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A METHOD FOR OPTIMAL RECONSTRUCTION OF VELOCITY RESPONSE USING EXPERIMENTAL DISPLACEMENT AND ACCELERATION SIGNALS

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Abstract

An algorithm is developed which takes advantage of the positive characteristics of integration and differentiation operations which can produce an optimal time-domain estimate of a velocity signal when a displacement and an acceleration signal are measurable. Simulations are carried out with Matlab where synthetic data is generated through solving the equations of motion of a multi-degree of freedom vibrating system. The velocity optimisation technique for velocity estimation is applied to the acceleration and displacement signals and the result is compared to the true velocity signal. A parametric analysis is carried out on the technique, as a function of extraneous noise on the simulated signals, as well as the integration and differentiation operator employed. Based on the results of the simulations, an experiment is performed where a reconstructed velocity is compared to a velocity signal measured directly using a laser vibrometer.

1. INTRODUCTION

Modern experimental identification techniques, although varying in algorithm, generally require high quality excitation and response signals - see Rice & Fitzpatrick [1], for a representative account. These signal; force, displacement, acceleration etc., should ideally be acquired directly from their respective transducers. However, due to constraints in size or budget, this is not always possible and quite often it is necessary to transform signals by the use of differentiation or integration algorithms, whether in the time or the frequency domain. These differentiation and integration operators introduce errors into the transformed signals when there is extraneous noise present, which is usually the case. Specifically, noise is amplified with a bias towards high

frequencies for a differentiated signal and low frequencies for an integrated signal.

This being recognized as a significant problem by both researchers and field-engineers, it is somewhat surprising to notice the lack of attention it receives by the community dealing with system identification, at least in the open literature. Nevertheless, if not explicitly, this issue can often be perceived in the background of experimental identification papers - see, for instance, Richard et al. [2] or Zhao et al. [3].

Basically, the differentiation of a signal in the frequency domain is achieved by multiplying the Fourier transform of the time history by $i\omega$. Similarly the integration can be determined by dividing by $i\omega$ (taking symmetrical properties of the FFT into account). If noise is contained in the original time histories then it also will be multiplied or divided by $i\omega$. Due to this, for the case of differentiation, the corruptive influence of noise will increase with frequency whereas for integration it will decrease. Hence differentiated displacement signals will typically exhibit undue high-frequency energy, while integrated acceleration signals will display unacceptable low-frequency excursions, both of them clearly unphysical. This algorithm uses both estimates of velocity from integrated acceleration and differentiated displacement and combines the more accurate regions of both to produce an optimum velocity signal. Being based on a frequency-domain criterion, the method proposed here is aimed at post-processing of experimental signals for off-line identification purposes and is thus not suited for real-time control implementations.

1.1. Basic Algorithm for Velocity Reconstruction

Consider any data record $x(t)$ of total length T_r that is stationary with zero mean ($\mu_x = 0$). Let the record be divided into n_d contiguous segments, each of length T . Each segment of $x(t)$ is termed $x_i(t)$, where $i = 1, 2, \dots, n_d$. In digital terms, each record segment $x_i(t)$, is represented by N data values $\{x_{in}\}$, with $n = 0, 1, 2, \dots, N - 1$. Given this definition for digitised signals (the sampled acceleration and displacement signals for example), a discrete, finite, single sided, autospectral density function estimate for any signal $x(t)$ may be written as

$$\hat{G}_{xx}(f_k) = \frac{2}{n_d N \Delta t} \sum_{i=1}^{n_d} |X_i(f_k)|^2 \quad k = 0, 1, \dots, \frac{N}{2} \quad (1)$$

Figure 1 shows a block diagram of the basic algorithm. The noisy displacement and acceleration signals, whether simulated or experimental, are differentiated and integrated respectively. Averaged autospectra are calculated from these signals using a sufficiently high number of averages and ensuring an adequate frequency resolution. The presence of noise is assumed to increase the energy in the signal, and so, by comparing the two spectra, a cut off frequency (f_{cut}) is determined which chooses the sections of the two which have the lowest energy. Based on this cut-off frequency (and corresponding bin number), the FFTs of the full length displacement and acceleration signals are filtered in the frequency domain. The filtering is simply a case of multiplying the unwanted Fourier coefficients by zero, taking the symmetrical properties of the FFTs into account. The two filtered FFTs are added and by performing an inverse FFT operation, the optimum estimate of the velocity signal in the time domain may be obtained.

1.2. Variant on the Basic Algorithm

This is very similar to the previous method but instead of picking a single cut-off frequency, the two power spectra are compared frequency by frequency for the one with the least energy and

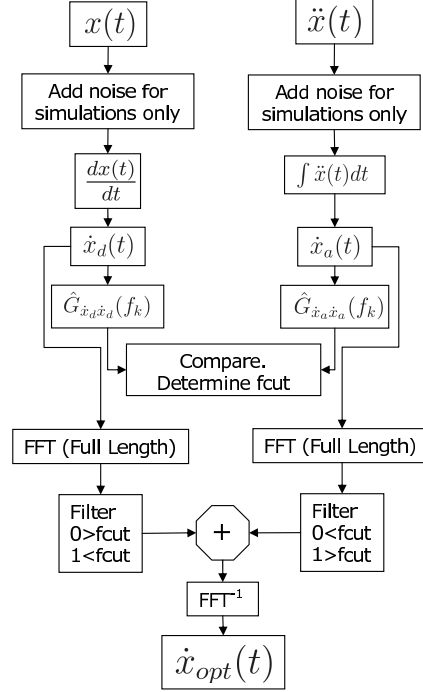


Figure 1. Block Diagram of Optimal Velocity Algorithm

the corresponding FFT's are combined based on this. Although mentioned, this variant of the basic algorithm will not be further explored here.

2. NUMERICAL SIMULATIONS

2.1. Test Problem

Using MATLAB, synthetic data was generated to simulate a linear, two degree of freedom system. This simple oscillator had the characteristics of both masses being equal to 1 Kg, natural frequencies being 10Hz and 200Hz, and damping being set at each dashpot in order to obtain modal damping values of 5% and 1% respectively. This system was excited with a single random force with a white noise excitation spectrum.

The signals, (excitation and response), could have uncorrelated, gaussian noise added before beginning the algorithm. The objective was to use the noisy displacement and acceleration signals to solve for velocity which could then be compared to the “true” velocity produced by the generator. Noise was added to each of the signals using the following equation,

$$x(t) + \left[\left(\sigma_x \frac{n}{100} \right) r(t) \right] \quad (2)$$

where n is the noise in percent and $r(t)$ is a random signal of normalised standard deviation. The three signals of interest are shown in figure 2.

2.2. Differentiation and Integration Operators

For this paper, it should be noted that all differentiation and integration was done in the time domain. However frequency domain techniques are available which eliminate frequency de-

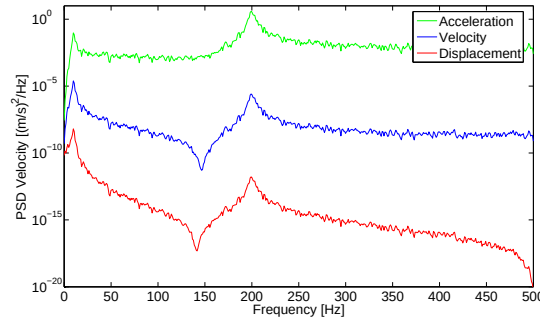


Figure 2. Response signals for simulated data. No noise case.

pendent biases on the operators which can be present even in the absence of noise. In order to highlight these frequency dependent biases, two (time domain) integration and differentiation algorithms were implemented and the results of their implementation are to be seen in figures 3(a) and 3(b). No noise has been added in these two cases. For the case of differentiation, it is seen that for both of the algorithms implemented, the operators tend to bias the result downwards at higher frequencies. The better performing first algorithm calculated simply according to the finite difference scheme

$$f'_{1-2} = \frac{f_1 - f_0}{h} \quad (3)$$

is chosen subsequently over the “three-point formulas” algorithm of the second technique. This downward bias of the operators invalidates the assumption that noise adds energy to the spectrum of the signal, as a small amount of additional noise may result in a differentiated signal with less energy at high frequencies than the true signal. However, it takes only 0.5% of added noise to counteract the downward bias and so, for real signals, this bias may be ignored.

For the integrated signals, the rectangular rule

$$J = \int_a^b f(x)dx \approx h[f(x_1) + f(x_2) + \dots + f(x_n)] \quad \left(h = \frac{b-a}{n}\right) \quad (4)$$

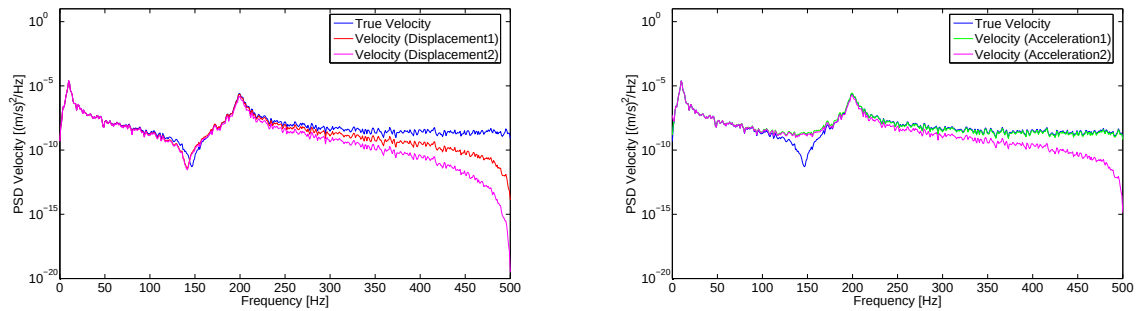
results in practically no bias in the integrated signal, and so is the preferred algorithm over the trapezoidal rule.

2.3. Parametric Analysis

In order to study the algorithms performance, a noise sensitivity analysis was performed on the signals. Tests were carried out with equal and different percentages of noise superimposed on the acceleration and displacement signals. Both scenarios are possible in an experimental environment. An error was calculated between the noisy reconstructed velocity and the true generated velocity in each case according to

$$\text{Error} = \frac{\sigma_{[\text{true velocity}(t) - \text{processed velocity}(t)]}}{\sigma_{[\text{true velocity}(t)]}} * 100 \quad (5)$$

Table 1 shows the results for these tests. The table is divided into three main blocks where they show the error between the true velocity and the integrated acceleration, differentiated



(a) Differentiated displacement versus true velocity. (b) Integrated acceleration versus true velocity. Calculated from, 1: the 2-point finite difference scheme, 2: the rectangular summation rule, 3: the trapezoidal rule.

Figure 3. Comparison between different integration and differentiation algorithms. Results are compared with “true” simulated velocity.

displacement and optimal velocity respectively, each as a function of noise.

From the table it is seen that for all combinations of noise level the optimal velocity algorithm results in the lowest error. However, it is also seen due to the operator bias, that even with zero noise, a 27% error results. Once there is a realistic level of noise present in the signals, it is seen how the algorithm far outperforms the sole implementation of either the integration or differentiation algorithms, with $\approx 30\%$ error for most noise levels.

Figure 4 illustrates how the cut-off frequency is chosen for the case of 10% noise, with figure 5 comparing the optimal velocity with the true noiseless velocity. The large dc component in the integrated acceleration due to low frequency amplification is the principle cause of its error whereas it is the high frequency region where the differentiated displacement fails. Both of these characteristics are seen in the time domain reconstruction of the signals in figure 6.

Table 1. Sensitivity analysis to noise. Synthetic data.

Percentage Noise		Error From True Velocity		
A	D	Integrated acceleration	Differentiated displacement	Optimal Velocity Estimation (Basic Algorithm)
0	0	28	28	27
1	1	106	34	28
2	2	206	49	28
5	5	510	105	28
10	10	1020	205	29
20	20	2038	406	34
50	50	5095	1014	59
0	10	28	205	28
10	0	1019	28	27
5	10	510	205	31
10	5	1019	105	29

3. EXPERIMENTAL ANALYSIS

A useful application for the technique is in the modal analysis of continuous systems. Figure 7 shows a laboratory rig used to experimentally examine the modes of vibration of a model aero-plane wing. The rig was modified as indicated in the schematic of figure 8. A 2-D aluminium

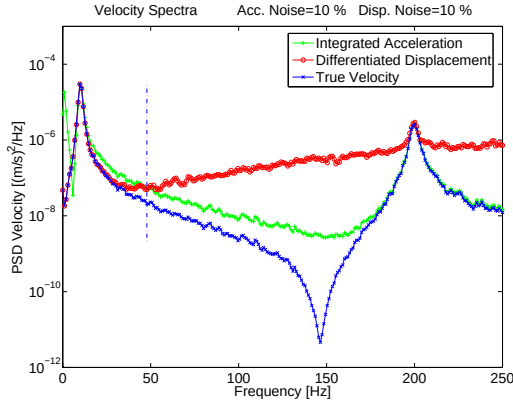


Figure 4. Choice of cut-off frequency for 10% noise.

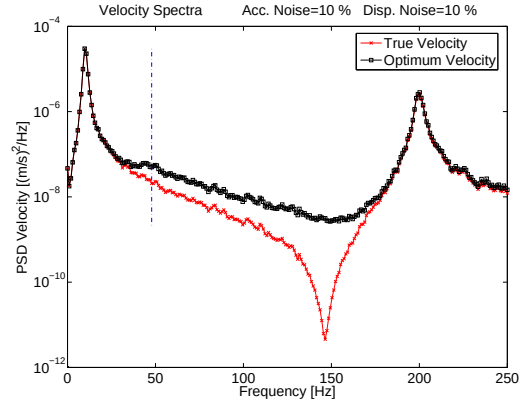


Figure 5. Optimal Velocity versus “true velocity”. 10% Noise.

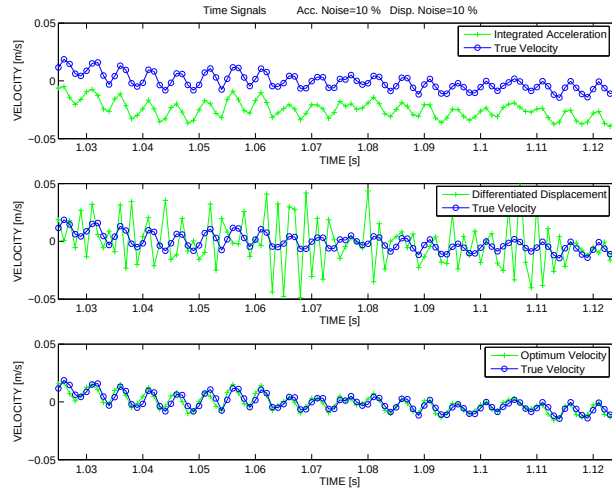


Figure 6. Time domain reconstruction of signals. 10% Noise.

wing shape, supported with a cantilever connection, is excited by an electromagnetic shaker with broad band noise. The response, near the tip of the wing, is acquired using a non-contact displacement transducer, an accelerometer and a laser vibrometer. The displacement transducer is located to the rear of the wing with the same coordinates as the accelerometer, and the laser vibrometer reflects off the accelerometer housing. This set-up ensures that each sensor records the motion of the same point of the wing.

Figure 9 shows the spectra from the three transducers. It shows the chosen cut-off frequency, below which the displacement only is used. As a result of the employment of the algorithm, the optimal velocity estimate is seen, in figure 10, to be a great improvement on either the integrated acceleration signal or the differentiated displacement signal.

4. CONCLUSIONS

A simple algorithm has been presented which can produce a velocity signal when it is not possible to do so directly by the use of a dedicated transducer. If a displacement and an acceleration signal are available, then by employing simple calculus, the technique will use the best qualities



Figure 7. Experimental Set-up.

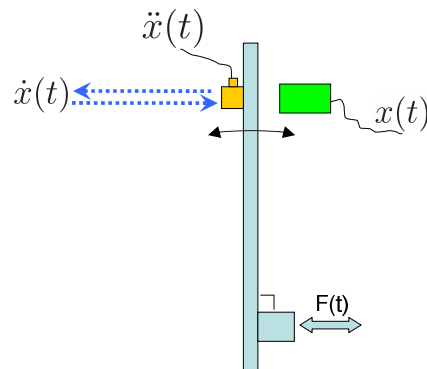


Figure 8. Experimental schematic. The accelerometer and displacement transducers are located at opposite sides of the wing at the same position. The laser vibrometer reflects off the accelerometer housing.

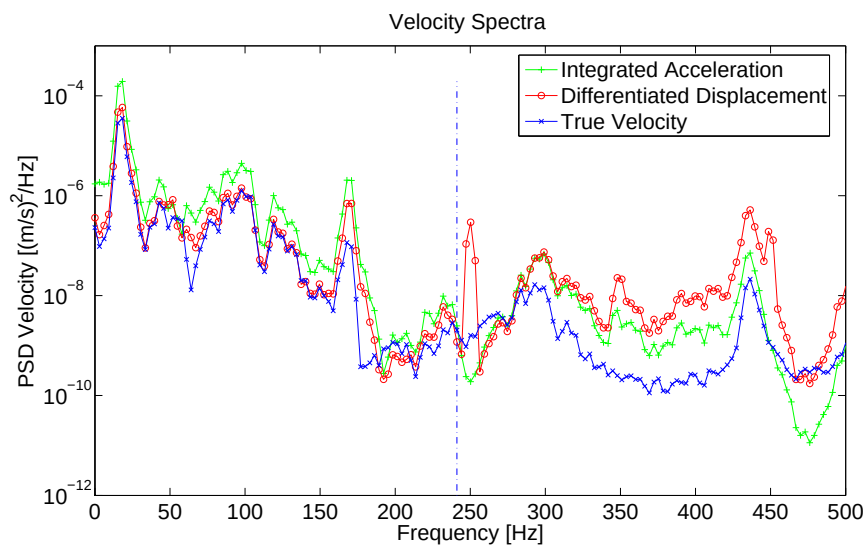


Figure 9. Choice of cut-off frequency for experimental data.

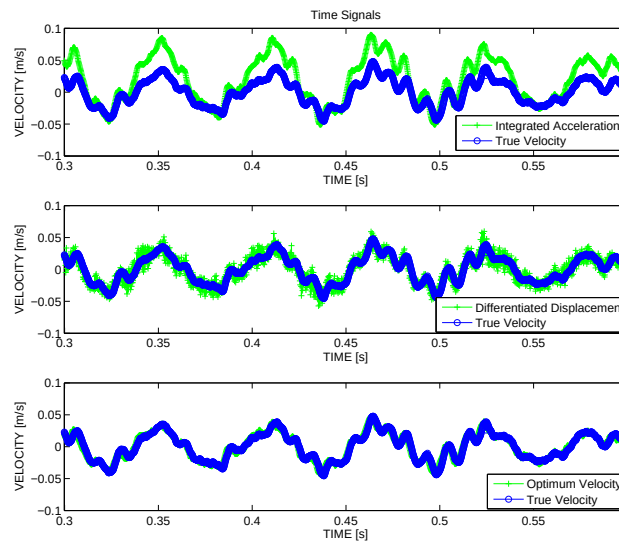


Figure 10. Time domain velocity reconstruction for the experimental data.

of each to form a velocity signal, which, in an experimental environment will be more accurate than if it had been determined from only either acceleration or displacement signals.

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