



Discussion to the article Undrained shear strength prediction of clays using liquidity index, by Q. Wang, S. Qiu, H. Zheng, R. Zhang

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Abstract

This communication presents a critical discussion of the article “Undrained shear strength prediction of clays using liquidity index”, authored by Q. Wang, S. Qiu, H. Zheng, R. Zhang, and recently published in *Acta Geotechnica* (<https://doi.org/10.1007/s11440-023-02107-9>). Various inaccurate claims and flaws regarding the Authors’ newly developed three-parameter strength–liquidity index (S_u-I_L) models/correlations are highlighted and discussed herein. In particular, comparing existing two-parameter and their newly developed three-parameter S_u-I_L models, contrary to the Authors’ claims, no improvement in prediction accuracy is achieved over the two-parameter model by introducing a third model parameter. Rather, it is shown herein that the existing two-parameter and the Authors’ three-parameter S_u-I_L models are mathematically identical. Furthermore, two of the Authors’ newly developed S_u-I_L correlations, i.e., relating the triaxial-compression and shearbox derived strengths to the liquidity index, are shown to be inaccurate, forecasting gross under- and over-predictions of the measured strengths, respectively. This highlights the need for a reassessment of the proposed correlations, and emphasizes the importance of accurate and reliable correlations in geotechnical engineering.

Keywords Clays · Liquidity index · Prediction model · Undrained shear strength

Abbreviations

FC Fall-cone
 SB Shearbox
 TC Triaxial compression
 VS Vane shear

List of symbols

a, b Fitting parameters of the two-parameter S_u-I_L model
 A, B, C Fitting parameters of the three-parameter S_u-I_L model
 I_L Liquidity index
 LL Liquid limit

$LL_{cup,ASTM}$ Casagrande liquid-limit determined according to the ASTM standard
 $LL_{FC,BS}$ British standard fall-cone liquid limit
 PL Plastic limit
 S_u Undrained shear strength
 $S_{u,FC}$ Undrained shear strength for fall-cone test
 $S_{u,SB}$ Undrained shear strength for shearbox test
 $S_{u,TC}$ Undrained shear strength for triaxial compression test
 $S_{u,VS}$ Undrained shear strength for vane shear test

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From the Mohr–Coulomb failure criterion and the critical state theory, the Authors [12] proposed a three-parameter exponential model to relate the undrained shear strength (S_u) with the liquidity index (I_L) for clay samples; i.e., $S_u = A \exp(B - CI_L)$, where A , B , and C are model parameters. They state that “the function form of this model is greatly different from the previous forecasting models” (page 4327 in their paper), which included some two-parameter exponential S_u-I_L models (e.g., [3, 11, 13, 14]) expressed in the form of $S_u = a \exp(-bI_L)$, where a and b

are the associated model parameters. However, these two- and three-parameter models are mathematically identical; i.e., $S_u = A \exp(B - CI_L) = A \exp(B) \exp(-CI_L)$, giving $b = C$ and $a = A \exp(B)$. Note, $a = A \exp(B)$ is the predicted strength at the plastic limit (PL) water content (i.e., for $I_L = 0$). Accordingly, although presented in different forms, the S_u – I_L relationships listed as Eqs. (3), (6), and (7) in Table 1 of the Authors' paper, as well as the Authors' three-parameter models given by their Eqs. (34)–(37), can all be rearranged to have a consistent form, with varying a and b parameter values, as shown in Tables 1 and 2 here. In other words, no improvement in fit (prediction accuracy) is achieved over the two-parameter model by introducing the third model parameter. The Discussers hence disagree with the Authors' conclusions that: (1) in comparing the two- and three-parameter models, *"the prediction accuracy of I_L – S_u model is limited by the form of model function and the number of parameters"*; (2) *"the three controlled parameters in the proposed model allowed a more accurate prediction results than two controlled parameters in the existing prediction methods"* (page 4336 in their paper).

Similar to the approach taken in the Authors' paper, in earlier studies the model parameters were usually determined by fitting the model to measured strength data for various water contents within the plastic range. The strength data can be obtained by different laboratory strength test methods, e.g., vane shear (VS), fall cone (FC), triaxial compression (TC) and/or shearbox (SB), as employed by the Authors in calibrating their three-parameter S_u – I_L model to obtain four separate correlations (i.e., their Eqs. (34)–(37)) for linking I_L to $S_{u,VS}$, $S_{u,FC}$, $S_{u,TC}$, and $S_{u,SB}$. Owing to the dissimilar shearing modes and rates of the different strength tests, slightly different strengths may be mobilized for a given fine-grained soil tested at the same water content (I_L) value ([4, 6–8]). Consequently, depending on the strength measurement approaches adopted, different S_u – I_L data curves can be obtained for the same investigated soil, resulting in dissimilar values of the deduced fitting parameters. However, the large variations seen in the original paper between Eqs. (34)–(37) seem strange. Specifically, the Authors' Eqs. (34) and (35) produce reasonable/good $S_{u,VS}$ and $S_{u,FC}$ predictions, equivalent to those obtained via the Vardanega and Haigh [11] and Leroueil et al. [5] S_u – I_L correlations (listed as Eqs. (4)

and (6), respectively, in the Authors' paper) when restricted to their respective I_L calibration ranges. However the Authors' Eqs. (36) and (37) appear to differ more from conventional understanding than might be expected.

To obtain their Eq. (36), the Authors employed the experimental $S_{u,TC}$ – I_L curves plotted for the Horton, London, Gosport and Shellhaven clays, originally reported in the paper by Skempton and Northey [10] and later reproduced in the paper by Wroth and Wood [13]. Close examination of the smooth experimental $S_{u,TC}$ – I_L curves for these four clays presented in [10, 13] indicates that the strength axis has units of lbf/in² [i.e., pounds force per square inch (PSI)], which the Authors mistakenly took as kilopascals when digitizing the plots for use in deducing their Eq. (36). In other words, the strength values used by the Authors in calibrating their $S_{u,TC}$ – I_L model are too low by a factor of 6.895. Correcting this error would lead to $S_{u,TC} = 98.6 \exp(-5.55I_L)$. Consequently, the solid black $S_{u,TC}$ – I_L data curve for the London clay deduced by the Authors' Eq. (36) and plotted in Fig. 5b of their paper represents a gross under-prediction of the undrained shear strength; e.g., forecasting strengths at the liquid limit (LL) and PL water contents of 0.056 and 14.3 kPa, respectively, rather than values of 0.38 and 98.6 kPa predicted by the revised/corrected equation presented herein.

The Authors' Eq. (37) also appears questionable; e.g., predicting an unrealistically high $S_{u,SB}$ value of 35.9 kPa for the LL state transition (i.e., at $I_L = 1$), and a more reasonable 98.5 kPa for the PL. This equation was obtained from calibrating their three-parameter model using the Yilmaz [15] dataset that considered fine-grained alluvial soils (clays) sampled from depths of between 2 and 16 m. Yilmaz [15] did not describe the sample preparation and shearbox testing procedures, other than reporting that the testing was performed in accordance with the ASTM standards. However, at least some of the sampled soils were heavily over-consolidated, with measured values of $S_{u,SB}$ increasing from 36 to 224 kPa for I_L reducing from 0.91 to -0.52 . One possible explanation for the super-high $S_{u,SB}$ value of 35.9 kPa predicted by Eq. (37) for the LL water content is that the shearbox calibration data included testing by [15] of some undisturbed/intact test-specimens for water contents at the higher end of the I_L range investigated.

Table 1 Equivalent two-parameter form of the Authors' three-parameter S_u – I_L models

Three-parameter model		Equivalent two-parameter form
$S_{u,VS} = 1.9 \exp(3.8 - 4I_L)$	Authors' Eq. (34)	$S_{u,VS} = 84.9 \exp(-4I_L) = 84.9 \times 10^{-1.73I_L}$
$S_{u,FC} = 1.92 \exp(3.8 - 4.65I_L)$	Authors' Eq. (35)	$S_{u,FC} = 85.8 \exp(-4.65I_L) = 85.8 \times 10^{-2.02I_L}$
$S_{u,TC} = 0.32 \exp(3.8 - 5.55I_L)$	Authors' Eq. (36)	$S_{u,TC} = 14.3 \exp(-5.55I_L) = 14.3 \times 10^{-2.41I_L}$
$S_{u,SB} = 0.35 \exp(5.64 - 1.01I_L)$	Authors' Eq. (37)	$S_{u,SB} = 98.5 \exp(-1.01I_L) = 84.9 \times 10^{-0.44I_L}$

Table 2 Examples of earlier reported two-parameter S_u – I_L relationships expressed here in the form $S_u = a \exp(-bI_L)$

Reference	S_u – I_L expression	Equivalent two-parameter model form
Wroth and Wood [13]	$S_u = 1.7 \times 10^{(2-2I_L)}$ [reported as Eq. (3) in Authors' paper]	$S_u = 170 \exp(-4.6I_L)^a$
Vardanega and Haigh [11]	$S_{u,FC} = 1.7 \times 35^{(1-I_L)^b}$ [reported as Eq. (6) in Authors' paper]	$S_{u,FC} = 59.5 \exp(-3.55I_L)$
Kayabali et al. [3]	$S_{u,VS} = 96 \times 0.187^{I_L^c}$ [reported as Eq. (7) in Authors' paper]	$S_{u,VS} = 96 \exp(-1.68I_L)$
Yang et al. [14]		$S_u = 159.6 \exp(-3.97I_L)$

^aWroth and Wood [13] reported their model in this format; ^b for $0.2 < I_L < 1.1$; ^cfor $46\% < LL < 91\%$

Furthermore, regarding the model prediction comparisons presented in Figs. 4–6 of the paper under discussion, the Authors used best-fit values of the A , B , and C parameters in applying their models, but they employed the published values of the parameters for the other models (fit functions) investigated. This approach does not allow fair comparisons.

Other observations by the Discussers are presented briefly, as follows:

- The fall-cone to Casagrande-cup derived LL (i.e., $LL_{FC,ASTM}$ to $LL_{cup,ASTM}$) correlation report as the second part of Eq. (10) in the Authors' paper actually pertains to measurements obtained in accordance with the British standard LL (i.e., $LL_{FC,BS}$) and ASTM standard LL (i.e., $LL_{cup,ASTM}$). That is, the ASTM standards (i.e., ASTM D4318 [1]) only allows the Casagrande-cup test, *not* the fall-cone test, for LL determination. Furthermore, both correlations given as Eq. (10) in the Authors' paper were deduced by O'Kelly et al. [9] from statistical analysis of a large database assembled for dissimilar fine-grained soils having LL values of up to 600%, with other power relationships having slightly different coefficients also reported in [9] for $LL_{FC,BS}$ ($LL_{cup,ASTM}$) $< 120\%$.
- The value of the cone factor (K) used in Hansbo's [2] fall-cone equation (presented as Eq. (11) in the Authors' paper) is affected by the cone's apex angle and surface texture, but not (as was reported in page 4327 of the Authors' paper) on the cone's volume.
- The plots for Fig. 6g, h are missing from the Authors' paper.

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Data availability No datasets were generated during the current study.

Declarations

Conflict of interest The authors declare no competing interests.

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