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STATISTICS IN THE SECOND LEVEL CURRICULUM

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1 INTRODUCTION

Statistics as a subject has been formally included in the second level curriculum in Ireland for a number of years, primarily as part of the mathematics syllabus, but also to a lesser extent in other syllabi Professional statisticians who are familiar with the way in which statistics is treated at this level have expressed dissatisfaction with it The existence of the Curriculum and Examinations Board, started in January 1984 by the Minister for Education, provides an opportunity for reviewing the position of statistics in the second level curriculum and making appropriate proposals for change

The terms of reference of the board call for the investigation of (among other topics)

(i) assessment of the junior level,
(ii) the Leaving Certificate as a measure of general education,
(iii) alternative senior cycle programmes,

and call for wide consultation on these and other topics

The Board has already published consultative documents, (1), (2), and has set up a number of committees and working parties with members largely drawn from outside the Board itself It is clear from its work to date that the Board is taking a very broad view, particularly with respect to innovation and change

In particular, the Board has explicitly or implicitly, raised possibilities of

(i) syllabus integration,
(ii) project work,
(iii) technological aids in teaching,
(iv) more relevant assessment

As will be seen, the appropriate development of statistics in the second level curriculum can be greatly facilitated by general developments in each of these areas It must be recognised from the outset, however, that appropriate teacher development is a fundamental requirement for any successful development of statistics in the second level curriculum The Board explicitly recognises this aspect in general (1, p 11)

Up to now it appears that statistics has not received much explicit attention from the Board Statistical understanding is mentioned as one of the elements of Mathematical Studies, one of a tentative set of categories in the junior cycle curriculum Statistics has also been considered at least implicitly by the Science and New Technologies Working Party However, given the forward looking perspective of the Board and the increasing relevance of statistics and acceptance of statistical ideas, the opportunity should not be missed of presenting to the Board a well argued case for the appropriate inclusion of statistics and related ideas in the second level curriculum

In the rest of this paper, the current position in Ireland is reviewed and criticized, recent developments in the UK and elsewhere are reviewed and some ideas and examples which may be useful in curriculum design are presented The paper concludes with discussion of key issues and proposals for future development

The paper is written from the point of view of a professional statistician with experience (but no training) in teaching at third level, but whose acquaintance with second level education is limited to that of a secondary school student of some years ago One con sequence of this is that no attention is paid directly to other types of second level schools, such as vocational or community schools It seems clear, however, that many of the ideas and issues discussed here would be relevant also for curricula designed for such schools

2 THE CURRENT POSITION IN IRELAND

The most substantial and explicit involvement of Statistics in the curriculum is through the mathematics syllabus for the Intermediate and Leaving Certificates Statistics is involved explicitly to a minor extent in the geography syllabus and is implicitly involved in one or two others Details of syllabi given in this section have been extracted from the Rules and Programme for Secondary Schools (3)

2 1 The Mathematics Syllabus

For the Intermediate Certificate, there are separate lower and higher courses in mathematics, with each being set out as a three year programme There is no statistics in the first year of either course For the second and third years, the statistics content is as follows

INTERMEDIATE CERTIFICATE MATHEMATICS

LOWER COURSE

2nd year Drawing and interpreting pictograms, bar charts, pie charts, trend graphs

3rd year Expression of discrete array as a frequency table Mean and mode

HIGHER COURSE

2nd year Same as lower course, plus moving averages

3rd year Same as lower course

This suggests a very elementary treatment of the simplest ideas of descriptive statistics, with emphasis on the graphical A review of recent examination papers suggests a very computational and technical approach to the topics listed with little emphasis on inter pretation and none at all on the idea of statistical variation On this basis, the distinction between the lower and higher courses seems to be that the former covers only the second year part of the programme, excluding trend graphs At the higher level, trend graphs and moving averages have not been examined since 1981

For the Leaving Certificate, there are two levels, ordinary and higher The ordinary level syllabus is as follows

LEAVING CERTIFICATE MATHEMATICS

ORDINARY LEVEL

Elementary ideas of frequency and cumulative frequency distributions Graphical treat ment Median Simple treatment of standard deviation given an array of numbers or a frequency distribution, including grouped frequencies (Use of assumed mean not required) Weighted averages

This appears to be a continuation of the Intermediate Certificate syllabus As for the Intermediate Certificate, the examinations tend to be artificial tests of knowledge of definitions and computation with no sign of testing what things mean in context, in what circumstances they might be used, how they might be interpreted

The higher level syllabus is quite different in nature

HIGHER LEVEL

Statistics

Sample point (e), sample space (S), event (E) Mutually exclusive events ($E \cap F = \phi$) The probability, P(E), of an event is a real number satisfying

Axiom 1 $P(E) \geqslant 0$

Axiom 2 $P(S) = 1, P(\phi) = 0$

Axiom 3 $P(E) = P(E_1) + P(E_2) + \quad + P(E_k)$,
where $E = E_1 \cup E_2 \quad \cup E_k$
and where $E_1, E_2 \quad , E_k$ are singletons

Theorem S1 If E and F are mutually exclusive events, then
$P(E \cup F) = P(E) + P(F)$

Theorem S2 If E and F are subsets of S, then
$P(E \setminus F) = P(E) - P(E \cap F)$

Theorem S3 If E and F are subsets of S, then
$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Definition E and F are equally likely events $\Longleftrightarrow$
$P(E) = P(F)$

Theorem S4 If E is a subset of S, then

$$P(E) = \frac{\#(E)}{\#(S)}, \text{ where the singletons are equally likely}$$

Definition E and F are independent events <==>
$P(E \cap F) = P(E) \quad P(F)$

Theorem S5 If E and F are independent events, then E and F are also independent events (sic)

Binomial distribution ($\bar{X} = np$, $\sigma = \sqrt{npq}$) (proof for σ not required)

Standard units $z = \dfrac{x \quad \bar{X}}{\sigma}$

Use of Tables to test a null hypothesis at the 5 per cent level of significance

This "Statistics" syllabus is followed by a syllabus on Vectors and Linear Transfor mations and these are alternatives In the examination, students may answer a question on one or the other, but not both

The higher level syllabus is followed, in the Rules and Programme for Secondary Schools by "Notes on the Revised Syllabus", the first part of which sketches out a course based on the above sullabus, including some examples

2 2 Criticism of the Mathematics Syllabus

In making any criticism of the statistics component of the second level mathematics syllabus, it must be remembered that statistics constitutes only a very small part of the entire syllabus and, consequently, can have only a correspondingly small amount of time devoted to it

Leaving aside the Higher Level Leaving Certificate for the moment, the other syllabi appear to be adequate as an introduction to the technical, mathematical and compu tational aspects of descriptive statistics although, as such, they strike me as rather dull The text books written for this syllabus which I have examined (4,5) closely reflect this situation, although there are some curious lapses

Thus, while these syllabi are unlikely to do any harm, by way of conveying erroneous material, they are unlikely to do much good either, and may have the effect of turning students away from statistics

The Higher Level Leaving Certificate syllabus is a different matter The bulk of the syllabus description is taken up with unexceptional if rather dull elementary probability theory At the end, there is reference to the Binomial distribution, standardization, the Normal distribution and hypothesis testing On reading the Notes in the Rules and Programme for Secondary Schools (3, pp 194 204), it emerges that the definition of the Binomial distribution is not linked in any way to the preceding probability theory and that the only role for the Normal distribution is as an approximation to the standardized binomial distribution Furthermore, it is inconceivable how the ideas surrounding hypothesis testing could be introduced in any meaningful way in the time implicitly available, so that all the statistical (as distinct from probabalistic) material in the syllabus is introduced in an entirely mechanical fashion, without motivation other than that there will be a question on it in the examination

Again, the text books written for this syllabus to which I had reference largely reflect this situation In one of them, at least, the motivation for hypothesis testing is considerably better than that in the Rules and Programme Notes However, there is a sprinkling of more or less serious errors throughout both

More serious than this, however, is the occurrence of errors and manifestations of mis conceptions in examination papers To quote just one example, question 8(b) of the 1983 Higher Level Mathematics Paper II reads in part as follows

> Let x be a random variable denoting the number of householders in a certain town who own a T V set Say why x has a binomial distribution

Clearly, in the context of this syllabus, x is not a random variable If "a certain town" was not specified, then x could conceivably be regarded as random, given a suitable population of towns However, there is no reason at all for saying that it should have a Binomial distribution A more sophisticated approach would be to regard x as having a subjective probability distribution, the prior distribution of a Bayesian analysis Pre sumably, this is not intended

More fundamental than lapses such as this, however, and perhaps partly responsible for them, is the complete absence, in syllabus, syllabus notes, examinations or texts, of any explicit reference to the idea of sampling variation and the idea of modelling empirical sampling variation by theoretical probabalistic variation While it is notoriously difficult to convey a reasonable grasp of these difficult concepts to beginning (and even more advanced) students, to omit any explicit reference at all must leave students asking the questions "why all the fuss?", "what has probability got to do with data?" etc

Unfortunately, my criticisms at this stage are rather negative I make more constructive suggestions later For the moment, I would recommend immediately dropping the material on the Binomial and Normal distributions and hypothesis testing from the Leaving Certificate Higher Level Mathematics syllabus, and rename as "Probability Theory" the remainder of what is now named "Statistics" Alternatively, replace the existing Higher Level syllabus by the existing Ordinary Level syllabus, perhaps giving it a somewhat more sophisticated treatment in the teaching This at least would have the virtue of preserving continuity from the Intermediate Certificate syllabus

I would also recommend the establishment of adequate procedures for reviewing examinations With the best will in the world, it is easy to make slips and give inadequate descriptions when setting examination papers in statistics In the Statistics Department in Trinity College (where lecturers set their own examinations), we have rigorous procedures for reviewing each others' examination papers and regularly give each other a hard time in this regard

2 3 Other Subjects
Statistics enters implicitly into subjects such as Agricultural Economics, Business Organization, Economics, in the Leaving Certificate programme However, there is no evidence that it receives any explicit treatment as a subject area within those subjects However, aspects of Statistics, mainly descriptive, are mentioned explicitly in the Geography syllabus

The Intermediate Certificate syllabus in History and Geography contains the following in the section on Practical Geography

> Simple presentation and reading of geographical information as illustrated by line graphs, bar graphs, circular graphs Such information would be derived from field work or secondary sources

Also, in the Leaving Certificate Geography syllabus, under each of the headings Social Geography, Economic Geography, Regional Geography, there is included the following

> Attention is drawn to the fact that much valuable information on Social (or Economic, or Regional) Geography may be gained from the study and construction of line graphs, bar graphs, pie charts and photographs as well as from other carto graphic forms which use varying scales of intensity dots and isopleths It is recommended that students should study the construction, interpretation and functionality of these

There is also a reference to the possibility that the use of "statistical maps and diagrams" may be tested in examinations

Here is a far more positive approach to the use of statistics There is even a reference, if veiled, to the concrete representation of empirical statistical variation, a fundamental idea conspicuously missing from the mathematics syllabi, as noted earlier There is also the very positive pedogogical device of having students collect their own data In fact, there is provision in the rules for 10% of the Geography mark in the Intermediate Certificate to be awarded on the basis of school based assessment of practical work, giving a valuable opportunity for practising statistics Unfortunately, only a small per centage of schools take advantage of this provision (See (2, Appendix 1), for further details of school based assessment of practicals and projects)

3 DEVELOPMENTS ELSEWHERE

3 1 General
There has been considerable effort in other countries in developing syllabi, teaching material, etc , for second level and even primary level school students By far the greater part of this development has taken place in the UK and this will be described later

In the USA, a number of small scale efforts have been made to bring statistics to second level students For example, the text by Mosteller, Rourke and Thomas (6) and the series of four books under the general title Statistics by Example (7) have met with some success The American Statistical Association has taken an interest in this are in varying degrees at different times At the present, the ASA Quantitative Literacy Project is organizing a three year project to train teachers in the strategies for teaching statistics and probability at the pre college level 200 mathematics and social science teachers are being put through three day "workshop" sessions, during which they are introduced to specially developed teaching materials designed to incorporate statistics and probability into the mathematics curriculum These materials are concurrently being field tested in a number of schools The teachers participating in these "training of trainers" workshops will be expected to replicate the workshops among other teachers in their own districts The project is funded by the National Science Foundation and endorsed by the National Council of Teachers of Mathematics

Smaller scale efforts are taking place in other countries, for example, Sweden and New Zealand A survey of the state of probability and statistics teaching in schools worldwide has been published by the International Statistical Institute (7)

3 2 Developments in the UK

In the UK at present, statistics as a subject enters into the curriculum to a much greater extent than in Ireland or elsewhere, and in many different ways For those not familiar with the organisation of second level teaching in the UK, it should be noted that the curriculum is not centrally set, as in Ireland Instead, this is done by a number of examination boards, some, but not all, geographically based To give some indication of the variety of courses available, the following tables give some details on examination syllabi incorporating statistics in 1980, both as a mathematical subject and as a user subject The examination levels are A, Advanced, AO and O, Ordinary Level As a reference guide, A level is more advanced than Leaving Certificate, and approaches first year University level, while O level is, generally, less advanced than Leaving Certificate but more advanced than Intermediate Certificate

(These tables and much of the review that follows are drawn from a report prepared by the Centre for Statistical Education, Sheffield (8))

As might be expected, the styles of statistical content vary from purely mathematical statistics to predominantly applied statistics within the "mathematical subjects" syllabus and from "background material" to technical in the "user subjects" Among the "user subjects" syllabi, some include so much material that it could only be covered at the most superficial of levels, with emphasis on computational technique Others link the statistics to practical projects, thereby giving some idea of its relevance and meaning

It may be worth noting that, for the most part, the "user subjects" comprise of the social sciences – economics, sociology, business studies, psychology and geography, with biology and computer studies featuring less frequently

One of the most promising developments in the UK has been the establishment of the Centre for Statistical Education, jointly based in the University of Sheffield and Sheffield City Polytechnic This centre has continued with earlier work done in connection with the Schools' Council Project on Statistical Education Its work includes curriculum

UK Examination Syllabi Incorporating Statistics 1980
Mathematical Subjects

	Level of Examination A	AO	O
Examining Board			
Associated	F, H, OH, OL	F, OH	F
Cambridge Local	F, OL	F, OL	L
Joint Matriculation	F, H, L	H	L
London University	H, L	F, H	NA
Oxford and Cambridge	L	H, L	NA
Oxford Local	L	NA	NA
Southern	L	L	L
Northern Ireland	L	L	F, H
Scottish	NA	NA	F
Welsh	L	NA	F
School Maths Project	L	NA	L

Key F - Full subject OH - Optional Half
H - Half subject OL - Optional less than Half
L - Less than half NA - Not Applicable

development, promoting the use of project work as part of the curriculum and developing an approach to the assessment and examination of such projects It has also worked on preparing in service training courses for teachers where not only must statistics *per se* be taught to the teachers, but also the teachers must be taught how to teach statistics This requires the development of teaching materials including text books, case studies, equip ment, project guides, and a range of pedagogic devices for use in class and outside

The Centre publishes a journal, *Teaching Statistics,* aimed at second level teachers, where articles on all aspects of statistical teaching appear

Another significant development in the UK has been the production of the Cockcroft Report (9) This has, I understand, been hailed world wide as a major contribution to the study of the teaching of mathematics It contains some important and incisive observations on the role of statistics, not only in a mathematics curriculum but also as a subject in its own right and as a part of syllabi in applications areas

UK Examination Syllabi Incorporating Statistics 1980
User Subjects

	Level of Examination A	AO	O
Examining Board			
AEB	<u>B</u>, CO, EG, E	-	-
Cambridge Local	<u>BS</u>, <u>S</u>, E, CS	-	-
Joint Matriculation	<u>B</u>, E, <u>ES</u>, G, S	HB, IS	<u>SS</u>
London University	<u>B</u>, E, G, S	-	G, BE
Oxford and Cambridge	<u>E</u>, G	-	-
Oxford Local	<u>EG</u>, G, CS	-	-
Southern Universities	E, G	-	-
Northern Ireland	G	-	G
Scottish	-	-	<u>G</u>
Welsh	<u>G</u>	-	-

Key:
B Biology
BS Business Studies
CO Communication Studies
CS Computer Studies/Science
E Economics
EG Economic Geography
G Geography
HB Human Biology
IS Industrial Studies
P Psychology
SS Social Studies
S Sociology
BE The Modern British Economy 1919 to the Present

Note:
Some syllabuses call for data interpretation with no indication of the level of sophistication required other specify a substantial technical content of the syllabus The latter are indicated by a line below the key letter thus <u>B</u>

One of the ideas developed in the report is that of a "Foundation List" of areas of mathematics that all students at second level should know about before they leave the school system (The report then considers developing a full mathematics curriculum "from the bottom upwards", with this list as a base) The list includes the following

STATISTICAL IDEAS

One aim should be to encourage a critical attitude to statistics presented by the media

Appreciate basic ideas of randomness and variability, know the meaning of pro bability and odds in simple cases

Emphasis should be placed on the relevance of probability to occurrences in every day life as well as to simple games of chance For many pupils it will not be appro priate to undertake work involving combined probability

Understand the difference between the various measures of average and the purpose for which each is used

Attention should be paid to the different uses of the word 'average' in newspaper reports, but it is not intended that pupils should necessarily be expected to use the words mean, median and mode

In a section devoted to the teaching of statistics, the report refers to the view, expressed in the submissions of the Schools Council Project on Statistical Education and of the Joint Education Committee of the Royal Statistical Society and the Institute of Statisticians, that, although statistics is commonly taught within mathematics courses, it should not be regarded solely as part of mathematics The former submission is quoted as stating that "Statistics is not just a set of techniques, it is an attitude of mind in approach ing data In particular it acknowledges the fact of uncertainty and variability in data and data collection It enables people to make decisions in the face of this uncertainty "

The report advocates an empirical basis for statistics teaching, as well as allowing time for basic ideas to sink in and develop, with a gradual introduction of ideas from early years through to advanced level It recommends the provision of text books and teaching materials which emphasize a practical approach, the use of calculators and micro computers and the use of project work It notes that project work in subjects such as biological science, geography and economics can contribute to the learning and under standing of statistics In relation to this, it advocates the nomination of a member of staff who would identify the use which was made of Statistics in the teaching of a variety of subjects and act as co ordinator for the teaching of statistics It also stresses the need for in service training courses on the teaching of statistics not only for mathematics teachers but also for teachers of other subjects

4 *A BACKGROUND FOR STATISTICAL CURRICULUM DEVELOPMENT*

In this section, some ideas relevant to the problem of statistics curriculum development and revision are put forward These are by no means comprehensive or definitive, rather they are intended as indicators of possible developments and stimuli for alternative developments

For this purpose, four distinct aspects of statistics are considered, descriptive, computational, inferential, mathematical

4 1 *Descriptive Statistics*

Traditionally, courses on descriptive statistics have tended to consist of somewhat un critical presentations of elementary data, summary devices such as frequency distributions, bar charts, mean, median, mode, etc

An example of the uncritical approach is the tendency to present mean, median and mode as measures of much the same kind of thing, to be used in much the same kinds of circumstances, and with no recommendations as to when to use which It is almost never suggested that a mean is useful when comparisons, aggregations, etc , are to be evaluated in terms of the basic unit of measurement, or converted into related units of measurement, a median is useful when comparisons, etc , are to be made in terms of frequency distributions, a mode is useful when comparisons, etc , are made in the context of well defined discrete categorizations For example, given mean consumption of a product, per head of population in each of several sales regions in a country, and given the population of each region, a sales manager can quickly work out the total consumption of the product in the country The median and mode are not suited to this purpose

Again, the median age in a population is a useful and easily assimilated rough indicator of the "youthfulness" or otherwise of the population It may be sued for rough comparisons between populations (e g , rural vs urban) or within populations at different times The mean is not directly suited to this purpose The modal class (but not the mode *per se*) could conceivably have a role in this situation, but would tend to describe a somewhat different phenomenon and, in any case, the necessary classification of ages is arbitrary

As regards the mode of a continuous frequency or probability distribution, neither I nor any of my colleagues that I consulted have ever used the mode in practice, or have seen it used, except in the context of Bayesian inference The use of the modal class of a discrete frequency or probability distribution may be sensible It may be used in population studies, as indicated above For example, in fertility studies, the modal child bearing age group is frequently quoted

The usual approach seems to derive from the general mathematical property that these quantities share of being "measures of location" or "measures of central tendency" The mathematical influence also seems to be evident in the frequently found method of evaluating a median as an inverse cumulative distribution function evaluated at 1/2 I understand that students have difficulty with the idea of a median This is not surprising if it is evaluated in this way

A more modern approach to "descriptive statistics" would refer to some of the non inferential techniques of data analysis developed in recent years by such as Ehrenberg (10) or Tukey (11), (although there is some danger that the techniques developed by the latter would be studied for themselves rather than their application) One way of viewing such techniques is as ways of uncovering the systematic structure in a set of data and thereby permitting a meaningful and relevant description of its main characteristics A modern approach would also take account of the recent interest in graphical displays of data The remarkable book by Tufte (12) contains much useful and innovative material on this subject

4 2 Statistical Computation

Statistical computation occupied a central place in most statistical curricula until fairly recently, and still does in too many Before the advent of electronic calculators and computers, computational methods that could be implemented with paper and pencil were a *sine qua non* for most statistical work However, in an age when most secondary schools (and many primary schools) now have micro computers as powerful as the main frames of ten years ago, there should be no place in a statistical curriculum for formulae such as those for variances which involve the "correction for the mean" They are com putationally inefficient and, because of rounding error, can be disastrously inaccurate Their use and proof takes up time which would be much better spent on other things A concentration on computation also leads to the phenomenon of mindless churning out of many calculated quantities based on few data Who knows what such quantities mean? Typically not the churner

4 3 Inferential Statistics

The step from descriptive and computational statistics to inferential statistics is absolutely critical Within the standard mathematical statistics currriculum, it tends to be done in a rather mechanical way which treats methods of statistical inference as problems in applied probability In courses which are not mathematically oriented, statistical inference tends to emerge as a "deus ex machina", leading to mysterious sets of rules whose application is interpreted in terms of mysterious jargon

The truth is that the concepts underlying statistical inference are far from trivial and far from easy to grasp In a sense, the suggestion that statistics is "an attitude of mind in approaching data" (as in the submission of the Schools Council Project on Statistical Education to the Crockroft committee, quoted earlier), encompasses what is required to come to terms with classical concepts such as statistical significance and con fidence Probability models provide an imaginary backdrop which can be used to create the required frame of mind Data can be fitted into this framework If it fits nicely, a particular hypothesis about a probability model (and hence about a practical problem) can be accepted, if it does not fit, the hypothesis can be rejected Alternatively, the framework can be adjusted to fit data, confidence limits limit the extent of adjustment that is reasonable, in terms of the fit of the model to the data

The application of probability to statistical data analysis in this way requires a reversal of the usual view of a frequency based probability model, whereas the view from a probability model is forward towards potential data, the view from data is backward towards probability models

The completion of the prescription of classical statistical inference entails a quantification of the backward view, in terms of statistical significance levels and confidence levels, through quantities originally derived from the forward view The crucial element in coming to terms with all of this is an appreciation of the status of these quantities Teachers of statistics have long recognised the elusiveness of these concepts and the difficulty in communicating them to students, to such an extent that some would recommend that inference be excluded from a second level curriculum

4 4 *Mathematical Statistics*

Unlike any of the three preceding forms of statistics, mathematical statistics can proceed without any reference to data and hence without reference to practical problem solving It may quite reasonably be viewed as a branch of mathematical probability theory As such, it may well be thought of as being suitable for inclusion in a pure mathematics syllabus Whether or not it should be included in a statistics syllabus is debatable Given the confusion which notions such as independence and mutual exclusion cause in elementary probability courses, it may be questioned as to whether explicit inclusion of elementary probability theory is warrented in a statistics syllabus Clearly, if inference is to be included then some notion of probability is essential It may be possible, however, to introduce these notions implicitly, by example, and somewhat more explicitly via computer simulation

5 *EXAMPLES*

In this section, suggestions for case studies and examples which may be useful for class room purposes are made Again, these suggestions are not intended to be comprehensive or definitive, but rather illustrative They are considered under four headings, opinion polls, experimentation, using computer technology, projects

5 1 *Opinion Polls*

Opinion polls, and particularly pre-election opinion polls, are increasingly becoming a feature of discussion of social and political problems They arouse feelings of different kinds, from complete distrust through regarding them as entertaining but not much more to having an individual's voting decisions or a party's electoral strategy being influenced by them Because of this and because they tend to deal with high profile issues, they are likely to stimulate the interest of students and hence be useful as a vehicle for introducing a number of statistical ideas Three examples which I use as part of the introduction to a first year University course are concerned with opinion polls

On Saturday, October 17th, 1981, the lead story in the *Irish Independent* dealt with an opinion poll on Dr Garret FitzGerald's "constitutional crusade" The first paragraph read

> The Taoiseach's crusade for a new Republic is being met with widespread public support throughout the country despite entrenched opposition from Fianna Fail

On the following day, Sunday, October 18th, 1981, the lead story in the *Sunday Tribune* dealt with another opinion poll on the same topic Its first paragraph read

> The Taoiseach, Dr FitzGerald, faces an uphill battle in his Constitutional "crusade", according to a Sunday Tribune/I M S poll published today

How could this be? The simple approach would be to dismiss opinion polls A more careful approach would be to compare the responses displayed first by each paper, as shown below

The contrast is a clear example of the well known phenomenon that people may react in one way to a general issue but differently to a specific instance of it The reporting of both polls also reflects interpretations being put on questions and on respondents answers to them which may not be at all justified The example clearly highlights the care needed in interpreting the results of opinion polls if anything is to be learned from them

On Tuesday, February 16th, 1982, *The Irish Times* front page lead headline read

"FIANNA FAIL TAKES LEAD IN POLL "

and the lead story started out

"Fianna Fail took a lead of five points "

On the same day, the *Irish Independent* main headline read

"COALITION LEADING"

with sub headline

"Garret opens up a 5 percent margin over Fianna Fail "

Here, on the face of it, are further directly contradictory results from different polls, with the contradictory percentages this time strikingly balanced Again, more careful inquiry reveals that the announcement of Fianna Fail's "alternative budget" proposals took place towards the end of the two day polling period for the second poll but that the first poll was conducted entirely after that announcement Also, the second poll incorporated a simulated ballot paper, tailored to relevant constituencies, as part of its interviewing, while the first used a conventional question to elicit voting intentions

While no one seemed to pay too much attention to the seeming contradictions in these two examples, a more recent example aroused some controversy On the night of Friday, February 3rd, the Fianna Fail T D Mr Padraig Flynn accused *The Irish Times* of misrepresenting the state of public opinion on the subject of divorce He also called for "searching examination of the methods and practices of opinion poll companies so that the public can know what degree of validity or reliability can be attached to future findings " Mr Flynn was referring to seeming contradictions between the results of two polls, one published in December, 1983 and a second published in February, 1984

Mr Flynn's main criticism was of the claim made by *The Irish Times,* on the basis of the second poll, that "two thirds of the electorate, a clear majority among the supporters of all parties in the Republic, favour divorce in certain circumstances, ", while the previously published poll indicated that less than half of the electorate were in favour of a law to permit divorce

Subsequently, the two opinion poll companies concerned, I M S and M R B I , made lengthy comments on the issue The I M S comment contrasted the two questions used

M R B I – "Divorce is not legal in Ireland but a legal separation where neither party is free to remarry is permitted Do you feel that divorce should be permitted in certain circumstances or do you feel that it should never be permitted here in Ireland?"

I M S – "If you were asked to vote on a law which would permit divorce in this country, would you vote in favour of or against such a law"?

The contrasts are clear, one refers to the existence of legal separations, and to FEELINGS about divorce IN CERTAIN CIRCUMSTANCES, the other makes no reference to the existence of legal separation and refers to VOTING INTENTIONS about A LAW which would permit divorce, UNQUALIFIED AS TO CIRCUMSTANCES It is not surprising that the percentages differ as they do

More than this, however, the I M S comment goes on to look at the results of an opinion poll it carried out in 1977 where, as well as asking a general question similar to that asked in December 1983, it also asked would respondents be in favour of or oppose divorce in a variety of marital circumstances, and elicited percentages in favour varying from 29 per cent to 73 per cent, depending on the circumstances

The M R B I comment included an analysis of the accuracy of some of its polls in the past, by comparison with subsequent election results, which indicated a very favourable comparison

Examples such as these may well be useful for introducing statistics in general and particular aspects of statistics to school students Some of the particular aspects that could be included would be

- data description and summarisation,
- definition, validity and reliability of data,
- statistical variation,
- sampling,
- inference

How far down this sequence one goes is a matter for serious consideration These examples, however, have sufficient depth to provide a variety of topics for investigation at each level of the sequence

5 2 Experimentation

One of the key contributions of statistics to the sciences is the design and analysis of statistical experiments Here I suggest two examples that may be useful in introducing students to this area

The first emphasises the dangers of drawing conclusions from ill designed experiments It is presented in the text by Freedman, Pisani and Purves (13) from which the following is quoted directly

> In acute cases of cirrhosis of the liver, the patient may start to hemorrhage and bleed to death One treatment involves surgery to redirect the flow of blood through what is called a PORTACAVAL SHUNT The operation to create the shunt is long and hazardous Do the benefits of this surgery outweigh the risks? Over fifty studies have been carried out to assess the effect of this surgery The results are summarised in the table below *A study of studies The conclusions of 51 studies on the protacaval shunt are related to their designs The well designed studies show the surgery to have little or no value The poorly designed studies exaggerate the value of the surgery*

	Degree of Enthusiasm		
Design	**Marked**	**Moderate**	**None**
No controls	24	7	1
Controls, but not randomised	10	3	2
Randomised controls	0	1	3

> There were 32 studies without controls, 75 per cent of these studies were markedly enthusiastic about the shunt, concluding that the benefits definitely out weighed the risks In 15 studies there were controls, but assignment to treatment or control was not randomized Only 67 per cent were markedly enthusiastic about

> the shunt But the 4 studies that were randomized controlled show the surgery to be of little or no value The badly designed studies exaggerated the value of this risky surgery
>
> One explanation is that in an experiment without controls, or an experiment where patients are assigned to treatment or control according to clinical judgement, there is a natural tendency to treat only the patients who are in relatively good shape This biases the study in favour of the treatment For the experiments summarized in the table, in all three types of study about 60 per cent of the patients in the treatment group were still alive three years after the operation The percentage of controls in the randomized controlled experiments who survived the experiment by three years was also about 60 per cent But only 45 per cent of the controls in the nonrandomized experiments survived for three years The most dangerous thing of all was to be a control in a bad experiment This shows the bias in the nonrandom ized experiments

Incidentally, the book by Freedman, Pisani and Purves is recommended for many ideas and examples concerned with teaching statistics at an elementary level It represents a break with the fiarly well established tradition of texts on elementary statistics and has received almost uniquely exceptionally favourable reviews in the statistical literature

A second interesting example in this area concerns an observational study which has within it the seeds of randomized experimentation This is concerned with the discovery of the transmission mechanism of cholera by John Snow in London in the 1850s A full account and discussion may be found in Goldstein and Goldstein (14)

By careful observation of the circumstances surrounding the outbreak of cholera in London around that time, Snow convinced himself that the disease was transmitted in the water supply However, he needed experimental proof so as to convince the authorities of the necessity for action Having failed with one intervention (for which there is a clear statistical explanation), Snow happened on a naturally randomized controlled experiment In one area of London, two private companies were in competition for supplying piped water to the individual houses One company took its water from the Thames at a point where it was contaminated by the sewage of the city, the other took its water from a place upstream, free of London sewage Snow wished to make a proper comparison between the companies with regard to the cholera incidence in houses served by the different companies To quote from his own account

> The pipes of each Company go down all the streets, and into nearly all the courts and alleys A few houses are supplied by one Company and a few by the other, according to the decision of the owner or occupier at that time when the Water Companies were in active competition In many cases a single house has a supply different from that on either sides Each company supplies both rich and poor, both large houses and small, there is no difference either in the condition or occupation of the persons receiving the water of the different companies
>
> As there is no difference whatever, either in the houses or the people receiving the supply of the two Water Companies, or in any of the physical conditions with which

> they are surrounded it is obvious that no experiment could have been devised which would more thoroughly test the effect of water supply on the progress of cholera than this, which circumstances placed ready made before the observer

The data he subequently collected clinched the argument

The success of this natural experiment may be used to support an argument for designed control and randomization in experiments

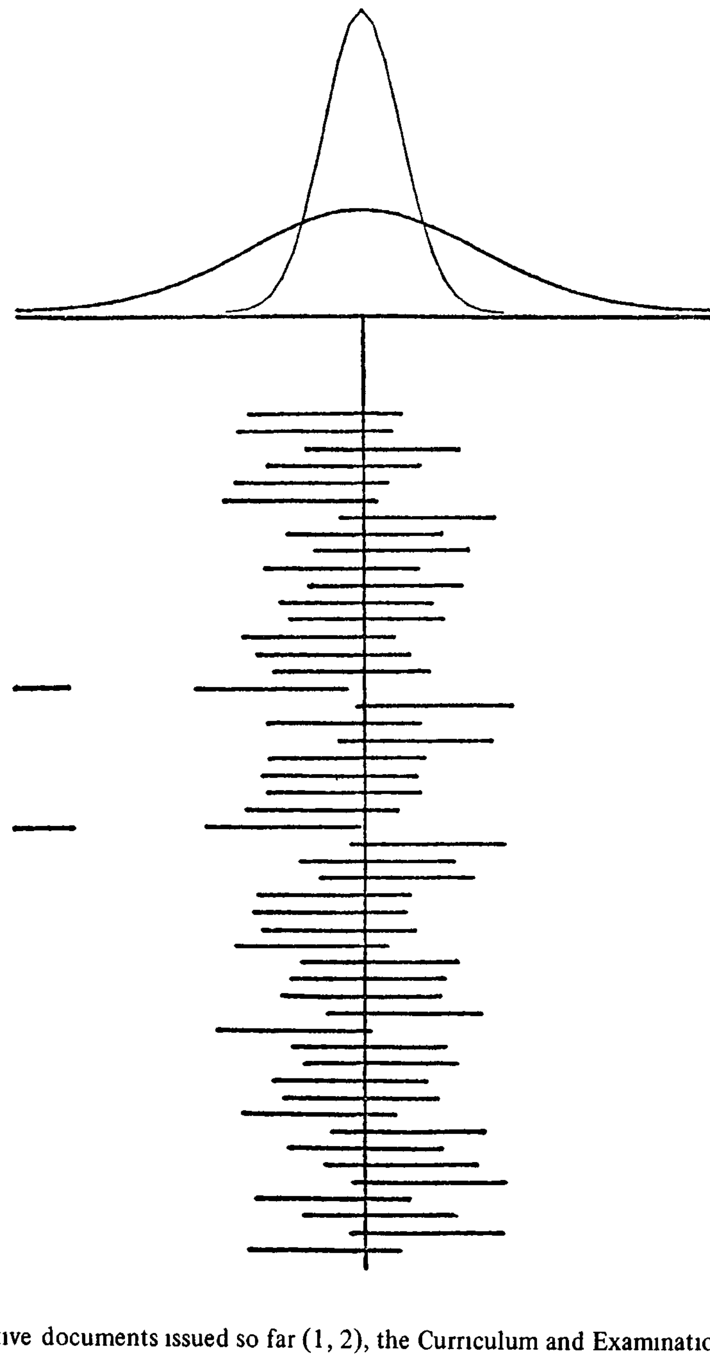
5 3 Using Computer Technology

Apart from number crunching which is, in itself, a secondary consideration from the point of view of teaching statistics, computers have two uses which can play a key role in teaching One is the use of the graphics capabilities of modern micro computers For example, the author is engaged in writing a programme in BASIC for the BBC micro computer which graphically illustrates the computation of various probabilities associated with the Normal distribution, using symmetry and complementation Many other examples suggest themselves, including "what if" – type analysis to look at the effects of "dirty data" on standard statistical displays, the effect of increasing sample size on sampling distributions, the adequacy of various approximations, for example the dependence of the Normal approximation to the Binomial distribution on the success probability and the number of trials, as illustrated by comparisons of corresponding pairs of binomial bar charts and Normal frequency curves

The second key use of computers for teaching statistics is in simulation of the frequency interpretation of probability distributions This approach could conceivably be used to dispense with most of the mathematical statistics that may be associated with statistics courses and allow concentration on the statistical concepts and ideas instead For example, the excellent book by A Stuart (15) uses simulation by paper and pencil, with very small numbers, to illustrate the basic ideas of sampling and of sampling distributions The ideas in this book could be very simply programmed and thereby allow more realistic numbers to be experimented with

As an example which combines the power of graphics and of simulation, consider the following Fifty samples of size 10 are generated from the standard Normal distribution The standard 95 per cent confidence interval for the population mean is computed from each sample A graph such as that which follows is generated, illustrating the original frequency curve, the frequency curve for the sampling distribution of the sample mean, and the computed intervals arranged around a vertical line representing the true mean value

In this example, two of the 50 calculated intervals do not cover the mean This empirical verification of the frequency basis for confidence intervals can be linked to a statement about a particular confidence interval that has been calculated from some real data, to the effect that we hope (or believe) that this one interval is one of the 95 per cent of all intervals that could have been calculated in theory which cover the true mean value, and hence we are 95 per cent confident that it does cover the true value



5 4 Projects

In its consultative documents issued so far (1, 2), the Curriculum and Examination Board has stressed the value of project work It has pointed out (in 2) that provision already exists for substituting marks based on in school assessment of project work for a portion of the written examination mark in some subjects, but that this option is taken up by less than 20 per cent of schools

Statistics, more than most subjects, lends itself to project work, both in the study of the subject itself and also of its application to other subjects The Cockcroft report has already been quoted referring to the value of project work and also the desirability of a "co ordinator' within a school, who would stimulate and co ordinate the inter disciplinary activities involved

As mentioned earlier, the UK Centre for Statistical Education has been engaged in developing curricula that involve project work and also in developing approaches to conducting and assessing such projects Holmes (16) has reported on experiences with implementing such curricula in three different examination boards, primarily with the A level Statistics syllabus of the Joint Matriculation Board The following is quoted directly from Holmes' paper

> Experience with the Joint Matriculation Board's syllabus shows that many teachers are unwilling to incorporate practical or project work within their teaching This may stem from a feeling of inadequacy – either that they have not taught this way before so they don't know what to advise or expect or they may feel that there would be insufficient suitable topics for them to use with their pupils To help with these problems there are guidelines for teachers in formula ting problems These guidelines partly take the form of questions such as 'Does the topic indicate some appreciation of the importance of statistics as a means of solving practical problems?', 'Is the topic trivial and likely to be unrewarding?', 'Will the candidate have access to material to enable him to carry out the project?', 'Does the topic offer promise of success within the time available and within the candidate's statistical expertise?' and so on Clearly teaching in this way throws a greater burden on the teacher, but it is ultimately far more rewarding to both the student and the teacher The fears about lack of suitable topics is largely unfounded Experience has shown that given the opportunity to be imaginative and use their own many interests, students come up with a wide variety of topics In the first year of the Joint Matriculation Board's A level Statistics candidates chose topics from Mathematics, Science, Economics, Languages, Geography, History, Leisure Activities and Sport, Commerce and Industry Titles included
>
> Testing for random numbers,
> The effect of soil conditions on the distribution of wild plants,
> Human reaction times and learning ability,
> Quality control of blood testing,
> Consumption of gas in the UK,
> Did Bacon write Shakespeare?,
> Do meander wave lengths vary with distance?,
> Age group populations of Huddersfield 1901 to 1971,
> A game of golf,
> Predicting football scores,
> The number of tills required in a particular sports shop,
> Petrol sales at self service and attended garages,
> Mistakes in newspapers,
> The effect of an overhaul on a caning machine,
> Lifetime of lightbulbs compared with manufacturer's claims

Although the use of projects causes problems in both teaching and in assessment, the gains far outweigh the disadvantages The obvious enthusiasm and commitment of many of the students to their projects needs to be seen to be believed The insight they gain in working on projects is also evident in the more formal written papers The introduction of such work into school courses is to be wholeheartedly welcomed and encouraged

In Ireland, many students participate in project work through the Aer Lingus Young Scientists Exhibition Earlier this year, I visited the most recent exhibition with a view to evaluating the statistical content of projects Given the statistical content of the school curriculum, I was quite impressed by a number of the projects, where the students concerned took a sensible approach towards data collection, analysis and display On the other hand, I was not at all impressed by the standard of judging which tended to reward projects where formal techniques of analysis were applied, not always validly, where the "STATISTICS ALL" syndrome familiar to SPSS users tended to predominate, and where award winners clearly did not understand the statistical techniques they were using

6 *DISCUSSION AND CONCLUSIONS*

A number of issues relating to second level statistics curriculum development have been raised in this paper, from the point of view of a professional statistician I have suggested that a broad view of statistics should be taken, as a starting point for any curriculum revision or development More precise suggestions may be summarized as follows

(i) Empirical Basis
Use examples and case studies to introduce and demonstrate statistical concepts and methods,

(ii) Descriptive Statistics
Use a modern approach emphasising simple analyses leading to adequate description and effective display, making use of modern graphical techniques,

(iii) Computers in Statistics
Use computers as teaching aids, to demonstrate/display various technical aspects of statistics and to facilitate simulation in explicating probability and inference,

(iv) Projects
Can provide a valuable learning resource and also facilitate more realistic assessment,

(v) Mathematical Probability and Statistics
Are best treated as branches of mathematics, although the case for including mathematical statistics in isolation from other statistics is weak,

(vi) Statistical Inference
The degree to which this can be handled at second level needs careful investigation,

(vii) Syllabus Integration
Statistics lends itself more than most subjects to integration into other subjects, statistics can benefit teaching and learning of the other subject particularly in project work, and vice versa

(viii) Teacher Training
Is vital to any successful development of statistics at the second level

(ix) Centre for Statistical Education
Is a valuable resource for documentation, experience and advice

The question of how much statistics should be included in the second level curriculum has not been addressed in this paper, apart from brief references to statistical inference and mathematical statistics I understand that there is a proposal to introduce statistics as a half subject in the Leaving Certificate This would give considerable scope for developing statistical ideas, although questions of whether statistical inference should be included and, if so, to what extent, would still need to be addressed

In a course which did not include or emphasize inference, examples and case studies based on census type data could be used It could be argues, however, that some reference to the problem of inference should be included in such a course, suitably illustrated by examples and case studies

I should also note that discussion has been confined to syllabi for secondary schools Consideration should be given to the statistical requirements of other types of second level schools The report of the Centre for Statistical Education (8) already referred to does discuss the requirements of such schools in the UK context It specifically considers the needs of prospective employees, and how these might be reflected in a statistics syllabus

I should stress that the views I have expressed are those of a professional statistician whose only contact with the second level system was in passing through it some years ago Clearly, any further developments would require input from people more directly involved with second level education

I believe it is important that any developments of statistical curricula should be based on a firm foundation of statistics I would suggest that this Society should make an authoritative submission to the Curriculum and Examinations Board concerning the development of statistics at second level The Society represent a wide spectrum of theoreticians in, practitioners of and users of statistics and is well placed to advise on the statistical needs of the community and how these might be reflected in the second level curriculum Naturally, I would hope that any such submission would pay due heed to the ideas that I have put forward in this paper

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DISCUSSION

M A Moran It is a pleasure to be invited to discuss this paper and to congratulate Dr Stuart on his initiative in presenting it As might be expected I endorse many of the sentiments he has expressed and indeed I responded in a similar spirit to a questionnaire from the Curriculum and Examinations Board earlier this year While sharing a common perspective of second level with Dr Stuart, I have had my contact renewed by my daughters taking the Intermediate and Leaving Certificate courses respectively!

One could add to the specific comments made in the paper on the current syllabi in Statistics The axiomatic presentation appears somewhat muddled and considerable confusion must result from the use of notation such as $\bar{X}$ for a population mean Some of the basic descriptive elements in the syllabi for the Intermediate seem appropriate to primary level where bar charts, pie charts, etc , would allow children to exercise the concepts of length, ratio and angle At Leaving Certificate level one wonders what educational criteria underlie the decision to treat Statistics and Vectors and Linear Transformations as alternatives An enquiry made on my behalf at a meeting of the Cork branch of the Irish Mathematical Teachers' Association last September showed that only a small minority chose to teach Statistics Vectors were considered shorter easier to teach and related to other topics I contacted the Examinations Section of the Department of Education last week to find out what proportion of candidates for Mathematics in the Leaving Certificate Examination attempt the questions in Statistics but it appears no such analyses are made It would seem that Statistics is neither practised nor preached at second level!

This is very serious in an era of information technology when statistical reasoning in the organisation, management and interpretation of information has become more and more important Dr Stuart rightly emphasised the practical nature of the subject and how it lends itself to project work Sadly the position of Statistics is typical of a curriculum where children's first introduction to drama is not a short play acted in class but Shakespeare, and where abstract concepts of Mathematics are introduced before a sound intuitive and practical understanding based on concrete examples has been allowed to develop Vectors may indeed relate to other topics such as reflections, translations, axial symmetries and isomorphisms, but also unfortunately, many pupils leave school and indeed enter third level with serious gaps in basic numeracy An obvious lesson is that syllabi for second level should not be the exclusive province of specialists in the nominal subjects

Dr Stuart devoted a considerable portion of his paper to describing developments in the United Kingdom Statistics at academic and professional levels is indeed highly developed there and moreover has a strong empirical and computational base However their second level educational structure and social structure are different from ours and we should hesitate before copying them in such matters In view of the specialisation they allow, it is also unclear what basic statistical education the majority of school leavers there receive or subsequently use For example, the most systematic and widespread use of the simple but important, descriptive and graphical techniques discussed in Section 4 1 of Dr Stuart's paper has not been in the UK or the US, but in Japan where they have been credited in part for the remarkable standards of quality and reliability achieved by Japanese goods

Of Dr Stuart's suggestions for case studies, I liked his own based on the opinion polls best I am less enthusiastic about the portacaval shunt example My experience with pre medical students at third level is that such examples often fail to motivate until students develop an interest and understanding of their relevance in later clinical work There is a danger that one might introduce at second level statistical concepts and methods which are best appreciated in a narrower and specialist context Dr Stuart seems aware of this in his reservations regarding the degree to which inference can be handled at second level

He excluded for his paper any suggestions as to how Statistics should be included in the second level curriculum This is, however, where the real difficulties start Practical subjects are often more difficult to teach and require more time and initiative on the part of the teacher There are competing demands from so many other subjects, a problem which will be compounded if Computer Studies is introduced as a full subject If Statistics enters as a half subject, perhaps within the existing Applied Mathematics, it will be very unfortunate In such an arrangement pupils, or more likely schools, will have to decide between it and Computer Studies and the latter will surely be selected The choice, moreover, is educationally unsound as the two subjects should complement and reinforce each other The scope for co operative work in planning a study, collecting data, processing it, including report writing on a computer should be considerable This is one use of the computer I would like to see at second level

I sometimes feel that subjects at second level merely reflect departmental structues at older third level institutions rather than any coherent educational objectives Surely the foundation list of topics, which Cockcroft envisaged that all students should know before they leave school, is a more acceptable approach In such a list statistical concepts and methods have an essential place whether they constitute all, half or less of any examination paper The unity of subjects rather than their various components should be emphasised and terms used in tonight's paper such as "mathematical statistics" and "pure mathematics" ought not arise at all at second level

Dr Stuart ended his paper with a proposal that this Society should make a submission on behalf of the subject to the Curriculum and Examinations Board I support his proposal, not alone for its own sake, but also because I would like to see the Society assume a role as the natural base for statisticians of all persuasions on this island I have often wondered, in this regard, why it has never recognised the considerable developments in Statistics at third level in Ireland or actively sought to increase its membership within that sector

However, this question is only tangentially related to the subject of Dr Stuart's presen tation tonight His paper is an important contribution to the further development of Statistics in Ireland If his intentions are realised, and I hope they will, it can only be good for second level as well I congratulate him and propose the vote of thanks

C C O Caoimh I am very glad to second the vote of thanks to Dr Stuart for his stimul ating paper He has, indeed, highlighted a lacuna in our second level curriculum, a lacuna that must be filled as soon as possible In Statistics the pupil is studying a subject that has relevance not only in other school subjects but also in very many courses at third level and even in society itself I have been endeavouring for some time to persuade the Depart ment and school organisations to give serious thought to the introduction of Statistics into the curriculum but so far I have been unsuccessful In this context I warmly support the proposal of Dr Stuart to make a submission to the Curriculum and Examinations Board

Dr Stuart's paper is a constructive criticism of the present state of Statistics in the Leaving Certificate syllabus and from a purely statistical point of view this criticism is justified Unfortunately one cannot take Statistics in isolation The curriculum as a whole must be considered and educators are of the opinion that the curriculum is already over crowded Presently I am pressing for the inclusion of Statistics and Computer Studies Others feel that Philosophy and Russian should be considered Religious Knowledge is about to be included

Some of the difficulties which mitigate against the inclusion of further subjects are TIME, TEACHERS and MANAGEMENT In the case of TIME, the period spent by a pupil in school in any one year is a finite fixed quantity During this time it is difficult for a pupil to study more than six or seven subjects in any reasonable depth Hence the addition of one other subject must displace an existing one from the pupil's curriculum The pupil/teacher ratio is also fixed so that if a mathematics teacher is time tabled to teach a new subject such as Statistics, who will teach the Mathematics? Management is involved in finding a room to take the new extra class in Statistics Very often all available space is in use in a school during the day So where does the Statistics class go? I am not arguing that these difficulties are unsurmountable but rather that the introduction of a new subject is not as simple as it seems In fact I will be suggesting a possible solution later on

Let me dwell for a moment on the present situation The updating of the Mathematics syllabus is carried out by a Syllabus Committee that makes proposals to the Department At the last review in 1978 I, as Chairman of the Syllabus Committee, was very anxious that Statistics be included The Committee, however, felt there was sufficient material already on the syllabus and that if more was to be included, it would have to be as an alternative to some existing topic Vectors and Linear Transformations were chosen as the alternative to Statistics The content of the Statistics section was obviously circum scribed by the teaching time available and so it was important to make as much use as possible of the Mathematics the pupils had already studied And so Venn diagrams were a good introduction to Events and Sample Space The Binomial Distribution followed from the Binomial Theorem Use is made of the Normal Tables to calculate $P(-1\ 96 < Z \leqslant 1\ 96)$ for example, where the random variable Z is normally distributed The fact that Z is continuous is referred to but not dealt with in any great detail Using the linear transformation

$$Z = \frac{X - \mu}{\sigma}$$

Where X is taken as normally distributed and testing if $Z \geqslant 1\ 96$ allows us to reject or accept our Null Hypothesis A brief discussion of the Type I error rounds off the course Dr Stuart argues for the abolition of ALL this material from the present higher level syllabus and suggests that our most able pupils be confined to drawing bar charts and finding means and standard deviations of frequency distributions My experience is that pupils very soon tire of such work At the higher level they demand more than this from Statistics Motivated teachers have no difficulty in firing pupils' enthusiasm with suitable hypothesis testing using coins, dice and experimental results

How then do I see the future? It seems to me that it is going to remain very difficult to introduce a large body of knowledge, such as a subject like Statistics or Computer Studies, into the existing curriculum As a means of overcoming this difficulty I suggest we choose a subject which is already in the curriculum and add on some extra modules Take Applied Mathematics, for example Suppose we define Applied Mathematics as the subject with the following modules STATICS, DYNAMICS, STATISTICS, COMPUTER STUDIES, NUMERICAL ANALYSIS, GRAPH THEORY (NETWORK ANALYSIS, CRITICAL PATH ANALYSIS)

This subject is determined by choosing two of the above This arrangement would have the advantage of not requiring an extra teacher (i e , in the schools already offering

Applied Mathematics) and would extend the pupils' choice Leaving aside for the moment the problem of inservice training of teachers, this arrangement would have the dis advantage of precluding pupils in schools where Applied Mathematics is not at present being offered

It is premature at this stage to go into details of syllabus content for such a Statistics module In general I would favour some Distribution theory (Binomial, Poisson and Normal) and some Applied Statistics such as SAMPLING I give a course in sampling every year to teachers of higher level Mathematics and I also use one of the letters from MRBI referred to by Dr Stuart to motivate the course

The sentence I use is

> "The examples of accuracy levels shown below are very considerably within the claimed plus or minus three per cent range "

(Note that the sample size was 1,000)

I have spoken at some length because I felt in seconding the vote of thanks to Dr Stuart I should seriously address the many points he raised in his paper One point I did not address was his reference to an examination question If the survey referred to was carried out at a particular time, then X could not obviously be a random variable It was under stood by pupils and by teachers that many surveys were carried out over a period of time It would have been better if this had been said explicitly

I formally second the vote of thanks

E Mullins I am sure that Dr Stuart is right in emphasizing the need for informal methods, use of computers and the encouragement of project work in schools I am less enthusiastic about Mr O Caoimh's support for teaching formal statistical inference at second level To argue for such a curriculum is to ignore our singular failure to teach statistical inference adequately at third level The absence of statistical methodology as a working tool in Irish industry and the misuse of statistical tests in the international scientific journals are abundant evidence of this failure The motivation of having one's own data to analyse is a key factor in the learning of statistics for most people Under graduates generally do not have this motivation and without project work it is totally lacking in second level schools My own experience teaching self selected postgraduate Diploma in Statistics to students and scientists and engineers in short courses in industry is that, even where motivation is present, people find the conceptual framework of statistical inference difficult It would be unreasonable, therefore, to expect that secondary school pupils are taught to do more than reproduce the mechanical motions of statistical tests I hope that this Society would consider the learning of such a skill at best a waste of time

Reply by Dr Stuart I am gratified by the positive way in which my paper was received, as indicated by tonight's speakers I wish to make just a few remarks in response to some of them

As regards Professor Moran's remarks, I am pleased to receive his support I, in turn, endorse most of what he has to say I was struck by his remark that "some of the basic descriptive elements in the syllabi for the Intermediate seem appropriate to primary level" On checking my own children's primary school geography books, I found in one of them several examples of statistical tables, maps and charts Generally, these were informative and well presented, although unfortunatley there was one involving oil barrels in the "gee whiz" category Further, on examining a primary school mathematics book, I found a whole section on graphs and charts which presents the basic ideas in a much more exciting, realistic and fundamental way than anything I have seen at second level The emphasis on ideas and interpretation in the primary school text was a refreshing contrast to the concentration on the dry mechanistic approach of the second level texts

Professor Moran recommends hesitation in copying the United Kingdom in their teaching of statistics at second level I agree that their siuation is different However, I wish to emphasise the benefits for syllabus development that can be gained by studying the work of the Centre for Statistical Education in Sheffield

I share Professor Moran's feeling about the influence of third level departmental structures One can only hope that the opportunity provided by the "clean sheet" apparently given to the Curriculum and Examinations Board will be used to break down the subject area barriers which become more and more irrelevant Statistics can play a key role in this In this context, I have to reject the suggestion, implicit in Mr Ó Caoimh's account of the development of the existing syllabus, that the statistical content of a mathematics syllabus must be dictated by the mathematics already in the syllabus On the contrary, I suggest that a broad genuinely interdisciplinary approach is likely to be more productive in the end

My suggestion that the present Leaving Certificate Higher Course syllabus be replaced by descriptive statistics did not imply confining attention to bar charts, means and standard deviations, as Mr O Caoimh suggests Rather, I had in mind more serious developments of descriptive statistics such as those suggested in the last paragraph of Section 4 1, developments which, I am glad to see, were endorsed by Professor Moran I also take issue with the suggestion that honours mathematics students are "our most able pupils" In my experience of teaching statistics to both beginning and advanced university students of mathematics, prowess at mathematics is no guarantee of prowess at statistics On the contrary, while they have little trouble with the relatively low level mathematical content of my courses, they typically have considerable difficulty with the conceptual and interpretative aspects, those aspects which are most conspicuously missing from the second level syllabi

As regards Mr O Caoimh's defence of the examination question which I criticised, it is quite clear that population data were referred to, not sample data resulting from a survey Also, reference to a series of surveys adds considerably to the complexity of the problem, taking it far out of the reach of Leaving Certificate students In any case, the series idea in itself has no bearing on whether or not x is binomial The question was misconceived

It is a pleasure to acknowledge the contribution of Mr Mullins, whom I hold in the highest respect as a teacher of statistics I support almost everything that he says, perhaps not surprisingly as he and I are close colleagues at TCD However, in spite of my acknowledgement of his teaching ability, I am not as sure as he appears to be that there is no place for statistical inference in the second level curriculum At the same time, I am equally unsure of how to include statistical inference and how much

Professor Moran notes that the real difficulties start when one considers how to include statistics in the second level curriculum This is certainly true, and it was deliberate on my part to exclude consideration of this question, because I do not have the answer Equally, I did not attempt to answer the questions concerning the inclusion of statistical inference It does seem to me, however, that students should be brought to understand the sentence quoted by Mr Ó Caoimh concerning opinion polls "The examples of accuracy levels shown below are very considerably within the claimed plus or minus three per cent range " There is strong evidence to suggest that many of the people who regularly make statements of this kind do not understand them in any operational way It is, I believe, a considerable challenge to statisticians to explain such statements to statistically unsophisticated audiences Further progress, by sullabus developers, statiticians and statistics users, will be difficult if the challenge is not met