Glueball matrix elements on anisotropic lattices

Kentucky Glueball Collaboration: S.J. Dong,[a] T. Draper[a], W.C. Kuo,[b] K.F. Liu[a], C. Nenkov[a]*, M. Peardon[a], J. Sloan[a], and B.L. Young[c] †

[a]Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506, USA.

[b]Department of Physics, National Sun Yat-Sen University, Kaohsiung 804, Taiwan.

[c]Department of Physics and Astronomy, Iowa State University, Ames, IA 50011, USA.

We describe a lattice calculation of the matrix elements relevant for glueball production in J/ψ radiative decays. The techniques for such a calculation on anisotropic lattices with an improved action are outlined. We present preliminary results showing the efficacy of the computational method.

1. MOTIVATION

Glueballs, apart from being one of the most exotic states predicted from QCD, are crucial in completing the picture of the strong interaction, since these mesonic states go beyond the standard quark model. Unfortunately there is a considerable controversy as to their phenomenological identification. Even for the low-lying states there are a lot of experimental candidates:

- 0^{++} — $\left\{ \begin{array}{l} f_0(1500) \text{ from } p\bar{p} \text{ annihilation,} \\ f_J(1710) \text{ from radiative } J/\psi \\ \qquad\qquad \text{decay} \end{array} \right.$

- 0^{-+} — $\eta(2225)$

- 2^{++} — $\left\{ \begin{array}{l} f_2(1710) \text{ and } f_2(2230) \text{ from} \\ \text{radiative } J/\psi \text{ decay} \end{array} \right.$

Recently, there appears to be strong experimental evidence that glueballs have been detected [1]. For years radiative J/ψ decays,

$$J/\psi \longrightarrow \gamma + G \tag{1}$$

have been considered [2] the best hunting ground for low lying glueballs (i.e. those below 3 GeV).

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The main objective of this work is to measure vacuum-to-glueball transition matrix elements which are needed for estimating the partial widths of J/ψ radiative decays into glueballs [3]. We employ an $O(a_s^2)$-improved $SU(3)$ gauge action on an anisotropic lattice [4], with the temporal lattice spacing much less than the spatial one. This has been shown [5] to be an efficient means of studying the masses of glueballs, as clearer resolution of the decay of correlators is allowed. The use of this technique in a matrix element calculation seems highly advantageous as the local current operator introduces more UV noise into Monte-Carlo measurement.

2. METHOD

The matrix elements we compute are of the form $\langle G|F^a_{\mu\nu}F^a_{\rho\sigma}|0\rangle$, where G is the $0^{++}, 0^{-+}$ or 2^{++} glueball and $F_{\mu\nu}$ is the gluon field strength operator.

Specifically, we would like to extract the matrix elements for the three lightest glueball states that can be created by dimension-4 gluonic operators. These are;

- 0^{++} — Scalar

$$\langle G|\mathrm{Tr}E^2|0\rangle \quad \text{and} \quad \langle G|\mathrm{Tr}B^2|0\rangle, \tag{2}$$

- 0^{-+} — Pseudoscalar

$$\langle G|\mathrm{Tr}E\cdot B|0\rangle, \tag{3}$$

- 2^{++} — Tensor

$$\langle G|\mathrm{Tr}E_iE_j|0\rangle \quad \text{and} \quad \langle G|\mathrm{Tr}B_iB_j|0\rangle. \tag{4}$$

Previous studies have shown that smooth interpolating fields give better overlap with the glueball states. For each lattice irrep of interest, we compute the correlation matrix for a large basis, $\{\phi_i\}$, of smeared and fuzzed operators. For every member of this basis, the correlation with the appropriate gluon field-strength operators of Eqns. (2–4) is also computed. Variational techniques are then employed to find the coefficients, c_i, needed to make an optimal ground-state creation operator, $\Phi(t) = \sum_i c_i\phi_i(t)$. The relevant correlators required to extract both the glueball mass and creation matrix element are then formed. These are the smeared-smeared operator

$$\langle 0|\Phi(t)\Phi(0)|0\rangle - \langle 0|\Phi|0\rangle^2 \xrightarrow{t\to\infty} \langle 0|\Phi|G\rangle\langle G|\Phi|0\rangle e^{-M_G t} \tag{5}$$

and the local-smeared operator

$$\langle 0|\Phi(t)F^2(0)|0\rangle - \langle 0|\Phi|0\rangle\langle 0|F^2|0\rangle \xrightarrow{t\to\infty} \langle 0|\Phi|G\rangle\langle G|F^2|0\rangle e^{-M_G t}, \tag{6}$$

where $|G\rangle$ is the lowest glueball state in the irrep. The vacuum subtraction denoted here need only be performed for the A_1^{++} irrep. A three-parameter fit to these correlators then extracts the glueball mass M_G and the desired glueball creation matrix element $\langle G|F^2|0\rangle$, as well as the glueball interpolation field matrix element $\langle 0|\Phi|G\rangle$.

The construction of the appropriate improved gluonic operators on the lattice proceeds as follows. For a set of prototype small Wilson loop shapes (for example, the plaquette, 2×1 rectangle, etc.) a linear combination of the 48 orientations P_j (some will be degenerate for loops with symmetry) of each prototype, P, is taken which transforms under the appropriate irrep, R, of the cubic group (including parity). The irreps we need to include are thus the A_1^+, A_1^-, E^+ and T_2^+. The real part of the trace on these loops is then taken, as all the glueball states of interest are positive under charge conjugation. The

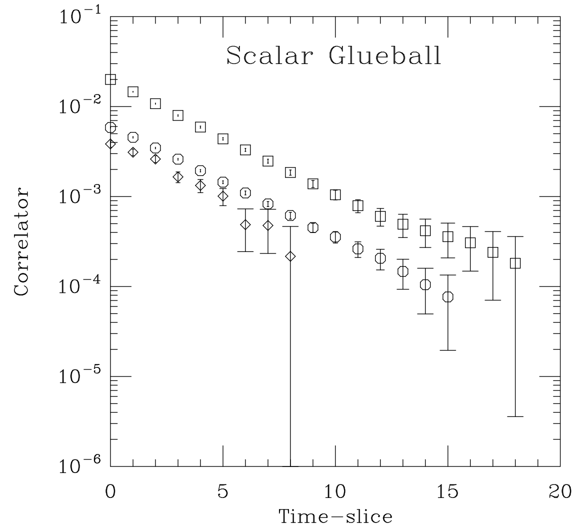

Figure 1. A_1^{++} (scalar) correlators. The $\langle\Phi|\Phi\rangle$, $\langle\mathrm{Tr}B^2|\Phi\rangle$ and $\langle\mathrm{Tr}E^2|\Phi\rangle$ data are indicated by $\square$, $\circ$ and $\diamond$ respectively. The $\langle\Phi|\Phi\rangle$ correlator has been rescaled by 2×10^{-2} for presentation.

lattice operators are

$$C_i^{(R)}(P) = \sum_{j=1\ldots 48} a_{ij}^{(R)}\mathrm{ReTr}(1-U(P_j)), \tag{7}$$

where the index i runs over the dimension of the irrep. Then, using the standard mapping

$$U_\mu(x) = \mathcal{P}\exp\{-ig\int_x^{x+\hat\mu} A_\mu(s)\, ds_\mu\}, \tag{8}$$

the operators, $C_i^{(R)}(P)$, are expressed at tree-level in perturbation theory in terms of the dimension 4 and 6 continuum gluon field operators transforming under the same irrep of the cubic group. Once this expansion is performed for all prototypes, a linear combination of the lattice operators that removes the $\mathcal{O}(a_s^2)$ discretization errors is found. For the scalar and E^{++} polarizations of the tensor currents, this search is unnecessary, as conveniently, improving the action generates the required operators (Tr E_i^2 and Tr B_i^2).

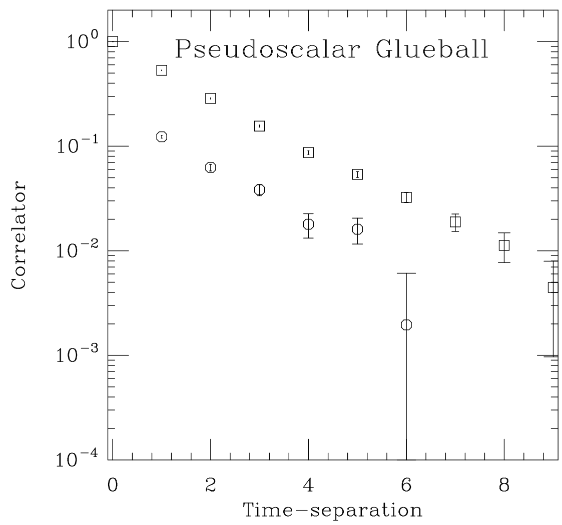

Figure 2. A_1^{-+} (pseudoscalar) correlators. The $\langle\Phi|\Phi\rangle$ and $\langle \mathrm{Tr} E\cdot B|\Phi\rangle$ data are indicated by $\Box$ and $\circ$.

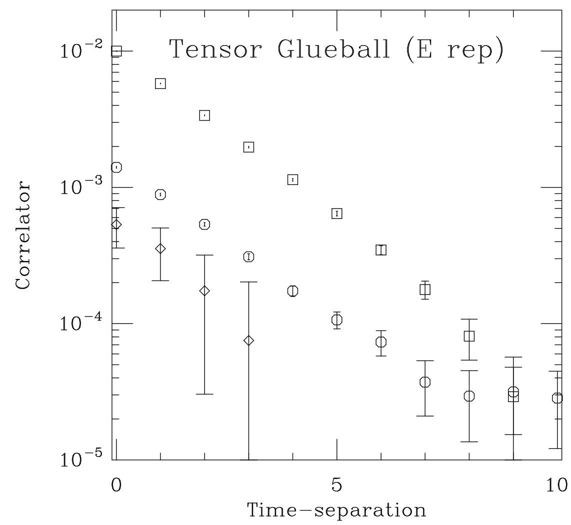

Figure 3. E^{++} (tensor) correlators. The $\langle\Phi|\Phi\rangle$, $\langle \mathrm{Tr} B_iB_j|\Phi\rangle$ and $\langle \mathrm{Tr} E_iE_j|\Phi\rangle$ data are indicated by $\Box$, $\circ$ and $\Diamond$ respectively. The $\langle\Phi|\Phi\rangle$ channel has been rescaled by 10^{-2}.

3. PRELIMINARY RESULTS

In Figs. 1, 2 and 3 we present results from a feasibility study on an $8^3\times 40$ lattice at $\beta = 2.4$ and anisotropy $a_s/a_t = 5$. These run parameters correspond [5] to a spatial lattice spacing of 0.221(11) fm and temporal cut-off of 4.5(2) GeV. For all channels considered, a signal for the smeared-local correlator can be observed over a sufficiently wide range of time slices to extract the relevant observables mentioned in section 2. Correlators with chromoelectric fields are seen to be significantly more noisy, however.

4. CONCLUSIONS

A calculation of the matrix elements relevant for predicting the branching ratio for radiative decays of J/ψ into glueballs has been proposed as a useful handle on the phenomenological identification of the lighter glueball candidates. This decay should be a glueball-rich channel, with fairly clear experimental signature. We have performed exploratory studies of the 0^{++}, 0^{-+} and 2^{++} (in the E irrep) correlator channels, on a lattice with 5:1 anisotropy and with spatial lattice spacing of $a_s \approx 0.2$ fm. They show that anisotropic lattice techniques will allow a Monte-Carlo calculation of the lattice matrix elements with reasonable computational resources.

In the future, we will calculate with various lattice sizes and spacings and take the infinite-volume and continuum limits. We will include heavier glueball states and all 20 irreps of O_h^{PC}. We will calculate the renormalization constants.

REFERENCES

1. See for example F. Close, these proceedings.
2. K.F. Liu, B.A. Li, and K. Ishikawa, Phys. Rev. D **40**, 3648 (1989).
3. Y. Liang *et al.*, Phys. Lett. B **307**, 375 (1993).
4. C. Morningstar, Nucl. Phys. B (Proc. Suppl.) **53**, 914 (1997); M. Alford *et al.*, in preparation.
5. C. Morningstar and M. Peardon, Phys. Rev. D **56**, 4043 (1997).