

Fixed Capital Stock Estimation: An Empirical Exercise using Irish Data

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INTRODUCTION

The present paper arose from the need to obtain estimates of the stock of fixed capital (excluding dwellings and roads) in the Irish economy during the period 1953 to 1972. The two existing estimates (based on capital census estimates) are restricted for the most part to the manufacturing sector, exhibit a wide margin of divergence and are open to doubt on a number of issues. It was decided therefore to start afresh.

So far as the author could ascertain there is no capital census information for the non-manufacturing sectors and for reasons to be discussed below, the information that is available for the manufacturing sector is of doubtful merit. In addition appropriate capital expenditure data are not available earlier than 1953 - though there are some fragments of information relating to pre-1953 which as will be seen, prove useful.

The method of estimation proposed - a derivation of perpetual inventory - is in the circumstances an approximate one and is reported in the hope that it may stimulate further interest in the problem of capital stock estimation particularly at the level of national accounting^[1]. It is possible however that the results might serve as a temporary makeshift pending the accumulation of further information which would facilitate the development of more reliable capital stock estimates.

Before presenting the proposed method and results it will be useful to look briefly at the perpetual inventory and capital census methods and at the existing estimates of capital stock. To facilitate future references a start is made by noting the problems which arise in the preparation of constant price estimates.

CONSTANT PRICE ESTIMATES

The economic concept of capital stock is not an unambiguous one. It may be regarded as the quantity of productive capacity in existence at a point of time or as the present embodiment of a prospective flow of productive services, the difference being a result of the durability of capital goods. An asset acquired some years previously has a shorter prospective life and hence offers a lower stock of prospective services than one which is newly acquired even though the

older asset may have been maintained so as to render its productive efficiency equal to that of the newer. Consequently while the productive capacity of an asset may remain constant over a period, the prospective flow of services is constantly eroded as the age of the asset increases.

The stock of capital, however defined, in existence at any given time is the result of capital formation in the past and may be regarded as a two-dimensional summation - over types of assets, and since the assets are durable, over time. Because capital stock is not normally a collection of homogeneous assets a composite measure can be obtained only by aggregating money values. However, because the assets have been bought at different times and at different prices a summation of capital expenditures at original costs will not provide a meaningful measure of quantity. To remove the effect of price changes it is necessary to deflate capital expenditures by means of a price index.

The preparation of a suitable price index is fraught with difficulty particularly because assets do not normally remain identical over time. For example a new version of an existing machine may embody the same quantity of factors of production but because of technical improvements may be twice as efficient as the older version. The question then arises, should the price index relate to the costs of producing the machines or should it relate to the relative capacities of the machines. If the former, then the price-adjusted values of the two machines are identical; if the latter then the new machine is equivalent to two of the older. Ideally while the latter form of price index is desirable, price indices of capital goods tend in practice to be of the former type, being based on information about costs of material and labour.[2] As a result the price index will tend to rise too quickly and the price-adjusted capital expenditure too slowly omitting some if not all the increases resulting from increased efficiency.[3] Subject to this caveat, deflated capital expenditures may be regarded as homogeneous and the summation over time as a measure of quantity.

There are two generally accepted methods of estimating capital stock - the perpetual inventory method and the capital census (or direct) method. Both are of relevance to the present study and it is useful at this point to review them briefly and to examine their limitations.

PERPETUAL INVENTORY

In the perpetual inventory method[4] the gross stock of a capital asset in existence at a given time is estimated by summing all previous gross capital expenditures on that asset and deducting the gross value of assets which have been retired. In principle the computations may be conducted in original or price-adjusted cost terms, however, in what follows it will be assumed that the expenditures are price-adjusted.[5] If the price index is based on say, year t , the gross capital stock is said to be valued at year t replacement cost new, i.e. assets irrespective of their vintage are valued at the cost of replacing them with identical new assets in year t . [6]

In view of the price index problem outlined above it is salutary to quote the warning given by Barna[7] about the interpretation of the term, replacement cost new.

"Replacement cost new is a clearcut concept only if it is intended to replace assets with identical units, but this is seldom the case in the strict sense. It is perhaps a valid generalisation to suggest that as a result of inventions capital assets undergo an almost continuous and steady improvement and that hardly any two assets produced at different times are identical. This generalisation is applicable to buildings as well as plant. In these circumstances the concept is never understood in the literary sense but as replacement by a substitute. If the substitute has the same output and inputs but costs less, the lower price is taken. A further allowance may be necessary to take

account of the lower running costs of the substitute. The art of evaluating replacement cost in fact consists in finding that amount of modern assets which can be regarded as the equivalent of the old, in the economic rather than in the engineering sense."

The term gross is used to indicate that no deduction has been made for ageing of assets, although gross capital expenditure is defined as net of amounts realised from the sale of assets.[8] When adjusted for ageing (or capital consumption) capital expenditure and capital stock are referred to as net. The amount of an asset consumed in the process of production cannot be measured directly since it represents services which are used up. Information from depreciation accounts operated by firms is not in general useful since these are normally based on original cost and cumulation would therefore suffer from the same deficiency as capital stock estimates based on original cost.

Estimates of capital consumption can be made only on the basis of assumptions about the expected lives of assets and the pattern of utilisation. The later development of a more efficient version of an existing asset may render the older asset obsolete thereby making the actual life shorter than had been expected.[9] However, beyond making a broad guess, based perhaps on past experience, regarding prospective technical advance and its likely impact on the life of an existing asset, it is difficult to see how an allowance for obsolescence may be incorporated into the expected life of an asset.[10]

There are two widely used methods of estimating capital consumption. Both operate a system of attrition over the life of an asset where the rate of writing-off is intended to approximate to the rate of reduction in value due to physical wear and tear and to obsolescence. The first, the straight-line method, assumes that an asset renders its services in equal arithmetic amounts throughout its working life and estimates annual capital consumption by dividing the difference between the replacement cost and the scrap value (if any) of the asset by its life in years. The second, the declining-balance method assumes that capital services, apart from those in the last year, are rendered in equal geometric amounts. Annual capital consumption is estimated as a constant proportion of the written-down value of the asset at the end of the previous year and hence decreases as the age of the asset increases.[11] In the final year the remaining written-down value represents the scrap value, or if zero scrap value is assumed the final year's written-down value is attributed to capital consumption.[12]

The two methods can lead to considerable differences in estimated capital consumption and in the present state of information the choice seems to be arbitrary. In the present study the straight-line method will be used partly because of its simplicity and partly because it is the method adopted in previous studies of capital stock in Ireland. Following Redfern[13] and Dean[14] it is also used in the compilation of the official estimates of capital stock in the UK.[15]

Net capital expenditure in any year is estimated as gross capital expenditure less capital consumption. Net capital stock is found by cumulating net capital expenditure and measures the written-down replacement cost of the stock. If the rate of writing-off is realistic and if the price index is accurate the written-down replacement cost should accord with its second-hand price making allowance for any installation costs.

Neither of these estimates of capital stock corresponds exactly with the theoretical concepts of productive capacity or the embodiment of a flow of expected services. With perfect knowledge of the length of life of assets, their cost of replacement and their pattern of utilisation, then gross stock would be a measure of productive capacity and net stock of the flow of expected services. Given the lack of information under these headings a considerable divergence is likely to exist between the theoretical concept and the corresponding empirical estimate. Nor are the estimates of capital consumption described above, entirely satisfactory. Ideally capital consumption should be

measured at the cost of replacing capacity but due to the problems of compiling a suitable price index for capital goods it tends to be measured at the cost of replacing an existing asset with an identical one, thus failing to allow for increases in efficiency.

The methodology of perpetual inventory may be illustrated by means of the example in Table 1 below.[16] All vintages of the asset in question are assumed to have a working life of five years. In addition, retirements are assumed to take place at exactly the time when the working life expires. This latter assumption which implies that there is no distribution of retirements greatly simplifies the operation of perpetual inventory and is normally adopted in empirical studies. It means that capital retirement in any year is recorded as the gross capital formation undertaken T years previously where T is the assumed working life. As will be shown below the cost of this assumption in terms of loss of accuracy is usually very small. The straight line method of estimating capital consumption is used and the asset is assumed to have a scrap value of zero at the end of its working life.

TABLE 1
EXAMPLE OF PERPETUAL INVENTORY

£ constant prices

Year	Gross capital formation (1)	Gross value of assets retired (2)	Annual change in gross capital stock (3)	Gross stock of capital (4)	Capital consumption (5)	Net capital formation (6)	Net stock of capital (7)
1	10	0	10	10	2	8	8
2	10	0	10	20	4	6	14
3	10	0	10	30	6	4	18
4	0	0	0	30	6	-6	12
5	0	0	0	30	6	-6	6
6	10	10	0	30	6	4	10
7	0	10	-10	20	4	-4	6

The figures in column (1) represent the value of gross capital formation at constant prices in each of seven consecutive years. For the present it is assumed that there was no capital formation prior to year 1. The gross value of assets retired (col. (2)) is as explained above, column (1) lagged by five years which is the assumed length of working life of the asset. For example in year 6 the assets acquired in year 1 are retired. Column (3) shows the difference between annual gross capital formation and retirements and measures the annual gross increment in the stock. The gross

stock of capital at the end of each year (col. (4)) is found by cumulating column (3) beginning in year 1. Annual capital consumption shown in column (5) is found by dividing the gross stock of capital in each year by five - the life in years of the asset. Thus in year 1, capital consumption is one-fifth of gross capital formation in year 1, i.e. £2. In year 2, it is a further one-fifth of gross capital formation in year 1 plus one-fifth of gross capital formation in year 2.[17]

Annual net capital formation (col. (6)) is gross capital formation (col. (1)) less capital consumption (col. (5)) and measures capital formation in excess of that required to maintain the stock of capital. For example in year 2, of gross capital formation of £10, £4 is required to compensate for capital consumption leaving £6 as the net addition to stock. The net stock of capital at the end of each year (col. (7)) is found by cumulating net capital formation beginning in year 1.

The table draws attention to the distinction between capital retirement and capital consumption and between gross and net capital stock. Capital retirement indicates that assets of a particular vintage have reached the end of their working lives and have been taken out of use. Capital consumption on the other hand, records the gradual decline in the value of assets which are still in use due to the reduction in the expected flow of future services resulting from increasing age.

Gross capital stock, in an ideal world, measures productivity capacity at a given time and in Table 1 shows that between years 3 and 5 inclusive, productive capacity remained constant.[18] It does not indicate, however, that the average age of the stock was increasing during this period. In order to see this, it is necessary to examine net capital stock which declined throughout the period, indicating (again in an ideal world) the resultant fall in the prospective stream of services. In year 6 the gross value of assets retired is exactly compensated by gross capital formation so that productive capacity is again unchanged. The replacement of assets of an earlier vintage by new assets however, reduces the average age of stock thereby increasing the expected flow of future services and these changes are shown by the increase in net capital stock.

It should be noted that an increase in net stock could also indicate an increase in productive capacity. In the period from year 1 to year 3 inclusive, productive capacity increased and so too did net capital stock. Similarly a decrease in the net stock of capital may be due to an increase in the average age of the stock of assets in use or to a decline in productive capacity.[19]

In practice data on capital formation are rarely available over a period of years dating back to the beginning of the process of capital formation. However, this does not preclude the estimation of gross capital stock provided data on gross capital formation are available for a number of years at least equal to the assumed working life of the asset. This follows from the assumption that capital retirement in year t is gross capital formation in year $(t-T)$. Hence gross capital stock at the end of year t is determined solely by the summation of gross capital formation in year t and the preceding $(T-1)$ years.

Net capital stock may also be determined in these circumstances. Because assets are assumed to be wholly retired at the end of their working lives only those assets which are less than $(T-1)$ years old contribute to capital consumption. Hence net capital stock at the end of any year may be estimated by subtracting capital consumption cumulated over the life of the asset from the gross capital stock at that time or by cumulating written-down capital formation over the life of the asset.[20]

REVIEW OF THE ASSUMPTIONS OF PERPETUAL INVENTORY

In the previous section the perpetual inventory method was facilitated by a number of assumptions, viz. constant length of life over all vintages of the asset, zero retirement distribution, the availability of a sufficiently long time-series of gross capital formation and zero scrap value and it will be useful to examine the effect of these assumptions on the accuracy of perpetual inventory estimates of capital stock. It is apparent that perpetual inventory relies heavily on the length of working life which is assumed for an asset: if for example, the assumed life exceeds the true life,

gross stock will be overestimated because the summation of past capital formation is extended back over too many years, [21] and if capital formation has been increasing over the assumed life annual changes in gross stock will also be overestimated.

The problem is compounded by the possibility that the working life of an asset may not be constant through time. Redfern found that the number of buses in use at the end of 1938 was equivalent to the vehicle intake of the preceding 9.3 years, while at the end of 1947 and of 1952 the number in use was equivalent to the intake of the preceding 16.3 years. As a result, his estimates of gross stock which had assumed a life of 10 years were biased downwards in 1947 and 1952 [22].

Capital consumption estimated by the straight-line method is less sensitive to errors in the length of life since overestimation of the gross stock is partially offset by overestimation of the assumed life. For example in Table 1 above if the true life is five years but is erroneously assumed to be seven years, then at the end of year 7 gross stock would be estimated as £40 instead of the true value of £20 - an error of 100 per cent. However, capital consumption in year 7 using a life of seven years would be $£40/7 = £5.7$ compared with the true figure of £4 - in this case an error of 42.8 per cent. [23] Consequently although the level of net stock would be overestimated to the same extent as gross stock, annual changes in net stock (i.e. net capital formation) would be less affected. [24]

The effect on perpetual inventory estimates of assuming that an asset is retired at exactly the time when its working life expires has been examined by Schiff [25] and by Barna. [26] By assuming that the distribution of ages at retirement was bell-shaped and symmetrical with mean T years and range zero to $2T$ years, [27] and that gross capital expenditure over the preceding $2T$ years had increased at an exponential rate of $100g$ per cent per annum, Schiff found that gross stock estimates which made allowance for retirement distribution (K_1) were less than those based on the assumption that all retirements took place at exactly the mean age (K_2). The percentage divergence between the two (measured as $100 (K_2/K_1 - 1)$) was a unimodal function of the product Tg , being positive for all values of Tg except $Tg = 0$ at which point it was zero. For a given value of T the divergence first increased but later decreased as the assumed growth rate of capital expenditure (g) increased. Similarly for a given value of g as T increased. The maximum deviation of approximately 5.25 per cent occurred when the product Tg was equal to 2, which could mean a 50 years average life associated with a 4 per cent growth of capital expenditure, or 20 years average life and a 10 per cent growth of capital expenditure, and so on. For values of Tg greater than 5.5 (or less than 0.5) the divergence was less than 2 per cent.

On the basis of these results Schiff concluded, "If the retirement distribution can be assumed to resemble the (normal) curve and the trend of installations over a past period of double the average service life of the assets can be roughly represented by a constant rate of growth, then, whatever that growth rate and whatever the life average, the gross survivor value computed by the "crude" method [28] does not deviate by more than 6 per cent from what would be obtained by the correct method The margin between 6 per cent and the 5.26 per cent we have derived is certainly sufficient to allow for any possible difference of our result from what would be obtained if annual rather than continuous functions were used Minor erratic oscillations of actual installations around a generally realistic ... trend, or of actual retirements around a generally realistic (normal) curve, will hardly affect the reliability of the results, except perhaps in the case of very short average service lives." [29]

Barna also concluded that the error ($K_2 - K_1$) in gross stock estimates which erroneously assumed no distribution of retirements was slight and that its magnitude depended on the shape of the retirement distribution, the choice of an assumed average age at retirement and the trend growth of past gross capital expenditure. He was of the opinion however, "from the little empirical evidence which we have" that the retirement distribution had a slight positive skew and on the basis of experiments with hypothetical figures concluded that ($K_2 - K_1$) could be positive or negative and was likely

to be of the order of 5 to 10 per cent of K_1 [30]

From these findings it seems reasonably well established that for empirical purposes the assumption of no retirement distribution is a very useful one since it greatly simplifies the perpetual inventory calculations at the cost of only minor inaccuracy.

With regard to the availability of historical data the work by Goldsmith [31] and later by Jefferson [32], both of whom examined the relationship between the net and gross capital stock of an asset under varying assumptions regarding the pattern of gross capital expenditure in the preceding T years and the average length of life of the asset, is relevant.

From the definition of gross capital stock given earlier it will be recalled that if data on gross capital expenditure are available over a period of T consecutive years then, the gross stock of capital is independent of the time pattern of expenditures since it merely records the total. However, this is not true of net capital stock which because of the allowance for cumulated capital consumption, will be high or low depending on whether capital expenditure was increasing or decreasing throughout the period [33]

Goldsmith developed formulae for gross and net capital stock and for the ratio of net to gross stock at the end of T years under assumptions of a stream of gross capital expenditures continuously increasing (or decreasing) at 100g per cent per year during the preceding T years; a constant working life of T years; no retirement distribution; zero scrap value; and straight-line writing-off. Using the notation adopted above, the net-gross formula is,

$$\frac{N}{K} = \frac{I}{1 - e^{-gT}} - \frac{1}{gT}$$

where it will be noted, the ratio depends not on the level of capital expenditure, but on its growth rate and on the life of the asset. [34] It may also be noted that the ratio has an upper limit of unity as T increases for a given value of g since an asset of infinite life does not suffer depreciation or obsolescence [35] and hence net and gross capital stock are identical.

In an accompanying table the net-gross ratio and gross and net stock as multiples of the current level of gross capital expenditure were shown for selected values of the length of life and the growth rate of gross capital expenditure. [36] The multiples displayed wide variation, increasing for a given growth rate as the asset life increased and decreasing for a given asset life as the growth rate increased. [37] On the other hand the net-gross ratio was less sensitive to variations in the growth rate and the length of life - increasing slowly as T and/or g increased - and Goldsmith concluded that, "most actual values for this ratio will be between 0.50 or slightly less, and approximately 0.70". [38]

This conclusion was also reached by Jefferson who considered the case where the working life of the asset was eighty years. This was divided into two equal sub-periods and annual gross capital expenditure was assumed to remain constant during the first forty years. In the second forty years it was again constant but a multiple of the annual level in the first. Using straight-line writing-off he demonstrated by a series of numerical examples that the net-gross ratio at the end of the eightieth year ranged from approximately 0.5 when the multiple was unity to approximately 0.7 when the multiple was 10. [39]

Since in practice, capital expenditures do not follow a strict exponential time-path, Goldsmith examined the differences between the net-gross ratio and net and gross capital stock calculated using trend values in the formulae and calculated by summing actual capital expenditure when cyclical movements of capital expenditure about the exponential trend were introduced. He noted that the differences were small [40] provided that the cyclical fluctuations of capital expenditure were fairly regular, and concluded that, "insofar as expenditures in constant prices show less marked deviations from their growth trend than expenditures in current values as is usually the case, calculation by formula will be more reliable for the former than the latter The danger in approxima-

ting capital stock estimates on the basis of the trend in gross capital expenditures lies not so much in the cyclical movements of capital expenditures around their trend as in sharp, discontinuous, or long deviations from the trend".[41]

Goldsmith also found that adjusting the formulae to allow for a non-zero scrap value did not lead to substantial differences in the estimated net-gross ratio. For example, if the ratio without scrap value allowance was 0.50 and the ratio of scrap to original value s was 0.10, the net gross ratio with allowance for scrap value would be 0.55. Similarly, a net-gross ratio (without scrap allowance) of 0.60 rose to 0.64 and a ratio of 0.80 to 0.82. In general the difference declined for given s the larger the net-gross ratio without scrap allowance,[42] and for a given net-gross ratio the smaller the value of s .

Since an allowance for scrap value affects only net stock it was not necessary to adjust the gross stock formula. The amended net-gross and net stock formulae respectively are,

$$\frac{N(2)}{K} = \frac{N(1)}{K} + s \left(1 - \frac{N(1)}{K} \right)$$

and

$$N(2) = (1 - s)N(1) + sK$$

where the superscripts (1) and (2) denote respectively estimated without scrap value allowance and estimated with scrap value allowance.

Translating Goldsmith's results into terms of the error involved in the net stock estimate by erroneously omitting a scrap value allowance shows that in the first example cited above, the error $(N(2) - N(1))$ is equivalent to 9 per cent of $N(2)$ and in the second, 6 per cent of $N(2)$. [43]

There is the possibility therefore that two simplifications which are used in the perpetual inventory calculation of capital stock - the omission of scrap value and of retirement distribution - may introduce errors tending in the same direction. The omission of scrap value allowance leads to an underestimate of net stock and depending on the shape of the retirement distribution, its omission may also result in underestimation.[44]

The importance of these results lies in the fact that where capital expenditure data are not available even over a period equal to the life of the asset it may nevertheless be possible to estimate capital stock using information about the growth rate of capital expenditure and the average length of life of the asset. Goldsmith suggested that this is particularly true when dealing with annual data and capital expenditure aggregates such as national income statistics because the aggregates display milder and more regular cycles about their trend than expenditures on individual assets.[45]

One final point may be made about Goldsmith's formulae. They use an exponential growth path for capital expenditure which arguably, is more appropriate for dealing with monthly or quarterly rather than annual data. When a discrete growth path is used[46] the formula for the net-gross ratio becomes

$$\frac{N}{K} = \frac{1}{1 - (1 + g)^{-T}} - \frac{(1 + g)}{gT}$$

Like Goldsmith's formula, this latter form indicates that the ratio has an upper limit of unity for any growth rate as T increases. However, for low values of T the two formulae yield substantially different values. Table 2 below compares values of the net-gross ratio obtained using Goldsmith's formula and using the discrete growth formula for a number of values of T when the growth rate is 2 per cent and 10 per cent per annum. As expected, the ratio obtained using discrete growth is the lower, though for both growth rates the difference becomes negligible as T increases.

TABLE 2

COMPARISON OF NET-GROSS RATIOS BASED ON EXPONENTIAL AND DISCRETE GROWTH PATHS OF GROSS CAPITAL EXPENDITURE

Asset Life (years)	Growth rate of capital expenditure	2 per cent per annum		10 per cent per annum	
		exponential	discrete	exponential	discrete
5		.51	.42	.54	.44
10		.52	.47	.58	.53
20		.53	.51	.66	.62
50		.58	.57	.81	.79
100		.66	.65	.90	.89

The corresponding formulae for gross and net stock respectively are

$$K_t = \frac{k_t(1+g)}{g} \left[1 - (1+g)^{-T} \right]$$

and

$$N_t = \frac{k_t(1+g)}{Tg^2} \left[(T-1)g + (1+g)^{-(T-1)} - 1 \right]$$

where k_t denotes gross capital formation in the current year. As the life of the asset increases for a given value of g both formulae approach a limiting form given by

$$K_t = N_t = \frac{k_t(1+g)}{g}$$

One final aspect of perpetual inventory may be noted. It has been shown by Nevin^[47] that if the gross or net capital stock of an asset is known - or has been estimated by a method not involving assumptions about its length of working life - and if gross capital formation data are available for a sufficient number of preceding years, then the average life of the asset can be uniquely determined.

In brief the procedure is as follows,^[48] Assuming zero scrap value, and straight-line depreciation,^[49] the net stock of an asset at the end of year t is,

$$N_t = \frac{T-1}{T} k_t + \frac{T-2}{T} k_{t-1} + \dots + \frac{T-(T-1)}{T} k_{t-T+2}$$

from which, if N_t is known (say, N) and if T is not greater than the number of years (n) for which

the observations on capital formation (k_t) are available, T may be determined. If, using the k_t for all the n years available the summation yields a value equal to N , then T is equal to n . If the summation is greater than N , k_t are eliminated starting from the earliest until equality (or as near as possible) is achieved.

When the summation yields a value less than N i.e. the number of observations (n) available is less than T , it becomes necessary to estimate an average for k_t during the years prior to $(t-n+1)$. Denoting this average by $\bar{k}$, the expression for the net stock becomes

$$N_t = \frac{T-1}{T}k_t + \frac{T-2}{T}k_{t-1} + \dots + \frac{T-n}{T}k_{t-n+1} + \bar{k} \left[\frac{T-(n-1)}{T} + \dots + \frac{T-(T-1)}{T} \right]$$

where the first n terms in the summation are as before. An estimate of T can then be obtained by a process of successive approximation.[50]

It will be apparent from the nature of perpetual inventory that errors in the estimated value of T are inversely related to errors in the estimate of average capital formation in the years before $(t-n+1)$ i.e. if $\bar{k}$ overestimates the true average then T will be underestimated and vice versa.[51] Nevin also found that errors in the average life estimates increased with the true length of life and therefore were likely to be greater for say, buildings than for plant and machinery.[52]

Having obtained an estimate for T , capital consumption in year t may then be estimated as

$$c_t = \frac{1}{T} \left[\sum_{i=t-n+1}^t k_i + \bar{k}(T-n) \right]$$

and net capital formation as the difference between gross capital formation and capital consumption.

CAPITAL CENSUS

The principal alternative method of estimating capital stock is that of a capital census based on a sample of valuations of assets for fire insurance or balance sheet purposes.[53] The two sources are not of equal merit and it is best to consider them separately.

Insurance against fire is a general practice in which the insurer contracts to compensate the insured for losses incurred. There are two main types of insurance policy: insurance for reinstatement and insurance for indemnity. Under the former the insurer undertakes to replace damaged assets with equivalent new ones without deduction for depreciation. Under an indemnity policy the insurer undertakes to compensate the insured by paying a sum equivalent to the market value of the asset taking into account physical wear and tear and its expected future life.[54]

In all cases a definite value is placed on the assets insured and it is in the insured's interests that this value be kept at a realistic figure in each year. If it is less than the current value of the assets the amount paid in compensation is correspondingly reduced; if it is excessive he does not receive more than full compensation as affirmed by the policy. As a result, and subject to the methods used by the insured to obtain up-to-date valuations, valuations based on a reinstatement policy correspond to replacement cost new and valuations based on an indemnity policy to written-down replacement cost, both at the current year's prices.[55] It should be noted that since the purpose of a reinstatement policy is to replace an older asset with an equivalent new one the valuation of the insured asset should reflect changes in productive efficiency. If new assets are more efficient than old ones the correct valuation of the old asset should be lower than one intended simply to compensate for purchase price changes.

While fire insurance is general, a number of factors must be taken into account in deriving a capital census. In the first place, not all assets are insured against fire. Normally those assets which are regarded as indestructible are omitted - general examples are land and foundations of buildings while most other uninsured assets are in heavy industry and include blast furnaces, coke ovens and other assets which are not inflammable. Secondly, despite what was said above about the incentives for the insured to maintain realistic valuations the possibility of over or under-valuation remains. Apart from errors, some firms may deliberately under-value in order to reduce their insurance premiums while over-valuation may be the result of a decision to allow for a margin of error.^[56] This of course assumes that firms regularly revalue existing assets. Nevin^[57] found that many firms in Ireland initially insured assets at their original cost but did not adjust these valuations in succeeding years. Barna^[58] reported similarly on British manufacturing industry pointing out that in general, in periods of rising prices values were not regularly revised upward or were not revised by a sufficient amount.

In attempting a capital census based on fire insurance valuations it is therefore highly desirable to supplement the insurance figures by additional information on uninsured assets and to explore the methods which have been used to adjust the valuations.

A number of advantages have been claimed for the capital census (based on fire insurance valuations) method compared with the perpetual inventory method.^[59] First, a capital census does not depend so heavily on historical data some of which, depending on the length of life of the asset, are likely to be unreliable. Second, expenditure on smaller assets and improvements to existing assets may be recorded in revenue accounts thus leading to an underestimate in the capital expenditure figures used in perpetual inventory though not in the capital census since these would be included for fire insurance. Third, perpetual inventory relies on a mathematical formula which may not accord with reality.

It may be said in favour of perpetual inventory that once estimates of capital stock have been obtained it is a relatively simple matter to continue with a time series, and that with the passage of time and improvement in statistics the reliance on historical data is reduced. Furthermore while estimates of capital stock from a perpetual inventory of capital expenditures which omit certain items will be understated, the understatement is likely to be mitigated by two factors - first the omitted assets are likely to have short lives, and second, to the extent that expenditure on improvement of assets affects their working lives this will be (in principle at least) taken account of in the average lives used in perpetual inventory. There is little empirical data on the amount of understatement that may occur in the estimates of capital expenditure. A study of Great Britain for 1958 found that the understatement was as high as 11 per cent plus a further improvement element in repairs for which no method of measuring could be found.^[60]

Ideally it would be desirable to utilise both methods using capital census estimates as periodic checks on a time series of perpetual inventory estimates. In the UK manufacturing sector for example, Barna's estimates of the gross capital stock in 1955 based on a sample of fire insurance valuations were about 50 per cent higher than those obtained by Redfern using the perpetual inventory method.^[61] In principle, Barna's estimates should have been the smaller, since he used valuation at the cost of replacement with new, and presumably technically more advanced assets, while Redfern's valuation was at original cost multiplied by a price index based on changes in the price of assets embodying a constant quantity of factors of production.^[62] Dean's later estimates using the perpetual inventory method and data which had not been available to Redfern correspond much more closely with Barna's.^[63] It was this confirmation of the estimates which presumably led to the inclusion in the UK national accounts statistics of estimates of gross capital stock which first appeared in 1964.^[64]

Since all business firms are required for tax purposes or by company law to place a value on their capital assets at the end of each accounting year it might appear that balance sheet valuations would provide a useful source of data on capital stock. However, balance sheet valuations

tend to be expressed in terms of original cost, the summation of which is a mixture of real changes, price changes and technological changes. In addition some firms may revalue existing assets expressing them in terms of the prices at the time of revaluation while subsequent acquisitions of assets are added on at original costs. As a result, the figures relating to different firms are not compatible and since the aggregates for each firm relate to a variety of price levels it is not possible to reconcile them. The same is true of cumulated depreciation entries in balance sheets.

In view of these difficulties Barna preferred to base his census on fire insurance valuations and Jefferson described balance sheet valuations as "economically meaningless". Some doubt must therefore be cast on the official estimates of "provision for depreciation" which are used in the estimation of gross national product for the Republic of Ireland.[65] The depreciation estimates are largely based on allowances permitted by the Inland Revenue and presumably are derived from balance sheet valuations.

SOURCES OF CAPITAL FORMATION DATA IN IRELAND

There are four main sources of published statistics on annual capital formation in Ireland. The Census of Industrial Production (CIP) publishes changes at current prices in fixed capital assets by each industry from 1945 onward.[66] Six categories of asset are distinguished: passenger vehicles, work vehicles, plant and machinery, buildings, land and other fixed assets.

An analysis by six other categories of asset, but not by industry, of gross domestic fixed capital formation at current and constant prices from 1953 onward is published in National Income and Expenditure (NIE). In this case the asset categories are dwellings, roads, other building and construction (including land rehabilitation and plantation and orchard development), transport equipment, agricultural machinery and other machinery and equipment. Estimates of total gross domestic fixed capital formation are also available at current and constant prices back to 1938 in the Irish Statistical Survey and two early pamphlets on national income estimates.[67]

The United Nations Yearbook of National Accounts publishes estimates of gross domestic fixed capital formation at current and constant prices from 1953 onward classified by type of asset in one table and by industrial use in another. Unfortunately it is not possible to reconcile the two to obtain industrial capital formation by asset type. Five types of asset are distinguished: dwellings, non-residential buildings, other construction and works,[68] transport equipment[69] and machinery and other equipment. The categories non-residential building and other construction and works combined correspond to the combination of other building and construction and roads as published in the NIE and the UN category machinery and other equipment comprises the NIE categories agricultural machinery and other machinery and equipment. Finally the National Accounts of OECD countries presents gross domestic fixed capital formation at current and constant prices disaggregated by industrial use (but not by type of asset) for the years from 1957 onward. From 1960 the public and private sectors are distinguished. This is also the case in the UN Yearbook. The national accounts sources do not include a separate entry for the transfer costs of land and buildings.[70]

PREVIOUS STUDIES OF CAPITAL STOCK IN IRELAND

The obvious lack of historical statistics has severely hampered attempts to estimate fixed capital stock in Ireland leading to a heavy reliance on ingenuity on the part of researchers. Two such attempts have been made - both for the manufacturing sector - and of necessity they have taken the results of a capital census as their starting points. The first was by Nevin who prepared estimates of the mid-year net stock of capital of each industry in the manufacturing sector for the

years 1947 to 1959.[71] Nevin's starting point was provided by the results of a sample inquiry by the Central Statistics Office into the financial capital structure of Irish industry in 1958 which contained supplementary information on the balance sheet and insurance valuations of fixed assets.[72]

The fact that the valuations were supplementary is important. No explanations were sought from the respondents to the inquiry about the methods used in valuing the assets. Nevin was of the opinion that most insurance policies were likely to be of the reinstatement type,[73] but that the original valuations were seldom adjusted in later years. Consequently he equated the sample insurance valuations not with replacement cost new (at 1958 prices) but with written-down replacement costs at 1958 prices on the assumption that the rate of depreciation in real terms was approximated by the rate of increase in prices. In contrast, Barna's capital census in the UK was based on a sample inquiry undertaken specifically to obtain information on insurance valuation of assets and requested respondents to give information on their methods of valuation. It was found that most respondents used replacement cost new or a close approximation to it.[74]

In Nevin's study, estimates were made of the insurance valuation of some assets which appeared only in the balance sheet data and the sample totals for each industrial grouping were then grossed-up (using the ratio of the "remainder of net output"[75] accounted for by the sample to the comparable total for each industrial grouping) and summed to yield an estimate of the net stock of capital in the manufacturing sector at the end of 1958.[76]

In order to generate a time series of net capital stock it was necessary to estimate annual net capital formation. Gross capital formation data at current prices were based on the CIP series described above. Because the insurance valuations had distinguished only two categories of assets, land and buildings on the one hand and plant and machinery on the other, Nevin sub-totalled the CIP data into "buildings" comprising buildings, land and other fixed assets and "plant" comprising vehicles, plant and machinery. To take account of non-respondents[77] the CIP figures for each year were grossed-up to the level of the national accounts total for gross fixed capital formation in manufacturing[78] by allocating the national accounts total between buildings and plant and among industries in the same proportions as the CIP figures.

The two series were then converted to constant (1958) prices using the official wholesale price indices for capital goods. For buildings the index was that for building and construction, for plant that for transportable capital goods for use in industry. For years prior to 1953 the official price indices did not distinguish between buildings and plant and both index numbers had to be projected backward using a wholesale price index for a composite, capital equipment.

To move from gross to net capital formation required estimating capital consumption which entailed determining the average working lives of both buildings and plant in each industry.[79] As has been shown above Nevin had demonstrated elsewhere that given an estimate of gross or net capital stock of an asset obtained by any method not involving assumptions about its length of working life, and given gross capital formation data for a sufficient number of preceding years, the average life of the asset at the date of valuation of the stock could be uniquely determined using the perpetual inventory formulae. Where gross capital formation data were available for a period less than the life of the asset, it was necessary as a preliminary step to estimate annual average gross capital formation for the unknown years thus introducing an element of uncertainty the estimated life being sensitive to variations in the estimated average capital formation.

Gross capital formation data were available for only twelve years preceding the date of the census (i.e. from 1947 to 1958 inclusive) and Nevin attempted estimates of annual average gross capital formation by industry and by asset for the years prior to 1947. The estimates for buildings were based on building output and for plant on imports of industrial machinery. For manufacturing as a whole Nevin estimated that annual average gross capital formation in buildings at 1958 prices in the period 1926 to 1946 was 40 per cent of the 1947-59 average and used this percentage for all earlier years. The corresponding average for plant was estimated at 70 per cent of the 1947

level. The aggregates were then distributed among the individual industries on the basis of numbers employed and the relative magnitudes of assets per employee obtaining in 1958.

Using these estimates of pre-1947 annual gross capital formation along with the post-1947 data and the direct estimate of net stock in 1958 it was possible to estimate the corresponding average lives and capital consumption using the technique outlined above. Nevin was then able to compute a time series of net capital formation from 1947 to 1959 (the latest year for which data were available) which when linked to the capital stock estimate for 1958 yielded estimates of net stock in each year from 1947 to 1959.

Estimates of the mid-year capital stock of manufacturing industries for the years 1953 to 1968 were later prepared by Henry.[80] Henry chose as his starting point end-of-year balance sheet valuations of (i) buildings, land and other fixed assets and (ii) plant, machinery and vehicles which were published for CIP industries (old listing) for each year from 1945 to 1950. It was assumed that all CIP respondents had made returns and that no grossing-up was necessary. For 1945 or 1950[81] so called starting values for each of six categories of asset were estimated by disaggregating the balance sheet valuations. For most industries the balance sheet valuation of buildings, land and other fixed assets was attributed to buildings giving zero starting values to land and other fixed assets. In general the balance sheet valuation of plant, machinery and vehicles was allocated among plant and machinery, passenger vehicles and work vehicles in proportion to subsequent purchases. The starting values were then adjusted to constant (1958) prices using separate price indices for buildings, land, plant and machinery and vehicles.[82]

The equivalent-new capital stock[83] of each asset in the initial year was then estimated as a multiple of the starting value at constant prices. For buildings 1.5 was used; for other fixed assets and plant and machinery, 2.0; and for both kinds of vehicles, 1.8. No adjustment was made to the starting value for land. The adjustments were intended to compensate for any writing-off included in the balance sheet valuations.

If it is assumed that the starting values at 1958 prices correspond to net capital stock and equivalent-new to gross stock then the multiples are in keeping with the range of net-gross ratios estimated by Goldsmith using perpetual inventory and straight-line writing-off[84] though Henry does not give any information on the origin of the multiples.

Annual equivalent-new capital formation for the period 1945 (or 1950) to 1968 was derived from the CIP estimates of changes in the value of fixed capital assets by adjusting to 1958 prices using the same price indices as for the starting values of capital stock and inflating the figures for purchases and sales of used assets by the same multiples as the starting stock.[85] The CIP gave sales of land and buildings as a composite item all of which was attributed by Henry to buildings.[86] Purchases of other fixed assets did not appear in the CIP as a separate item until 1963 and purchases prior to that year were assumed to be nil.[87] No attempt was made at reconciliation with the national income estimates of gross fixed capital formation in manufacturing.[88]

Henry's categorisation of fixed assets by six types compared with Nevin's two necessitated a re-examination of the problem of average lives. Henry's investigations which were based partly on consultations with industrialists and partly on the work by Nevin[89] and by Kuh[90] led to the following estimates: buildings and land were taken as having an infinite life; passenger and work vehicles a life of eight years; plant and machinery a life which varied from 13 to 50 years depending on the industry concerned; and other fixed assets a life of 24 years. Except for buildings and land, the equivalent-new stock of each asset in the initial year and second hand assets were assumed to have a half-life remaining.

Time series of equivalent-new capital stock starting in 1945 or 1950 as appropriate were then constructed as follows. The initial equivalent-new values of the stock of buildings and land were retained as positive amounts in the first and all subsequent years. Purchases[91] were added on in the year in which they occurred and in all subsequent years and sales were deducted in a

similar fashion. This is the perpetual inventory method of estimating gross capital stock assuming for the moment that the initial equivalent-new capital stock is the same as gross stock. Since land and buildings had been each assigned an infinite working life retirements do not have to be taken into account.[92]

For the other four types of assets, the initial equivalent-new values were entered as positive amounts for each year of the half-life but subjected during this period to straight-line writing-off down to 85 per cent. No further account was taken of the starting values in subsequent years. Purchases of new assets were added on at their equivalent-new value in the year of acquisition, subjected to straight-line writing-off to 80 per cent over the appropriate working life and then omitted. Purchases of second-hand assets were similarly treated being written-down to 85 per cent of the initial equivalent-new value over a half-life and then omitted. Sales, apart from being negative changes, were treated in the same way as purchases of second-hand assets i.e. introduced in the year of sale at their equivalent new value and gradually written-down to 85 per cent of that value. The resultant estimates of capital stock at year-end were converted to mid-year by averaging.

The gradual writing-down over the asset life to 80 or 85 per cent of the equivalent-new value was intended to simulate a reduction in capacity due to increased repair and maintenance time. This represents a divergence from the perpetual inventory approach of estimating either gross or net capital stock. Under perpetual inventory an allowance for ageing of assets is not made in estimating gross stock;[93] on the other hand, the introduction of a multiplication factor, (the purpose of which was to compensate for any writing-off that had been included in the balance sheet valuations) into the estimation of equivalent-new capital stock in the starting year implies that the initial stock was gross capital stock.[94]

Before comparing the estimates obtained by Nevin and Henry it is useful to reflect on what the two estimates were intended to measure. Nevin undoubtedly was attempting to measure written-down replacement value at 1958 prices, i.e. net capital stock[95] while Henry's objective was to obtain a measure of productive capacity,[96] which despite the small allowance for ageing is probably closer to gross stock. In principle therefore, it is to be expected that Henry's estimates should be greater than those of Nevin. However, as can be seen from Table 3 below (which also includes Kennedy's amended and extended version of Nevin's results) the converse is the case. In the early years Henry's estimates are substantially lower than Nevin's with the difference diminishing in later years.

The prime source of the problem would seem to be Henry's use of balance sheet valuations as a basis for his capital census. As was mentioned earlier, balance sheet valuations are a summation of the money values of assets at different prices so that conversion to a constant price basis can not be properly effected by adjusting the sum as a whole as was done by Henry. A proper price adjustment would require details of the original cost and date of acquisition of each asset. If the price of capital goods has been rising - and this appears to have been the case[97] - then price-adjusting the balance sheet valuations is likely to lead to a serious underestimate of the capital stock. For example if the balance sheet valuation for 1945 is adjusted to 1958 prices using the 1958 based price index for 1945, this would only be correct if the money value of assets acquired before 1945 had already been adjusted to 1945 prices. Otherwise no account is taken of price changes prior to 1945. If prices have been rising prior to 1945 the money value of pre-1945 assets should be inflated to compensate for the price rise.[98]

The discussion in the previous paragraph relates to the relative orders of magnitude of the two estimates. It does not imply that Nevin's estimates are necessarily an accurate measure of the net stock of capital particularly since the methods used in obtaining the fire insurance valuations on which Nevin's estimates were based are not recorded. In this context it may be noted that for mid-1955 Nevin found that his estimate of capital per head in Irish manufacturing was equivalent to 74 per cent of that estimated for the UK by Barna.[99] The comparison was based on net stock in Ireland and gross stock in the UK thereby reducing the Ireland-UK ratio although the

TABLE 3
COMPARISON OF ESTIMATES OF CAPITAL STOCK IN IRISH MANUFACTURING (MID-YEAR)

	£ million at 1958 prices													
	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966
Nevin	211.9	216.4	220.6	224.5	227.3	230.1	232.1	-	-	-	-	-	-	-
Kennedy	216.0	220.3	224.4	228.3	230.9	233.8	236.7	241.7	251.2	264.8	281.5	298.7	316.0	335.9
Henry	121.1	127.8	136.9	145.8	151.9	158.6	167.5	175.8	188.1	204.4	222.9	245.5	267.7	292.9

Sources: Nevin, op.cit.(1963) Table F; Kennedy, op.cit., Appendix Table 4; Henry, op.cit., Appendix 4.

downward bias was offset to some extent by the fact that Barna's estimates excluded the stock of vehicles and other fixed assets. More recent estimates for the UK show that vehicles and other fixed assets consistently account for less than 4 per cent of total gross stock in manufacturing so that the offset was marginal. On the other hand the range from 0.5 to 0.7 of net-gross ratios estimated by Goldsmith indicates that had gross stock in Irish manufacturing been used in the comparison the result would have been substantially in excess of 74 per cent.

Some indication of how much higher the Ireland-UK would have been was obtained by making rough estimates of the gross stock implied by Nevin's method. The gross stock of each of plant and buildings (Nevin's definitions) at the end of 1947 was estimated for each industrial group by multiplying Nevin's average pre-1947 gross investment by the corresponding working life (as amended by Kennedy). Net-gross ratios were then calculated for each industrial group for the two asset groups combined since neither Nevin nor Kennedy published separate estimates. The ratio lay within the expected range of 0.5 to 0.7 for all industrial groups (except textiles and clothing) and for manufacturing as a whole was 0.57.^[100] On this basis a comparison of gross capital stock per head would yield the result that in Irish manufacturing there was almost 1-1/3 times as much capital per worker as in the UK, which seems scarcely plausible. There arises the suspicion therefore that Nevin's figures for net capital stock are overestimated.

Since the basic data on which to base a capital census - fire insurance valuations and details of the methods of valuation used - are not available for the non-manufacturing sectors, and since the data which are available for manufacturing are of doubtful merit, the capital census method cannot be used to obtain capital stock estimates for the whole economy. Nevertheless it is useful to examine the existing studies a little further to ascertain if some guidance can be obtained regarding the working lives of assets which may help in attempting to extend the scope of the capital estimates.

It will be apparent from the preceding discussion that so far as is possible it is desirable to deal with homogeneous groupings of assets when considering average working lives. It is difficult for example, to interpret Nevin's average life of buildings (his definition) when one of the components - land - has an infinite life. Henry criticised Nevin's use of aggregates of assets on the grounds that it led to implied overestimates of the average life of plant and machinery,^[101] however as Kennedy pointed out, any errors on this score were apparently compounded by a series of mistakes.^[102] In compiling annual investment data from the CIP Nevin apparently misclassified a number of industries compared with the capital census classification and miscalculated in two other instances. Kennedy's corrections yielded substantial reductions in the average lives of plant for three industries and would bring the weighted average over all manufacturing industries more in line with Dean's average for plant and machinery of 34 years. This, of course, does not entirely solve the problem since Nevin's and Kennedy's results relate to vehicles, plant and machinery combined.^[103]

The greater disaggregation introduced by Henry goes some way towards providing more meaningful estimates of the average lives of assets. Table 4 below compares his results with those used for manufacturing industry by Dean^[104] in the UK and Goldsmith^[105] in the US. Nevin's estimates as amended by Kennedy are also shown.

Henry's estimates for plant and machinery and for vehicles accord closely with those used by Dean. Goldsmith's estimates tend to be lower partly perhaps due to swifter technological change in the US inducing earlier retirement and partly to a more aggregative approach - Goldsmith does not include an industrial analysis.^[106] The range for Nevin's composite group of plant, machinery and vehicles would seem to be on the high side. In the case of other fixed assets the estimates by Henry and Goldsmith are reasonably close.

The widest variation occurs in the case of building. Henry's use of an infinite term of life

seems extreme since buildings cannot be regarded as entirely free of depreciation and Nevin's estimates include land and other fixed assets. Again Goldsmith's estimated range seems on the short side and a more realistic figure would probably be around Dean's estimate of 80 years.[107]

TABLE 4

COMPARISON OF ESTIMATES OF AVERAGE WORKING LIVES (IN YEARS) OF ASSETS

	Type of Asset				
	Buildings	Plant and machinery	Passenger vehicles	Work vehicles	Other fixed assets
Henry	infinite	13 - 50	8	8	24
Dean	80	16 - 50	10	10	no comparable estimate
Goldsmith	40 - 50	20	6	6	20
Nevin	54 - 126 ¹	← 15 - 42 →			see footnote

¹includes land and other fixed assets.

ESTIMATES OF CAPITAL STOCK

Because appropriate census data are not available it was decided to use a variant of perpetual inventory to obtain estimates of the capital stock for the whole economy (excluding dwellings and roads) in an initial year, 1953.[108] The stock in subsequent years was then estimated using the published statistics of gross capital formation. All calculations were at 1958 prices.

Lack of capital formation data prior to 1953 meant that the gross capital stock at the end of 1953 had to be estimated using the formula in which the stock of an asset at the end of a given year is determined by gross capital formation in that year, the length of life of the asset and the average growth rate of gross capital formation over a preceding period equal to the life of the asset. It need hardly be repeated that the results must be regarded as highly tentative.

Because of the problems (outlined earlier) involved in ascertaining the average working lives of assets it was decided to use capital formation statistics where disaggregation is by type of asset rather than by type of industry. For non-manufacturing this meant using National Income and Expenditure as the source. Unfortunately it is not possible to reconcile these statistics with those published for manufacturing industries in the Census of Industrial Production because the asset classifications in the former overlap both sectors.[109] It was decided therefore to estimate capital stock for the whole economy on the basis of the asset classifications in National Income and Expenditure. These classifications are not ideal since the average working life must be regarded as relating not only to time but also as an average over the component assets within the group. However, they probably represent more homogeneous groupings than those adopted by Nevin,[110] and using information available for Ireland, the UK and the US it was possible to obtain a reasonably plausible average working life for each classification.[111]

Following Dean,[112] an average life of 80 years was assumed for other building and construction. It was felt that since some depreciation takes place this was more realistic than Henry's

assumption of an infinite life. Other building and construction expenditure includes the estimated full cost of work under the land rehabilitation project. Land rehabilitation constitutes an asset of indefinitely long life however since the annual capital expenditure involved represents a small part of the total it was felt that little injustice would be done by treating it as an integral part of other building and construction.[113]

Expenditure on transport equipment covers aircraft, ships and boats, rail vehicles and road vehicles for business use including the proportion of private cars believed to be purchased for business use. Some indication of the range of working lives involved may be obtained from Redfern who estimated the average lives of the component assets at 30 years for railway rolling stock, 25 years for ships, 14 years for aircraft and 10 years for road vehicles.[114] Goldsmith's estimates for the US were broadly similar.[115] The CSO (Dublin) provided the author with current price capital expenditure estimates on each asset for the period 1960 to 1972.[116] The distribution of the expenditure, both absolute and in percentage terms, is shown in Table 5 below. Using the percentages as weights and Redfern's working life estimates an average working life of 11 years for the group as a whole was obtained.[117]

TABLE 5
GROSS CAPITAL EXPENDITURE ON TRANSPORT EQUIPMENT: 1960-1972

	Aircraft	Ships and boats	Rail vehicles	Road vehicles	Total
£ million (current prices)	54.2	35.4	26.8	730.2	846.6
Percentage of total	6.4	4.2	3.2	86.3	100

The average life of agricultural machinery was assumed to be 12 years. The PEP Report on Agricultural Machinery[118] lists estimated working lives for twenty-five categories of machinery. No information on the breakdown by type of machinery was available for Irish data so the average life used is the mean of all the categories. Any error is likely to be small since with one exception the working lives are close to 12 years. Goldsmith identified only four categories of machinery and the mean life was 11 years.

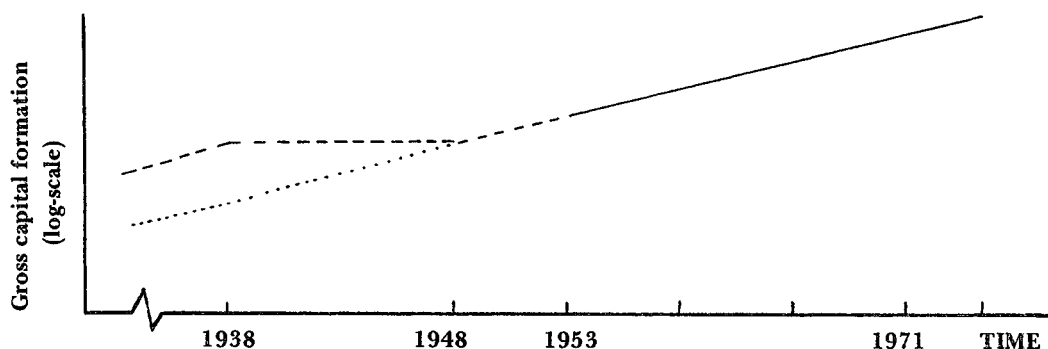
The asset category, other machinery and equipment must be regarded as a residual covering everything from office equipment to heavy machinery. It is reassuring that Henry in considering the CIP category, other fixed assets which contains "furniture, office machines, containers and so on" was able to conclude on the basis of his consultative method that such assets "might be taken to have a life similar to plant and machinery".[119] Furthermore, as can be seen in Table 4, Goldsmith also assigned the same average life to both plant and machinery and other fixed assets. There is the further problem that the aggregation also extends across industries. However, despite the range of lengths of life exhibited by plant and machinery in various industries there is evidence to suggest that in most industries the average life is in the region of 30 years. In his study of the UK Dean presented a table[120] showing the distribution of plant and machinery according to length of life by industry. For most of the individual orders the weighted average life is just over 30 years and for manufacturing, construction, distribution and other services as a whole the weighted average is 33 years with almost 80 per cent of plant and machinery having lives between 25 and 34 years. In the absence of information on the asset composition or the industrial spread of the category 'other machinery and equipment' it was decided to assume that they were similar

to the UK and to take the average life as 33 years.

Having decided on an assumed average working life for each group of assets the next step was to estimate the annual average growth rate of gross capital formation over that life. In each case this was done basically by fitting a trend to gross capital formation for the period 1953 to 1971 inclusive and assuming that the same trend rate of growth had occurred prior to 1953. However, some adjustment was necessary to take account of the depressed levels of gross capital formation during the period of World War II. In 1938 total gross fixed capital formation was at virtually the same level as in 1948 but fell sharply in the intervening years. If the 1953-71 trend growth rate were extended backward without amendment this would understate the level of gross capital formation prior to 1938 by failing to take account of the break in trend between 1938 and 1948 (see Figure I below).

Consequently the time path of pre-1953 gross capital formation was regarded as falling into three phases. The first was from 1948 to 1953 inclusive during which capital formation grew at the post-1953 trend rate. As a check, projecting backward to 1948 for each asset group and adding the estimates yielded a figure very close to the published total gross capital formation figure for 1948.[121] The second was from 1938 to 1947 inclusive during which annual capital formation was assumed to remain constant. This undoubtedly involves an overestimate, particularly important for those assets with relatively short lives. However the original official estimates of total capital formation for the years 1938 to 1943 were later found to be overstated but revised figures were published only for 1938[122] so that it was not possible to establish the magnitude of the error involved and in any event adjustments to individual asset groups would have been little more than guess-work. The third phase extended from 1937 backward during which capital formation was again assumed to have grown at the post-1953 trend rate.

FIGURE I



The phases are shown diagrammatically in Figure I where the broken line indicates the three phases of capital formation. The dotted line continuing backward from 1948 shows the lower level of capital formation which would result from ignoring the break in trend.

The 1953-71 trend rates of growth were estimated using the constant growth model,

$$Y_t = Y_0 (1 + g)^t$$

where Y_t is gross capital formation in year t , Y_0 is the value of Y_t in the initial year (taken here as 1953 where $t = 0$) and 100g per cent is the annual growth rate. Transferring to logarithms gives,

$$\log Y_t = a + bt$$

$$\text{where } a = \log Y_0 \text{ and } b = \log(1 + g)$$

When $\log Y_t$ is regressed on a time trend t , the coefficient of t is an estimate of $\log (1 + g)$ from which g can be determined and the regression intercept is the logarithm of the trend value of Y_t at time zero, i.e. in 1953. The regression results, estimated growth rates and trend values of gross capital formation in 1953 are shown in Table 6 below.

TABLE 6
TREND GROWTH RATES OF GROSS CAPITAL FORMATION
BY ASSET CATEGORY: 1953-71

Asset category	Intercept (a)	$Y_0 =$ antilog(a)	Coefficient (b)	100g = 100 (antilog (b)-1)
Dwellings	1.1618	14.52	0.0181	4.2
Roads	0.7199	5.25	0.0017	0.4
Other building and construction	1.4361	27.30	0.0217	5.2
Transport equipment	0.9616	9.15	0.0338	8.1
Agricultural machinery	0.6705	4.68	0.0160	3.8
Other machinery and equipment	1.1564	14.33	0.0394	9.5

One final step was necessary before the gross capital stock at the end of 1953 could be estimated. The division of pre-1953 capital formation into three phases required an amendment to the gross capital stock formula which now became[123]

$$K = k \left[\frac{1+g}{g} \left\{ 1 - (1+g)^{-T-10} \right\} + 10(1+g)^{-5} \right]$$

where K is gross capital stock at the end of 1953, k is the trend value of gross capital formation in 1953,[124] T is the average life of the asset and 100g per cent the annual average rate of growth of gross capital formation. It was assumed that each asset had zero scrap value, that there was no distribution of retirements and that the working life of each asset remained constant over all vintages.

A similar amendment was necessary for the net stock formula which now became

$$N = \frac{k(1+g)^2}{Tg^2} \left[Tg(1+g)^{-1} - 1 + 5g(1+g)^{-7} \left[2Tg - 23g - 2 \right] + (1+g)^{-(T-10)} \right]$$

where N is net capital stock at the end of 1953 and the other symbols are as before.[125]

The summations represented by the two above formulae were of course shortened in the cases of transport equipment and agricultural machinery where the respective lives of 11 and 12 years meant that only the first and part of the second phases were relevant.

The estimates of gross and net capital stock at the end of 1953 for each asset group are shown in Table 7 below. From these starting values it was then possible using the published series of annual gross capital formation at constant prices and the perpetual inventory method to estimate gross and net capital stock and capital consumption in each year after 1953.

Estimates of the gross value of retirements were obtained using the appropriate trend value of

gross capital formation in 1953 and the pre-1953 growth rate of capital formation. For agricultural machinery, published gross capital formation was used for retirements from 1965 onward and for transport equipment from 1964 onward. The results are set out in the appended tables.

TABLE 7
ESTIMATED GROSS AND NET CAPITAL STOCK
(AT 1958 REPLACEMENT COST) AT END - 1953

	(£ million)				
	Other building and construction	Transport equipment	Agricultural machinery	Other machinery and equipment	Total
Gross stock	748.3	76.6	48.9	235.7	1109.5
Net stock	547.1	37.7	23.2	144.6	752.6
Average working life (years)	80	11	12	33	

RELIABILITY OF THE ESTIMATES

The capital stock estimates depend jointly on the estimates of trend annual gross capital formation prior to 1953 and on the assumed average working lives. Errors in both gross and net stock vary directly with errors in the assumed lives and in pre-1953 gross investment. These sources of possible errors are not independent (though they may be compensating) since the longer the life of an asset the greater the danger of estimated pre-1953 investment being unrepresentative. The reliability of the stock estimates therefore probably bears an inverse relationship to the lives of the assets. At one end of the reliability spectrum is agricultural machinery where the assumed life can be taken with reasonable certainty as around 12 years. Consequently after 1965 the capital stock estimates no longer depend on estimates of pre-1953 investment. The same is also true of transport equipment where after 1964 the stock estimates no longer depend on pre-1953 investment estimates, although in this case the average life of the group as a whole is less reliable. The stock estimates for other building and construction lie at the other end of the spectrum and must be viewed with considerable caution while the estimates for plant and machinery may be regarded as lying somewhere between the two extremes in terms of reliability.

The average working lives were decided upon in the light of experience in Ireland, the UK and the US, and although the asset groupings are not entirely satisfactory, the lives do not seem unreasonable. Regarding pre-1953 capital formation, it is perhaps worth recalling that the capital stock estimates do not require that the actual pattern of annual capital formation adhere strictly to the trend. The results by Goldsmith quoted earlier indicate that the margin of error in the stock estimates due to using trend rather than actual figures of capital formation is small even under conditions of wide cyclical fluctuations, provided the fluctuations are smooth. The fact that the error is inversely related to the length of life also provides some small reassurance. Furthermore if the assumption of a trend growth of capital formation is reasonably accurate then as Schiff and

Barna have shown, the assumption that an asset is retired at exactly the end of its life is unlikely to produce an error in excess of ten per cent. This latter point is of particular relevance to the asset category, transport equipment, since the heterogeneous nature of the component assets with working lives ranging from 10 to 30 years superimposes a retirement distribution about the overall average life of 11 years. On the basis of Barna's results it would seem that there is a danger that the stock estimates have some upward bias.^[126] However, since this category accounts for a small part of the total it is unlikely to have a substantial effect. Two other possible sources of error may be noted. First, to the extent that the capital formation price indices do not take account of improvements in asset efficiency there will be a tendency towards underestimation in the growth rates of the stock estimates. Secondly, the assumption adopted here, that assets have zero scrap value will, if incorrect, result in an underestimate of net capital stock.^[127] As was shown earlier the magnitude of the error depends, among other things, on the ratio of scrap to original value about which there is no information.

In the absence of comparable estimates the next step is to examine how closely the present capital stock estimates accord with economic experience during the period in question. Between 1953 and 1971 output in Ireland increased at an annual average rate of 3.2 per cent while employment declined on average by 0.4 per cent.^[128] Thus the growth in output was achieved by an annual increase of 3.6 per cent in output per head employed. This long-term growth of labour productivity suggests a tendency towards increasing capitalisation of production resulting in long-term increases in capital per head and in the ratio of capital to output.^[129]

As can be seen from columns (1) and (2) of Table 8 below, the present capital stock estimates suggest that such was the case, with capital per head increasing from £931 (at 1958 replacement cost) in 1953 to £2,857 in 1971 and the capital output ratio from 1.8 to 2.7

During the same period a similar pattern emerges for the UK - output increased by an average 2.9 per cent per annum, employment by 0.3 per cent and output per head by 2.6 per cent. Capital per head increased from £1997 in 1953 to £3633 in 1971 and the capital-output ratio from 2.2 to 2.7. (See columns (3) and (4) of Table 8). A striking feature emerges from a comparison of the two countries - the substantially faster growth in Ireland of labour productivity (Table 8, column (6)). Comparisons of labour productivity are hazardous: it has been noted elsewhere that part of the increase in Ireland reflects the movement of employment out of agriculture and more generally there are problems raised by differences in the industrial structures of the two countries. Nevertheless increases in capital per head and the capital-output ratio in Ireland relative to the UK would be consistent with the observed relative increase in labour productivity since for example the long-term movement of employment into more capital intensive sectors would require corresponding increases in the capital stock with which to equip the labour inflow. This appears to have been the case: column (5) of Table 8 shows that capital per head in Ireland as a percentage of the UK increased from under 50 in 1953 to 70 in 1971, and a comparison of columns (2) and (4) shows that the capital-output ratio in Ireland grew faster than in the UK.

While suggesting that the long-term trends are in the right direction, this does not of course necessarily imply that the levels of capital per head and the capital-output ratio are correct. However the range of values in each case does not seem implausible and may be put forward as tentative evidence that the capital stock estimates for Ireland are of the correct order of magnitude.

It would be unrealistic to place too heavy an emphasis on broad comparisons of the kind made here; capital per head and capital-output ratios vary considerably among sectors of the economy and among industries within the same sector.^[130] However, in view of the reservations outlined earlier in the review of previous estimates of capital stock in Ireland there would be little to gain from attempting more detailed comparisons. In conclusion it is perhaps salutary to repeat the caution expressed in the introduction that the estimates presented here are at best indicators of orders of magnitude and to stress the need for further development of Irish capital stock statistics.

TABLE 8

CAPITAL PER HEAD, CAPITAL-OUTPUT RATIO AND OUTPUT PER HEAD: IRELAND AND THE UK¹

	Ireland		UK		Ireland as percentage of the UK	
	Gross capital stock per head	Gross capital - output ratio	Gross capital stock per head	Gross capital - output ratio	Gross capital stock per head	Output per head
	£(1958 replace- ment cost)		£(1958 replace- ment cost)		%	%
	(1)	(2)	(3)	(4)	(5)	(6)
1953	931	1.81	1966	2.25	47.4	59.0
1954	980	1.88	1997	2.24	49.1	58.4
1955	1050	1.95	2033	2.24	51.6	59.2
1956	1123	2.07	2060	2.25	54.5	59.2
1957	1215	2.14	2119	2.27	57.3	60.8
1958	1280	2.28	2220	2.34	57.7	59.3
1959	1338	2.24	2271	2.31	58.9	60.7
1960	1395	2.24	2361	2.27	59.1	59.9
1961	1456	2.22	2459	2.31	59.2	61.5
1962	1515	2.26	2455	2.35	61.7	64.1
1963	1586	2.31	2541	2.33	62.4	62.8
1964	1666	2.31	2608	2.28	63.9	63.1
1965	1764	2.39	2687	2.31	65.6	63.5
1966	1865	2.49	2784	2.37	67.0	63.9
1967	1965	2.48	2950	2.44	66.6	65.5
1968	2069	2.43	3118	2.46	66.4	67.4
1969	2189	2.50	3271	2.55	66.9	68.3
1970	2357	2.61	3439	2.61	68.5	68.4
1971	2543	2.66	3633	2.65	70.0	69.7

¹ Sources: Irish data: employment; Trend of Employment and Unemployment; capital; see text; output, National Income and Expenditure. UK data: employment, British Labour Statistics, Historical Abstract, HMSO, London, (1971) and Monthly Digest of Statistics; capital, Dean, op. cit., and National Income and Expenditure; output, National Income and Expenditure. End-year capital stock estimates were adjusted to mid-year by averaging consecutive years.

TABLE 9
GROSS CAPITAL STOCK (1958 REPLACEMENT COST)
BY ASSET GROUP: END OF YEAR

	Other building and construction	Transport equipment	Agricultural machinery	Other machinery and equipment	TOTAL
<i>£ million</i>					
1953	748.3	76.6	48.9	235.7	1,109.5
1954	782.2	82.6	52.7	253.4	1,170.9
1955	820.0	88.2	55.0	271.8	1,235.0
1956	854.7	96.2	54.6	287.3	1,292.8
1957	884.5	101.9	55.8	299.9	1,342.1
1958	910.3	109.8	57.0	315.3	1,392.4
1959	938.7	116.6	58.1	331.1	1,444.5
1960	967.8	122.7	58.6	350.3	1,499.4
1961	999.5	131.9	60.5	374.4	1,566.3
1962	1,038.3	141.4	61.9	403.9	1,645.5
1963	1,082.7	150.5	64.1	438.3	1,735.6
1964	1,132.8	161.8	66.6	472.3	1,833.5
1965	1,185.0	173.8	68.7	510.8	1,938.3
1966	1,237.1	183.5	67.8	548.8	2,037.2
1967	1,290.6	192.0	68.4	589.4	2,140.4
1968	1,348.0	205.9	75.0	637.8	2,266.7
1969	1,411.7	235.2	81.6	701.7	2,430.2
1970	1,474.3	260.2	87.3	772.8	2,594.6
1971	1,547.1	292.1	90.8	841.7	2,771.7
1972	1,620.9	306.5	97.8	916.8	2,942.0

TABLE 10
NET CAPITAL STOCK (1958 PRICES) BY ASSET GROUP:
END OF YEAR

	Other building and construction	Transport equipment	Agricultural machinery	Other machinery and equipment	TOTAL
<i>£ million</i>					
1953	547.1	37.7	23.2	144.6	752.6
1954	572.0	42.4	26.5	156.5	797.4
1955	600.3	46.2	28.1	168.8	843.4
1956	625.2	51.7	27.0	177.9	881.8
1957	644.9	54.3	27.4	183.9	910.5
1958	660.3	58.4	27.7	192.4	938.8
1959	678.1	60.8	27.9	201.2	968.0
1960	696.2	62.4	27.4	213.1	999.1
1961	716.6	66.8	28.3	229.5	1,041.2
1962	743.6	71.2	28.7	250.7	1,094.2
1963	775.8	75.1	29.9	276.1	1,156.9
1964	813.1	80.9	31.3	300.5	1,225.8
1965	851.9	89.3	32.4	328.7	1,302.3
1966	890.0	94.1	33.5	355.8	1,373.4
1967	929.0	99.3	34.6	384.7	1,447.6
1968	971.2	106.4	38.4	420.6	1,536.6
1969	1,019.1	128.4	43.3	470.6	1,661.4
1970	1,065.2	142.7	46.8	526.4	1,781.1
1971	1,120.7	160.8	47.7	578.7	1,907.9
1972	1,176.3	163.7	50.9	634.9	2,025.8

TABLE 11

GROSS DOMESTIC FIXED CAPITAL FORMATION (1958 PRICES) BY ASSET GROUP

	Other building and construction	Transport equipment	Agricultural machinery	Other machinery and equipment	TOTAL
<i>£ million</i>					
1953	30.8	8.6	6.1	20.6	66.1
1954	34.7	12.2	7.7	19.6	74.2
1955	38.7	11.8	6.2	20.5	77.2
1956	35.6	14.2	3.5	17.8	71.1
1957	30.8	11.9	5.1	15.1	62.9
1958	26.8	14.1	5.1	18.1	64.1
1959	29.5	13.0	5.0	18.8	66.3
1960	30.2	12.8	4.4	22.5	69.9
1961	32.9	16.4	5.9	27.7	82.9
1962	40.0	17.3	5.6	33.4	96.3
1963	45.7	17.6	6.5	38.7	108.5
1964	51.5	20.5	7.0	38.7	117.7
1965	53.6	24.2	6.8	43.7	128.3
1966	53.6	21.5	6.8	43.7	125.6
1967	55.1	22.7	6.8	46.8	131.4
1968	59.1	25.8	10.1	55.2	150.2
1969	65.5	43.4	11.7	71.3	191.9
1970	64.5	38.0	10.8	79.2	192.5
1971	74.8	44.7	8.5	77.8	205.8
1972	75.9	30.8	11.4	84.0	202.1

TABLE 12

CAPITAL CONSUMPTION (1958 PRICES) BY ASSET GROUP

	Other building and construction	Transport equipment	Agricultural machinery	Other machinery and equipment	TOTAL
<i>£ million</i>					
1953	9.4	7.0	4.1	7.1	27.6
1954	9.8	7.5	4.4	7.7	29.4
1955	10.3	8.0	4.6	8.2	31.1
1956	10.7	8.7	4.6	8.7	32.7
1957	11.1	9.3	4.7	9.1	34.2
1958	11.4	10.0	4.8	9.6	35.8
1959	11.7	10.6	4.8	10.0	37.1
1960	12.1	11.2	4.9	10.6	38.8
1961	12.5	12.0	5.0	11.3	40.8
1962	13.0	12.9	5.2	12.2	43.3
1963	13.5	13.7	5.3	13.3	45.8
1964	14.2	14.7	5.6	14.3	48.8
1965	14.8	15.8	5.7	15.5	51.8
1966	15.5	16.7	5.7	16.6	54.5
1967	16.1	17.5	5.7	17.9	57.2
1968	16.9	18.7	6.3	19.3	61.2
1969	17.6	21.4	6.8	21.3	67.1
1970	18.4	23.7	7.3	23.4	72.8
1971	19.3	26.6	7.6	25.5	79.0
1972	20.3	27.9	8.2	27.8	84.2

TABLE 13

NET DOMESTIC FIXED CAPITAL FORMATION (1958 PRICES) BY ASSET GROUP

	Other building and construction	Transport equipment	Agricultural machinery	Other machinery and equipment	TOTAL
<i>£ million</i>					
1953	21.4	1.6	2.0	13.5	38.5
1954	24.9	4.7	3.3	11.9	44.8
1955	28.4	3.8	1.6	12.3	46.1
1956	24.9	5.5	-1.1	9.1	38.4
1957	19.7	2.6	0.4	6.0	28.7
1958	15.4	4.1	0.3	8.5	28.3
1959	17.8	2.4	0.2	8.8	29.2
1960	18.1	1.6	-0.5	11.9	31.1
1961	20.4	4.4	0.9	16.4	42.1
1962	27.0	4.4	0.4	21.2	53.0
1963	32.2	3.9	1.2	25.4	62.7
1964	37.3	5.8	1.4	24.4	68.9
1965	38.8	8.4	1.1	28.2	76.5
1966	38.1	4.8	1.1	27.1	71.1
1967	39.0	5.2	1.1	28.9	74.2
1968	42.2	7.1	3.8	35.9	89.0
1969	47.9	22.0	4.9	50.0	124.8
1970	46.1	14.3	3.5	55.8	119.7
1971	55.5	18.1	0.9	52.3	126.8
1972	55.6	2.9	3.2	56.2	117.9

NOTES

1. While some roadwork needs to be renewed from time to time (some lasts indefinitely and does not depreciate) no allowance for roadwork depreciation is included in the national accounts estimates of "provision for depreciation" which are used to transform net national product (at current factor cost) into gross national product. (See for example, *National Income and Expenditure*, (1972), CSO, (Dublin), Table A2 and p.41). This is in accordance with the practice recommended by the United Nations. See "A System of National Accounts and Supporting Tables", UN, (New York) 1964.
2. See for example, *Irish (Trade Journal and) Statistical Bulletin*, March, 1955, p.3. The description of the then new wholesale price index numbers (including series for capital goods) states, "Every effort has been made to ensure that price quotations relate at the different inquiries to identical qualities of goods. At the same time it has to be recognised that ... it would be impossible to guarantee that as time goes on, the ideal of identical quality can be maintained."
3. See for example, Goldsmith, R.W., "The Growth of Reproducible Wealth of the USA from 1805 to 1950" in Kuznets, I., (ed.) *Income and Wealth Series II*, Bowes and Bowes (Cambridge) 1952, p.261. In discussing his capital stock estimates Goldsmith commented, "The results ... remain subject to the tendency common in virtually all deflation procedures of understating improvements in the quality of durable assets The deflators therefore, have a tendency to rise more over time than they should. Consequently rates of growth calculated from the deflated figures are likely to represent minima." A similar point is made by Kendrick, J.W., "Productivity Trends: Capital and Labour", *Review Economics and Statistics*, vol. 38, (1956), p.250, " ... the price deflators do not make allowance for changes in efficiency of the capital items ... Since quality changes are not taken into account, the associated capital input measures what the real services of the stock would have been had base period technology prevailed in all periods."
4. The method is due to Goldsmith, R.W., "A Perpetual Inventory of National Wealth", *Studies in Income and Wealth*, vol. 14 (1951). It has been used in the UK notably by Redfern, P., "Net Investment in Fixed Assets in the UK 1938 - 1953", *Journal of the Royal Statistical Society, Series A (Gen.)*, vol. 118 (1953) and Dean, G., "The Stock of Fixed Capital in the UK in 1961", *Journal of the Royal Statistical Society, Series A (Gen.)* vol. 127 (1964).
5. Goldsmith (1952) *op. cit.*, p.263 suggests that when changes in the structure rather than growth are the object of study it may be preferable to use undeflated original figures.
6. Replacement cost should include the cost of installation as well as the purchase cost.
7. Barna, T., "The Replacement Cost of Fixed Assets in British Manufacturing Industry in 1955", *Journal of the Royal Statistical Society, Series A (Gen.)*, vol. 120 (1957), p.6.
8. In the preparation of national accounts, when an asset is sold as scrap, or to a consumer, or for export, the sale proceeds are deducted from purchases of assets to obtain gross domestic fixed capital formation.
9. Capital consumption is not merely the result of physical wear and tear. Obsolescence also enters in because, "the quality and quantity of existing facilities declines in relation to the

services of new and technically superior capital goods even though the quantity and quality of the services of the existing facilities do not decline absolutely. Hence the appearance on the market of technically superior capital items implies an increase in the cost of operating existing equipment and it is this increase in opportunity cost that is represented in the obsolescence component of depreciation charges". See Brown, M., *On The Theory and Measurement of Technological Change*, Cambridge University Press (1968) p.82.

10. Goldsmith (1952) *op. cit.*, p.252 suggested that obsolescence could be defined as the result of a difference between the actual and anticipated useful life of an asset.
11. For example, if the replacement cost new of a given asset is £100 and the value of the asset is assumed to decline at 10 per cent per year, then capital consumption during the first year is 10 per cent of £100 = £10, and the written-down value at the end of the first year is £90. Capital consumption during the second year is 10 per cent of £90 = £9 and the written-down value at the end of the second year is £81; and so on.
12. An alternative, though less satisfactory method of allowing for capital consumption is the "one-hoss shay", i.e. to assume that an asset can if suitably maintained, retain its full value throughout its life until it is finally retired. Annual capital consumption is therefore measured by the gross value of assets retired in that year and net capital formation is the difference between gross capital formation and retirements. The method is unsatisfactory since as Redfern (*op. cit.*, p.142) has pointed out it misses the point of attempting to allow for the ageing of assets which are still in use by ignoring, "the 'two-dimensional' nature of capital: the essence of a productive asset is that it continues to render services over a long period; its value is related not only to the service it renders in a given year, but also to the number of years over which it can be expected to remain productive." The origin of the name, "one-hoss shay", is the poem by Oliver Wendell Holmes in which a chaise was constructed without a weakest spot to avoid the problem where "a chaise breaks down but doesn't wear out". The result was that after carefree running for one hundred years it suddenly "went to pieces all at once". A list of other possible methods is as given in Brown, M., *op. cit.*, p.83.
13. *op. cit.*, p.142. Redfern chose the straight-line method because, "in a great number of cases the productive services rendered by an asset are likely to be more nearly constant over time than to reduce exponentially".
14. *op. cit.*, p. 340.
15. See *National Accounts: Sources and Methods*, CSO (London) (1968), p.383.
16. The Table is derived from Redfern, *op. cit.*, Table 1, p.144.
17. If the asset had a scrapping value of £5 (at constant prices) then capital consumption in year one would be $\text{£1} = \frac{1}{5} (\text{£10} - \text{£5})$; in year two it would be £2, and so on.
18. The "ideal world" qualification is important. Nevin maintained that totals for capital stock obtained by the perpetual inventory method are "not estimates in the ordinary sense of the term at all, but are merely the numerical expression of an assumption that the life of a particular class of asset is x years". See Nevin, E., "The Life of Capital Assets: An Empirical Approach", *Oxford Economic Papers*, Vol. 16 (1964).

19. See for example, Redfern, *op cit.*, p.144 where it is pointed out that, "positive or negative net capital formation may occur without any change in the quantity of assets in use. Positive net investment may imply an increase in the quantity of assets in use or capacity, or it may be the consequence of a large renewals programme .. without any current (or prospective) increase in capacity; in other words, net investment represents either an increase in capacity or an increase in the average future life of the assets in use. Similarly, negative net investment may mean a reduction in capacity or it may mean a decrease in the average future life of the assets in use."
20. The relationships may be demonstrated using the notation: k_t, n_t, c_t, r_t are respectively gross capital formation, net capital formation, capital consumption and gross value of assets retired, all in year t ; K_t and N_t are respectively gross and net capital stock at the end of year t ; T is the assumed constant life of the asset.

Perpetual inventory defines K_t as

$$K_t = \sum_{i=1}^t k_i - \sum_{i=1}^t r_i$$

Since by assumption assets are not retired until they are T years old,

$$r_i = k_{i-T} \text{ and}$$

$$K_t = \sum_{i=1}^t k_i - \sum_{i=1}^{t-T} k_i = \sum_{i=t-T+1}^t k_i$$

Thus K_t is determined by summing k_i over the life of the asset.
Using the straight-line method, c_t is,

$$c_t = \frac{1}{T} K_t = \frac{1}{T} \sum_{i=t-T+1}^t k_i$$

and n_t is,

$$n_t = k_t - c_t = k_t - \frac{1}{T} \sum_{i=t-T+1}^t k_i$$

Again, by definition,

$$N_t = \sum_{i=1}^t n_i = \sum_{i=1}^t k_i - \sum_{i=1}^t c_i$$

which on substitution yields,

$$N_t = K_t - \frac{1}{T} \left[k_{t-T+1} + (k_{t-T+1} + k_{t-T+2}) + \dots + (k_{t-T+1} + \dots + k_t) \right] \\ - \frac{T-1}{T} k_t + \frac{T-2}{T} k_{t-1} + \dots + \frac{T-(T-1)}{T} k_{t-T+2}$$

Thus N_t is determined by deducting capital consumption accrued over the life of the asset from K_t since the terms in the square bracket are gross capital stock in each year from $(t - T + 1)$ to t if previous years' gross capital formation is ignored.

21. For assets of long life this would be mitigated by the fact that the level of capital formation in the past was generally lower than in more recent years.
22. *op. cit.*, p.169.
23. If annual gross capital formation were constant throughout the assumed life there would be no error in capital consumption.
24. These points are dealt with in more detail in Redfern, *op. cit.*, pp.146 ff. Nevin (1964) *op. cit.*, has shown using Redfern's figures for the UK that varying the assumed life of assets by one-third changes the annual growth of net capital stock between 1947 and 1952 from 5.3 per cent to 7.0 per cent.
25. Schiff, E., "Gross Stocks Estimated from Past Installations", *Review of Economics and Statistics*, Vol. 40 (1958).
26. Barna, *op. cit.*
27. The properties of the curve are similar to those of curves fitted to empirical data reported in Winfrey, R., "Statistical Analyses of Industrial Property Retirements", *Iowa Experiment Station, Bulletin 125* (1936).
28. i.e. without allowance for retirement distribution.
29. *op. cit.*, p.177.
30. For example, for a positively skewed retirement distribution (the modal retirement age being less than the mean), if the mean is taken as the assumed constant retirement age, the perpetual inventory method would overestimate gross stock if capital expenditure had an increasing trend and underestimate if capital expenditure had a decreasing trend.

31. *op cit* (1962) Ch. 3.
32. Jefferson, C. W., "A Method of Estimating the Stock of Capital in Northern Ireland Manufacturing Industry", *The Economic and Social Research Institute* (Dublin), Paper No. 44 (1968).
33. Capital replacement will also be affected. Maddison quotes an example. "If the life of capital is fixed and is the same in all countries a country which has had a steady growth rate (of capital stock) of 2 per cent a year and an average life of capital of thirty years will need to replace 1.8 per cent of its capital stock each year. A country which has had a steady growth of 5 per cent a year will need to replace only 0.8 per cent of its capital stock annually as it will have a newer stock. If the capital output ratio is 2.5, then the first country would spend 4.6 per cent of GNP each year on replacement and the second 1.9 per cent." See Maddison, A., *Economic Growth in the West*, George Allen and Unwin (London) 1964, p.85. The explanation is as follows: a capital stock of K will grow to 1.8K in 30 years at 2 per cent. Replacement in the thirtieth year will be one-thirtieth of K which is equivalent to 1.8 per cent of 1.8K.
34. The corresponding formulae for gross and net capital stock respectively are,

$$K = k \left(\frac{1 - e^{-gT}}{1 - e^{-g}} \right)$$

$$\text{and } N = k \left[\frac{1 - \frac{1}{gT}(1 - e^{-gT})}{1 - e^{-g}} \right]$$

where k is the level of gross capital expenditure in the current and not the initial year. The formulae for net stock and the net gross ratio are rather more complicated if use is made of the declining balance method of writing-off. An interesting comparison of net stock calculated by both methods of writing-off based on the same capital expenditure data for a number of categories of assets may be found in Garland, J. M., and Goldsmith, R. W., "The National Wealth of Australia, The Measurement of National Wealth", *Income and Wealth Series VIII* (1959).

35. This is perhaps more easily seen if Goldsmith's formula is rearranged to give,

$$\frac{N}{K} = \frac{e^{gT}}{e^{gT} - 1} - \frac{1}{gT}$$

36. *op. cit.*, (1962), Table 1, p.19. The values for the length of life were 5, 10, 20, 50 and 100 years; for the growth rate, 0, 1, 2, 3, 5 and 10 per cent per annum.
37. For example, when the growth rate was 1 per cent, gross stock as a multiple of current capital expenditure varied from 4.90 to 63.20 as the asset life was increased from 5 to 100 years. With an asset life of 100 years the multiple fell from 63.20 to 10.50 as the growth rate increased from 1 to 10 per cent.
38. *op. cit.*, p. 19.
39. The results utilise two assumptions which were not made explicit: that the asset has zero

scrap value, and that there is no distribution of retirements. An interesting result emerges when Jefferson's calculations are generalised. Dividing T (the asset life) into two equal sub-periods and assuming that in the first gross capital expenditure is constant at k per year and in the second at nk per year (where n is a positive number) yields the following expression for the net-gross ratio at the end of the Tth year,

$$\frac{N}{K} = \frac{1 + 3n}{4(1 + n)} - \frac{I}{2T}$$

where as before, the ratio is determined by the length of life and the multiple by which gross capital expenditure differs between the first and second halves of the preceding T years. From the formula, if $n = 1$ (gross capital expenditure is constant throughout the life of the asset), the net-gross ratio has an upper limit of 0.5, the proximity to the limit improving as T increases. For given T, the ratio has an upper limit of 0.75 as n increases and a lower limit of 0.25 as n approaches zero.

This formula may be used to correct an error in Goldsmith's Table 1 in which the net-gross ratio is given as 0.5 for all values of T when the growth rate (g) is zero i.e. annual gross capital expenditure is constant. Since Goldsmith's formulae are based on the sum of a geometric progression, the use of which is not appropriate when $g = 0$, it is necessary to use the formula given above for this case. The correct values for the net-gross ratio when $g = 0$, for the range of asset lives used by Goldsmith are given below.

Length of life (years)	5	10	20	50	100
Net-gross ratio	0.40	0.45	0.475	0.49	0.495

40. For example, when $T = 10$, $g = 0.04$ and the length of cycle was 4 years, the two estimates of gross capital stock differed by a maximum of 6 per cent for cyclical fluctuations of capital expenditure ranging from 50 to 150 per cent of trend. When $T = 50$, the maximum difference was reduced to less than 2 per cent and was not much greater than 3 per cent when capital expenditure fluctuated between zero and twice the trend value. The differences were substantially increased if actual capital expenditure in the current year rather than its trend value was used in estimating from the formula.
41. *op. cit.*, (1962), p.21.
42. The value of the net-gross ratio varying directly with changes in T and/or g.
43. Since $N^{(1)}/K = 0.50$ and $N^{(2)}/K = 0.55$,

$$(N^{(2)} - N^{(1)}) = 0.05K = 0.05N^{(2)}/0.55 = 0.09N^{(2)}$$
44. Goldsmith (1962), *op. cit.*, takes a different view stating (see p.27), "Thus fortunately, two deviations that are introduced by the use of simplifying formulae - the disregard of scrap value and of retirement distribution - tend in opposite directions." However, this seems to be an error. In quoting Schiff's results on retirement distributions Goldsmith noted that (see p.26), "If account is taken of the distribution of actual retirement dates - assumed to be normal, i.e., symmetrical and bell-shaped - the gross stock will be above the value found by the formulae which disregard retirement distribution". This may be compared with Schiff (1958) *op. cit.*, p.176, "... the gross stock value derived by the short-

cut method exceeds the value derived by the correct method for all rn products except $rn = 0$ (at which point the two values are equal). In Schiff's notation n is the average working life and r the continuous growth rate of capital formation.

45. *op. cit.*, (1962), p.28.

46. Taking gross capital expenditure in the current year as k , then capital expenditure in the preceding years (counting backwards) is

$$k(1+g)^{-1}, k(1+g)^{-2}, \dots$$

and so on.

47. *op. cit.* (1964).

48. The exposition here is restricted to the case where the net stock is known.

49. Assuming also that there is no distribution of retirements.

50. Alternatively the expression can be rearranged into a quadratic form the positive solution of which yields a formula from which T can be calculated for given N and values of k_i for the years from

t to $(t - n + 1)$. The formula is,

$$2\bar{k}T = (2n+1)\bar{k} + 2(N-C) + [\bar{k}^2 + 4(N-C)^2 + 4\bar{k}\{(2n+1)(N-C) + 2D\}]^{1/2}$$

$$\text{where } C = \sum_{i=t-n+1}^t k_i$$

$$\text{and } D = k_t + 2k_{t-1} + \dots + nk_{t-n+1}$$

51. For example using hypothetical data Nevin (*op. cit.* (1964), p.234) found that if the true value of T were 10 years, and if k_i were known for only the last three years, an underestimate of 14 per cent in $\bar{k}$ (the average for the first seven years of the life) would result in T being overestimated by 9 per cent; if $\bar{k}$ were overestimated by 35 per cent, T was overestimated by 25 per cent. In all cases a larger n relative to T substantially reduced the error.

52. See Nevin, E., "Capital Stock of Irish Industry", *The Economic and Social Research Institute* (Dublin) paper No. 17 (1963), p.7.

53. It has been used in the UK by Barna, *op. cit.*, and in Ireland by Nevin, E., *op. cit.*, (1963) and by Henry, E. W., "Estimation of Capital Stock in Irish Industry 1953 to 1968", *Journal of the Statistical and Social Inquiry Society of Ireland*, vol. 22(4) (1971/2).

54. Some large firms may operate an internal insurance scheme, however their accounting records would follow the same lines as if they insured with an insurance firm. Separate

insurance may be undertaken to cover loss of profits etc. as a result of fire. On insurance practice in the UK see Barna, *op cit.* for Ireland see Jefferson, C. W., "Capital Statistics for Irish Manufacturing Industry", *The Economic and Social Research Institute* (Dublin) Paper No. 60 (1971).

55. Barna, *op cit.*, p.9, reported that the technique of valuation can be divided into three categories: (a) detailed and expert valuation by a firm of valuers or by the manufacturer's own engineers; (b) the replacement cost of an asset may be estimated from records detailing the original cost and date of acquisition of each item and price indices applied to bring the original costs up to date; (c) a short-cut method may be used in which previously revalued costs are brought up to date using a price index.
56. Barna, *op. cit.*, p.9 felt that over-valuation was unlikely to be a problem since the insured was not permitted to benefit from it, and that if it occurred it was due to error. However, Jefferson, C. W., *op. cit.*, (1971) concluded that on the 'margin of error' argument, over valuation was a distinct possibility.
57. *op. cit.* (1963) p.2.
58. *op. cit.* p.9.
59. See for example, Barna, *op. cit.*
60. See Dean, G. A., "Fixed Investment in Britain and Norway", *Journal of the Royal Statistical Society, Series A*, Vol. 127 (1964).
61. See Barna, *op. cit.*, p.21. The differences were greater for buildings (66 per cent of Redfern's estimate) than for plant and machinery (44 per cent).
62. This is the price index problem referred to above in the section, "Constant Price Estimates".
63. See Dean, *op. cit.*, *The Stock of Fixed Capital*, p.335.
64. See *National Accounts Statistics, op. cit.*, p.384, official estimates of capital consumption were first published in 1956 and of net capital stock in 1966.
65. See for example, NIE (1972) CSO, (Dublin), Table A.2.
66. Purchases of new and second-hand assets are distinguished and sales of assets are also shown. The difference between purchases and sales corresponds to gross fixed capital formation at current prices.
67. See *National Income and Expenditure 1938-44* (Pr. No. 7356) and *Tables of National Income and Expenditure 1938 and 1944-50* (Pr. 350).
68. From 1960 disaggregated into other construction (except land improvement) and land improvement and plantation and orchard development.
69. From 1960 subdivided into passenger cars and other.

70. When all sectors of the economy are considered jointly purchases less sales of land and existing buildings net out to equal the costs involved in the transfer of ownership, i.e. stamp duties, legal fees, agents' commissions etc.
71. *op cit*, (1963). Nevin's estimates were later extended with some modifications to cover the years 1946 to 1966. See Kennedy, K. A., *Productivity and Industrial Growth*, Clarendon Press (1971).
72. The results of the inquiry are reported in Henry, E. W., and Heelan, L. J., "Capital in Irish Industry", *Journal of the Statistical and Social Inquiry Society of Ireland*, vol. 21 (1962-3).
73. This was later confirmed by Jefferson, *op. cit.*, (1971) p.22.
74. *op. cit.*, p.14.
75. i.e. net output less wages and salaries. The grossing-up factors ranged from 1.04 for both buildings and plant in Drink and Tobacco to 3.52 for buildings in Wood. See Nevin, *op. cit.*, (1963), Table B.
76. An estimate was also made of rented assets in the clothing industry.
77. Enterprises which have undertaken initial capital formation but not yet begun production and enterprises employing fewer than three persons are excluded from the census.
78. Data for the period 1947-59 were made available by the Central Statistics Office, Buildings and Plant were not distinguished.
79. Due to the wide asset groupings used by Nevin, average working life must be interpreted here as an average over types of assets as well as over lives.
80. *op. cit.*, (1971/2). Estimates were also made for coal and peat-processing, for mining and "selected construction".
81. The starting year depended on whether or not it was possible to reconcile a particular industrial sector on the old and new listings of the CIP.
82. The price index for plant and machinery was also applied to other fixed assets and from 1953 to 1968 was the simple average of the official wholesale price indices for imported producer capital goods and transportable capital goods for use in industry. Prior to 1953 the old official wholesale price indices for metal and manufactures and for capital equipment were used. One price index also served for both categories of vehicles: the implicit price deflator obtained from the ratio of the CIP value and volume indices for motor vehicles, land and road (CIP 49). The price index for buildings was the official capital goods wholesale price index for building and construction. The price index for land was taken from the CSO work-files for 1958 to 1968 and was estimated for the earlier years on the assumption of an annual fall (moving backward from 1958) of 3 per cent. No attempt was made to allow for changes in efficiency. For details see Henry *op. cit.*, Appendix 1.
83. Equivalent-new is the term used by Henry. Its relation to gross and net will be discussed below.

84. These were discussed earlier in the section, Review of Assumptions. It will be recalled that over a wide range of growth patterns of gross capital formation and length of asset life the ratio of net to gross capital at the end of T years lay in the range of 0.5 to 0.7.
85. The list of CIP purchases and sales used was that for 1963 and later years which have ten entries for purchases and five for sales. Some items for earlier years were more aggregated and did not distinguish new and used purchases. These aggregates (mainly vehicles) were distributed between new and used using the proportions observable for the post-1963 years. All sales were assumed to be of used assets.
86. As Henry (*op. cit.*, p.11) pointed out this could result in an overestimate of the capital stock of land, although in compensation the starting value of land was usually taken to be zero.
87. Consequently the estimated capital stock of other fixed assets was regarded by the author as 'dubious'. See Henry, *op. cit.*, p.10.
88. No estimate was made for rented assets because of the lack of reliable data.
89. *op. cit.* (1963).
90. Kuh, E., *Capital Stock Growth: A Micro-Economic Approach*, Contributions to Economic Analysis, vol. 31, North-Holland, Amsterdam (1963).
91. In equivalent-new terms. Sales were also in equivalent-new terms.
92. Retirement occurs when an asset has reached the end of its working life. The sale of an asset implies that the asset has some working life still remaining.
93. Assets are retired at their gross (i.e. undepreciated) value.
94. An interesting point arises here. If the series of multipliers which Henry used to translate the starting values of capital stock into equivalent-new capital stock were based on Goldsmith's net-gross ratios, the fact that the scrap values are assumed to be as high as 80 or 85 per cent of equivalent-new value would mean that Henry's multipliers are probably too large.
95. See Nevin, *op. cit.* (1963) p.7.
96. See Henry, *op. cit.* p.6.
97. See Henry, *op. cit.*, Appendix 1 on price deflators for equivalent-new capital stock calculations.
98. This viewpoint is shared by Jefferson who in the discussion of Henry's paper suggested that a further multiplication factor of between 1.5 and 2.0 might be applied to the balance sheet valuations to obtain an adequate base figure for equivalent-new capital stock. See *Journal of the Statistical and Social Inquiry Society of Ireland*, vol. 22 (1971/2), p.28.
99. See Nevin (1963) *op. cit.*, p.8.

100. The table below shows the results in greater detail.

Gross and Net Capital Stock Estimates (1958 prices) in Manufacturing in 1974.

	Gross Stock of Capital £ million	Net Stock of Capital £ million	Net-gross Ratio %
Food	86.7	44.6	51
Drink & Tobacco	50.4	25.4	50
Textiles	30.1	24.0	80
Clothing	19.5	14.2	73
Wood	9.8	5.8	59
Paper	46.4	24.1	52
Chemicals	18.0	9.1	51
Minerals	13.8	7.4	54
Metal Products	18.3	10.3	56
Other Manufacturing	10.9	7.3	67
Total	303.9	172.2	57

Sources, net stock, Kennedy, op. cit., Table 4; gross stock, see text.

101. *op. cit.*, p.11. Henry points out that in three industrial groupings, textiles, metals, and other manufacturing, Nevin estimated the average life of plant as greater than 60 years. Since vehicles which are included in Nevin's definition of plant, have a life of around 10 years, the implication is that plant and machinery has a life in excess of 60 years compared with for example, Dean's maximum for any industry of 50 years.

102. *op. cit.*, p.263.

103. It may be noted that apart from the clothing industry, the corrections led to very small changes in the capital stock estimates. Some indication of this may be obtained by comparing Nevin's and Kennedy's estimates of total capital stock in manufacturing in Table 3 above. The average lives (in years) estimated by Nevin are shown below. Kennedy's revisions are shown in parentheses.

Industry	Buildings	Plant
Food	91	23
Drink & Tobacco	99	15
Textiles	208 (126)	63 (42)
Clothing	67 (110)	13 (30)
Wood	92	24
Paper	125	36
Chemicals	92	33
Minerals	126	31 (27)
Metals	89	61 (30)
Miscellaneous	74 (54)	62 (24)

Source: Kennedy, *op. cit.*, p.264.

104. *op. cit.*, The Stock of Fixed Capital, p.331.

105. *op. cit.*, (1951), pp. 22-23.

106. An indication that the working lives used by Goldsmith and quoted in the text may be on the low side is given in Goldsmith (1952) *op. cit.* Here estimates of the stocks of "structures" and "equipment" in the US were made for broad sectors of the economy using working lives similar to those quoted above - 50 years for structures and 12 years for equipment. The author commented (see p.252) that, "the rates of depreciation are generally on the high side, i.e. in many cases imply an average useful life below the probable actual one".

107. Dean's average lives for assets in the manufacturing, construction, distribution and other services sectors are used in the UK official estimates of capital stock and capital consumption. See *National Accounts Statistics, Sources and Methods*, HMSO (London) 1968, p.386.

108. A third method was proposed for Northern Ireland by Jefferson, *op. cit.* (1968). With gross investment data available only from 1949 onward and no capital census data, Jefferson attempted estimates of the net capital stock in Northern Ireland manufacturing industries for the period 1949 to 1964. The net stock in 1964 was regarded as consisting of two parts; that which had been accumulated prior to 1949 and that which had been accumulated after 1949. The annual post-1949 net stock was estimated by applying the perpetual-inventory method to the available gross investment statistics. The pre-1949 net stock in existence at the end of 1949 was estimated by assuming that the percentage changes between 1949 and 1964 in the net capital-output ratios of Northern Ireland industry groups were the same as for corresponding groups in the UK as a whole. By further assuming that pre-1949 net stock represented 60 per cent of the gross stock at the end of 1949 it was then possible to estimate capital consumption in 1949 using average lives of assets from Dean, (*op. cit.*).

The method did not seem appropriate here, there being no grounds for assuming either similar capital-output ratio changes in Ireland and the UK or similar changes in capital utilisation.

109. As detailed earlier the asset classifications are dwellings, roads, other building and construction, transport equipment, agricultural machinery and other machinery and equipment. In principle it would seem that a reconciliation of the various published statistics on gross capital formation is possible since presumably they all emanate from the same source. However, the Central Statistics Office (Dublin) informed the author that reconciliation was not possible.
110. It may be noted that Christensen, L. R., and Jorgenson, D. W. "The Measurement of U.S. Real Capital Input 1929-1967", *Review of Income and Wealth* 1969, also used similar broad categories of assets - non-residential structures, producers' durable equipment (along with land, residential structures, non-farm business inventories, farm inventories and consumers' durable equipment) - divided among three sectors, corporate business, non-corporate business and households and institutions. See also Goldsmith (1952) *op. cit.*, who differentiated only equipment and structures for the sectors, agriculture, non-agriculture and government.
111. As was shown earlier the assumption of a constant average life over time is unlikely to affect perpetual inventory estimates of capital stock to a substantial degree. It will also be recalled that the formula assumes zero scrap value at the end of an asset's life and that there is no distribution of retirements.
112. *op. cit.* (1964).
113. Statistics of capital expenditure on land rehabilitation, which relates to land drainage and the land project are not published. However, an indication of the annual amounts involved was obtained from the Public Authorities' accounts under land project grants and arterial drainage (see Tables A.19 and A.21; *National Income and Expenditure* (1972)). The entries relate to financial years and are at current prices. Adjusted proportionately to calendar years for the period 1966 to 1972 the amounts under these headings amounted to 3.6 per cent of the current price expenditure on other building and construction.
114. *op. cit.*, p.175.
115. *op. cit.* (1951), pp. 22-23. The estimates were as follows: railways and transit equipment, 30 years; ships and boats, 30 years; aircraft, 5 years; road vehicles, 6 years.
116. Suitable price index numbers are not available to convert the individual current price series to constant prices.
117. An alternative method which suggested itself was to estimate the average life for this asset grouping from gross capital stock and capital consumption data for a similar grouping in the UK. (See Tables 64 and 58 respectively, *National Income and Expenditure* (1973), HMSO (London)). Since capital consumption is estimated by dividing gross stock by the average life, the procedure can be reversed and average life obtained by dividing gross stock by capital

consumption. The resultant average life was 16 years. However, using this method would imply that the distribution of assets in the group is the same in Ireland and the UK. On the basis of capital expenditure data this does not appear to be the case. The table for the UK comparable to that for Ireland shown in the text is shown below. Comparison of the two indicates the preponderance of capital expenditure on road vehicles in Ireland with relatively little on the other three groups, while in the UK capital expenditure on ships also constitutes a large part of the total. Consequently it is to be expected that the overall average life of the group would be shorter in Ireland. Application of the method cited in the text to the UK yielded an overall average life of just over 15 years.

UK Gross Capital Expenditure on Transport Equipment · 1960-1972

	Aircraft	Ships	Railway rolling stock	Road vehicles	Total
£ million (current prices)	695	2537	548	6022	9802
Percentage of total	7.1	25.9	5.6	61.4	100

118. See *Agricultural Machinery* (PEP Report, 1949) Table 11.

119. *op. cit.*, p.10.

120. *op. cit.*, (1964) p.346.

121. For the purpose of checking the 1948 estimates, post-1953 trends were fitted to all six categories of assets. See text Table 6 below.

122. See *Tables of National Income and Expenditure for 1938 and 1944 to 1950*, (Pr. 350), p.2. The estimates were originally published in the White Paper on *National Income and Expenditure, 1938-44* (P. No. 7356).

123. The sum of gross capital formation in the first, second and third phases respectively is

$$k \left[(1+g)^0 + \dots + (1+g)^{-5} \right],$$

$$10k(1+g)^{-5},$$

$$\text{and } k(1+g)^{-5} \left[(1+g)^{-1} + (1+g)^{-2} + \dots + (1+g)^{-(T-16)} \right]$$

Summing over the three phases gives the expression for gross capital stock in the text.

124. It will be recalled from a previous section (*Review of the Assumptions of Perpetual Inventory*) that Goldsmith *op cit* (1962) found that the accuracy of this method was improved when the trend rather than the actual value of gross capital formation was used.
125. Because of the break in the trend growth of capital formation it seemed preferable to estimate net capital stock separately rather than to use the net-gross ratio appropriate to the combination of the growth rate of capital formation and the asset life. The resulting net-gross ratios were, Other Building and Construction (0.73), Plant and Machinery (0.61), Transport Equipment (0.49) and Agricultural Machinery (0.48).
126. Since the asset lives are 30, 25, 14 and 10 years for rail vehicles, ships, aircraft and road vehicles respectively and since their relative importance in terms of capital stock is weighted very heavily towards road vehicles, with aircraft, ships and rail vehicles coming in decreasing order, the distribution of retirements may be expected to be positively skewed. In these circumstances, use of the mean as the average working life in conjunction with an increasing trend of capital formation is likely to produce an overestimate of the capital stock.
127. and an overestimate of capital consumption.
128. Output is GNP at constant (1958) market prices. At the time of writing, employment figures were not available for 1972. Growth rates were calculated using the constant growth model outlined in the previous section.
129. See for example, Kendrick, *op. cit.*, who noted a significant positive rank correlation between annual percentage changes in capital per head and output per head for the US private domestic economy for the period 1899 to 1955. In the case of Ireland the growth of labour productivity would also have benefited from the movement of labour out of a low productivity industry - agriculture. The percentage of total employment accounted for by agriculture, forestry and fishing fell from 39.3 in 1953 to 26.3 in 1971.
130. Some indication of inter-sectoral variation is given below in the table - which shows gross capital stock per head in the UK Manufacturing and non-manufacturing sectors for selected years. The employment figures are employees in employment and to maintain consistency manufacturing excludes textiles and includes construction. Capital stock (measured at 1958 replacement cost) refers to plant, machinery and buildings.

	Manufacturing	Non-manufacturing
	£	£
1953	1553	2432
1958	1821	2661
1963	2339	2766
1968	2833	3558

Source: See Table 8 above.