Efficient Computational Methods for Probabilistic Damage Tolerance Design

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ABSTRACT: Aircraft engine safety is influenced by material anomalies that can form during manufacturing melt and machining operations. These anomalies may occur at any location within a component and have led to catastrophic engine failure events and loss of life. A probabilistic damage tolerance (PDT) approach is used to assess the probability of fracture associated with these events. It involves probabilistic treatment of material anomaly size, frequency, and location as well as other variables that are related to applied loads and material properties and the effectiveness of non-destructive inspections. During the design process, PDT is applied to finite element (FE) component models to assess the suitability of components for use in commercial aircraft engines. The design process is directly impacted by the computation time of the PDT assessment. In this paper, two approaches are presented to address the computational efficiency of PDT assessments. Anomaly frequency affects the size of the largest anomaly that may be found in a component. A conditional probability approach is used to compute the fracture failure cumulative distribution function considering the expected number of anomalies in each zone. This approach can reduce the number of simulations that are needed to satisfy computational accuracy requirements for rare anomaly materials. A strategy for the optimal discretization of a FE component model into regions called zones is also described to address the uncertainty in the location of material anomalies. This approach can also reduce computation time significantly. An example is presented to illustrate the approach. These approaches are applied to the efficient certification of aircraft engines according to United States Federal Aviation Administration (FAA) requirements.

1. INTRODUCTION

The use of probabilistic methods for risk assessment is an established practice in the international gas turbine engine industry. Over the past several decades, the U.S. Federal Aviation Administration (FAA) and the aircraft engine industry have been developing enhanced life management methods to address rare but significant threats posed by material anomalies. For commercial aircraft engines, regulatory agencies recommend probabilistic approaches as part of the certification process, as summarized in several FAA Advisory Circulars (Federal Aviation Administration 2001, 2009a,b). These approaches were developed for aircraft engine applications with a single rare anomaly type such as hard alpha inclusions in titanium (Aerospace Industries Association Rotor Integrity Subcommittee 1997, Leverant et al. 2003) or nicks and scratches associated with machined surfaces (Enright et al. 2012). For these materials, the probability of more than one anomaly in a part is assumed to be negligible (Wu et al. 2002).

Some materials may contain more than one rare type of anomaly. The probability of fracture of a component with several types of anomalies can be predicted using established system reliability methods (Thoft-Christensen & Murotsu 1986, Hoyland & Rausand 1994, Melchers 1999). On the other hand, components made from some rotor-grade alloy materials may contain hundreds or even thousands of anomalies. For these multiple anomaly materials, failure can be modeled as a series system of anomalies. In either case, the unconditional probability of fracture p_i for a zone can be expressed as a cumulative distribution function (CDF) that considers both the average occurrence probability of anomalies *P(d)* and the conditional probability of fracture at a location i given that an anomaly is present $P(F_i|d)$. This enables $P(F_i|d)$ to be computed independently for each anomaly type, and the number of Monte Carlo samples for each anomaly type to be based on different anomaly occurrence rates. This concept is described in Section 2.

A zone-based risk assessment methodology has been developed under the guidance of the aircraft turbine engine industry to estimate component probability of fracture based on groupings of finite elements (FEs) called zones. The methodology has been incorporated into DARWIN® (Design Assessment of Risk With Inspection), a probabilistic fracture mechanics software program developed for fracture risk assessment of gas turbine engine components (Millwater et al. 2000, Leverant et al. 2003, McClung et al. 2004, Enright et al. 2012).

Zoning enables the computation of p_i at discrete locations in a component. This is in contrast with software that has been developed in other industries (e.g., nuclear, aerospace, and power-generation (Dillström 2000, Cavallini & Lazzeri 2007, Ocampo et al. 2011, Beretta et al. 2015)) that do not use zones and can only perform conditional fracture risk assessments.

Zones represent regions of a component with similar $P(F_i|d)$ values. The assignment of FEs to zones is one of the key technologies associated with the zone-based risk assessment methodology. This task can be performed manually but is time consuming and requires human judgment. Algorithms have been developed for assigning FEs to zones based on adaptive refinement of manually created zones (Millwater et al. 2002, 2007) or automatically created zones that require approximate fracture risk estimates at each of the nodes in a component FE model (Moody et al. 2013).

For small FE models (i.e., 10^3-10^4 FEs), these algorithms can assign FEs to zones within minutes. However, for complex 3D FE models (typically containing 10^6-10^7 or more FEs), the computation time can be measured in days or even weeks.

An improved algorithm has been developed that partitions the FE model into "pre-zones" (Enright et al. 2016). Pre-zones are groups of elements that are expected to have similar $P(F_i|d)$ values based on similar values of stress, temperature, and distance from the crack to the nearest surface. The initial crack is placed at the location of maximum $P(F_i|d)$ within each pre-zone. Maximum $P(F_i|d)$ locations are identified in each pre-zone via a Gaussian Process model fit of estimated $P(F_i|d)$ values at a small number of locations. This algorithm enables FE models containing millions of elements to be represented using only hundreds of pre-zones. The algorithm is described in Section 3 and illustrated in Section 4.

2. PROBABILITY OF FRACTURE

The fracture event is defined as the presence of an anomaly and the nucleation and growth of the associated crack that exceeds the material fracture toughness prior to the design target life. The anomaly occurrence probability $P(d_j)$ for a specified number of anomalies j is based on the average number of anomalies λ_i per unit volume V_i of a zone. $P(d_j)$ is modeled as a Poisson point process (Roth 1998, Haldar & Mahadevan 2000):

$$P(d_j) = \frac{(\lambda_i V_i)^j}{j!} exp(-\lambda_i V_i) \tag{1}$$

$P(F|d)$ is dependent on a number of random variables related to the applied stress values and the fatigue nucleation and growth processes (Wu et al. 2002).

For materials with rare material anomalies, the probability of fracture at a specified location p_i is based on the average occurrence probability of a single rare anomaly $P(d_1)$ and the probability of fracture at a location i given that a single anomaly is present $P(F_i|d_1)$:

$$p_i = P(F_i|d_1)P(d_1) \tag{2}$$

For multiple anomaly materials, $P(F|d)$ is dependent on the number of anomalies j and is expressed as $P(F_i|d_j)$ (Enright & Huyse 2006):

$$P(F_i|d_j) = 1 - \left[1 - P(F_i|d_j)\right]^j \quad (3)$$

The influence of j on $P(F_i|d_j)$ and $P(d_j)$ is shown conceptually in Figure 1 (Enright & McClung 2011). Each of the members in the system shown in Figure 1 represents a discrete number of anomalies that could be present in a given component. These members have a mutually exclusive relationship (i.e., it is not possible for a zone in a selected component to have only two anomalies and only three anomalies simultaneously). p_i is equal to the sum of the failure probabilities associated with each of the events in the system (Hoyland & Rausand 1994):

$$p_i = \sum_{j=1}^{n} \left[\left\{1 - \left[1 - P(F_i|d_j)\right]^j\right\} \cdot \frac{(\lambda_i V_i)^j}{j!} exp(-\lambda_i V_i)\right] \quad (4)$$

Eqn. (4) reduces to (Enright & Huyse 2006):

$$p_i = 1 - exp[-\lambda_i V_i \cdot P(F_i|d_1)] \quad (5)$$

For materials with rare material anomalies, the component is discretized into a series system of zones. p_i is assessed using Eqn. (2) for anomalies located within each zone (Leverant et al. 2003). The component probability of fracture p_F is defined as:

$$p_F = P[F_1 \cup F_2 \cup \cdots \cup F_m] = 1 - \prod_{i=1}^{m}(1 - p_i) \quad (6)$$

If multiple anomalies are present, p_i can be estimated for each zone using Eqn. (5). In this case p_F is defined as (Enright & Huyse 2006):

$$p_F = 1 - \prod_{i=1}^{m} exp[-\lambda_i V_i \cdot P(F_i|d_1)] \quad (7)$$

When more than one anomaly type is present, the multiple anomaly types can be modeled as additional members of a series system of zones (Enright & McClung 2011) as shown conceptually in Figure 2.

When Monte Carlo (MC) simulation is used to compute p_i, the accuracy of the computation is dependent on the value of p_i and the number of MC samples N. A minimum of $N = 1/ p_i$ MC samples would be required to obtain a single failed sample. It is desirable to have at least ten failed samples in a MC simulation.

For a component with 100 zones containing rare material anomalies, the value of p_i could be on the order of 10^{-5} (Wu et al. 2002). For this example, at least one million MC samples would be required to obtain ten failed samples in a zone.

When p_i is expressed as a conditional failure probability as in Eq. (2), $P(d_1)$ can be treated as a deterministic variable. This effectively reduces the number of MC simulations that are required for a specified level of accuracy. For rare anomaly materials, the value of $P(d_1)$ is on the order of 10^{-3} (Wu et al. 2002). For the previously mentioned 100 zone component containing rare material anomalies, the value of $P(F_i|d_1)$ would be on the order of 10^{-2}. This reduces the minimum number of samples to obtain ten failed samples

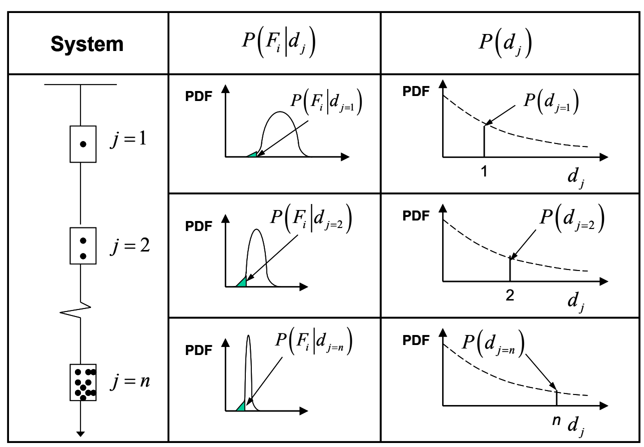


Figure 1. For multiple anomaly materials, both the probability of fracture given an anomaly $\boldsymbol{P(F_i|d_j)}$ and the anomaly occurrence probability $\boldsymbol{P(d_j)}$ are dependent on the number of anomalies present (Enright & McClung 2011).

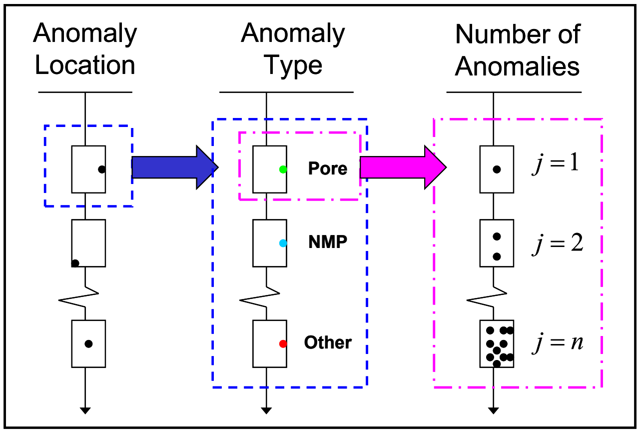


Figure 2. For components with multiple anomalies of multiple types at multiple locations, the probability of fracture can be modeled using several nested series systems to represent the various failure events (Enright & McClung 2011).

from one million to one thousand, or about three orders of magnitude fewer samples for the example component.

3. AUTOZONING ALGORITHM

An autozoning algorithm has been developed that is based on groups of finite elements called pre-zones that have similar fatigue crack growth (FCG) lives and $P(F_i|d)$ values (Enright et al. 2016). FCG lives are strongly dependent on applied stress, temperature, and the distance from the anomaly to the surface of the component. Pre-zone boundaries are therefore dependent on these variables.

The pre-zone-based autozoning algorithm has been implemented in the DARWIN software. Pre-zone boundaries are based on contours of applied stress, temperature, and distance-to-surface values shown conceptually in Figure 3 (Enright et al. 2016). DARWIN creates a set of bins over the range of values for each of these three variables. For example, one of the applied stress bins could consist of all contiguous finite elements with maximum stress levels within a specified range. A Cartesian or cylindrical grid is used to define the physical locations within the FE model associated with the bins for each of these variables. Pre-zone boundaries associated with applied stress, temperature, and distance-to surface for an example FE model are shown in Figure 4.

The $P(F_i|d)$ value is assumed to be constant within a pre-zone. To ensure a conservative solution, anomalies are placed at the location with the maximum $P(F_i|d)$ value in each pre-zone. This location is identified via a Gaussian Process (GP) model fit (Rasmussen & Williams 2006, Diggle 2010) of $P(F_i|d)$ values that are computed at a small number of locations (training points) within each pre-zone. The advantage to the GP model is that it can capture complex nonlinear relationships without needing a priori information regarding the functional form of the response surface.

The locations of initial training points within each pre-zone are defined using Latin Hypercube Sampling (LHS). The number of training points in each pre-zone is based on the dimensionality of the model (i.e., 10 initial training points for 2D geometries, 20 initial training points for 3D geometries).

The GP response surface is then interrogated to identify the maximum $P(F_i|d)$ location using Efficient Global Optimization (EGO) (Jones et al. 1998). The improvement I at any point x is a random variable:

$$I = max(f_{min} - Y, 0) \tag{8}$$

where $-Y$ represents a random variable from the GP model and $-f_{min}$ is the current maximum value of $P(F_i|d)$ for all training points in the pre-zone. (Note that EGO is a minimization technique, so negative signs are applied to the variables to obtain a local maximum). The expected improvement at this point is then:

$$E[I(\boldsymbol{x})] = (f_{min} - \hat{y})\Phi\left(\frac{f_{min} - \hat{y}}{s}\right) + s\phi\left(\frac{f_{min} - \hat{y}}{s}\right) \tag{9}$$

where $\hat{y}$ and s are estimates of the mean and standard deviation from the GP at the location $\mathbf{x}$, respectively. φ and Φ are the standard normal density and standard normal distribution functions, respectively.

DARWIN computes the expected improvement for each node that is not already a training point within each pre-zone. Training points are added if the expected value of the improvement is above a predefined tolerance or if the standard deviation of the GP model is above a predefined tolerance. The algorithm continues until the training point satisfies the predefined tolerances or the maximum number of training points for the pre-zone is reached.

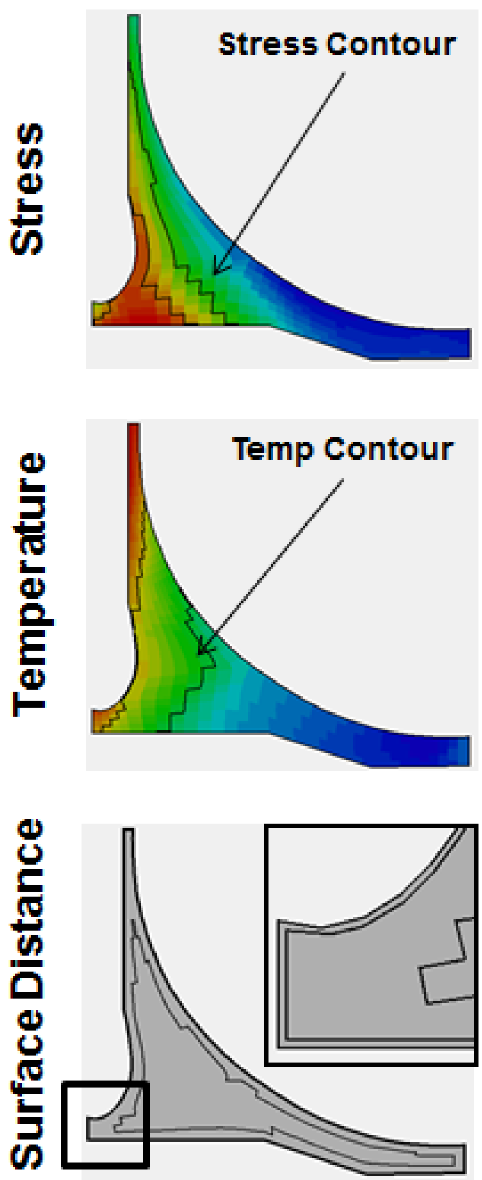


Figure 3. Contours of applied stress, temperature, and distance-to-surface values are used to define pre-zone boundaries (Enright et al. 2016).

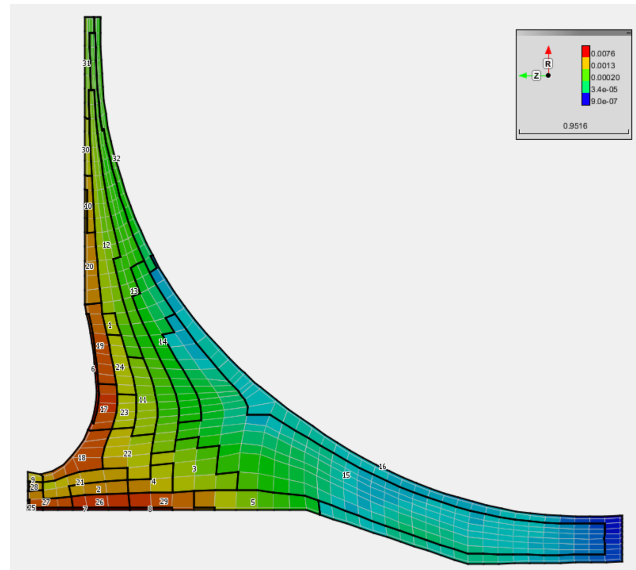


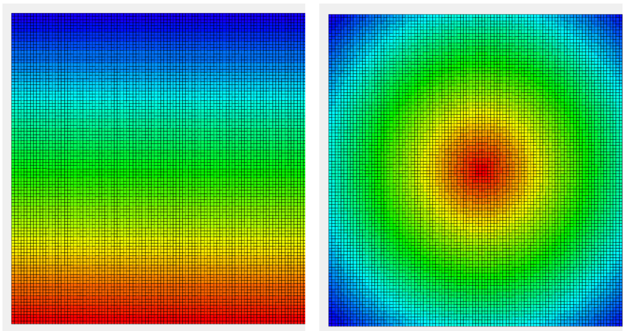
Figure 4. Pre-zone boundaries associated with applied stress, temperature, and distance-to-surface for an example FE model (Enright et al. 2016).

4. APPLICATION EXAMPLE

The pre-zoning-based autozoning algorithm is illustrated for the square axisymmetric component FE model shown in Figure 5 (Enright et al. 2016). FE model boundary conditions were applied to provide linearly varying stresses (Figure 5a) and radially varying temperatures (Figure 5b). The number of FEs in the model was adjusted to demonstrate several autozoning strategies: (1) exhaustive (one zone per element), (2) optimal (Moody et al. 2013), and (3) pre-zoning.

The computational performance of the three autozoning strategies is shown in Figure 6 (Enright et al. 2016). For FE models with 10,000 elements, optimal and pre-zoning are about 10X and 100X faster than exhaustive autozoning, respectively. For FE models with 10^6 elements, optimal and pre-zoning are about 100X and 1,000X-10,000X faster than exhaustive autozoning, respectively.

As shown in Figure 7 (Enright et al. 2016), the component probability of fracture p_F (also



(a) **(b)**

Figure 5. Variation of stress (a) and temperature (b) in a square axisymmetric component (Enright et al. 2016).

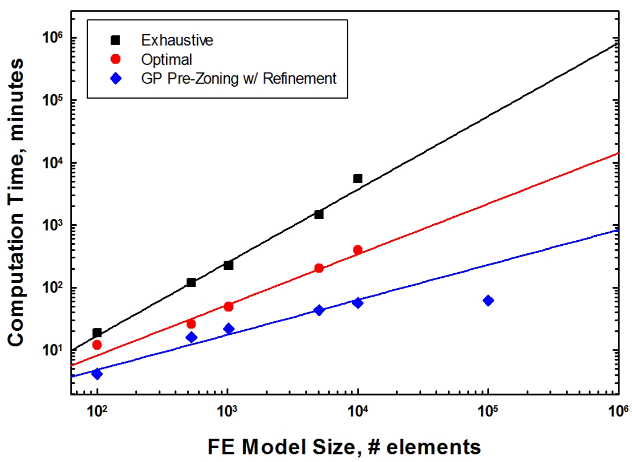


Figure 6. Computation time versus finite element model size for three autozoning strategies (Enright et al. 2016).

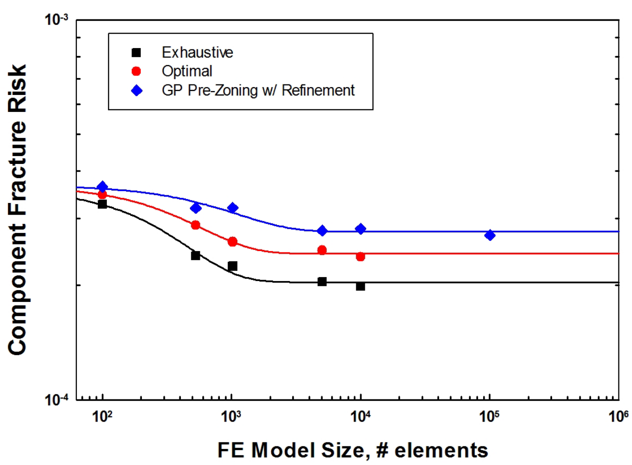


Figure 7. Component fracture risk versus FE model size for three autozoning strategies (Enright et al. 2016).

referred to as fracture risk) decreases with increasing number of FEs and converges to a steady value as expected (Wu et al. 2002). Note that the optimal and pre-zoning autozoning strategies predict higher p_F values than the exhaustive strategy. This is due to the error in computing approximate $P(F_i|d)$ values and the GP fit that are associated with the optimal and pre-zoning strategies, respectively.

5. CONCLUSIONS

Two approaches were presented for improving the computational efficiency of PDT assessments. A conditional probability approach was presented for PDT assessment of rare anomaly materials that can reduce the number Monte Carlo simulations that are required to achieve a specified accuracy by several orders of magnitude. A strategy was presented for the optimal assignment of FEs to zones that can reduce computation time associated with PDT assessments by several orders of magnitude or more. These approaches can be used to improve the overall efficiency of certification assessment of aircraft engines.

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