECE damage states based on binned intensity representation. Then, they used Bayesian inference to derive the model parameters for an ensemble of fragility curves. Empirical tsunami fragility curves are usually constructed using generalized linear models based on probit, logit, or the complementary loglog link functions (Charvet et al. 2014, Lahcene et al. 2021). They proposed approaches based on the likelihood ratio and Akaike Information Criterion for assessment of goodness of fit, model comparison, and selection. For estimating confidence intervals of empirical tsunami fragility curves, they employed bootstrap resampling.

This paper presents a simulation-based Bayesian method for inference and model class selection for the ensemble modelling of the tsunami fragility curves for MECE damage states for a given class of buildings (Jalayer et al. 2023). By fitting the (positive definite) fragility link function to the conditional probability of being in a certain damage state, given that building is not in any of the preceding states, the method ensures that the fragility curves do not cross (i.e., they are "hierarchical"). The method uses adaptive Markov Chain Monte Carlo Simulation (MCMC, Beck and Au 2002), based on likelihood estimation using point-wise intensity values, to infer the ensemble of the fragility model parameters. Alternative link functions are compared based on log evidence which considers both the average goodness of fit (based on log likelihood) and the model parsimony (based on relative entropy). This way, among the set of viable models considered, it identifies the simplest model that fits the data best. By "simplest model", we mean the model having maximum relative entropy (measured using the Kullback-Leibler (Kullback and Leibler 1951) distance) with respect to the data. This usually means the model has a small number of parameters.

2. METHODOLOGY

2.1. Basic definitions

- Intensity measure (*IM*): an intermediate variable that conveys information about an event from the hazard level to the fragility level (herein, we used tsunami flow depth)
- Damage level (D_j): a discrete random variable defining the j^{th} *damage level (threshold)*; the vector of damage levels is $\{D_j, j=0:N_{DS}\}$, where N_{DS} is the total number of damage levels considered (depending on the damage scale being used). Normally, D_0 denotes the *no-damage* threshold, while $D_{N_{DS}}$ defines the total *collapse* or *being totally washed away* (see Figure 1).
- Damage state (DS_j): is defined by the logical statement that D is between the two damage thresholds D_j and D_{j+1}; i.e., $DS_j \equiv (D \geq D_j) \cdot (D < D_{j+1})$. Obviously, for the last damage state, we have $DS_{N_{DS}} \equiv D \geq D_{N_{DS}}$ (see Figure 1). Damage states are mutually exclusive and collectively exhaustive (MECE); i.e., being in one damage state excludes all others and all the damage states together cover the entire range of possibilities in terms of damage (see Jalayer et al. 2023 for cases where observed damage data is not available for some of damage levels).

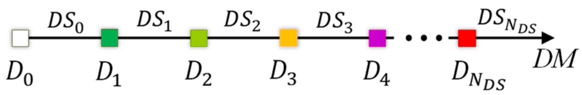


Figure 1: Graphical representation of D_j and DS_j.

- Link functions used in generalized regression models: the (conditional) probability of exceeding a damage level D_j given intensity level IM_i, denoted as π_{ij}, is linked to the linear logarithmic predictor $l_{ij} = \alpha_{0,j} + \alpha_{1,j} \ln IM_i$, where $\alpha_{0,j}$ and $\alpha_{1,j}$ are regression parameters. Three different link functions are: (a) logistic regression "logit", $\pi_{ij} = [1 + \exp(-l_{ij})]^{-1}$; (b) lognormal cumulative distribution function "probit", $\pi_{ij} = \Phi(l_{ij})$; (c) complementary log-log "cloglog", $\pi_{ij} = 1 - \exp[-\exp(l_{ij})]$.

2.2. Fragility modelling

2.2.1. MLE-based hierarchical fragility assessment

The fragility curves are constructed in a hierarchical manner by first constructing the fragility increment $P(DS_j|IM) = P[(D \geq D_j) \cdot (D < D_{j+1})|IM]$, which is the conditional probability of being in DS_j (see Jalayer et al. 2023 for derivation):

$$P(DS_j|IM_i) = (1-\pi_{ij})\cdot\left[1-\sum_{k=0}^{j-1}P(DS_k|IM_i)\right] \text{ for } j \geq 1$$
$$P(DS_0|IM_i) = P(D < D_1|IM_i)$$
$$P(DS_{N_{DS}}|IM_i) = P(D \geq D_{N_{DS}}|IM_i) = 1-\sum_{j=0}^{N_{DS}-1}P(DS_j|IM_i) \quad (1)$$

where $\pi_{ij} = P(D \geq D_{j+1}|D \geq D_j, IM_i)$. Hence, we ensure that the fragility curve of a lower damage level will not fall below the fragility curve of the subsequent damage threshold (the ill condition of $P(DS_j|IM) < 0$ does not take place). Note that for the last damage state $DS_{N_{DS}}$, the probability $P(DS_{N_{DS}}|IM_i)$, which is also equal to the fragility $P(D \geq D_{N_{DS}}|IM)$ for $D_{N_{DS}}$, can be estimated by satisfying the collectively exhaustive (CE) condition. Accordingly, the fragility for other damage levels $P(D \geq D_j|IM_i)$, where $0 < j < N_{DS}$, can be obtained by starting from the fragility of the higher threshold $P(D \geq D_{j+1}|IM)$, and adding successively $P(DS_j|IM)$ as follows:

$$P(D \geq D_j|IM_i) = P(DS_j|IM_i) + P(D \geq D_{j+1}|IM_i) \quad (2)$$

The set of hierarchical fragility models based on Eq. (2) has $2 \times N_{DS}$ model parameters $\boldsymbol{\theta} = [\{\alpha_{0,j}, \alpha_{1,j}\}, j = 0{:}N_{DS}-1]$. Obviously, no model parameter is required for the last damage level (by satisfying the CE condition, see Eq. 1). The model parameters $\boldsymbol{\theta}$ are obtained by fitting the link functions in Eq. (5) to conditional fragility term π_{ij}. Herein, we have used MATLAB as a statistical software package (developed by MathWorks) to estimate the maximum likelihood estimate (MLE) of $\boldsymbol{\theta}$ using the command:

$\text{glmfit}(\log(\mathbf{x}_j), \mathbf{y}_j, '\text{binomial}', '\text{link}', '\text{model}')$

The $'\text{model}'$ will be either $'\text{logit}'$, $'\text{probit}'$, or $'\text{comploglog}'$. For each damage level D_{j+1}, the vector $\mathbf{x}_j$ is the *IM*'s for which the condition $D \geq D_j$ is satisfied (partial damage data so that the domain of possible damage levels is reduced to $D \geq D_j$); $\mathbf{y}_j$ is the column vector containing one-to-one probability assignment to the *IM* data in $\mathbf{x}_j$ with zero (=0.0) assigned to those data corresponding to DS_j ($D_j \leq D < D_{j+1}$) and one (=1.0) to those related to higher damage states (with $D \geq D_{j+1}$). Since the fragility model parameters are estimated by employing the MLE, this approach is dubbed as "MLE-based hierarchical fragility assessment".

2.2.2. BMCS-based hierarchical fragility assessment

Instead of using MLE, this method of fragility assessment employs the Bayesian inference for model updating using adaptive MCMC to estimate the joint posterior distribution of fragility model parameters for a given model class *k*, $p(\boldsymbol{\theta}_k|\mathbf{D}, \mathbb{M}_k)$. Herein, we have considered *k*=1:3 based on the 3 link functions defined in Section 1.1, so that $\mathbb{M}_k$ refers to hierarchical fragility modelling based on logit (*k*=1), probit (*k*=2), and cloglog (*k*=3) link models. $\mathbf{D} = \{(IM, DS)_i, i = 1{:}N_{CL}\}$ defines the observed data (intensity and damage) for N_{CL} buildings surveyed for a specific class *CL*. For each realization of the vector of model parameters $\boldsymbol{\theta}_k$, the corresponding set of hierarchical fragility curves can be derived based on the procedure described in Section 1.2.1. Since we have realizations of the model parameters drawn from the joint probability density $p(\boldsymbol{\theta}_k|\mathbf{D}, \mathbb{M}_k)$ (based on samples drawn from adaptive MCMC procedure), we can use the concept of *Robust Fragility* to derive confidence intervals for the fragility curves (see Jalayer et al. 2023; moreover see Jalayer et al. 2015, 2017, and Jalayer and Erahimian 2020 for more discussion on robust fragility concept).

Moreover, we use the Bayesian model class selection (BMCS) herein to identify the best link model to use in the generalized linear regression scheme. BMCS (or model comparison) is essentially Bayesian updating at the model class level to make comparisons among candidate model classes given the observed data (e.g., Muto and Beck 2008). The proposed BMCS- and the MLE-based hierarchical fragility estimations lead to essentially the same set of parameters' estimates. Nevertheless, the latter does not readily lead to the confidence band and model selection. The posterior probability of the k^{th} model class, denoted as $P(\mathbb{M}_k|\mathbf{D})$ can be written as follows:

$$P\left(\mathbb{M}_k \middle| \mathbf{D}\right) = \frac{p\left(\mathbf{D} \middle| \mathbb{M}_k\right) P\left(\mathbb{M}_k\right)}{\sum_{k=1}^{N_{\mathbb{M}}} p\left(\mathbf{D} \middle| \mathbb{M}_k\right) P\left(\mathbb{M}_k\right)} \tag{3}$$

Equal weights can be assigned to each model for estimating the prior, i.e., $P(\mathbb{M}_k) = 1/3$. The term $p(\mathbf{D}|\mathbb{M}_k)$ is the likelihood or *evidence*. Jalayer et al. (2023) has shown that the logarithm of the evidence (called *log-evidence*) is the composition of two terms. The first term called "*Average Data Fit*" denotes the posterior expected value of the log-likelihood $\ln[p(\mathbf{D}|\boldsymbol{\theta}_k, \mathbb{M}_k)]$ (see Eq. 4) over the domain of $\boldsymbol{\theta}_k$ conditioned on the model class $\mathbb{M}_k$. The second term is the "*Relative Entropy*" (Kullback and Leibler 1959, Cover and Thomas 1991) between the prior $p(\boldsymbol{\theta}_k|\mathbb{M}_k)$ and the posterior $p(\boldsymbol{\theta}_k|\mathbf{D}, \mathbb{M}_k)$ of $\boldsymbol{\theta}_k$ given model $\mathbb{M}_k$, which is a measure of the distance between the two PDFs. It is the mean information gain of posterior compared to the prior (i.e., the expected value of $\ln[p(\boldsymbol{\theta}_k|\mathbf{D}, \mathbb{M}_k)/p(\boldsymbol{\theta}_k|\mathbb{M}_k)]$ over the domain of $\boldsymbol{\theta}_k$ conditioned on the model class $\mathbb{M}_k$), which is always non-negative (see e.g., Jalayer et al. 2012, Ebrahimian and Jalayer 2021). The relative entropy term measures quantitatively the amount of information (on average) that is "gained" about $\boldsymbol{\theta}_k$ from the observed data $\mathbf{D}$. In the estimation of log-evidence, this expression penalizes for model complexity, i.e., if the model extracts more information from data (which is a sign of being a complex model with more model parameters), the log-evidence reduces.

The likelihood $p(\mathbf{D}|\boldsymbol{\theta}_k, \mathbb{M}_k)$ (used in the first term of log evidence calculation) can be derived, based on point-wise intensity information, as the likelihood of $n_{CL,j}$ buildings being in damage state DS_j considering that $\sum_{j=0}^{N_{DS}} n_{CL,j} = N_{CL}$), according to data $\mathbf{D}$ defined before:

$$p(\mathbf{D} \,|\, \boldsymbol{\theta}_k, \mathbb{M}_k) = \prod_{j=0}^{N_{DS}} \prod_{i=1}^{n_{CL,j}} P\left(DS_j \middle| IM_i\right) \tag{4}$$

2.2.3. MLE-Basic fragility assessment

The so-called "*Basic method*" refers to the general methodology of fitting fragility model to data using one damage state at a time. In this method, $\pi_{ij} = P\left(D \geq D_j \middle| IM_i\right)$, and the fragility is obtained by using a generalized regression model. With reference to the MLE method described previously in section 1.2.1, the vector $\mathbf{x}_j$ herein is the *IM* associated to all damage data (and not partial, as in the hierarchical fragility method), and $\mathbf{y}_j$ is the column vector of one-to-one probability assignment to the *IM* data in $\mathbf{x}_j$ with zero (=0) assigned to those data with an observed damage threshold $D < D_j$, and one (=1) to those with $D \geq D_j$. Thus, for the empirical fragility associated with the damage threshold D_j , and based on the model $\mathbb{M}_k$, there are two model parameters to be defined, namely $\boldsymbol{\theta}_{\text{MLE}-Basic} = \{\alpha_0, \alpha_1\}_k$. The fragility curves obtained under the "MLE-*Basic*" method could potentially cross, leading to the ill condition that $P\left(DS_j \middle| IM\right) < 0$.

3. APPLICATION

3.1. General information

The 2009 south pacific tsunami was triggered by an unprecedented earthquake doublet (Mw 8.1 and Mw 8.0) on September 29, 2009. The tsunami seriously impacted numerous locations in the central South Pacific. Herein, the damage data related to the 1-storey brick masonry residential (Class 1) and Timber residential (Class 2) buildings associated with the reconnaissance survey sites of American Samoa and Samoa islands were utilized. Based on the observed damage regarding different indicators (see Reese et al. 2011 for more details on damage observation), each structure was assigned a damage state between (DS_0 and DS_5). The original data documented in Reese et al. (2011) reporting the tsunami flow depth and the attributed damage state to each surveyed building can be found on the site of the European Tsunami Risk Service

(ETRiS, https://eurotsunamirisk.org/). The five damage levels ($N_{DS} = 5$) and a description of the indicators leading to the classification of the damage states are given in Table 1 based on Reese et al. (2011). For the building class 1, 120 data is available in all damage levels from 0 to N_{DS}=5; however, for class 2, only 23 damage data are available in the damage levels ranging from 2 to 5. Note that the number of fragility curves derived is going to be equal to 5 for class 1, and equal to 3 for class 2.

Table 1: The damage scale used for 2009 South Pacific Tsunami (Reese et al. 2011).

Damage Level		Damage level description
D_0	None	no damage
D_1	Light	non-structural damage
D_2	Minor	significant non-structural and minor structural damage
D_3	Moderate	significant structural and non-structural damage
D_4	Severe	irreparable structural damage, will require demolition
D_5	Collapse	complete structural collapse

3.1.1. *BMCS- and MLE-based hierarchical fragility modelling.*

With reference to Section 1.2.2, the posterior probability of each model, $P(\mathbb{M}_k|\mathbf{D})$, where k=1:3, can be estimated based on the BMCS approach. For each model class $\mathbb{M}_k$, the model parameters $\boldsymbol{\theta}_k$ are estimated through the adaptive MCMC method described in detail in Jalayer et al. (2023), which yields the posterior distribution $p(\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k)$. Using the drawn samples from the joint posterior PDF $p(\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k)$ for a selected building class, the log-evidence $\ln[p(\mathbf{D}|\mathbb{M}_k)]$, can be estimated by subtracting the second term *Relative Entropy* term from the first term *Average Data Fit* (Table 2 for the two buildings classes).

Table 2: Model selection based on BMCS approach.

Class	Model	Average Data Fit	Information Gain	Log-Evidence	Posteriore $P(\mathbb{M}_k\|\mathbf{D})$ (Eq. 3)
1	$\mathbb{M}_1$	-124.4561	23.6853	-148.1414	0.055
	$\mathbb{M}_2$	-123.4659	23.9566	-147.4224	0.113
	$\mathbb{M}_3$	-120.6454	24.7810	-145.4264	0.832
2	$\mathbb{M}_1$	-20.4791	9.7549	-30.2340	0.315
	$\mathbb{M}_2$	-19.9106	10.4660	-30.3766	0.273
	$\mathbb{M}_3$	-19.7565	10.2117	-29.9682	0.411

The last column illustrates the posterior probability (weight) of the model $P(\mathbb{M}_k|\mathbf{D})$ according to Eq. (4) assuming that the prior $P(\mathbb{M}_k) = 1/3$ (where k=1:3). The best model for each building class is $\mathbb{M}_3$ shown with a blue color.

The robust fragility curves derived from the BMCS-based hierarchical fragility curves and the corresponding plus/minus one standard deviation ($\pm 1\sigma$) intervals are also plotted in Figure 2 corresponding to Classes 1-2, and for the best model $\mathbb{M}_3$ (see Table 2) and for the damage thresholds defined in Section 2.1. The colors of the hierarchical robust fragility curves, labled as D_j-BMCS, match closely those shown in Table 1. The corresponding $\pm 1\sigma$ confidence interval curve, which reflects the uncertainty in the model parameters, is shown as a light grey area with different color intensities. The BMCS-based hierarchical robust fragility and its confidence interval are compared with the result of MLE-based hierarchical fragility assessment, labeled as D_j-MLE. The latter fragility curves are shown with similar colors (and darker intensity) and with the same line type (and half of the thickness) of the corresponding D_j-BMCS fragility curves. The first observation is that the results of MLE- and BMCS-based fragilities are quite close in all damage thresholds (as expected, see Jalayer and Ebrahimian 2020). Moreover, the BMCS provides also the confidence bands for the fragility curves, which cannot be directly provided by the MLE method. Figure 3 illustrates the BMCS-based fragility curve associated with the ultimate damage threshold D_5 for both building classes together with all the sample fragilities, which are based on realizations of the model parameters drawn from the joint PDF $p(\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k)$. The *IM* values for which the damage level is not exceeded are shown with blue circles having the probability equal to zero. Other *IM*s that lead to the exceedance of the damage level are shown with red circles with a probability equal to one. Moreover, Figure 3 illustrates the fragility parameters including: the equivalent lognormal parameters median, η_{IM_C}, and logarithmic standard deviation (dispersion), β_{IM_C}; the epistemic uncertainty in the empirical fragility assessment β_{UF}; the intensities IM_C^{16}, IM_C^{84}, IM^{84}, and IM^{16} (the latter two are IMs at the median, i.e. 50% probability, from the robust fragility minus/plus one standard deviation, respectively.

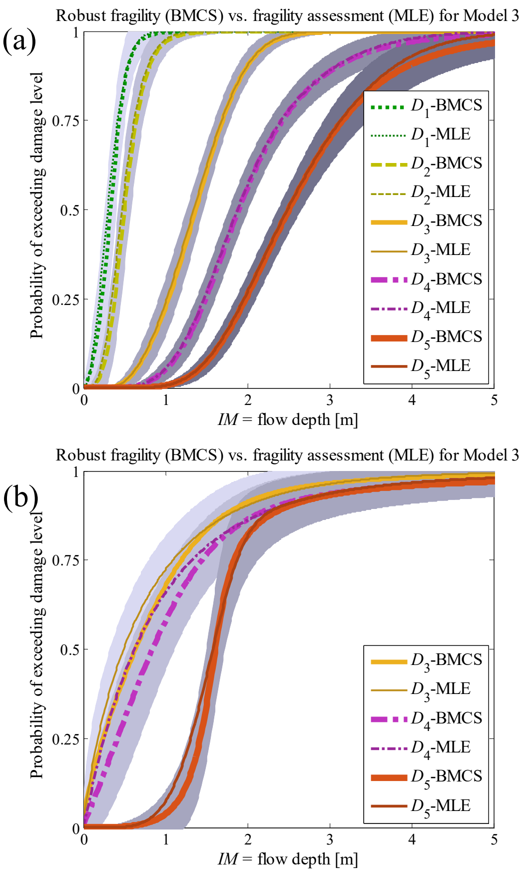
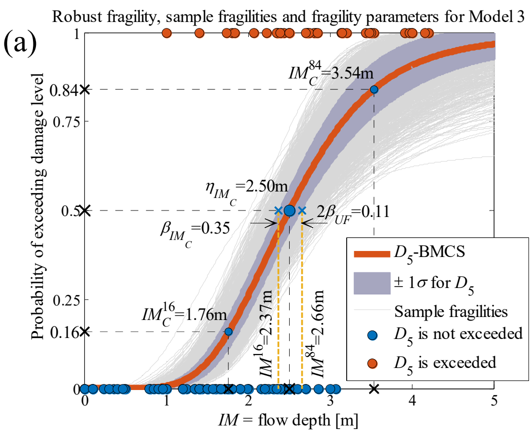


Figure 2: Hierarchical BMCS-based fragility and their $\pm 1\sigma$ confidence intervals in comparison with MLE method for building classes (a) 1 and (b) 2.



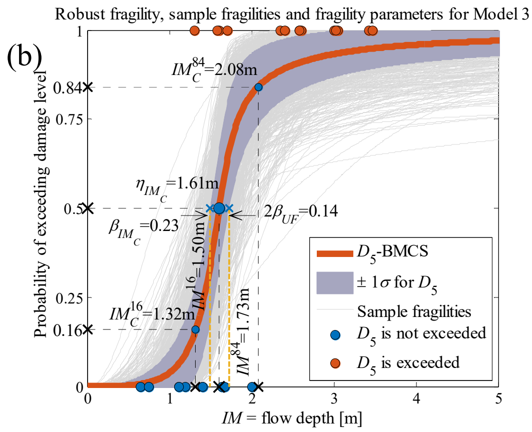


Figure 3: Robust fragility associated with D_5, together with all the sample fragilities, and the fragility parameters for building classes (a) 1 and (b) 2.

In order to define these parameters, it is first noted that the empirical fragilities (hierarchical or basic) are not necessarily attributed to a lognormal distribution (with the exception of using probit link function for MLE-*Basic* fragility assessment). Hence, we have derived equivalent lognormal statistics η_{IM_C} and β_{IM_C} for the resulting fragility curves. The median intensity, η_{IM_C}, for a given damage level, is calculated as the *IM* corresponding to 50% probability on the fragility curve. The dispersion β_{IM_C} is estimated as half of the logarithmic distance between the *IM*s corresponding to the probabilities of 16% (IM_C^{16}) and the 84% (IM_C^{84}) on the fragility curve; thus, the dispersion can be estimated as $\beta_{IM_C} = 0.50 \times \ln(IM_C^{84}/IM_C^{16})$. The overall effect of epistemic uncertainties on the median of the empirical fragility curve (due to the uncertainty in the fragility model parameters and reflecting the effect of limited sample size) can be seen by employing the BMCS-based hierarchical approach. This is considered through (logarithmic) intensity-based standard deviation denoted as β_{UF} (see Jalayer et al. 2020). β_{UF} can be estimated as half of the (natural) logarithmic distance (along the *IM* axis) between the median intensities (i.e., 50% probability) of the fragilities derived with 16% (denoted as IM^{84}) and 84% (IM^{16}) confidence levels, respectively; i.e., $\beta_{UF} = 0.50 \times \ln(IM^{84}/IM^{16})$.

3.1.2. MLE-based hierarchical and basic fragility modelling.

Figure 4 compares the fragility assessment obtained based on MLE-based hierarchical fragility modeling (see also the MLE-based curves in Figure 2) with the result of MLE-*Basic* method for the "best" model $\mathbb{M}_3$ outlined in the previous section. It is noted that the fragility functional form is different between the two methods. MLE-based fragility assessment uses Eq. (1) and Eq. (2) to construct hierarchical fragility curve given that the conditional fragility term $\pi_{ij} = P\left(D \geq D_{j+1} \middle| D \geq D_j, IM_i\right)$ based on the partial damage data is one of the link functions defined in Section 1.1. However, in the fragility assessment using MLE-*Basic* method, the fragility curve $\pi_{ij} = P\left(D \geq D_j \middle| IM_i\right)$ is directly one of the link expressions (based on the whole damage data). This difference manifests itself in the deviation between the two fragility models for higher damage thresholds D_4 and D_5 (see Figure 4a for class 1), which are particularly noticeable at higher *IM* values (with exceedance probability >50%); however, their medians are quite similar. In Figure 4b (class 2), we can observe that fragility curves do intersect in the case of MLE-*Basic* fragility assessment, while this condition does not exist in hierarchical fragility curves. The intersection points of the consequent damage states D_{j+1} with D_j when using the MLE-*Basic* fragility estimation method are shown with color stars on each figure.

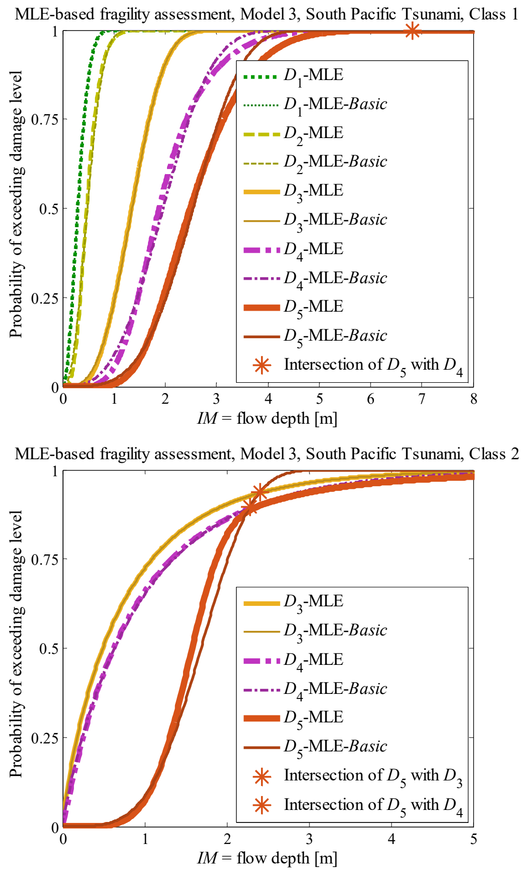


Figure 4: (a) Comparison between the fragility assessment by MLE-based hierarchical fragility modeling and MLE-Basic fragility assessment given (a) class 1, (b) class 2.

4. CONCLUSIONS

- A fully simulation-based Bayesian model class selection (BMCS) workflow for empirical hierarchical fragility modeling given a class of buildings or infrastructure is presented.
- The workflow is capable to: (1) perform hierarchical fragility modeling for MECE damage states; (2) estimate the confidence interval for the resulting fragility curves; (3) select the simplest model that fits the data best (i.e., maximizes log evidence) amongst a suite of candidate fragility models (herein, alternative link functions for generalized linear regression are considered).
- The proposed workflow is quite general and can be applied to any type of damage expressed by a set of MECE data, where the exceedance of a damage is expressed as a Bernoulli variable.
- The workflow is very robust to cases where few data points are available and/or where data is missing for some of the damage levels.
- Hierarchical fragility assessment can be done also based the MLE. For each damage level, the reference domain should be the subset of data that exceeds the consecutive lower damage level, instead of taking the entire set of data points as reference domain.
- The basic fragility estimation ("MLE-*Basic*", non-hierarchical fragility model) fits the damage data for each damage level at a time (the reference domain is set to all damage data). Using MLE-

Basic leads to results that are slightly different from the hierarchical fragility curves. The difference grows for higher damage levels, and may lead to ill-conditioned results (i.e., fragility curves crossing).

- The proposed workflow is provided as open-source software on the site of the European Tsunami Risk Service (ETRiS): https://eurotsunamirisk.org/tsunamirisktoolkit/
The code is also available at the following URL: https://github.com/eurotsunamirisk/computeFrag/tree/main/Code_ver2.

5. AKNOWLEDGEMENT

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