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Property of the intrinsic drift coefficient in globally-evolving-based generalized density evolution equation for the first-passage reliability assessment of non-linear systems

Jianbing Chen
Professor, College of Civil Engineering, Tongji University, Shanghai, China

Tingting Sun
Graduate Student, College of Civil Engineering, Tongji University, Shanghai, China

ABSTRACT: The recently developed globally-evolving-based generalized probability density evolution equation (GE-GDEE) combined with constructing the absorbing boundary process (ABP) has provided a powerful tool for assessing the first-passage reliability of stochastically excited high-dimensional systems. The intrinsic drift coefficient of the ABP, which plays an essential role as the "physically-driving force" in the GE-GDEE-based reliability evaluation, is studied in the present work. By investigating a typical nonlinear system excited by additive Gaussian white noise, it is found in this paper that the intrinsic drift coefficient of the ABP differs from that of the corresponding original process in the vicinity of boundaries and inclines to zero therein, which can be physically interpreted as a local reduction of damping. Accordingly, the failure probability given by employing the intrinsic drift coefficient of the corresponding original process is non-conservative but in the same order of magnitude as the true value and thus can be an acceptable approximation. The findings in this work provide a physical insight into constructing the intrinsic drift coefficient of ABPs for assessing the first-passage reliability of high-dimensional nonlinear systems via GE-GDEE.

Accurately assessing the first-passage reliability of structures under the action of stochastic excitation is critical to ensure the safety of structures and has long been the research focus (Lin, 1967; Ellingwood and Galambos, 1982; Li and Chen, 2009). Due to lacking analytical solutions of the first-passage reliability of general dynamical systems, a great number of numerical methods for first-passage analysis have been developed.

One category of approaches is the level-crossing theory method (Rice, 1944; Coleman, 1959). However, the Poisson (Coleman, 1959) or Markov crossing assumptions usually adopted in the level-crossing theory are only reasonable for the crossing event with rare occurrences and might result in conservative or non-conservative solutions, even with various associated modifications (Li and Chen, 2009; Vanmarcke, 2010). Besides, evaluating the mean crossing rate is rather difficult for high-dimensional nonlinear systems, which further limits the application of the level-crossing theory in reliability assessment. Another class of widely-used reliability method is the Monte Carlo simulation (MCS) (Shinozuka, 1972; Li and Chen, 2009) and its improved version based on efficient sampling strategies, such as the subset simulation (Au and Beck, 2001), the line sampling (Pradlwarter et al., 2007), and the directional importance sampling (Misraji et al., 2020), etc. Though these sampling strategies have increased the efficiency of MCS in varying degrees, the computational cost is still unbearable for capturing the small failure probability of real-world engineering systems because of the random convergence nature of MCS (Li and Chen, 2009; Li and

Wang, 2022). Besides, there is extensive research on reliability assessment based on diffusion processes. These methods estimate the reliability or life by numerically solving the FPK equation (Naess et al., 2011), the backward Kolmogorov equation (Spanos and Kougioumtzoglou, 2014), or the Pontryagin-Vitt equations (Xu and Jin, 2018). But the diffusion processes theory encounters enormous difficulties in solving the corresponding high-dimensional partial differential equations for complex engineering structures via currently available numerical methods, such as the finite difference method (Chang and Cooper, 1970), the finite element method (Bergman and Spencer, 1988), and the path integral solution (Naess et al., 2011; Di Paola and Bucher, 2016), etc. Meanwhile, the extreme value distribution (EVD) method has draw much attention. The recently developed augmented Markov vector method (Chen and Lyu, 2020; Lyu et al., 2020) and the corresponding probability evolution integral equation (Lyu et al., 2021) give the exact solution of EVDs for low-dimensional Markov systems. But finding EVDs for generic multi-dimensional or high-dimensional stochastic processes is still onerous (Naess and Moan, 2013). Other moment-based approaches to evaluate EVDs, such as the high-order moment method (Zhao and Lu, 2007) and fractional moment method (Xu, 2016), are efficient but problem-dependent.

Despite great efforts, the first-passage reliability evaluation, particularly accurately capturing small failure probabilities for high-dimensional stochastic dynamical systems, remains challenging. The recently developed globally-evolving-based generalized density evolution equation (GE-GDEE) and the corresponding reliability method (Lyu and Chen, 2022) show great potential in overcoming this difficulty. In this method, the absorbing boundary process (ABP) with respect to the quantity of interest and the prescribed failure criteria is constructed first. Then, the reliability is readily obtained from the GE-GDEE, usually of one or two dimensions, of the probability density function (PDF) of the ABP. In this GE-GDEE-based reliability method, the intrinsic drift coefficient of ABPs functions as the "physically-driving force" and is crucial to the accuracy of reliability results. The latest research by Sun et al. (2023) has investigated the properties of intrinsic drift coefficients of ABPs and gave the precise solution for a linear single-degree-of-freedom (SDOF) system. In this solution, a damping reduction is observed in the vicinity of boundaries. However, the accurate intrinsic drift coefficient of ABPs for nonlinear systems is still unknown. Especially whether the local damping reduction property also exists in nonlinear systems needs to be studied.

For this reason, the present paper investigates the intrinsic drift coefficient of the ABPs in the GE-GDEE for the first-passage reliability assessment of nonlinear systems. Firstly, the GE-GDEE-based reliability evaluation is briefly reviewed. Then, the intrinsic drift coefficient of the ABP for an SDOF Duffing oscillator excited additive by Gaussian white noise is obtained and verified by substituting it into the GE-GDEE and evaluating the corresponding reliability. The local damping reduction of the intrinsic drift coefficient in the vicinity of boundaries compared to that of the associated original process is also found in this nonlinear SDOF system. Consequently, the reliability given by adopting the intrinsic drift coefficient of the corresponding original process is non-conservative but in the same order as the solution by MCS. Tell me that story.

1. PHYSICALLY-DRIVEN GE-GDEE FOR THE FIRST-PASSAGE RELIABILITY ASSESSMENT

For clarity, the first-passage reliability assessment via GE-GDEE will be briefly reviewed in this section. Without losing generality, consider the following Itô stochastic differential equation

$$\mathrm{d}\boldsymbol{Z}(t) = \boldsymbol{A}(\boldsymbol{Z}(t), t)\,\mathrm{d}t + \boldsymbol{B}\mathrm{d}\boldsymbol{W}(t), \tag{1}$$

where $\boldsymbol{Z}(t)$ is an m-dimensional vector, $\boldsymbol{W}(t)$ is the vector of Wiener processes that satisfies $\mathcal{E}\left[\boldsymbol{W}(t)\boldsymbol{W}^{\mathrm{T}}(t+\Delta t)\right] = \boldsymbol{D}\Delta t$, and $\mathcal{E}(\cdot)$ refers to the expectation operator. The equation of motion of a discrete system is usually a second-order ordinary differential equation, which can be transformed into Eq. (1) by introducing state variables.

In practice, the critical component of the responses, for example, $Z_\ell(t)$, $\ell \in \{1, 2, \cdots, m\}$, is required not to exceed certain thresholds. The corresponding first-passage reliability is written as

$$R(t) = \Pr\{Z_\ell(\tau) \in \Omega_\mathrm{s}, \forall\, \tau \in [0,t]\}, \tag{2}$$

where Ω_s is the safe domain.

To estimate the reliability in Eq. (2) via the GE-GDEE, an absorbing boundary process (ABP) with respect to $Z_\ell(t)$ and Ω_s is introduced and formally defined as (Lyu and Chen, 2022)

$$\breve{Z}_\ell(t) = \begin{cases} Z_\ell(t), \text{ if } Z_\ell(\tau) \in \Omega_\mathrm{s}, \ \forall \tau \in [0,t], \\ Z_\ell(\tau), \text{ if } Z_\ell(\tau) \in \partial\Omega_\mathrm{s}, \ \exists \tau \in [0,t], \end{cases} \tag{3}$$

where $\partial\Omega_\mathrm{s}$ is the boundary of Ω_s.

As the definition in Eq. (3) implies, the first-passage reliability in Eq. (2) exactly equals the integral of the PDF of the ABP within the safe domain, i.e.,

$$R(t) = \int_{\Omega_\mathrm{s}} p_{\breve{Z}_\ell}(z_\ell, t)\,\mathrm{d}z_\ell. \tag{4}$$

Notice that in Eq. (3) and (4), the safe domain Ω_s excludes its boundary $\partial\Omega_\mathrm{s}$ because the failure probabilities are concentrated on the boundaries. According to Eq. (4), the reliability can be captured once the PDF of ABP is accurately evaluated. In other words, the first-passage reliability assessment is converted to seeking the PDF of the corresponding ABP, where the latter can be readily obtained via the GE-GDEE as long as the ABP is path-continuous (Lyu and Chen, 2022). As Eq. (3) implicates, the constructed APB remains path-continuous if the related original process is path-continuous. Therefore, the PDF of the ABP $Z_\ell(t)$, denoted as $p_{\breve{Z}_\ell}(z_\ell, t)$, follows the GE-GDEE (Lyu and Chen, 2022)

$$\frac{\partial p_{\breve{Z}_\ell}(z_\ell,t)}{\partial t} = -\frac{\partial a^{\mathrm{int}}_{\breve{Z}_\ell}(z_\ell,t)\, p_{\breve{Z}_\ell}(z_\ell,t)}{\partial z_\ell} + \frac{1}{2}\frac{\partial^2 b^{\mathrm{int}}_{\breve{Z}_\ell}(z_\ell,t)\, p_{\breve{Z}_\ell}(z_\ell,t)}{\partial z_\ell^2}, \tag{5}$$

where $a^{\mathrm{int}}_{\breve{Z}_\ell}(z_\ell,t)$ and $b^{\mathrm{int}}_{\breve{Z}_\ell}(z_\ell,t)$ are the intrinsic drift and diffusion coefficients of $\breve{Z}_\ell(z_\ell,t)$, respectively, and are written as,

$$a^{\mathrm{int}}_{\breve{Z}_\ell}(z_\ell,t) = I\{z_\ell \in \Omega_\mathrm{s}\} \cdot \lim_{\Delta t \to 0} \frac{1}{\Delta t} \mathcal{E}\left(\breve{Z}_\ell(t+\Delta t) - \breve{Z}_\ell(t) \middle| \breve{Z}_\ell(t) = z_\ell\right), \tag{6}$$

and

$$b^{\mathrm{int}}_{\breve{Z}_\ell}(z_\ell,t) = I\{z_\ell \in \Omega_\mathrm{s}\}\,\sigma_{\ell\ell}. \tag{7}$$

In Eq. (7), $\sigma_{\ell\ell}$ is the (ℓ,ℓ)-th component of the matrix $\boldsymbol{\sigma} = \boldsymbol{BDB}^\mathrm{T}$.

Apart from constructing ABPs, an alternative approach to evaluate the reliability in Eq. (2) via the GE-GDEE is to directly impose the absorbing boundary conditions on the GE-GDEE in terms of the original quantity of interest (Lyu et al., 2021) , namely

$$\begin{aligned} \frac{\partial q_{Z_\ell}(z_\ell,t)}{\partial t} &= -\frac{\partial a^{\mathrm{int}}_{Z_\ell}(z_\ell,t)\, q_{Z_\ell}(z_\ell,t)}{\partial z_\ell} + \frac{1}{2}\frac{\partial^2 b^{\mathrm{int}}_{Z_\ell}(z_\ell,t)\, q_{Z_\ell}(z_\ell,t)}{\partial z_\ell^2}, \\ q_{Z_\ell}(z_\ell,t) &= 0, \quad \text{for } z_\ell \notin \Omega_\mathrm{s}, \end{aligned} \tag{8}$$

where $q_{Z_\ell}(z_\ell,t)$ refers to the reliability density function (RDF) (Di Paola and Bucher, 2016) or the remaining PDF (Li and Chen, 2009); $a^{\mathrm{int}}_{Z_\ell}(z_\ell,t)$ and $b^{\mathrm{int}}_{Z_\ell}(z_\ell,t)$ are the intrinsic drift and diffusion coefficients of $Z_\ell(t)$, respectively,

$$a^{\mathrm{int}}_{Z_\ell}(z_\ell,t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \mathcal{E}\left(Z_\ell(t+\Delta t) - Z_\ell(t) \middle| Z_\ell(t) = z_\ell\right), \tag{9}$$

and

$$b^{\mathrm{int}}_{Z_\ell}(z_\ell,t) = \sigma_{\ell\ell}. \tag{10}$$

As Lyu and Chen (2021) suggested, integration of the RDF in the safe domain yields the reliability, i.e.,

$$R(t) = \int_{\Omega_\mathrm{s}} q_{Z_\ell}(z_\ell,t)\,\mathrm{d}z_\ell \tag{11}$$

The above two reliability methods are both based on the GE-GDEE but different in the implementation order of imposing the absorbing boundaries and reducing the dimension via the GE-GDEE. In the approach proposed by Lyu and Chen (2022), the trajectories first arriving at the boundaries of the safe domain are absorbed, and then the GE-GDEE is established (ABP-GEGDEE approach for short hereafter). But in the approach given by Lyu and Chen (2021), the absorbing boundaries are imposed after the GE-GDEE in terms of the quantity of interest is derived (thereby GEGDEE-ABC for short).

Lyu and Chen (2022) has theoretically proved that the absorbing boundary conditions should be imposed first; that is, the order of imposing absorbing boundary conditions and dimension reduction is non-exchangeable. Specific to the systems considered in this section, they are excited by additive Gaussian white noises, and exchanging the order of the two procedures merely makes the intrinsic drift coefficients different, whereas the other parts remain the same. Accordingly, the numerical implementation of the reliability assessment with the absorbing boundaries imposed first and later are the same except for the numerical estimation of the intrinsic drift coefficients. The numerical realization of the two approaches is briefly summarized in the following three steps:

(i) Generate a number of trajectories of the process of interest and its velocity process.
(ii) Estimate the intrinsic drift coefficient, which is in the form of conditional expectation [Eq. (6) and (9)], at each time step. Numerical estimators such as the local weighted smooth scatter plots technique (LOWESS) (Cleveland, 1979; Lyu and Chen, 2021), and the copula methods (Chen and Lyu, 2022) can be applied in this step. For the ABP-GEGDEE approach, the trajectories will be kicked out once they cross out the safe domain and will never be counted on in the subsequent time steps. For the GEGDEE-ABC approach, all trajectories are utilized at all time steps, whether they cross out or not.
(iii) Substitute the numerically estimated intrinsic drift coefficient into the GE-GDEE and solve the GE-GDEE via numerical methods, such as the path integral solution (Di Paola and Bucher, 2016) and the finite difference method (Chang and Cooper, 1970).

The variation of the intrinsic drift coefficient caused by exchanging the order of imposing absorbing boundaries and establishing the GE-GDEE underscores the essential role of the intrinsic drift coefficient as the "physically-driving force" in the GE-GDEE-based reliability assessment. Based on the relevant findings for linear systems in Sun et al. (2023), this work focuses on the properties of the intrinsic drift coefficient of ABPs for the nonlinear system. To this end, the intrinsic drift coefficients in the reliability assessment for a typical nonlinear SDOF system is investigated in the next section.

2. PROPERTIES OF THE INTRINSIC DRIFT COEFFICIENTS OF THE ABP OF TYPICAL NONLINEAR SYSTEMS

Consider an SDOF Duffing oscillator excited by a stochastic process. The equation of motion reads

$$\ddot{X}(t) + 2\zeta\omega_0\dot{X}(t) + \omega_0^2 X(t) + \alpha\omega_0^2 X^3(t) = \xi(t), \tag{12}$$

where $\xi(t)$ is the Gaussian white noise whose correlation function is $\mathcal{E}[\xi(t)\xi(t+\Delta t)] = D\delta(\Delta t)$. The system parameters are $\omega_0 = 1$ rad/s, $\alpha = 0.2$, $\zeta = 0.15$, and $D = 1\ \mathrm{m}^2/\mathrm{s}^3$. The first-passage reliability in terms of the velocity process is considered:

$$R(t) = \Pr\{|V(\tau)| < b,\ \text{for}\ \tau \in [0,t]\}, \tag{13}$$

where $V(t) = \dot{X}(t)$.

To evaluate the reliability in Eq. (13) through the ABP-GEGDEE approach, the ABP corresponding to the velocity $V(t)$, denoted as $\check{V}(t)$, is constructed according to Eq. (3), and its intrinsic drift coefficient is written as

$$\begin{aligned} a_{\check{V}}^{\mathrm{int}}(v,t) &= -I\{|v| < b\} \cdot \\ &\mathcal{E}\left\{\omega_0^2 X(t) + \alpha\omega_0^2 X^3(t) + 2\zeta\omega_0 V(t) \middle| \check{V}(t) = v\right\}. \end{aligned} \tag{14}$$

The analytical expression of $a_{\check{V}}^{\mathrm{int}}(v,t)$ is unavailable, therefore it is numerically estimated according to the equivalence between the ABP-GEGDEE approach and the diffusion process theory (Lyu and Chen, 2022). Rewrite the conditional expectation in Eq. (14) into the following integral form

$$a_{\check{V}}^{\mathrm{int}}(v,t) = \int_{-\infty}^{\infty} \left(-\omega_0^2 x - 2\zeta\omega_0 v\right) \frac{p_{X\check{V}}(x,v,t)}{p_{\check{V}}(v,t)} \mathrm{d}x. \tag{15}$$

where the PDFs in Eq. (15) are identified by

$$p_{X\check{V}}(x,v,t) = q_{XV}(x,v,t),\ \text{for}\ |v| < b, \tag{16}$$

and

$$p_{\check{V}}(v,t) = \int_{-\infty}^{\infty} q_{XV}(x,v,t)\,\mathrm{d}x,\ \text{for}\ |v| < b. \tag{17}$$

In Eq. (17), $q_{XV}(x, v, t)$ refers to the RDF of $X(t)$ and $V(t)$ that follows the FPK equation with the associated absorbing boundary condition,

$$\begin{aligned} &\frac{\partial q_{XV}(x,v,t)}{\partial t} = -\frac{\partial v q_{XV}(x,v,t)}{\partial x} \\ &+ \frac{\partial\left(2\zeta\omega_0 v + \omega_0^2 x + \alpha\omega_0^2 x^3\right) q_{XV}(x,v,t)}{\partial v} \\ &+ \frac{D}{2}\frac{\partial^2 q_{XV}(x,v,t)}{\partial v^2}, \\ &q_{XV}(x,v,t) = 0, \text{ for } |v| \geq b. \end{aligned} \tag{18}$$

For the GEGDEE-ABC approach, the intrinsic drift coefficient of velocity process is adopted, i.e.,

$$\begin{aligned} &a_V^{\mathrm{int}}(v,t) = \\ &-\mathcal{E}\left\{\omega_0^2 X(t) + \alpha\omega_0^2 X^3(t) + 2\zeta\omega_0 V(t) \middle| V(t) = v\right\}. \end{aligned} \tag{19}$$

The stationary intrinsic drift coefficient of $V(t)$ can be derived from the exact stationary solution of this SDOF Duffing oscillator (Zhu, 2006), where in this exact solution, the displacement is independent with the velocity. Therefore, the exact stationary expression of $a_V^{\mathrm{int}}(v,t)$, denoted as $a_{V,\mathrm{s}}^{\mathrm{int}}(v)$, is determined as

$$a_{V,\mathrm{s}}^{\mathrm{int}}(v) = 2\zeta\omega_0 v. \tag{20}$$

During the non-stationary phase, the intrinsic drift coefficient of the associated original process is estimated with the method offered by Chen and Rui (2018).

Figure 1 presents the intrinsic drift coefficient of the ABP concerning different thresholds and that of the corresponding original process at the stationary phase. In particular, for a typical high barrier, $b = 4.4$ m/s, the representative points employed to estimate the intrinsic drift coefficients are plotted in Fig. 2. Besides, Figure 3 displays the intrinsic drift coefficients evaluated from the representative points in Fig. 2 estimated by the LOWESS (Cleveland, 1979).

As Fig.1 shows, the intrinsic drift coefficient of the ABP departs from that of the corresponding original process and inclines to zero in the vicinity of the boundaries. But the two curves coincide in the center area away from the boundaries. Since the intrinsic drift coefficient is in the unit of damping force per mass, this trend can be interpreted as a local reduction of damping near the boundaries. A similar local damping reduction has been found in the linear systems investigated in work by Sun et al. (2023).

It is also noticed that the curve shape of the intrinsic drift coefficient of the ABP is alike under different barrier levels. However, as Fig. 2 and Fig. 3 indicate, the curving effect under high barriers might be concealed if adopting the purely data-driving estimation of the intrinsic drift coefficient, since the representative points are sparse nearing high obstacles.

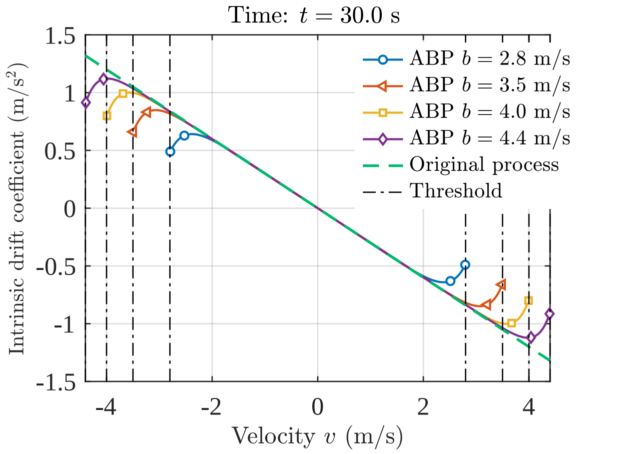

Figure 1: The stationary intrinsic drift coefficient of the ABP and that of the associated original process.

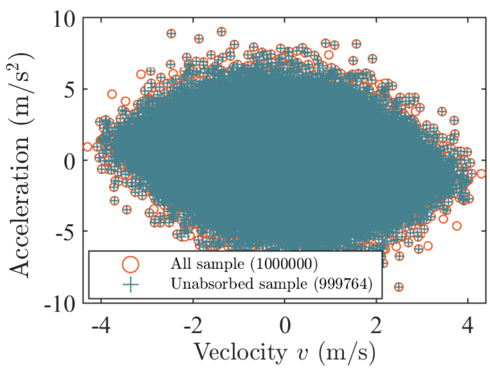

Figure 2: The representative points employed to estimate the intrinsic drift coefficient at 30 s.

The failure probabilities by the ABP-GEGDEE and GEGDEE-ABC approaches are compared in Fig. 4. The ABP-GEGDEE approach shows fair

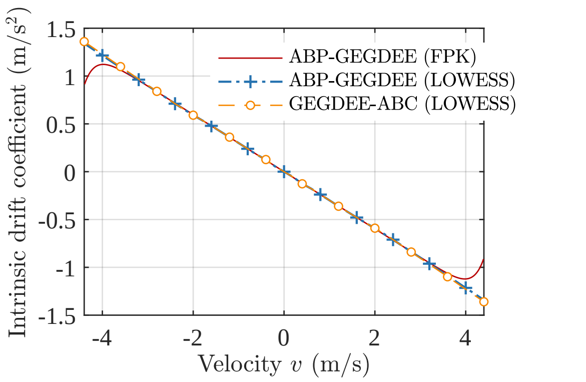

Figure 3: The stationary intrinsic drift coefficient of the ABP and that of the associated original process, where the "ABP-GEGDEE (FPK)" refers the precise results given by solving the FPK equation in Eq. (18), and "ABP-GEGDEE (LOWESS)" and "GEGDEE-ABC (LOWESS)" are the results numerically estimated from sample data.

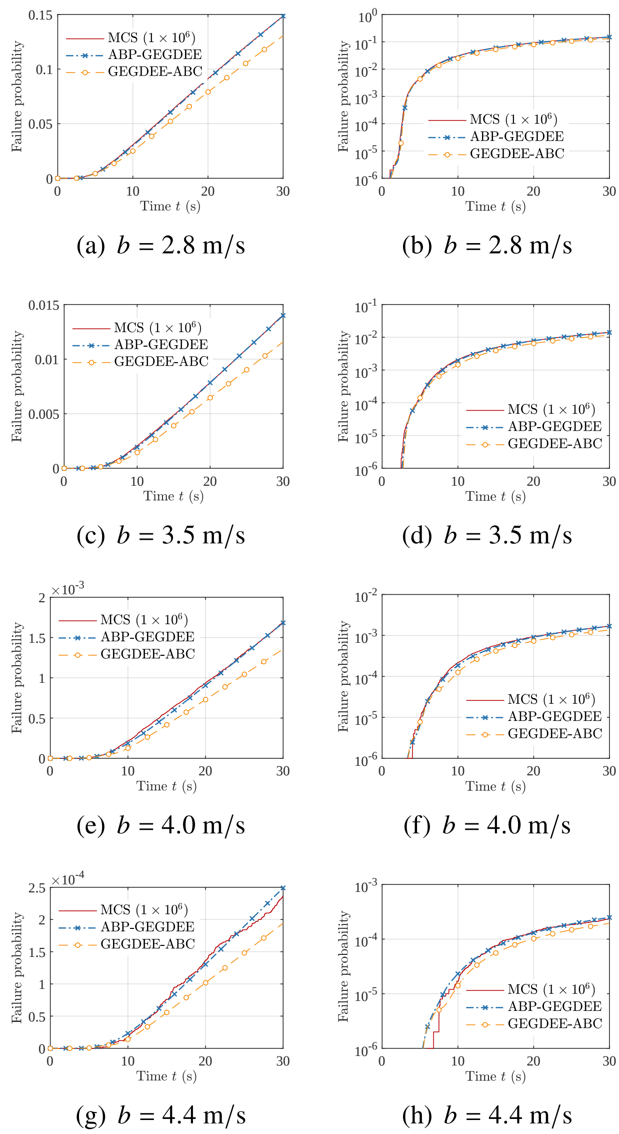

Figure 4: The failure probabilities results of the GE-GDEE-based method (the left column: in linear coordinate, the right column: in logarithmic coordinate).

accuracy even when the failure probability is in the order of magnitude $10^{-4} - 10^{-5}$, which validates the obtained solution of the intrinsic drift coefficient of ABPs. On the other hand, the GEGDEE-ABC approach, which has not captured the local reduction of damping force, gives non-conservative results. It is noticed that the failure probabilities predicted by the GEGDEE-ABC approach, despite errors, are in the same order of magnitude as those by MCS.

3. CONCLUSIONS

The intrinsic drift coefficients of the ABPs, which plays a crucial role in the GE-GDEE-based first-passage reliability assessment, were investigated in this paper. A typical nonlinear system was employed to illustrate the properties of intrinsic drift coefficients of ABPs relative to the corresponding original processes. Some important results have been found:

(1) For the nonlinear system, there is a local damping reduction in the intrinsic drift coefficient of the ABP, which is similar to the properties of that in linear systems found in previous research.

(2) The properties of the intrinsic drift coefficients of ABPs in the vicinity of the boundaries might be concealed by the purely-data-driven approach, for the representative points are usually sporadic near the boundaries. Besides, this research implicates a demand for a more accurate estimation of the intrinsic drift coefficient of the ABPs in the vicinity of boundaries where the data is sparse.

(3) The failure probabilities given by the GEGDEE-ABC with the semi-analytical intrinsic drift coefficients are non-conservative but in the same order of magnitude as the results by MCS, no matter the level of barriers. The error can be explained by the local

deviations of intrinsic drift coefficients in the GE-GDEE approach relative to those in the ABP-GEGDEE approach. In other words, the GEGDEE-ABC approach can be employed as an approximate method, especially for small failure probability problems.

This work only considers the system subjected to additive Gaussian white noise. Therefore, the intrinsic drift coefficient of ABPs for systems under other kinds of stochastic excitation should also be studied. Furthermore, a more rigorous theoretical explanation of the properties of the intrinsic drift coefficients is demanded to improve the construction of the intrinsic drift coefficient for more general systems.

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