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The yield behavior of Leighton Buzzard sand in a hollow cylinder apparatus

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Abstract

The yield behavior of Leighton Buzzard sand was investigated for different magnitudes of the three principal stresses and rotation of the major principal stress axis using a hollow cylinder apparatus. The magnitude of the deviator stress at yield was found to vary continuously, although in a well-defined pattern, under generalized stress conditions. The Lade yield criteria and Matsuoka-Nakai yield criteria were both found to adequately predict the onset of yielding under generalized stress conditions. The experimental data for the Mohr-Coulomb yield criteria deviated most from the theoretical values and did not follow a consistent pattern. The experimental data indicated that the magnitude of the deviator stress at yield and the magnitude of all three yield criteria were independent of the magnitude of rotation of the major principal stress axis.

Introduction

Most sedimentary soils are inherently anisotropic. Hence, their response under applied loading depends on both the magnitude and the orientation of the principal stresses in the ground. The principal stresses rotate during loading in most geotechnical engineering problems. As a result, ground deformations can occur due to both changes in the magnitudes and changes in the directions of the principal stresses.

The yield behavior of Leighton Buzzard sand was investigated for generalized stress conditions using a hollow cylinder apparatus. Stress paths were followed such that segments of yield surfaces could be determined from a single test specimen. The yield response was investigated for different rotations of the major principal stress axis, with the magnitude of the intermediate principal stress varying between that of the major and minor principal stresses. The experimental yield data was compared with the theoretical values predicted by the Mohr-Coulomb, Matsuoka-Nakai and Lade yield criteria to establish whether these criteria are suitable for predicting the onset of yielding in sand under generalized stress conditions.

Yield criteria

The Mohr-Coulomb yield criteria is a generalized form of the Tresca yield criteria and is written in terms of the stress state and two material properties, the effective cohesion c' , and the effective angle of internal friction ϕ' . Assuming the value of $c' = 0$, and a constant ϕ' for all stress states, the Mohr-Coulomb yield criteria can be written in terms of the effective major (σ_1') and minor (σ_3') principal stresses, as follows:

$$\frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = \text{Mohr-Coulomb constant}$$

The Matsuoka-Nakai yield criteria is a theoretical development of the concept of the compoundedly mobilized planes and spatially mobilized planes originally presented by Matsuoka (1974). The Matsuoka-Nakai yield criteria is expressed in terms of the effective stress invariants (Matsuoka & Nakai, 1985) and is given by:

$$\frac{J_1 J_2}{J_3} = \text{Matsuoka-Nakai constant}$$

where J_1 , J_2 and J_3 are the first, second and third effective stress invariants, respectively.

The Lade yield criteria was developed following an analysis of Cubical Triaxial test data for sand (Lade & Duncan, 1975). The Lade yield condition is best expressed in terms of the first and third effective stress invariants, as follows:

$$\frac{J_1^3}{J_3} = \text{Lade constant}$$

Overview of the hollow cylinder apparatus

The hollow cylinder apparatus used in this study was developed at University College Dublin, Ireland. The test apparatus subjects a hollow cylindrical test specimen (50mm outer radius x 35.5mm inner radius x 200mm long) to an inner confining pressure (p_i), an outer confining pressure (p_o), an axial load (W), and a torque (T), Figure 1a. The test apparatus was closed-loop controlled allowing precise regulation of the applied axial load, torque and the confining pressures. Figure 1b presents the resultant normal and principal stresses induced in an element of the test specimen wall. The deformational response of the test specimen is shown in Figure 1c.

The automated test apparatus is capable of regulating the induced stresses in the test specimen to 0.5 kPa during either drained or undrained test conditions. The

deformational response of the test specimen is recorded using internal and external instrumentation to a resolution of at least 5×10^{-5} strain and 6×10^{-5} strain, respectively. A more detailed description of the apparatus is given by O'Kelly & Naughton (2003). The control program automatically compensates for membrane penetration effects using the method developed by Sivathayalan & Vaid (1998) and for membrane restraint effects using corrections developed by Tatsuoka *et al.* (1986).

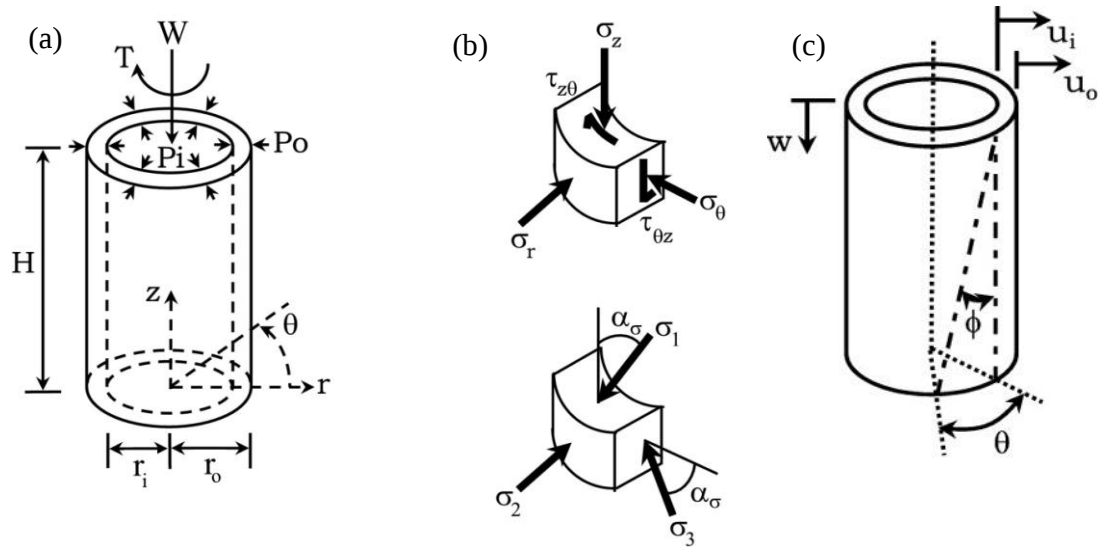


Figure 1. (a) System of confining pressures, axial and torsional loads applied to the test specimen. (b) Resultant normal and principal stresses acting on an element of the test specimen wall. (c) Deformational response.

Investigation of anisotropy

The study examined the yield behavior of Leighton Buzzard sand. Physically identical test specimens were anisotropically consolidated by increasing the effective major-to-minor principal stress ratio, R' ; re-orientating the direction of the major principal stress axis, α_σ ; and changing the magnitude of the intermediate principal stress relative to the magnitude of the major and minor principal stresses. The relative value of the intermediate principal stress was quantified in terms of the intermediate principal stress parameter, b . The b parameter has a range of 0 to 1. With $b = 0$, the intermediate and minor principal stresses are equal, while $b = 1$ results in the intermediate and major principal stress being equal.

Properties of the sand and sample preparation method

Fine-to-medium, white Leighton Buzzard sand was examined in the present study. Some physical properties of the sand are summarized in Table 1. The sand examined was selected so that the maximum particle diameter to sample wall thickness ratio of 1:20 was sufficiently large so as not to interfere with the development of potential shear bands (Wroth & Houlsby 1985). Hollow cylindrical test specimens were formed using a wet pluviation technique developed by O'Kelly (2000). Saturated test

specimens in a loose condition were prepared as a single layer by depositing sand between inner and outer annular moulds using wet pluviation. Tapping the sides of the moulds caused the test specimen to densify.

Table 1. Properties of the Leighton Buzzard sand.

Property	Coefficient of uniformity	Coefficient of curvature	Mean particle diameter D_{50} (mm)	Specific gravity	Maximum void ratio	Minimum void ratio
Value	1.32	0.96	0.52	2.64	0.77	0.50

The test specimens had an initial relative density of between 72 % and 78 %, Table 2. Naughton (2002) showed that this level of variation did not have a significant effect on the constitutive response of the test specimens for isotropic loading conditions. The inherent anisotropy of the test specimens was discussed by Naughton & O’Kelly (2003) while the stress-induced anisotropic response was discussed by Naughton & O’Kelly (2004).

Table 2. Properties of test specimens immediately following sample setup.

Test	Dry mass of sand (g)	Height (mm)	Inner radius (mm)	Outer radius (mm)	Relative density (%)
Y0000	1174.0	181.5	35.5	50.0	77.5
Y0500	1163.8	181.4	35.5	50.0	72.4
Y0030	1173.0	181.5	35.5	50.0	77.0
Y0530	1174.1	181.5	35.5	50.0	77.5
Y0060	1170.0	181.6	35.5	50.0	75.5
Y0560	1163.3	181.4	35.5	50.0	72.2
Y0090 ⁺	1165.6	181.5	35.5	50.0	73.3
Y0590 ⁺	1176.0	181.5	35.5	50.0	78.4

⁺Tests Y0090 and Y0590 consisted of an instantaneous rotation of α_σ at the end of isotropic consolidation. This was achieved by starting the anisotropic consolidation stage from $\alpha_\sigma = 90^\circ$, $b = 0$ and $R' = 1$.

Stress paths followed

The stress paths were designed using the same methodology as Tatsuoka *et al.* (1974) and consisted of three stages. The first stage of the stress path involved consolidating the test specimens isotropically from an effective stress of 50 kPa (at the end of sample saturation) to an effective stress of 100 kPa. This stage ensured that all of the samples were normally consolidated.

The second stage of the stress paths involved increasing R' , from $R' = 1.0$ to $R' = 1.5$, while at the same time rotating the major principal stress axis to $\alpha_\sigma = 0^\circ, 30^\circ, 60^\circ$ or 90° . The value of the b parameter was either maintained at $b = 0$ or increased from $b = 0$ to $b = 0.5$ during this anisotropic consolidation stage. The tests were designated

as Ybbaa, where bb denotes the final magnitude of the b parameter, with the decimal point removed, and aa denotes the final magnitude of α_σ , Table 2.

The third stage of the stress paths was designed to determine a segment of different yield surfaces in generalized stress space. After establishing an initial point on a yield surface, the stress path would search for another point on that yield surface at a different location in stress space, Figure 2. The mean effective confining stress and the rotation of the major principal stress axis were both held constant during this stage of the stress path. The initial yield point (Points B and F, Figure 2) was established by increasing the effective major-to-minor principal stress ratio, until significant non-linear straining of the test specimen commenced. After establishing the value of the initial yield point, the value of R' was reduced and the test specimen reconsolidated to a higher value of the b parameter. The magnitude of R' was increased again in order to locate another point on that yield surface. The stress path started from $b = 0$ or $b = 0.5$ at the beginning of the third stage, and the value of the b parameter was incremented in steps of 0.25. This procedure allowed a segment of two yield surfaces to be determined from a single sample.

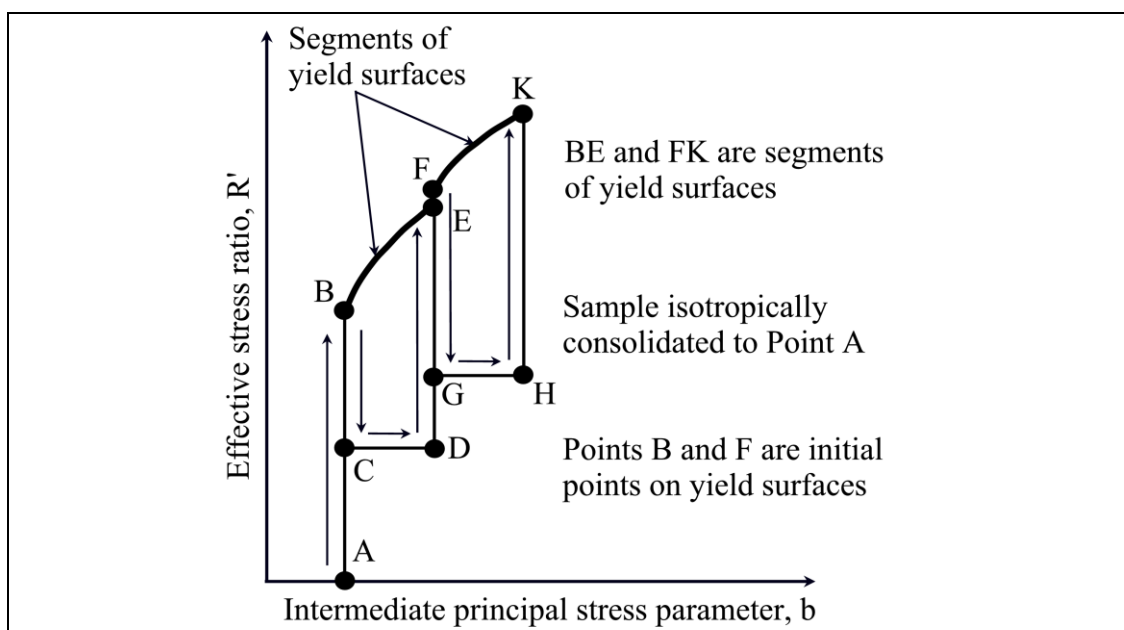


Figure 2. Typical stress path used to determine segments of the yield surfaces.

The stress-strain response of test specimen Y0530 during stage three is shown in Figure 3. The stress-strain response was generally linear for the unload part and the early stage of the reload part becoming slightly non-linear as the yield surface was approached. No definitive location for the yield point was evident from the stress-strain relationships with the actual location of a yield point established using a graphical procedure, Figure 4(a). The idealized linear-elastic and plastic sections of the stress-strain curves were extended with their point of intersection defining one point on the yield surface. A minimum of five stress-strain relationships could be plotted for each stage of the stress paths. However, significant variation in the

location of the yield point from different stress-strain plots occurred. When expressed in term of R' , the location of the yield points varied from 0.2 % to 10.9 % about the mean value, Figure 4(b). Changing the magnitude of the b parameter produced very small stains in the test specimens

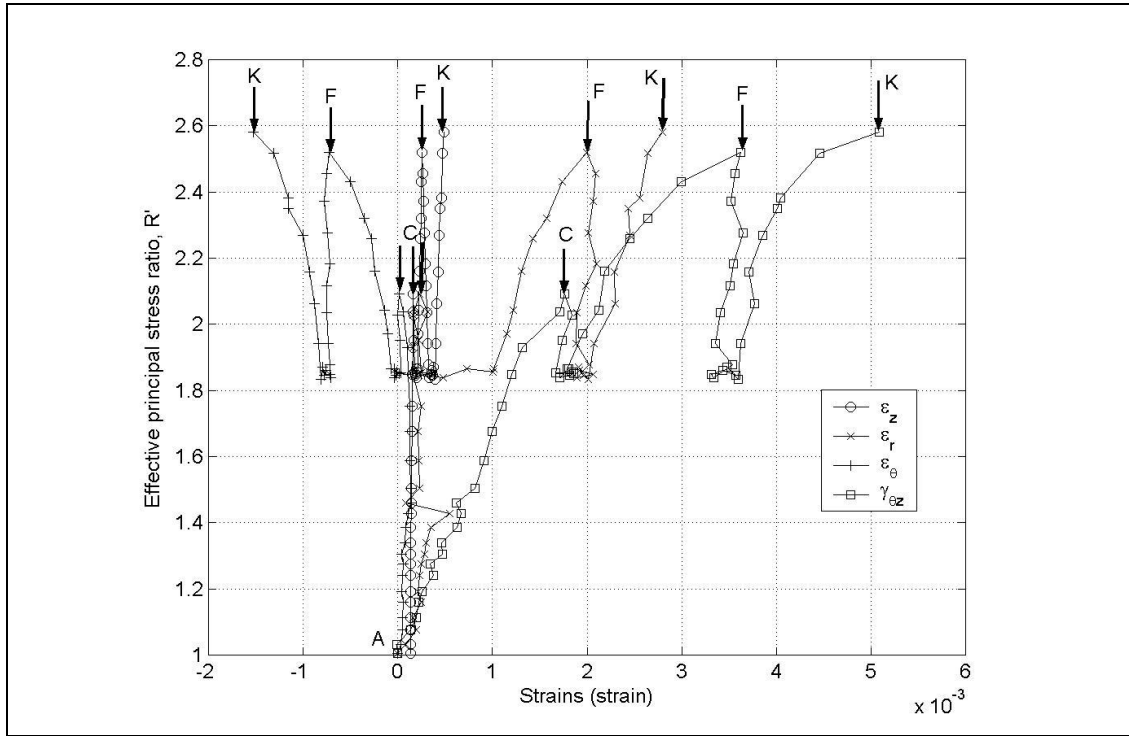


Figure 3. Stress-strain response of test specimen Y0530.

Experimental Results

Figure 5 presents the experimental data plotted in terms of α_σ , the b parameter, and the magnitude of the deviator stress at yield, q , given by:

$$q = \sqrt{(\epsilon'_1 - \sigma'_3)^2 + (\epsilon'_1 - \sigma'_2)^2 + (\epsilon'_2 - \sigma'_3)^2}$$

In general, the magnitude of the deviator stress at yield appears to follow a well defined pattern and is independent of α_σ . Two trends are observed:

1. Tests that start from $b = 0$, all display a decrease of approximately 15 to 20 % in the magnitude of the deviator stress as the magnitude of b increases
2. Tests that start from $b = 0.5$ display an increase in the magnitude of the deviator stress at yield or remain relatively constant as the b parameter approaches unity.

The minimum magnitude of the deviator stress on the yield surface was always located at $b = 0.5$.

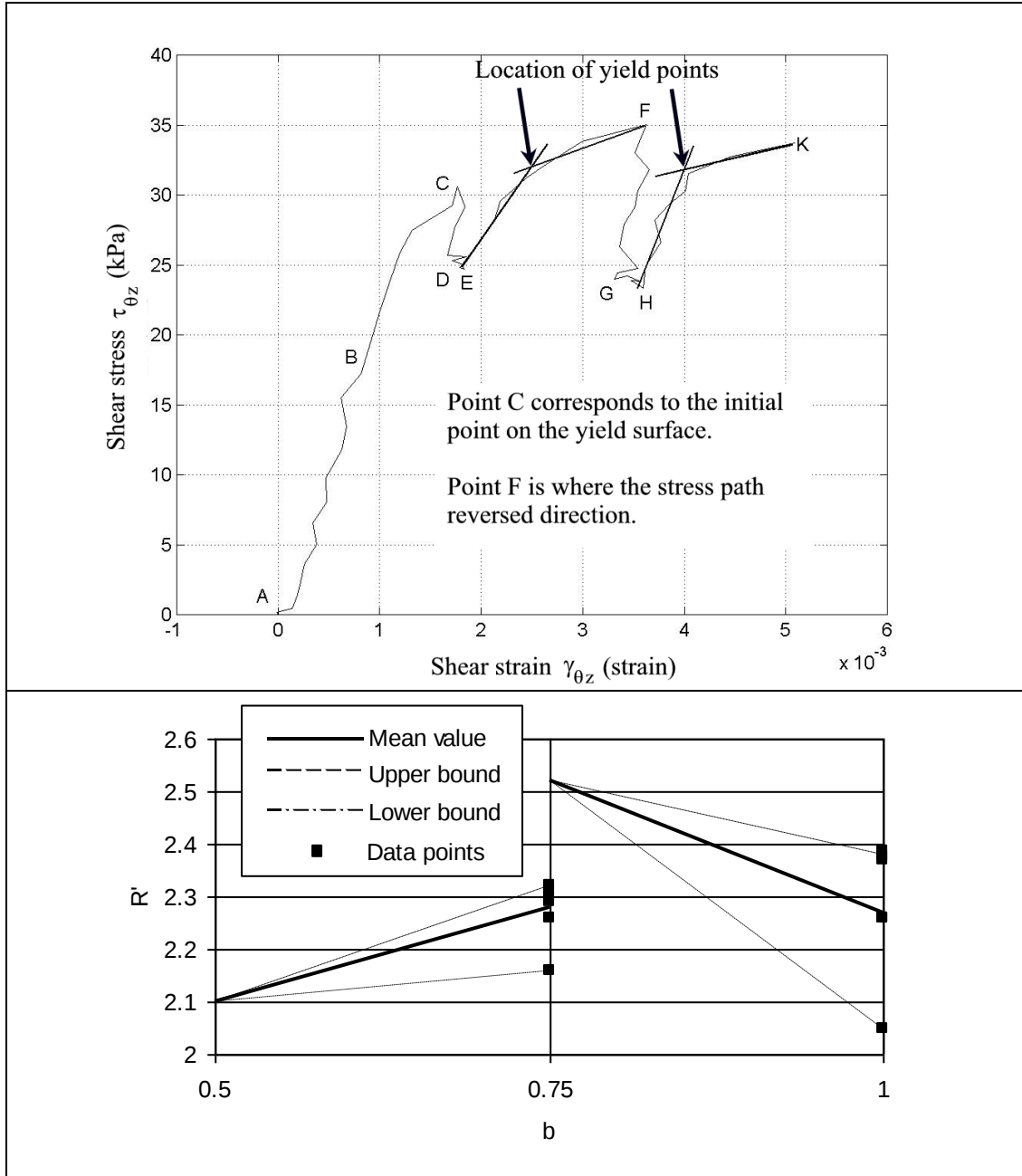


Figure 4. (a) Determination of yield point using graphical technique for Test Y0530 and (b) variation in the magnitude of the yield point for test Y0530.

Figure 6 presents the experimental yield data expressed in terms of the Mohr-Coulomb, Matsuoka-Nakai and Lade yield constants, versus the b parameter for different orientations of the major principal stress axis. The experimental data is normalized relative to the initial point on the yield surface at each stage of the stress path. The magnitude of the theoretical yield surface for each of the yield criteria is therefore unity. This allows a direct comparison between the experimental and theoretical yield surfaces defined using the different yield criteria.

In general, the experimental data for the Matsuoka-Nakai and Lade yield criteria follow the same pattern when compared with the theoretical yield values. The onset of yielding was slightly under-estimated for $0.0 \leq b \leq 0.5$ and, in general, slightly over-estimated for $0.5 < b \leq 1.0$. On the other hand, the Mohr-Coulomb yield criteria does not appear to follow a general trend as the magnitude of the b parameter changes.

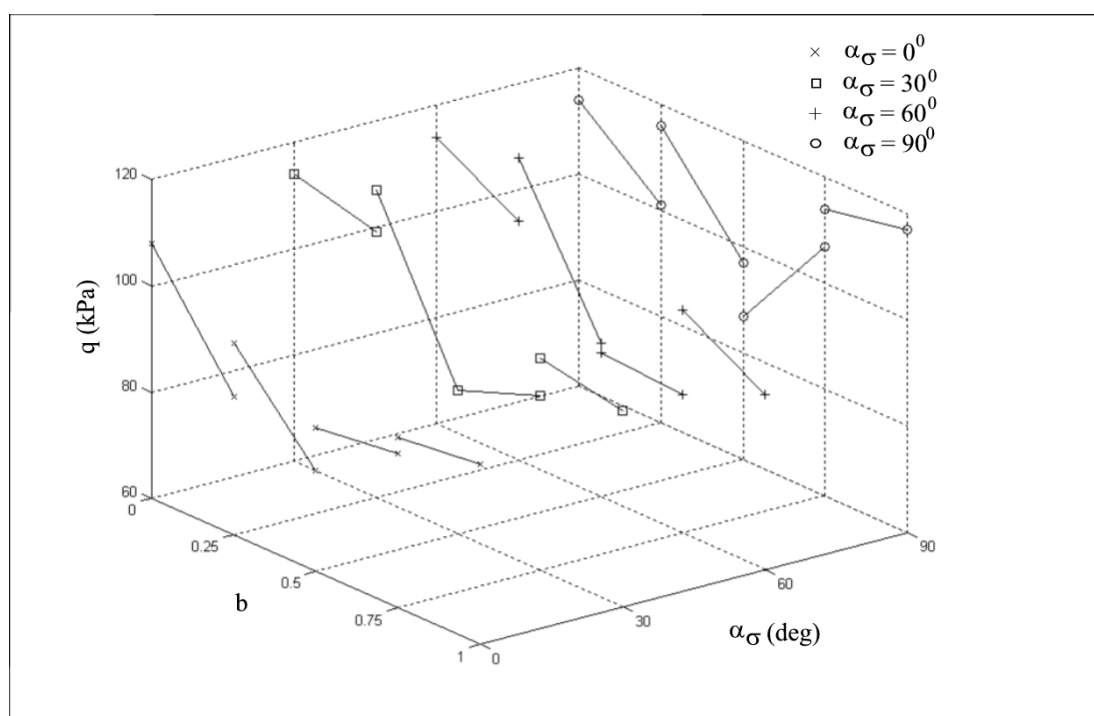


Figure 5. Variation of the deviator stress on the yield surface.

Discussion

The stress paths followed in this study successfully established segments of different yield surfaces from identically prepared sand test specimens which had initially undergone different rotations of the major principal stress axis. The shape of the experimental yield surface, expressed in terms of the deviator stress, Mohr-Coulomb, Matsuoka-Nakai and Lade yield criteria, appears to be largely unaffected by the magnitude of rotation of the major principal stress axis during the consolidation stage of the tests.

Conclusions

The following points are concluded from this investigation of yielding in Leighton Buzzard sand:

1. Stress paths successfully identify segments of different yield surfaces in generalized stress space from a single test specimen using a hollow cylinder apparatus.

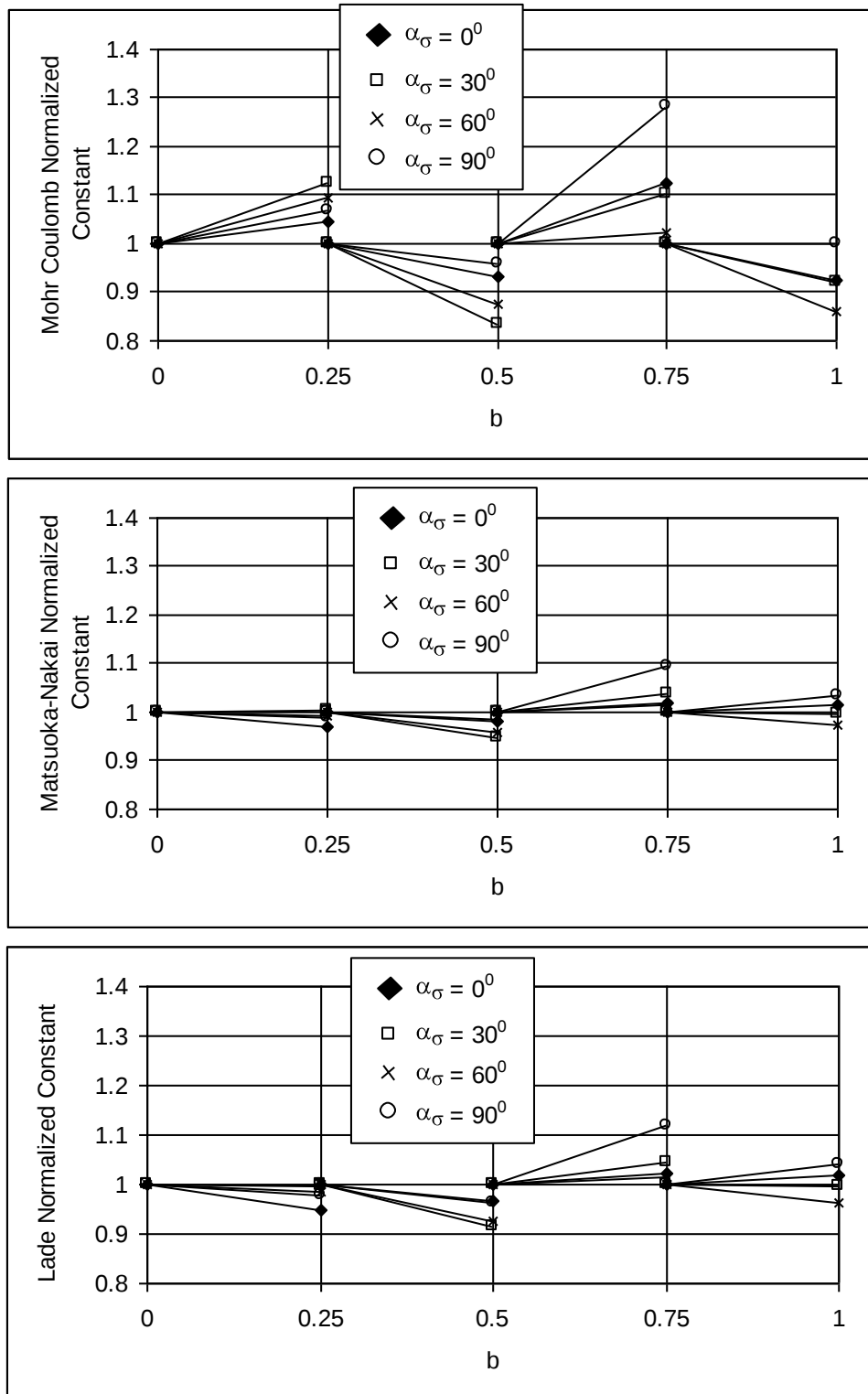


Figure 6. Experimental yield surfaces expressed in terms of the Mohr-Coulomb, Matsuoka-Nakai and Lade yield criteria.

2. The magnitude of the deviator stress at yield was dependent on the initial value of the b parameter, but was largely independent of the magnitude of the rotation of the major principal stress.
3. The Matsuoka-Nakai and Lade yield criteria were both found to adequately predict the onset of yielding under generalized stress conditions.
4. The Mohr-Coulomb yield criteria generally deviated more from the theoretical values than the other two criteria examined and did not appear to follow a consistent pattern.
5. The experimental yield surfaces indicate that the prediction of yielding by all three criteria is predominately independent of the magnitude of the rotation of the major principal stress during the initial consolidation stage.

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