

Unified framework of stochastic structural dynamics: direct probability integral method

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ABSTRACT: Stochastic dynamics of structures involves the topics of simulation of random excitation, random vibration analysis, reliability estimation and reliability-based design optimization. However, the existing methods only focus on parts of these topics, resulting in the complicated calculation and the absence of versatility. The limited scope of application of approximation methods, low accuracy and inefficiency of numerical methods and the expensive computational cost of stochastic sampling methods restrict the development of stochastic dynamics. In this study, the novel direct probability integral method (DPIM) as a unified framework is proposed to solve the uncertainty propagation and quantification and reliability-based design optimization problems of static and dynamic structures. This method decouples the computation of probability density integration equation (PDIE) and governing equation (dynamic or static equilibrium equation) of structures, and can achieve the probability density functions of stochastic responses and reliabilities for linear or nonlinear structural systems. Firstly, the PDIE governing the propagation of randomness from input to output is derived based on the principle of probability conservation. The two key techniques, i.e., the partition of probability space and smoothing of Dirac delta function, are introduced to solve the PDIE. Then, the first-passage dynamic reliability based on the equivalent extreme value mapping and design under uncertainty are addressed. Finally, numerical examples of static and dynamic structures illustrate that the DPIM is a unified, efficient and easy implementation methodology for uncertainty quantification and design optimization, especially for nonlinear stochastic dynamic analysis of large-scale complex structures.

1. INTRODUCTION

Uncertainty quantification becomes increasingly important which needs to be considered in the phase of structural design. Usually, the randomness in a dynamic structure comes from either material and geometric properties or external excitations (Roberts and Spanos 2003, Li and Chen 2009, Goller et al. 2013, Melchers and Beck 2018). Stochastic structural dynamics involves the topics of simulation of random excitation (e.g., nonstationary stochastic process), random vibration analysis, reliability assessment and reliability-based design optimization (RBDO). However, the existing methods only focus on parts of these topics, resulting in the complicated calculation and the absence of versatility. For

instance, there are three kinds of approaches for nonlinear random vibration analysis such as approximation methods (e.g., stochastic equivalent linearization method, stochastic average method), numerical methods (e.g., path integral, probability density evolution method) and stochastic sampling methods (e.g., Monte Carlo simulation) (Zhu 2006, Naess et al. 2011, Kougiumtzoglou and P. D. Spanos 2013, Di Paola and Alotta 2020, Valdebenito et al. 2020). Nevertheless, the limited application scope of approximation methods, low accuracy and inefficiency of numerical methods and the expensive computational cost of stochastic sampling methods hamper the advance of stochastic dynamics.

Recently, in terms of principle of probability conservation, the authors (Chen and Yang 2019, 2021) derived the probability density integral equations (PDIEs) representing the randomness propagation for static and dynamic structures. By solving the governing equation (e.g. equilibrium equation) of systems with the PDIE in a decoupled way, the direct probability integral method (DPIM) was proposed to perform stochastic response analysis and reliability estimation as well as RBDO of static and dynamic systems uniformly and efficiently (Chen et al. 2022, Li et al. 2022a, Li et al. 2022b, Chen et al. 2023). In what follows, this paper will describe the relevant advances of DPIM, especially in the aspect of unified and efficient analysis of stochastic structural dynamics.

2. DIRECT PROBABILITY INTEGRAL METHOD FOR RANDOM VIBRATION AND RELIABILITY ANALYSES

The direct probability integral method established the integral equation for describing the randomness propagation for stochastic static and dynamic structures, which breaks through the traditional paradigm of differential equation (e.g., FPK equation, probability density evolution equation) for charactering the uncertainty propagation for a long time. This method not only opens up a novel path for revealing the randomness propagation in the temporal and spatial domain, but also offers a universal and efficient tool for uncertainty quantification, random vibration, dynamic reliability analysis and optimum design of large engineering structures such as aerospace aircraft, high-rise buildings and long-span bridges.

2.1. Probability density integral equation of stochastic dynamic system

For stochastic dynamic system with input random vector Θ and output random vector $\mathbf{Y}$, it satisfies the principle of probability conservation (Li and Chen 2009), which is

$$\int_{\Omega_Y} p_Y(\mathbf{y}, t) d\mathbf{y} = \int_{\Omega_\Theta} p_\Theta(\boldsymbol{\theta}) d\boldsymbol{\theta} \quad (1)$$

where $p_Y(\mathbf{y}, t)$ means probability density function (PDF) of $\mathbf{y}$ at instant t ; Ω_Y indicates the total sample space of output random vector $\mathbf{Y}$; $p_\Theta(\boldsymbol{\theta})$ denotes PDF of input random vector Θ ; Ω_Θ denotes the input sample space. Specifically, Eq. (1) means that the total probability measure of output random variables equals to that of input random variables, and is invariant in the evolution of dynamic system (Chen and Yang 2019).

In Eq. (1), the physical evolution process of output response vector $\mathbf{Y}$ is governed by the motion equation of system, which can also be expressed by following deterministic mapping $\mathcal{G}$: $\mathbf{Y}(t)=\mathbf{g}(\boldsymbol{\theta}, t)$. In order to explicitly describe the randomness propagation from the input random vector Θ to output response vector $\mathbf{Y}$, PDIE is derived through the deterministic mapping $\mathcal{G}$ and Dirac delta function (Chen and Yang 2019)

$$p_Y(\mathbf{y}, t) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \delta[\mathbf{y} - \mathbf{g}(\boldsymbol{\theta}, t)] p_\Theta(\boldsymbol{\theta}) d\boldsymbol{\theta} \quad (2)$$

which can achieve the joint PDF of output random vector $\mathbf{Y}$. Furthermore, by using the property of Dirac delta function and performing marginal integral, the dimension reduction of PDIE for MDOF systems is realized and PDF of concerned response $y_\ell(t)$ is then given by

$$p_{Y_\ell}(y_\ell, t) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \delta[y_\ell - g_\ell(\boldsymbol{\theta}, t)] p_\Theta(\boldsymbol{\theta}) d\boldsymbol{\theta} \quad (3)$$

Equation (3) is termed as PDIE of concerned response $y_\ell(t)$. Since the multiple dimensional integrals with Dirac delta function in Eq. (3), it is hard to solve this equation analytically. To deal with this problem, the numerical process of Eq. (3) is presented in next subsection.

2.2. Numerical procedure for solving random vibration response

In this subsection, the numerical solution of PDIE is achieved based on DPIM. Two key techniques are the partition of probability space and the smoothing technique of Dirac delta function (Chen and Yang 2019). The corresponding numerical formula of PDIE is

$$p_{Y_\ell}(y_\ell, t) = \sum_{q=1}^N \left\{ \frac{1}{\sqrt{2\pi}\sigma} e^{-[y_\ell - g_\ell(\boldsymbol{\theta}_q, t)]^2 / 2\sigma^2} P_q \right\} \quad (4)$$

where $\boldsymbol{\theta}_q$ represents q th representative point in input sample space; N indicates the total number of representative points; P_q means the assigned probability of q th representative point; $g(\boldsymbol{\theta}_q, t)$ denotes the dynamic response of q th representative point, obtained by adopting a classical fourth-order Runge-Kutta method in this paper; σ is the smoothing parameter of Dirac delta function, which can be determined by an adaptive formula based on kernel density estimation (Chen and Yang 2021). More details of the numerical procedure of DPIM are referred to Ref. (Chen and Yang 2021). As illustrated in Eq. (4), DPIM achieves PDF and further mean and standard deviation of random vibration response of multi-degree-of-freedom (MDOF) system under a general random excitation by solving the deterministic motion equation and PDIE in turn with a decoupled way.

2.3. Dynamic reliability analysis

In structural reliability analysis, the failure of structure is determined by the performance function, which is consisted of structural resistance and load effects (Melchers and Beck 2018). The performance function can be also expressed a function to the random variables $\boldsymbol{\Theta}$ involved in the structure. For a dynamic structure, the performance function can be written as

$$Z(t) = g(\boldsymbol{\Theta}_s, \boldsymbol{\Theta}_f, t) = g(\boldsymbol{\Theta}, t) \quad (5)$$

where $\boldsymbol{\Theta} = [\boldsymbol{\Theta}_s, \boldsymbol{\Theta}_f]$, and $\boldsymbol{\Theta}_s$ and $\boldsymbol{\Theta}_f$ are the random variables associated to structural parameters and excitations, respectively.

For a static structure, the reliability can be easily obtained by integrating the PDF over the safe domain after obtaining the PDF of performance function. In a dynamic system, however, the first-passage failure criterion is adopted, and the dynamic reliability is defined as

$$P_r(t) = \Pr\{Z(t) = g(\boldsymbol{\Theta}, \tau) > 0, \tau \in (0, t)\} \quad (6)$$

which is a monotonically decreasing function with respect to the time t , since the random event that falls into the failure domain never returns.

Note that the dynamic reliability in Eq. (6) presents a time cumulative effect with $[0, t]$, that

is, the reliability at time instant t is that the structure lies in the safe domain in the time interval $[0, t]$. To assess dynamic reliability of first-passage problem (Chen and Yang 2021), an extreme value mapping is constructed, i.e.,

$$Z_{\min}(t) = h(\boldsymbol{\Theta}, t) = \min_{0 \leq \tau \leq t} \{g(\boldsymbol{\Theta}, \tau)\} \quad (7)$$

which decouples the cumulative effect of time.

The DPIM can be used to achieve the PDF of extreme value mapping $Z_{\min}(t)$, i.e.,

$$p_{Z_{\min}}(z_{\min}, t) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} p_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) \delta[z_{\min} - h(\boldsymbol{\theta}, t)] d\boldsymbol{\theta} \quad (8)$$

Since the time cumulative effect has been decoupled by the extreme value mapping, the dynamic reliability can be directly obtained by integrating the PDF of extreme value mapping over safe domain $\Omega_{Z,r} = \{z | z > 0\}$, namely

$$P_r(t) = \int_0^{\infty} p_{Z_{\min}}(z_{\min}, t) dz_{\min} = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} p_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) \mathcal{H}[-h(\boldsymbol{\theta}, t)] d\boldsymbol{\theta} \quad (9)$$

where $\mathcal{H}[\cdot]$ is Heaviside function, and there is

$$\mathcal{H}[-h(\boldsymbol{\theta}, t)] = \begin{cases} 0, & h(\boldsymbol{\theta}) > 0 \\ 1, & h(\boldsymbol{\theta}) \leq 0 \end{cases} \quad (10)$$

It is seen that in Eq. (9) the Dirac delta function has been analytically integrated as Heaviside function. Consequently, the first-passage dynamic reliability can be solved only by using the partition of probability space which avoids the smoothing of Dirac delta function, as expressed by

$$P_r(t) = \sum_{q=1}^N \left\{ \mathcal{H}[-h(\boldsymbol{\theta}_q, t)] P_q \right\} \quad (11)$$

Although the smoothing of Heaviside function can account for the contribution of non-representative points in a subdomain, the simplification of DPIM directly adopting Eq. (10) will be more useful for reliability analysis of large-scale structures (Chen et al. 2022, Li et al 2022a).

2.4. System reliability analysis

In reliability analysis for a large-scale structure, there are multiple failure modes, which is a system reliability problem. The performance functions of multiple failure modes for a dynamic structure are written as

$$Z_i = g_i(\boldsymbol{\Theta}, t), \quad i = 1, 2, \dots, m \quad (12)$$

For a dynamic series system, an extreme value mapping can be constructed by

$$Z_{i,\text{ext}}(t) = g_{i,\text{ext}}(\boldsymbol{\Theta}, t) = \min_{0 \leq \tau \leq t} \{g_i(\boldsymbol{\Theta}, \tau)\}, \quad i = 1, 2, \dots, m \quad (13)$$

As a result, the joint PDF of this extreme value mapping of performance functions can be formulated by the PDIE

$$p_{\mathbf{Z}_{\text{ext}}}(\mathbf{z}_{\text{ext}}, t) = \int p_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) \delta[z_{1,\text{ext}} - g_{1,\text{ext}}(\boldsymbol{\theta}, t)] \dots \delta[z_{m,\text{ext}} - g_{m,\text{ext}}(\boldsymbol{\theta}, t)] d\boldsymbol{\theta} \quad (14)$$

The first-passage dynamic reliability of series system can be directly obtained by an integral for Eq. (14) over the safe domain

$$\begin{aligned} P_{\text{r},\text{sys}}^s(t) &= \int_0^\infty \dots \int_0^\infty p_{\mathbf{Z}_{\text{ext}}}(\mathbf{z}_{\text{ext}}, t) d\mathbf{z}_{\text{ext}} \\ &= \int_{\Omega_{\boldsymbol{\Theta}}} p_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) \prod_{i=1}^m \mathcal{H}[g_{i,\text{ext}}(\boldsymbol{\theta}, t)] d\boldsymbol{\theta} = \prod_{i=1}^m \mathcal{H}[g_{i,\text{ext}}(\boldsymbol{\theta}_q, t)] P_q \end{aligned} \quad (15)$$

For a parallel dynamic system, the dynamic reliability is also expressed by

$$\begin{aligned} P_{\text{r},\text{sys}}^p(t) &= 1 - \int_{-\infty}^0 \dots \int_{-\infty}^0 p_{\mathbf{Z}_{\text{ext}}}(\mathbf{z}_{\text{ext}}, t) d\mathbf{z}_{\text{ext}} \\ &= 1 - \prod_{i=1}^m \mathcal{H}[-g_{i,\text{ext}}(\boldsymbol{\theta}_q, t)] P_q \end{aligned} \quad (16)$$

According to above process, the system failure probability of static and dynamic structures have been derived from the PDIE of extreme value of performance functions (Chen et al. 2022).

3. RELIABILITY-BASED DESIGN OPTIMIZATION VIA DPIM

Reliability-based design optimization can fulfill performance-based design of static and dynamic structures considering various uncertainties. For time-invariant and time-variant RBDO problems with multiple most probable points (MPPs, also termed as design points), the existing approaches encounter some difficulties in searching for all MPPs and performing time-variant RBDO which requires complicated time-variant reliability analysis. To this end, the direct probability integral method combining with the change of probability measure (COM), i.e., DPIM-COM approach, was proposed to attack time-invariant and time-variant RBDO problems with multiple MPPs in a unified framework (Li et al. 2022a).

Firstly, the DPIM in conjunction with Heaviside function is developed to estimate time-invariant and time-variant probabilistic constraints directly and explicitly. A typical formulation of RBDO model with time-invariant and time-variant probabilistic constraints is depicted as follows in a unified way

$$\begin{aligned} &\text{find} \quad \boldsymbol{\mu}_{\boldsymbol{\Theta}^D}, \mathbf{d} \\ &\text{min} \quad f_{\text{obj}}(\boldsymbol{\mu}_{\boldsymbol{\Theta}^D}, \boldsymbol{\mu}_{\boldsymbol{\Theta}^P}, \mathbf{d}) \\ &\text{s.t.} \quad h_j = \ln(P_{f_j}) - \ln(P_{f_j}^T) \leq 0, \quad (\text{time-invariant system}), \\ &\quad h_j(t) = \ln[P_{f_j}(t)] - \ln(P_{f_j}^T) \leq 0, \quad (\text{time-variant system}), \\ &\quad P_{f_j} = P[g_j(\boldsymbol{\Theta}^D, \boldsymbol{\Theta}^P, \mathbf{d}) \leq 0], \\ &\quad P_{f_j}(t) = P[g_j(\boldsymbol{\Theta}^D, \boldsymbol{\Theta}^P, \mathbf{d}, t) \leq 0], \\ &\quad j = 1, 2, \dots, n_g, \quad \boldsymbol{\mu}_{\boldsymbol{\Theta}^D}^L \leq \boldsymbol{\mu}_{\boldsymbol{\Theta}^D} \leq \boldsymbol{\mu}_{\boldsymbol{\Theta}^D}^U, \quad \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U \end{aligned} \quad (17)$$

where all the randomness involved in the probabilistic constraints in Eq. (17) is uniformly expressed as a random vector $\boldsymbol{\Theta} = [\boldsymbol{\Theta}^D, \boldsymbol{\Theta}^P]$, $\boldsymbol{\Theta}^D$ represents a random design variable vector, and $\boldsymbol{\Theta}^P$ is a random parameter vector. The means of $\boldsymbol{\Theta}^D$ and $\boldsymbol{\Theta}^P$ are $\boldsymbol{\mu}_{\boldsymbol{\Theta}^D}$ and $\boldsymbol{\mu}_{\boldsymbol{\Theta}^P}$, respectively. $\boldsymbol{\mu}_{\boldsymbol{\Theta}^D}^L$ and $\boldsymbol{\mu}_{\boldsymbol{\Theta}^D}^U$ are the lower and upper bound vectors for $\boldsymbol{\mu}_{\boldsymbol{\Theta}^D}$. $\mathbf{d}$ stands for deterministic design variable vector, whose lower and upper bound vectors are $\mathbf{d}^L$ and $\mathbf{d}^U$, respectively. The objective function is denoted as $f_{\text{obj}}(\boldsymbol{\mu}_{\boldsymbol{\Theta}^D}, \boldsymbol{\mu}_{\boldsymbol{\Theta}^P}, \mathbf{d})$. P_{f_j} and $P_{f_j}(t)$ represent the time-invariant and time-variant failure probabilities for the j -th failure mode, i.e., the j -th performance function $g_j(\boldsymbol{\Theta}^D, \boldsymbol{\Theta}^P, \mathbf{d}) \leq 0$ or $g_j(\boldsymbol{\Theta}^D, \boldsymbol{\Theta}^P, \mathbf{d}, t) \leq 0$, respectively. Note that the evaluation of probabilistic constraints for time-variant RBDO problems involves the multidimensional integral and failure domain determination of time-variant system, which brings some computational challenges. $P_{f_j}^T$ refers to the admissible failure probability (i.e., the threshold of failure probability) for the j -th probabilistic constraint, and n_g is the number of probabilistic constraints. Then, an efficient formula for calculating the sensitivity of probabilistic constraints with respect to random design variables is derived based on the DPIM and

COM strategy, where the representative points for reliability computation keep unchanged and only the corresponding assigned probabilities need to be updated. Accordingly, the corresponding structural responses of representative points at the current design variables can be reused to evaluate the sensitivity of probabilistic constraints, which saves much computational cost, especially for time-variant RBDO problems. Finally, three numerical examples, including the ten-story building with tuned mass damper under deterministic earthquake ground motion and near-fault stochastic impulsive ground motions, demonstrate the effectiveness and versatility of the proposed DPIM-COM approach (its flowchart is shown in Figure 1) for addressing time-invariant and time-variant RBDO problems with multiple MPPs.

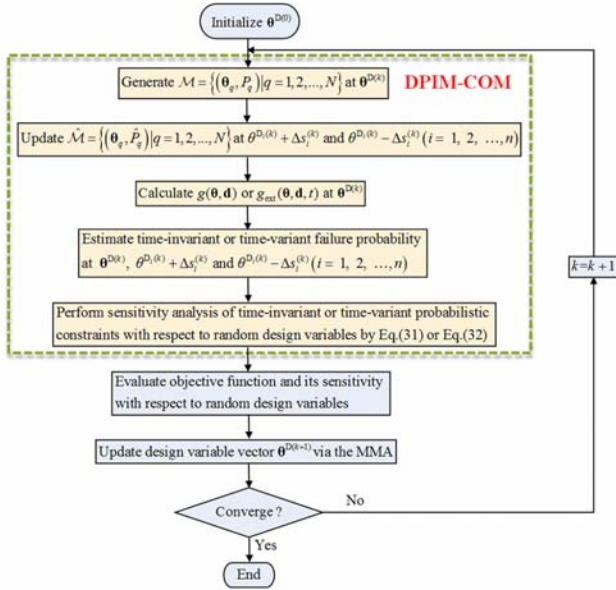


Figure 1: Flowchart of the proposed DPIM-COM approach for time-invariant and time-variant RBDO.

In addition, viscous dampers, as effective energy-dissipation devices, have been widely used for seismic mitigation of building structures. Due to the intrinsic uncertainties of structural parameters and earthquake ground motions, to obtain an optimal performance of structural control, the stochastic design optimization of viscous dampers for buildings is essential. Li et al. (2022b) established a new dynamic reliability-

based optimization framework to address the simultaneous layout and size design of nonlinear viscous dampers in frame buildings, considering the dual randomness of both nonstationary seismic excitations and uncertain structural model parameters. Firstly, a mixed integer optimization problem for nonlinear viscous dampers, including discrete and continuous design variables, is formulated, and the design target is to minimize the cost of the dampers subjected to the performance constraint on dynamic reliability of building structures. Then, sequential approximate mixed integer programming with trust region is proposed to solve the optimization problem. Moreover, direct probability integral method is suggested to assess the dynamic reliability and its sensitivity with respect to design variables. Finally, accounting for both stochastic nonstationary seismic excitations and random structural parameters, the optimized results of two numerical examples indicate that the proposed method is a competitive choice for realizing the layout and size optimization of limiting types of nonlinear viscous dampers in frame buildings. To ensure the same target performance of buildings in terms of the probability of exceeding the target value of inter-story drift, the larger peak ground acceleration of stochastic ground motions is, the more cost of dampers is required for seismic protection of buildings.

4. NUMERICAL EXAMPLE

A two-span 15-story frame building structure in Figure 2 with the Bouc-Wen hysteretic model is considered as an illustrative stochastic dynamic analysis example of application of DPIM, and the governing equation is formulated as

$$\mathbf{M}(\Theta_s)\ddot{\mathbf{X}} + \mathbf{C}(\Theta_s)\dot{\mathbf{X}} + \alpha\mathbf{K}(\Theta_s)\mathbf{X} + (1-\alpha)\mathbf{K}(\Theta_s)\mathbf{Z} = -\mathbf{M}(\Theta_s)\ddot{\mathbf{X}}_f(t) \quad (18)$$

where $\ddot{\mathbf{X}}$, $\dot{\mathbf{X}}$ and $\mathbf{X}$ indicate the lateral acceleration, velocity and displacement vector, respectively; $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{C}$ are separately the mass matrix, initial stiffness matrix and Rayleigh damping matrix. Θ_s represents the random vector from structural

parameters listed in Table 1; $\ddot{x}(\Theta_f, t)$ is the stochastic acceleration excitation, which is modeled by random vector Θ_f ; α is the ratio of the final tangent stiffness to initial stiffness; $\mathbf{Z}$ denotes the hysteretic displacement. Herein, the model parameters are adopted as: $\alpha=0.04$, $\beta=100$, $\gamma=180$, $A=1.0$, $N_c=1$, $d_v=d_\eta=1000$, $\zeta_s=0.2$, $\psi=0.05$, $d_\psi=5$, $\lambda=0.5$, $p=10$, $q=0.3$.

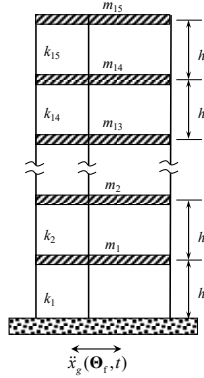


Figure 2: Two-span 15-story hysteretic frame subjected to stochastic earthquake ground motions.

Table 1: Probability distribution and statistical characteristics of structural parameters.

Floor i	Mass m_i (10^5 kg)			Stiffness k_i (10^5 kN/m)		
	Distribution	Mean	COV	Distribution	Mean	COV
1, 2	Lognormal	4.0	0.2	Lognormal	3.5	0.2
3~7	Lognormal	3.8	0.2	Lognormal	3.2	0.2
8~12	Lognormal	3.6	0.2	Lognormal	3.0	0.2
13~15	Lognormal	3.4	0.2	Lognormal	2.8	0.2

The stochastic acceleration excitations are generated by the random function based spectral representation (Chen and Yang 2021, Li et al. 2022b)

$$\ddot{x}(\Theta_f, t) = f(t) \sum_{k=0}^N \sqrt{S(\omega_k) \Delta \omega} \cdot [\cos(\omega_k t) R_k(\Theta_f) + \sin(\omega_k t) Q_k(\Theta_f)] \quad (19)$$

where $f(t)$ is the time modulated function

$$f(t) = 4(e^{-0.0995t} - e^{-0.1990t}) \quad (20)$$

and $S(\omega)$ is Clough-Penzien spectrum

$$S(\omega) = \frac{\omega^4}{(\omega^2 - \omega_f^2)^2 + 4\zeta_f^2 \omega_f^2 \omega^2} \cdot \frac{\omega_g^4 + 4\zeta_g^2 \omega_g^2 \omega^2}{(\omega^2 - \omega_g^2)^2 + 4\zeta_g^2 \omega_g^2 \omega^2} S_0 \quad (21)$$

in which $\omega_g = 5 \pi$ rad/s, $\zeta_g = 0.60$, $\omega_f = 0.5 \pi$ rad/s, $\zeta_f = 0.60$, $S_0 = 48.933 \text{ cm}^2/\text{s}^3$. In Eq. (35), $R_k(\Theta_f)$ and $Q_k(\Theta_f)$ are a pair of orthogonal random functions with respect to the basic random variables Θ_f , defined as

$$R_k(\Theta_f) = \cos(k\Theta_9) + \sin(k\Theta_9), \quad Q_k(\Theta_f) = \cos(k\Theta_{10}) + \sin(k\Theta_{10}) \quad (22)$$

where $\Theta_f = [\Theta_9, \Theta_{10}]$ are the random variables from ground motion excitation, and Θ_9 and Θ_{10} are both uniformly distributed within $[-\pi, \pi]$.

The inter-story drift $Y(\Theta, t)$ and roof displacement $X_{15}(\Theta, t)$ are considered as the key responses to determine the failure modes. Thus, the performance functions of the frame are formulated as:

$$Z_i(t) = g_i(\Theta, t) = 0.06 - |Y_i(\Theta, t)|, \quad i = 1, 2, \dots, 15 \quad (23)$$

$$Z_{16}(t) = g_{16}(\Theta, t) = 0.08 - |X_{15}(\Theta, t)|$$

The frame building is a series system consisting of 16 failure modes. Hence, the system performance function can be expressed by

$$Z_{\text{sys}}(t) = g_{t, \text{ext}}(\Theta, t) = \min_{0 \leq \tau \leq t} \left\{ \min_{1 \leq i \leq 16} [Z_i(\tau)] \right\} \quad (24)$$

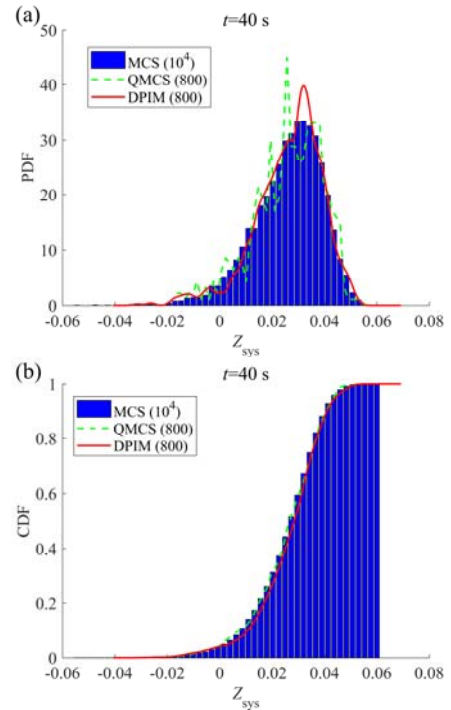


Figure 3: (a) PDF and (b) CDF curves of system performance function at $t=40$ s.

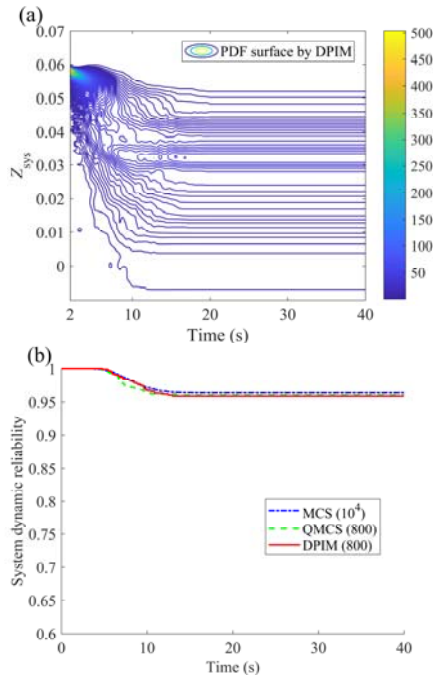


Figure 4: (a) Contour of PDF surface obtained by DPIM; (b) system reliability curves.

The PDF and cumulative density function (CDF) curves of system performance function at 40 s are shown in Figure 3, where both the DPIM and Quasi MCS (QMCS) adopt 800 deterministic integration points, and MCS uses 10⁴ random samplings. From these figures, one can observe that the PDF and CDF curves from DPIM are close to those of MCS than QMCS. The contour diagram of PDF surface from $t=2$ s to 40 s are illustrated in Figure 4(a). It is seen that the PDF of system performance function presents the skewness during the evolution from the time $t=2$ s to 40 s, and has an obvious stationary phase after 15 s. It is because the stochastic acceleration excitations acting on the base of structure have passed its peak and will tend to decrease after that moment, resulting in the responses no longer to be increased. From the dynamic reliability curves in Figure 4(b), it is observed that the same stationary phase occurs at the time after 15 s, and the final reliability is 0.9587.

5. CONCLUSIONS

This study establishes the PDIE involving multiple-dimensional integral with Dirac function

of concerned response for dynamic structure, and proposes a new formula with Heaviside function of DPIM to perform reliability analysis of structural system. Since the Dirac delta function can be analytically integrated as Heaviside function, the formula with Heaviside function is advanced for structural dynamic reliability and system reliability computation, in which only the partition of probability space is required. The intractable RBDO problem of dynamic system is also addressed via DPIM.

Finally, a two span 15-story frame building with stochastic parameters and random excitation demonstrates the versatility as well as high efficiency and accuracy of DPIM for stochastic dynamic responses and reliability analyses. In this study, the GF-discrepancy based point selection method is employed to generate representative points. The error of DPIM can be controlled by the GF-discrepancy of point set, and verified by comparing the results with those of MCS. In the future, the error estimation approach will be established by considering the number of representative points.

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