

Procurement Contracts under Limited Liability

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Abstract: This paper analyses procurement when contractors have limited liability and when the sponsor cannot commit to any specific form of future negotiation. It shows that introducing limited liability enhances competition and thus the likelihood of bankruptcy. Among efficient auctions in which only the winner gets paid, the commonly used first price auction is shown to give the lowest probability of bankruptcy. Finally, it shows that the characterisation of a mechanism minimising the project's cost results from trading-off bankruptcy costs with informational rents.

I INTRODUCTION

In procurement contracting, tenders must generally be submitted at a stage where there is still much uncertainty. Part of this uncertainty results from possible imperfect information regarding future costs or the exact characteristics of the product to be supplied. Another part is potentially associated with the sponsor being unable, or even unwilling, to commit to any specific future negotiation in the event of cost overruns. Indeed, the type of agreements that can be made in practice are often limited.

Under these circumstances, costs estimates and the level of liability are the only factors contractors can rely on when submitting tenders. Thus, in the absence of any form of future agreements, limited liability plays a crucial role as it determines the extent to which a contract can be enforced. It is then important, for those who decide on procurement rules, to understand how

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limited liability constraints affect the contractors' initial bidding decisions, as well as the outcome (e.g. the contracted price, the likelihood of bankruptcy) of the rules they set. This paper examines bidding under limited liability in the absence of specific future agreements. It highlights some possible consequences of limited liability constraints and compares the performance of different procurement rules. Although the analysis relies on a theoretical model, the results are informative from an applied point of view and of potential interest to policy makers.

In the literature, limited liability is often introduced as an ex post voluntary participation constraint. Formally, it states that the contract cannot be enforced whenever the contractor's profit falls below a certain level. Its implications then depend on the sponsor's ability and willingness to enforce the contract. In Riordan and Sappington (1988), the sponsor can either use contingent prices,¹ or can commit to negotiate a price in a second period, once all uncertainty is resolved. Contingent prices are set such that bankruptcy will not arise. Thus, they eliminate the possible distortions resulting from facing a potentially binding limited liability constraint. Unfortunately, in practice, it is often impossible for the sponsor to anticipate all future contingencies and therefore to write and rely on complete contracts. Without this possibility, limited liability constraints are likely to have a greater impact. When the price is negotiated ex post, an auction is used in a first period to select a contractor. To prevent the submission of very low tenders that would decrease the first period rent and make it impossible for the contractor to cover his cost, a minimal acceptable bid is imposed. These authors then show that the limited liability constraints then hold at the expense of efficiency. Indeed, in some cases, all contractors submit the minimal acceptable bid in the auction (this forms a so-called pooling equilibrium). In such a case, the auction does not allow one to identify and select the most efficient contractor. Committing to future negotiations is not always feasible and more importantly, not always desirable. A sponsor may not want to be locked in with a contractor who may then take advantage of the situation to increase his costs. Moreover, setting a minimal acceptable bid usually requires some information regarding future possible costs, the evolution of the project and so on. This is not always available to the sponsor. The following analysis discards both, the use of complete contracts and the possibility to commit to any *specific* form of future negotiation.

If all competitors have the same operating cost but differ in the level of their potential cost overruns, Spulber (1990) shows that the inability to enforce a

¹ Contracts setting contingent prices, i.e. prices for each and possible future events, are known as complete contracts.

contract can lead all bidders to propose the same bid. In such a situation, the efficiency and informativeness qualities of an auction are destroyed. The following analysis relies on a different model and shows in particular that an auction can preserve its qualities even in the absence of a specific renegotiation contract.

I consider a situation where a sponsor wants to hire a contractor to realise a project. The contractors are risk neutral and have limited liability. They differ in their relative efficiency, measured by the expected cost of realising the project. This expected cost is private information in the sense that each contractor knows only his own expected cost. The level of liability is the same for all; it is common knowledge and defined as the maximum amount of losses that can be sustained. As in Spulber (1990), it is assumed that the sponsor is unable to use complete contracts and cannot (or refuses to) commit to any form of future negotiation in the event of bankruptcy. Therefore, contractors have no information regarding what would happen in the event of a bankruptcy when submitting tenders. All they know is their expected cost and the maximum amount of losses they can face.

In the first part of this paper the performances of a first and second price sealed bid auction are analysed. In a first price auction, the selected contractor is the one proposing the lowest tender. He is paid his bid. It is the most common and frequently used form of auction in procurement. In a second price auction, the lowest tender still wins but the payment is equal to the second lowest bid. In a second part, I consider the problem of minimising the project's cost. The project's cost is given by the initial contracted price when it is high enough to avoid bankruptcy, otherwise, it is the contractor's realised cost to which are added some fixed bankruptcy costs for which the sponsor is accountable.²

The analysis of the first and second price auctions lead to the following conclusions. First, it shows that in the absence of any specific form of renegotiation, introducing limited liability enhances competition. In both auctions, tenders fall as the level of liabilities shrinks. This result is in accordance with the findings of Waehrer (1995) who analyses the consequences of imposing deposits paid by buyers who chose to default. He shows that a seller cannot gain by increasing the level of such deposits, as he would then lessen competition. Analogously, it is shown here that in the absence of limited liability, i.e. when the contract can be systematically enforced, competition is weaker. The intuition in either case is clear: as you oblige the contractors to face more risk (bear more losses) they will be more

² The project's cost should *not* be understood as the monetary transfer that the sponsor must pay to guarantee the project's completion. Analysing such an issue would require that the sponsor be able to commit to some sort of negotiation in the event of bankruptcy.

reluctant to be selected and therefore competition will be weaker. Still, in the present paper, less competition is shown not to be systematically negative as it also leads to a lower probability of bankruptcy (in expectation). Second, I prove that the expected contracted price will be higher in a first price auction. Thus, the probability of bankruptcy will be lower in a first price auction. Without limited liability the contracted price would have been the same in both auctions. However, because limited liability modifies the contractors' attitude towards risk (making them behave as risk lovers) the contracted prices will now differ. As the first price auction incorporates less risk (conditional on winning, uncertainty is eliminated) it generates less competition. The third conclusion states that in a second price auction there is always a strictly positive probability that the winning bidder will declare bankruptcy while in a first price auction, for a sufficiently low cost uncertainty, some low cost winning suppliers would never declare bankruptcy.

The analysis of cost minimisation is complex. I have been unable to characterise the exact characteristics of a cost-minimising auction. Still, it is shown that among all efficient mechanisms in which only the winner gets paid, the first price auction leads to the highest expected price. Thus, among efficient mechanisms with payment to the winner only, the first price auction is the one guaranteeing the lowest expected probability of bankruptcy. The feature of the first price auction triggering the result is its deterministic price. Because bidders behave as risk-lovers, they become more competitive (and thus lower their bids) as there is more uncertainty. Starting from a first price auction, any attempt to raise the expected contracted price by adding uncertainty in the payment will generate lower bids. Due to the risk-loving attitude the decrease in the bids will cause the expected price to fall. Thus, there is no mechanism including a random payment that will generate an expected price higher than the first price auction. Finally, a last result highlights an interesting consequence of bankruptcy. Because bankruptcy is informative, it permits saving on informational rents. These rents are the profits that must be left to a contractor to give him the proper incentive to reveal his cost through his bid (i.e. these rents permit to avoid a pooling equilibrium). The more efficient a contractor is, the more informational rents he can extract. By setting a lower bound to the contractor's profits, limited liability also sets an upper bound to the amount of informational rents that must be given away. However, bankruptcy may also generate costs for the sponsor due to delays in completion. Generally, it is shown that the cost minimising mechanism results from trading-off bankruptcy costs with informational rents.

We may now proceed with the analysis. The following section presents the model. Section II deals with the first and second price auctions. Section III

considers the cost minimisation problem. Finally Section IV presents a conclusion.

II THE MODEL

Suppose a sponsor (e.g. the government) wants to realise a project. He faces n risk-neutral, ex ante identical, contractors (or firms). The ex post cost of the project to each contractor, denoted by c_i with $i = 1, \dots, n$ is composed of a random component \tilde{s} and a firm's specific component. More precisely let:

$$c_i = \theta_i + \tilde{s}. \quad (1)$$

The random variable \tilde{s} captures the imperfect information a firm has about the cost of the project ex ante. It models the cost of all unpredictable events that may occur during the construction phase. Let \tilde{s} be distributed on the interval $[-S, +S]$, for some $S > 0$. Let $G(\cdot)$ denote the distribution function, and $g(\cdot)$ the density. Without loss of generality assume that

$$E(\tilde{s}) = \int_{-S}^S sg(s) ds = 0$$

The distribution of \tilde{s} is common knowledge.

The expected cost θ_i , referred to as firm i 's type, measures contractor i 's relative efficiency. The lower is θ_i the more efficient is contractor i with respect to its competitors. This variable is private information. Beliefs are formed considering that the variables θ_i ($i=1, \dots, n$) are independent draws from a common distribution function, $F(\cdot)$ defined on the interval $[\underline{\theta}, \bar{\theta}]$ such that $\underline{\theta} - S > 0$. The density function, $f(\cdot)$ is assumed to be continuous.

I assume that firms have limited liability. Formally, I consider that there exists a maximal amount of losses that they can be accounted for. In other words, their profits are bounded below. For tractability I assume that the minimum profit is given by $-A$ for all firms. The use of performance bonds provides a motivation for this symmetry. For projects large enough the sponsor generally requires that the competitors buy a performance bond. These are exercised in the event of non-performance. The cover required is generally left at the discretion of the contracting authority and is generally the same for all competitors.³ In that context A can be thought of as the value of the bond a

³ The rules for public procurement in Ireland stipulate the following: *For contracts over 200,000IRP Performance Bonds should generally be provided. The provision of the Bond, and the level of cover required, should be at the discretion of the Contracting Authority, having regard to all factors concerning the project and the need to protect the financial interests of the Contracting Authority.*

contractor must post, and therefore his maximum liability. The amount A is common knowledge.

The timing of the game is the following. Initially, the sponsor calls for tenders for a specific project. Even though she knows that competitors have imperfect information regarding the project's cost, the sponsor cannot commit to contingent prices or to any form of renegotiation at this stage. Then, the competing firms learn their expected cost and bid. Finally, the cost realises. In the absence of commitment ability all the contractors know is the lower bound of their profit, set by the limited liability constraint. Similarly, all the sponsor knows is that the cost will be covered up to a certain limit.

III FIRST AND SECOND PRICE SEALED BID AUCTIONS

In the absence of limited liability, the risk neutrality and the independent private value assumptions lead to the well-known revenue equivalence theorem. The introduction of limited liability breaks this result by modifying the contractors' attitude towards risk. Thus auctions, such as the first and second price sealed bid auctions, that would be otherwise equivalent lead to different outcomes. In this section I will analyse bidding in these two, commonly used, auctions. In both, the first price sealed bid auction (*FPA*) and the second price sealed bid auction (*SPA*) the winner is the lowest bid. The contracted price (denoted P) is equal to the winning bid in a *FPA*, and to the second lowest bid in a *SPA*.

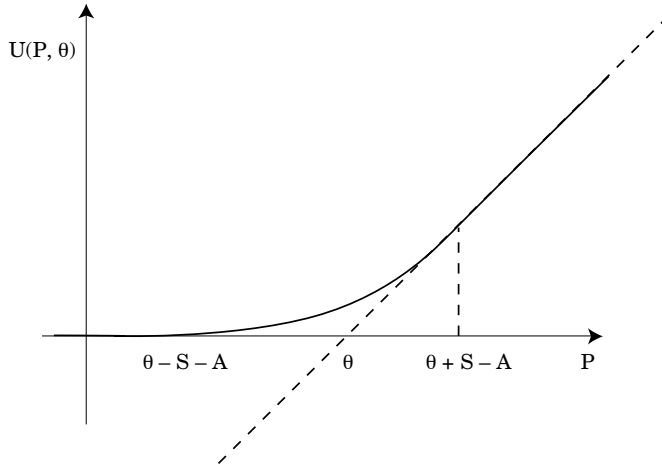
A contractor's bid maximises his expected profit where the expectation is taken over his competitors' types and over the cost uncertainty. Let $u(P, \theta)$ be the profits to a type θ firm when selected and when the contracted price is P . Under limited liability and in absence of commitment, we have:

$$u(P, \theta) = E_{\tilde{s}} [\max \{-A, P - (\theta + \tilde{s})\}] \quad (2)$$

where $E_{\tilde{s}}$ denotes the expectation taken over the cost uncertainty. This function is continuous and continuously differentiable in P and θ (see Appendix 1). The following notation will be adopted: u_k denotes the first derivative with respect to the argument k ($k = 1, 2$), u_{ks} denotes the first derivative with respect to argument k ($k = 1, 2$) and second derivative with respect to argument s ($s = 1, 2$). Figure 1 gives a clearer understanding how profits are modified under limited liability.

The dashed line corresponds to the profit function in absence of limited liability. As limited liability is introduced the function $u(P, \theta)$ becomes convex in P ($u_{11} \geq 0$). This means that the risk-neutral contractors *behave* as risk-lovers under limited liability.

Figure 1: $u(P, \theta)$ for a given θ .



Spulber (1990) shows how the absence of commitment can lead to a pooling equilibrium that destroys the benefits of using an auction. As shown in this lemma, this result cannot be generalised.

Lemma 1 *A pooling equilibrium cannot arise unless the minimum level of profit is zero ($A=0$).*

Proof: Suppose there exists a pooling equilibrium where all contractors submit the same offer, called b , and each wins with probability $1/n$. If for some θ , $u(P, \theta) > 0$, then such types would be better-off decreasing slightly their bid (to $b - \epsilon$) so as to be selected for sure and get a positive expected profit. If for some θ , $u(P, \theta) < 0$, then such types would be better-off increasing their bid not to be selected and get a zero profit instead of a negative expected profit. Thus in equilibrium we can only have $u(P, \theta) = 0$, for all θ . But the level of P satisfying $u(P, \theta) = 0$ depends on θ . Therefore a pooling equilibrium cannot exist.

When $A = 0$ bidding $b = \underline{\theta} - S$ (or anything below) can form a pooling equilibrium. In this case, contractors make no profits but cannot improve their situation by bidding more as they would never be selected if they did so.

Proposition 1 *In a SPA a monotone, symmetric dominant strategy equilibrium bidding function, denoted $B_2(\theta)$, is given by:*

$$u(B_2(\theta), \theta) = 0 \tag{3}$$

Proof: See Appendix 2.

In a SPA a bid only sets a lower bound for the price at which they may be contracted. Due to competition, the lowest price they can accept is the one that would generate zero expected profit.

Proposition 2 *In a FPA, if $\frac{u_1(P, \theta)}{u(P, \theta)}$ is increasing in θ , then there exists a monotone, symmetric equilibrium bidding function, $B_1(\theta)$, that is characterised by*

$$B_1'(\theta)u_1(B_1(\theta), \theta) - \frac{(n-1)f(\theta)}{1-F(\theta)}u(B_1(\theta), \theta) = 0 \quad (4)$$

and

$$u(B_1(\bar{\theta}), \bar{\theta}) = 0 \quad (5)$$

Proof: see Appendix 3.

A major difference between the first and second price auction is described in the following proposition.

Proposition 3 *In a SPA there is always a strictly positive probability that the winning bidder will declare bankruptcy (unless bidders have no liability constraint, i.e. unless $A > S$). In a FPA, for a sufficiently low cost uncertainty some low cost winning suppliers would never declare bankruptcy.*

Proof: see Appendix 4.

To illustrate the latter proposition consider a simple setting where \tilde{s} is uniformly distributed and $A = 0$. In such a case, the SPA equilibrium bidding function is given by:

$$B_2(\theta) = \theta - S.$$

In a FPA a monotone, symmetric equilibrium bidding function is given by:

$$B_1(\theta) = \theta + \frac{S(1-F(\theta^*))^{n-1} + \int_{\theta}^{\theta^*} [1-F(t)]^{n-1} dt}{[1-F(\theta)]^{n-1}} \text{ for } \theta \leq \hat{\theta}, \quad (6)$$

and

$$B_1(\theta) = \theta - S + \frac{\int_{\theta}^{\hat{\theta}} [1 - F(t)]^{\frac{n-1}{2}} dt}{[1 - F(\theta)]^{\frac{n-1}{2}}} \text{ for } \theta \geq \hat{\theta}, \quad (7)$$

where $\hat{\theta} = \max\{\underline{\theta}, \theta^*\}$ where θ^* satisfies $B_1(\theta^*) = \theta^*$. Thus, all types below θ^* will always complete the project for no more than the contracted price. For types above θ^* limited liability may bind depending on the realisation of the shock. Whether $\theta^* \geq \underline{\theta}$ depends on the value of S . As the cost uncertainty increases, more types offer tenders below their highest possible cost.

In the absence of limited liability we know that the sponsor would be indifferent between the *FPA* and the *SPA*. Both mechanisms lead to the same expected contracted price and therefore to the same expected cost. This equivalence breaks down under limited liability.

Proposition 4 *The FPA leads to a higher expected contracted price than the SPA.*

Proof: see Appendix 5.

The proof shows that the expected price to each type (but type $\bar{\theta}$) in a *SPA* is at most equal to their bid in a *FPA*, and strictly below for some types (on the left of $\bar{\theta}$). The intuition is obvious. Risk-lover bidders value more the marginal increase in the rent resulting from slightly increasing their bid than the resulting marginal decrease in the probability of winning. Thus, as they have more control over their rent in a *FPA*, bidders are less competitive than in a *SPA*.

From this proposition one must not conclude that the *SPA* should be systematically favoured. The contracted price is only one component of the cost of the project. Because the *SPA* leads to a lower expected contracted price, the likelihood of bankruptcy is higher (in expectation). If the sponsor faces high bankruptcy costs due to delays in completion, or the need of renegotiating, he would be better-off paying a higher price initially.

Finally, the following proposition formally states the implication of limited liability on the bidding behaviour.

Proposition 5 *Limited liability induces the firms to lower their bids in both the FPA and the SPA.*

Proof: See Appendix 6.

This result is obvious since losses are bounded below under limited liability. Thus, it gives all bidders an incentive to bid more aggressively. Still, as we stressed before, the change in the attitude towards risk triggers a countervailing incentive to the preceding one. (An increase in the marginal gain has more value than its associated decrease in the probability of winning.) This trade-off appears clearly when considering the close form solution given as an example. All bidders potentially facing a binding limited liability constraint bid according to:

$$B_1(\theta) = \theta - S + \frac{\int_{\theta}^{\bar{\theta}} [1 - F(t)]^{\frac{n-1}{2}} dt}{[1 - F(\theta)]^{\frac{n-1}{2}}} \quad (8)$$

This expression would be the equilibrium bidding function prevailing in absence of limited liability when a bidder has a (lower) cost equal to $(\theta - S)$ and faces (less) $\frac{n-1}{2}$ competitors. The first term reflects the greater competitiveness. Bidders somehow consider the lowest possible cost as their cost. The second reflects the countervailing incentive as it corresponds to the margin a risk neutral bidder would get when facing $\frac{n-1}{2}$ competitors instead of $n-1$. In equilibrium the first incentive dominates the second.

As bidders are more competitive, the contracted price falls when limited liability is introduced. Once again, this is not systematically positive for the sponsor, as it is then more likely that the contractor will not be able to cover the cost ex post. From that perspective, limited liability offers a rationale for the frequency and the size of cost overruns in public contracting.

IV COST MINIMISATION ANALYSIS

Consider now a situation where the sponsor initially designs a procurement contract, i.e. a rule that tells bidders whom will be selected and how much he will be paid as a function of the types they sequentially announce. Assume that the sponsor's goal is to minimise the (expected) cost of the project. This cost is given by the contracted price whenever it is sufficiently high to guarantee a profit above $-A$. Otherwise, the sponsor knows that the cost is given by the realisation of $(\theta_w + s)$ where θ_w , the winner's announced type and s are both initially random. For tractability, I consider mechanisms where only the selected firm gets paid.⁴

⁴ Extension to more complex mechanisms will be discussed.

Let $\hat{\Theta} = (\hat{\theta}_1, \dots, \hat{\theta}_n)$ be a vector of types (potentially different from the true types) and let $\hat{\theta}_{-i}$ be a vector of all types except bidder i 's type. In particular we have $\hat{\Theta} = (\hat{\theta}_{-i}, \hat{\theta}_i)$. Let Θ be the vector of true types. Formally, a contract is defined as a set of functions $\{x_i(\hat{\Theta}), P_i(\hat{\Theta})\}_{i=1, \dots, n}$. The function $x_i(\hat{\Theta})$ gives the probability of selecting firm i . The function P_i gives the price at which firm i is contracted if selected when $\hat{\Theta}$ is announced. Most rules will lead contractors to announce a type different from their true type. However, the revelation principle tells us that any rule has an equivalent direct contracting scheme in which contractors find it optimal to reveal their true type. Thus, among all mechanisms we only need to consider those who lead to revelations of true types: $\{x_i(\Theta), P_i(\Theta)\}_{i=1, \dots, n}$. Under incentive compatibility, the project's cost may then be written as:

$$T_i(\Theta) = \int_{-S}^{P_i(\Theta) - \theta_i + A} P_i(\Theta) g(\tilde{s}) d\tilde{s} + \int_{P_i(\Theta) - \theta_i + A}^S (\theta_i + \tilde{s} + C) g(\tilde{s}) d\tilde{s} \quad (9)$$

where C includes all bankruptcy costs for which the sponsor is accountable such as costs due to delays in completion.

Given a contracting scheme, the profit to firm i when announcing type $\hat{\theta}_i$ while others report truthfully is given by:

$$\pi(\hat{\theta}_i, \theta_i) = E_{\theta_{-i}} [x_i(\hat{\theta}_i, \theta_{-i}) u(P_i(\hat{\theta}_i, \theta_{-i}) \theta_i)]. \quad (10)$$

Let $\pi(\theta_i, \theta_i) \equiv \Pi(\theta_i)$. The cost-minimising contract must guarantee voluntary participation (VP) and incentive compatibility (IC). I will assume that outside opportunities are such that the reservation rents are zero. Thus VP holds if and only if

$$\Pi(\theta_i) \geq 0 \text{ for all } i.$$

IC holds if and only if:

$$\theta_i \in \arg \max_{\hat{\theta}_i} \pi(\hat{\theta}_i, \theta_i).$$

From (9) we can see that the cost-minimising contract depends on C . If bankruptcy is particularly costly to the sponsor, he should design a mechanism that limits bankruptcy, i.e. offer a high contracted price.

Proposition 6 *If $u_2(P, \theta)$ is convex in P then the FPA gives the highest expected contracted price among all efficient mechanisms in which only the selected firm gets paid.*

Proof: See Appendix 7.

The function $u_2(P, \theta)$ is convex in P when the distribution function $G(\cdot)$ is concave or linear. It means⁵ that the degree of risk lovingness increases with θ i.e. with inefficiency.

The *FPA* is not a direct mechanism in which it is optimal for bidders to tell their true type. However, as argued above, it has an equivalent incentive compatible direct mechanism that maximises the expected contracted price. The feature of the *FPA* (and of its equivalent *IC* mechanism) triggering the result is its deterministic price. In a *FPA* bidders know exactly what they will be paid if they win: their own bid. Therefore, conditional on winning, uncertainty is eliminated. Because bidders behave as risk-lovers, they are more competitive under more uncertainty. Starting from a *FPA*, any attempt to raise the expected contracted price by adding uncertainty in the payment will generate lower bids. Due to the risk-loving attitude the decrease in the bids will cause the expected price to fall. Thus, there is no mechanism including a random payment that will generate an expected price higher than the *FPA*'s expected price.

Unfortunately it is difficult to characterise the cost minimising mechanism for each possible value of C . Still, the following lemma highlights an interesting feature of such contracts.

Lemma 3 *The cost minimising contract results from trading off informational rents and renegotiation costs.*

Proof: The sponsor solves:

$$\min_{\{x_i(\Theta), P_i(\Theta)\}_{i=1, \dots, n}} \sum_{i=1}^n E_{\Theta}[x_i(\Theta) T_i(\Theta)] \quad (11)$$

subject to *(IC)*, *(VP)* and $\sum_{i=1}^n x_i(\Theta) \leq 1$ with $\forall i, x_i(\Theta) \in [0, 1]$.

By adding and substituting the cost, we can rewrite the objective function as:⁶

$$\min_{\{x_i(\Theta), P_i(\Theta)\}_{i=1, \dots, n}} \sum_{i=1}^n E_{\Theta}[x_i(\Theta) H_i(\Theta)]$$

where

$$H_i(\Theta) = \delta(P_i(\Theta), \theta_i) \left(\theta_i + \frac{F(\theta_i)}{f(\theta_i)} \right) + (1 - \delta(P_i(\Theta), \theta_i))(C + A + \theta_i) \quad (12)$$

and where $\delta(P_i(\Theta), \theta_i)$ is the probability of *not* facing a binding limited liability constraint:

⁵ See Lemma 1 in Maskin and Riley (1984).

⁶ See Myerson (1981) for a detailed procedure, as well as Appendix 8.

$$\delta(P, \theta) = G(P - \theta + A). \quad (13)$$

Interestingly, expression (12) tells us that with no commitment there is an upper bound to the informational rents that must be given away to guarantee incentive compatibility. When contracting under asymmetric information, a sponsor must give away some rents to guarantee a truthful revelation of types.

The expression $\left(\theta_i + \frac{F(\theta_i)}{f(\theta_i)}\right)$ is the so-called *virtual cost* from selecting bidder i .

It takes into account the informational rents to be left to firm i to guarantee incentive compatibility. Without limited liability these rents would systematically be paid. In the event of bankruptcy giving an amount A guarantees that the winner makes zero profits whatever he reveals. Thus, under bankruptcy paying A guarantees *IC*.

Finally the following results can easily be shown:

1. Setting a minimal acceptable bid in a first price auction would deter efficiency (provided the minimal acceptable bid is above $B_i(\underline{\theta})$). As a consequence the sponsor will not always benefit from using such a strategy since inefficient contractors are more likely to face bankruptcy.
2. The mechanisms allowing for payments to all competitors can increase competition (and thus lower the contracted price) if losers must pay a fee. Alternatively, giving a bonus to losers can increase the contracted price. This follows from the risk-loving attitude. To reduce competition the sponsor can provide some insurance to bidders. Lowering the gap between the gain upon losing and the gain upon winning is one way of doing so.

V CONCLUSION

Although limited liability together with the inability to commit to any form of future negotiation may still preserve the informativeness and efficiency qualities of an auction, they have non-negligible impacts. As it was shown, it leads to a more competitive behaviour in both a first and second price auction. Thus, bidders with limited liabilities face a higher risk of bankruptcy. This could explain the frequency of cost overruns and need of renegotiation in procurement. If we consider efficient auctions in which only the winner gets paid then a rationale for the use of a first price auction can be that it minimises bankruptcy (in expectation). It leads the sponsor to pay a higher price initially. Provided the cost uncertainty is low enough, it may lead the

most efficient bidders to never face bankruptcy. Finally, this paper shows that bankruptcy is not always negative as it permits savings on informational rents.

I have taken the view here that bidders differ in their efficiency level. Still, we know that other factors differentiate bidders. It would be interesting to extend this analysis to a model where contractors differ in their level of liabilities. If, as shown, the lowest offer is proposed by the one having the least liabilities, should he still be selected? How can auction rules be designed in such a case to make sure that renegotiation will never be needed?

VI APPENDIX

Appendix 1: Analysing the function $u(P, \theta)$.

We have:

$$u(P, \theta) = \begin{cases} -A & \text{if } P - \theta \leq -S - A \\ \hat{u}(P, \theta) & \text{if } P - \theta \in]-S - A, S - A[\\ P - \theta & \text{if } P - \theta \geq S - A, \end{cases}$$

where

$$\hat{u}(P, \theta) = \int_{-S}^{P-\theta+A} (P - (\theta + \tilde{s}))g(\tilde{s}) d\tilde{s} - A(1 - G(P - \theta + A))$$

$u(P, \theta)$ is continuous in P since

$$\lim_{P \rightarrow \theta - S - A} \hat{u}(P, \theta) = -A \text{ and } \lim_{P \rightarrow \theta + S - A} \hat{u}(P, \theta) = P - \theta.$$

Similarly, it is continuous in θ since

$$\lim_{P \rightarrow \theta + A + S} \hat{u}(P, \theta) = -A \text{ and } \lim_{\theta \rightarrow P + A - S} \hat{u}(P, \theta) = P - \theta.$$

Regarding first partial derivatives we have:

$$u_1(P, \theta) = \begin{cases} 0 & \text{if } P - \theta \leq -S - A \\ G(P - \theta + A) & \text{if } P - \theta \in]-S - A, S - A[\\ 1 & \text{if } P - \theta \geq S - A, \end{cases}$$

and

$$u_2(P, \theta) = \begin{cases} 0 & \text{if } P - \theta \leq -S - A \\ -G(P - \theta + A) & \text{if } P - \theta \in]-S - A, S - A[\\ -1 & \text{if } P - \theta \geq S - A. \end{cases}$$

Continuity of each partial derivatives is obvious.

Appendix 2: Proof of Proposition 1.

The function $u(P, \theta)$ is non-decreasing in P and non-increasing in θ . Therefore, the function $B_2(\theta)$, such that $u(B_2(\theta), \theta) = 0$, is non-decreasing. Consider now type θ . If she wins when bidding $B_2(\theta)$ her expected gain will be positive and there is no alternative bid that would make her better off. Indeed, a lower bid would have no effect while a higher bid increases the probability of losing and thus ending up with no revenue. If she loses when bidding $B_2(\theta)$ it means that the winning bid B^* is such that $u(B^*, \theta) \leq 0$. Therefore, undercutting that bid would make her worse-off as the expected gain could be negative. Once again, there is no alternative to $B_2(\theta)$ that could make her better off.

Appendix 3: Proof of Proposition 2.

Assume there exist a symmetric equilibrium bidding function $B_1(\theta)$. Assume it is an increasing and continuous function. Because type $\bar{\theta}$ never wins it must be the case that $u(B_1(\bar{\theta}), \bar{\theta}) = 0$ in equilibrium. Indeed if $u(B_1(\bar{\theta}), \bar{\theta}) > 0$, type $\bar{\theta}$ would be better off lowering its offer slightly so as to win a positive expected gain with a positive probability. If $u(B_1(\bar{\theta}), \bar{\theta}) < 0$ then, by continuity $u(B_1(\bar{\theta} - \varepsilon), \bar{\theta} - \varepsilon) < 0$ for ε small enough. Therefore, type $\bar{\theta} - \varepsilon$ would be better-off bidding $B_1(\bar{\theta})$ and never win.

If $B_1(\theta)$ is an equilibrium bidding function, the following must hold:

$$\theta \in \arg \max_t (1 - F(t))^{n-1} u(B_1(t), \theta)$$

The first order condition can be written as

$$(1 - F(t)) \left[\frac{u_1(B_1(t), \theta)}{u(B_1(t), \theta)} B_1'(t) - \frac{(n-1)f(t)}{1 - F(t)} \right] = 0 \text{ at } t = \theta$$

If $\frac{u_1(P, \theta)}{u(P, \theta)}$ is increasing in θ then

$$\frac{u_1(B_1(t), \theta)}{u(B_1(t), \theta)} B_1'(t) - \frac{(n-1)f(t)}{1 - F(t)} > 0 \text{ as } t > \theta$$

Therefore the function $(1 - F(t))^{n-1} u(B_1(t), \theta)$ reaches a maximum at $t = \theta$ as required.

Appendix 4: Proof of Proposition 3.

- The second price auction.

The equilibrium bidding function is such that $u(B_2(\theta), \theta) = 0$. Thus, any equilibrium bidding function is such that $B_2(\theta) \leq \theta$. Indeed, any bid greater than the expected cost ($b > \theta$) would be such that $u(b, \theta) > \theta$.

The only possibility to have, as a solution, $B_2(\theta) = \theta$ arises when $u(\theta, \theta) = 0$. Since the expected cost is equal to θ , $u(\theta, \theta) = 0$ when the bidder has enough liability to cover any potential loss (i.e. when $A \geq S$). If instead $A < S$, then $u(\theta, \theta) > 0$ and the equilibrium bid is such that $b < \theta$. In that case, $b - (\theta + S) < -A$ and the winning bidder will declare bankruptcy with a strictly positive probability.

- The first price auction.

Let $A = 0$. In a first price auction, the least efficient bidder still proposes a bid such that $u(B_1(\bar{\theta}), \bar{\theta}) = 0$. Thus, $B_1(\bar{\theta}) \in [\bar{\theta} - S, \bar{\theta}]$. The function $B_1(\theta)$ is such that $B_1'(\theta) < 1$ (see expressions (6) and (7) in the text). Thus, moving backwards from $\bar{\theta}$ it eventually cuts the function $(\theta + S)$. Provided S is small enough, there exists a unique $\theta^* \in [\underline{\theta}, \bar{\theta}]$ such that $B_1(\theta^*) = \theta^* + S$ and such that $B_1(\theta) > \theta + S$ for all $\theta < \theta^*$. As A increases, bids increase as bidders have to cover more losses. Thus, the range of bidders bidding above their highest cost increases.

Appendix 5: Proof of Proposition 4⁷

Let $b_2(\theta)$ be the expected contracted price paid to type θ in a SPA. We will show that $b_2(\theta) \leq B_1(\theta)$ for all θ and holds strictly for at least one type $b_2(\theta) < B_1(\theta)$. Formally:

$$b_2(\theta) = \int_{\theta}^{\bar{\theta}} B_2(t) \frac{(n-1)(1-F(t))^{n-2}f(t)}{(1-F(\theta))^{n-1}} dt$$

Differentiating both sides with respect to θ , we get

$$b_2'(\theta) = \frac{(n-1)f(\theta)}{1-F(\theta)} (b_2(\theta) - B_2(\theta)).$$

While, for a FPA we have

$$B_1'(\theta) = \frac{(n-1)f(\theta)}{1-F(\theta)} \frac{u(B_1(\theta), \theta)}{u_1(B_1(\theta), \theta)}.$$

Furthermore $B_1(\bar{\theta}) = B_2(\bar{\theta}) = b_2(\bar{\theta})$. Therefore, showing that $B_1'(\theta) \leq b_2'(\theta)$ whenever $B_1(\theta) = b_2(\theta)$ proves that $B_1(\theta)$ is (almost) everywhere above $b_2(\theta)$.⁸ We need to show that

$$\frac{u(B_1(\theta), \theta)}{u_1(B_1(\theta), \theta)} \leq B_1(\theta) - B_2(\theta). \quad (14)$$

⁷ This proof follows the steps of Maskin and Riley (1984) proof of theorem 4.

⁸ Starting at $\bar{\theta}$ we compare the functions for values to the left of $\bar{\theta}$.

Consider the right and left hand sides as functions of b where $b \equiv B_1(\theta)$. Note that at $b = B_2(\theta)$ both sides vanish. The derivative of the left-hand side (with respect to b) is $1 - \frac{u_{11}u}{u_1^2}$. The derivative of the right-hand side (with respect to b) is 1. Because $u_{11} \geq 0$, and because $b = B_1(\theta) > B_2(\theta)$, (14) holds. Thus, $b_2(\theta) \leq B_1(\theta)$ for all θ . Moreover for types facing a potentially binding limited liability constraint $u_{11} > 0$. This is the case for all types close enough to $\bar{\theta}$. Therefore there exists $\varepsilon > 0$ such that $b_2(\bar{\theta} - \varepsilon) < B_1(\bar{\theta} - \varepsilon)$.

Appendix 6: Proof of Proposition 5.

The *FPA* equilibrium bidding function without limited liability, $b_1(\theta)$ solves:

$$b'_1(\theta) = \frac{(n-1)f(\theta)}{1-F(\theta)} (b_1(\theta) - \theta)$$

and $b_1(\bar{\theta}) = \bar{\theta}$.

Type $\bar{\theta}$ always submits a bid \bar{b} such that $u(\bar{b}, \bar{\theta}) = 0$. Because limited liability gives a lower bound to $u(P, \bar{\theta})$ we have $B_1(\bar{\theta}) \leq b_1(\bar{\theta})$. Moreover, for all P and θ , $u_1(P, \theta) \in [0, 1]$ and $u(P, \theta) \geq P - \theta$. Thus, for all θ such that $B_1(\theta) = b_1(\theta) = b$, we have $B'_1(\theta) \geq b'_1(\theta)$. Thus it is true that $B_1(\theta) \leq b_1(\theta)$ for all θ .

Appendix 7: Proof of Proposition 6.

I show that if $u_2(P, \theta)$ is convex in P , then the first price auction is the one giving rise to the higher expected contracted price, among all mechanisms selecting the most efficient firm. The proof consists in showing that the *FPA* is the mechanism that maximises the bidders revenue. It is done in 5 steps. In what follows I refer to the expected contracted price as the price at which the selected firm is contracted given its type. Finally, the contracted price is the expectation of this price over the winner's type. At some stage, this proof relies on the necessary and sufficient conditions that guarantee *IC*. These will be found in Appendix 8.

Step 1: In a *FPA* the equilibrium bidding function is uniquely defined. Moreover, there does not exist any other efficient mechanism in which the expected contracted price is deterministic. Indeed, it can be shown that if the contracted price is, instead of the winning bid, any monotone transformation of this bid then we will have as an equilibrium bidding function the corresponding monotone transformation of the bidding function. Hence any rules that are monotone transformations of the *FPA* are equivalent to the *FPA*. No monotone transformations are excluded since I restrict attention to efficient rules.

Step 2: By construction we have

$$u_2(P, \theta) = -G(P - \theta + A)$$

Thus, $u_2(P, \theta)$ is convex in P if and only if $G(\cdot)$ is concave. This requirement is not satisfied around $P = \theta - S - A$. However, there is no loss in generalities from considering among all *IC* mechanisms including a random payment only the ones where $P_i(\theta) \geq \theta_i - S - A$.

Proof: Consider an *IC* payment scheme such that for some realisations of θ_i . Define $\hat{P}_i(\theta)$ such that:

$$\hat{P}_i(\theta) = \begin{cases} P_i(\theta) & \text{when } P_i(\theta) \geq \theta_i - S - A, \\ \theta_i - S - A & \text{when } P_i(\theta) < \theta_i - S - A. \end{cases}$$

We obviously have $\forall \theta_i, \pi(P_i(\theta), \theta_i) = \pi(\hat{P}_i(\theta), \theta_i)$, since any payment $P_i(\theta) < \theta_i - S - A$ gives a rent equal to $-A$ to the contractor.

Step 3: In any *IC* mechanism the bidders' revenue is continuous. This can be shown following Maskin and Riley (1984).

Step 4: The bidders' revenue is maximised in a *FPA*.

Let $\Pi_1(\theta_i)$ and $\tilde{\Pi}(\theta_i)$ be the gains of a type θ_i firm resulting from a *FPA* and any other *IC* mechanism including a random payment, $\tilde{P}(\theta)$, respectively. Let $P_1(\theta_i)$ and $E\tilde{P}(\theta_i)$ denote the expected contracted price of each mechanism. Note that in a *FPA*, the contracted price is deterministic and thus equal to the expected contracted price. We have:

$$E\tilde{P}(\theta_i) = \int_{\theta_i}^{\bar{\theta}} \tilde{P}(\theta_i) \frac{f(\theta_i)}{(1 - F(\theta_i))^{n-1}} d\theta_{-i}.$$

I will show that whenever $\Pi_1(\theta_i) = \tilde{\Pi}'_1(\theta_i) < \tilde{\Pi}'(\theta_i)$. Given that $\Pi_1(\bar{\theta}) = \tilde{\Pi}(\bar{\theta}) = 0$ in any *IC* mechanism, and given that the gains are continuous, this implies that $\Pi_1(\theta_i) > \tilde{\Pi}(\theta_i)$ for all θ_i . (Remember that we are interested in what happens to the *left* of $\bar{\theta}$.)

Since contractors act as risk lovers, the rents from a deterministic (*FPA*) mechanism equal the rents from a random mechanism only if the expected contracted price in the deterministic mechanism is higher. In other words:

$$\Pi_1(\theta_i) = \tilde{\Pi}(\theta_i) \Rightarrow P_1(\theta_i) > E\tilde{P}(\theta_i)$$

Recall that the *FOC* for *IC* gives (see Appendix 8):

$$\Pi'(\theta_i) = \int_{\theta_i}^{\bar{\theta}} u_2(P, \theta) f(\theta_{-i}) d\theta_{-i}. \quad (15)$$

Since $u_2(P, \theta)$ is decreasing in P , we have:

$$P_1(\theta_i) > E\tilde{P}(\theta_i) \Rightarrow u_2(P_1(\theta_i), \theta_i) \leq u_2(E\tilde{P}(\theta_i), \theta_i) \quad (16)$$

If, in addition, $u_2(P, \theta)$ is convex in P , we have, using Jensen's inequality:

$$u_2(E\tilde{P}(\theta_i), \theta_i) \leq E_{\tilde{P}}[u_2(\tilde{P}(\theta), \theta_i) / \theta_i = \min_j \theta_j] \quad (17)$$

Given (15), (16) and (17)

$$P_1(\theta_i) > E\tilde{P}(\theta_i) \Rightarrow \Pi'_1(\theta_i) < \tilde{\Pi}'(\theta_i)$$

Step 5: If $u(P, \theta)$ is convex in P and given that the bidders' gain increase with the expected contracted price, we have:

$$\begin{aligned} \Pi_1(\theta_i) > \tilde{\Pi}(\theta_i) &\Rightarrow u(P_1(\theta_i), \theta_i) \geq u(E\tilde{P}(\theta_i), \theta_i) \\ &\Rightarrow \forall \theta_i, P_1(\theta_i) \geq E\tilde{P}(\theta_i) \end{aligned}$$

Thus, at any point the *FPA* gives a higher expected contracted price to firm i .

Appendix 8: Necessary and Sufficient Conditions for IC.

Lemma 2: A mechanism $\{x_i(\Theta), P_i(\Theta)\}_{i=1, \dots, n}$ satisfies IC if and only if

$$\Pi'(\theta_i) = - \int_{\theta_{-i}} x_i(\Theta) \delta(P_i(\Theta), \theta_i) f(\theta_{-i}) d\theta_{-i}, \quad (18)$$

and

$$\int_{\theta_{-i}} \frac{\partial}{\partial r} [x_i(r, \theta_{-i}) \delta(P_i(r, \theta_{-i}), \theta_i)] f(\theta_{-i}) d\theta_{-i} \leq 0 \quad (19)$$

Proof:

(\Rightarrow) By definition, an IC mechanism is such that

$$\theta_i \in \arg \max_r \pi(r, \theta_i) \quad (20)$$

where

$$\pi(r, \theta_i) = \int_{\underline{\theta}}^{\bar{\theta}} x_i(r, \theta_{-i}) u(P_i(r, \theta_{-i}), \theta_i) f(\theta_{-i}) d\theta_{-i} \quad (21)$$

Using the envelope theorem, we have:

$$\Pi'(\theta_i) = \left. \frac{\partial \pi(r, \theta_i)}{\partial \theta_i} \right|_{r=\theta_i} \quad (22)$$

Since $u_2(P, \theta) = -\delta(P, \theta)$, where $\delta(P, \theta)$ is defined by (13), (22) is equivalent to (18).

The second order condition is:

$$\left. \frac{\partial^2 \pi(r, \theta_i)}{\partial r^2} \right|_{r=\theta_i} \leq 0.$$

Recall the first order condition:

$$\left. \frac{\partial \pi(r, \theta_i)}{\partial r} \right|_{r=\theta_i} = 0.$$

Differentiating the above with respect to r on both sides, we get:

$$\left. \frac{\partial^2 \pi(r, \theta_i)}{\partial r^2} \right|_{r=\theta_i} + \left. \frac{\partial^2 \pi(r, \theta_i)}{\partial r \partial \theta_i} \right|_{r=\theta_i} = 0.$$

Thus, the second order condition can be written as:

$$\left. \frac{\partial^2 \pi(r, \theta_i)}{\partial r \partial \theta_i} \right|_{r=\theta_i} \geq 0,$$

which leads to (19).

(\Leftarrow) Consider a mechanism such that (18) and (19) hold. Integrating (18) we get:

$$\Pi(s) - \Pi(r) = - \int_s^r \left[\int_{\underline{\theta}}^{\bar{\theta}} x_i(t, \theta_{-i}) \delta(P_i(t, \theta_{-i}), t) f(\theta_{-i}) d\theta_{-i} \right] dt \quad (23)$$

Moreover, by construction, we have:

$$\begin{aligned} \pi(r, s) - \Pi(r) = \\ \int_{\underline{\theta}}^{\bar{\theta}} x_i(r, \theta_{-i}) [u(P_i(r, \theta_{-i}), s) - u(P_i(r, \theta_{-i}), r)] f(\theta_{-i}) d\theta_{-i}. \end{aligned} \quad (24)$$

We can rewrite (24) as

$$- \int_r^s \left[\int_{\underline{\theta}}^{\bar{\theta}} x_i(r, \theta_i) \delta(P_i(r, \theta_i), t) f(\theta_{-i}) d\theta_{-i} \right] dt$$

Using (19) it is then trivial to show that, given r and s ,

$$\Pi(s) - \Pi(r) \geq \pi(r, s) - \Pi(r)$$

which implies *IC*.

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