Regression and the Rate of Change in Economic Time Series

P. D. PRAETZ

This note comments on Geary's (1972) use of regression to calculate the rate of change in economic time series. His paper refers to "the rate of change", but it does not distinguish between the instantaneous rate of change, with which he is concerned, and the common discrete rate, which is a rate per period. The bias and variance of the estimators of the discrete rate of change for the two methods discussed by Geary are derived, using his assumptions.

The time series (Z_i), $i = 1, \ldots, T$ has the rate of change (r) defined by

$$(1+r)^{T-1} = Z_T/Z_1 \tag{1}$$

for the method in common use. The series, ignoring the disturbance term, is assumed to have the form $\gamma \exp(\beta t)$, where β is the instantaneous rate of change, if it is significantly different from zero, i.e.,

$$\beta = (dZ_t/dt)Z_t \tag{2}$$

In log form, suitable for regression, the model is

$$\log Z_t = \alpha + \beta t + u_t \tag{3}$$

$t = 1, 2, \ldots, T$, where $\alpha = \log \gamma$ and the disturbances u_t have zero mean, constant variance σ^2 and are mutually uncorrelated.

Let ρ be the population discrete rate of change per period corresponding to r in (1), and let r_1 and r_2 be estimates of it from the common method (1) and the least squares regression methods on (3). Thus the data is represented by $\gamma(1+\rho)^t$ in the discrete case and $\gamma \exp(\beta t)$ in the continuous case, i.e., $1+\rho = \exp(\beta)$.

If b_1 and b_2 are estimates of β from the common and regression methods, the estimates of ρ are given by $r_1 = \exp(b_1)-1$ and $r_2 = \exp(b_2)-1$.

To derive the expectation (E), bias (B) and variance (V) of r_1 and r_2 we need

$$E(\exp(x)) = \exp(\tfrac{1}{2}\sigma^2\Sigma a^2_i) \tag{4}$$

$$= \exp(\tfrac{1}{2}V(x)) \tag{5}$$

where $x = \Sigma a_i u_i$ and the a_i are constants. This follows from the moment generating function of the u_i, assumed normally distributed.

Using Geary (2.5 and 2.6), namely

$$b_1 = \beta+(u_T-u_1)/(T-1) \tag{6}$$

and

$$V(b_1) = w = 2\sigma^2/(T-1)^2 \tag{7}$$

we obtain

$$E(1+r_1) = E(\exp(b_1)) \tag{8}$$

$$= \exp(\beta+\tfrac{1}{2}w) \text{ using (5), (6) and (7).}$$

$$\therefore \quad E(r_1) = (1+\rho)\exp(\tfrac{1}{2}w)-1 \tag{9}$$

Also

$$B(r_1) = E(r_1)-\rho$$

$$= (1+\rho)(\exp(\tfrac{1}{2}w)-1) \tag{10}$$

The variance of r_1 is given by

$$V(r_1) = V(r_1+1) = E(r_1+1)^2-(E(r_1+1))^2$$

$$= E(\exp(2b_1))-(1+\rho)^2\exp(w)$$

$$= \exp(2\beta+2w)-(1+\rho)^2\exp(w) \text{ as above}$$

$$= (1+\rho)^2(\exp(2w)-\exp(w)) \tag{11}$$

Similarly, for b_2, using Geary (2.10), namely

$$V(b_2) = 12\sigma^2/T(T^2-1) \tag{12}$$

we obtain E, B and V of r_2 by substituting (12) for (w) in (9), (10) and (11).

In Table 1 below, we have the bias and standard deviation for estimators r_1 and r_2 with $\sigma^2 = \cdot 04$ and $T = 5, 10, 15$ and 20; $\rho = \cdot 10$.

TABLE 1: *Values of bias and standard deviation (in parentheses) for estimators r_1 and r_2 when $\rho = \cdot 10$, $\sigma^2 = \cdot 04$ and $T = 5, 10, 15$ and 20*

	T (sample size)							
	5		*10*		*15*		*20*	
r_1	·0028	(·079)	·0005	(·035)	·0002	(·022)	·0001	(·017)
r_2	·0022	(·070)	·0003	(·024)	·0001	(·013)	·0000	(·009)

If T is small and σ^2 large, the bias could be embarrassing; but otherwise it is no problem, as in Table 1.

Thus, while the Geary estimators have no bias and slightly smaller variances than those defined above, they are not very operational as they are based on an instantaneous rate of increase. For the obvious practical reasons of ease of understanding and calculation, applied research workers use rates of growth of, say, five or ten per cent per annum, rather than of 4·88 . . . or 9·53 . . . per cent growing continuously. Geary elegantly demonstrated the superiority of the regression method and the inferiority of the common method, but his frame of reference is not quite the same as the traditional formulation of growth. Obviously his main conclusion still holds in this situation. However, the practicalities of rates of change or growth suggest, using either the common or regression method, biases and variances given by equations (10), (11) and (12).

REFERENCE

Geary, R. C. (1972): "Two Exercises in Simple Regression", *Economic and Social Review*, 3, 551–560.

Monash University,
Australia.