Polarisation distribution for Internal Conical Diffraction and the Superposition of Zero and First Order Bessel Beams

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ABSTRACT

Internal conical refraction leads to the formation of zero (J_0) and first order (J_1) Bessel beams in superposition. The (J_0) beam retains the input circular polarisation and the (J_1) has opposite polarisation but with a single phase change around the beam axis giving it ħ optical angular momentum per photon. This results in the conical beam having ½ ħ net optical angular momentum per photon. This provides a simple system in which a beam of 0, ½ and ħ optical angular momentum can be easily generated and selected with use of only a circular polariser. In the far field the characteristic Bessel beam structures are formed and can be made non-diverging with use of a lens. We report the formation of non-diverging Bessel beam of core diameter (a) of 5.7μm over a maximum non-diverging core length of 1(±.05)mm. However due to the fine structure of the conical beam at its beam waist position two cores are produces and are of opposite phase.

Keywords: Conical refraction, Biaxial, Bessel beams, optical orbital angular momentum

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THEORY

Internal conical refraction occurs when light propagates down the optic axis of a transparent biaxial medium. If the input beam is a planar Gaussian beam the biaxial crystal changes the electric field distribution of the beam to be a cone of light within the crystal and a cylinder of light upon exiting the crystal [(1) (2)]. A biaxial crystal is described by a direction dependent refractive index $n(\vec{k})$. It is possible to construct a wave surface and corresponding ray surface plots to describe the propagation of the light. The wave surface gives the phase and polarisation of a ray of light for that given direction in a medium [(4)]. The corresponding ray surface is defined as tangential to these points in space

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giving the resulting direction of propagation for the given ray, (one ray for each wave, two waves for each direction). This is more easily shown in fig (1-a) where the wave surface is shown along with the appropriate polarisations and a clearly defined optic axis. The cone of internal conical refraction is formed by a plane perpendicular to the direction of the wave surface optic axis. This plane touches an infinite amount of points on the ray surface tracing out a circle. This plane is shown in fig (1-b), as is the cone of internal conical refraction. Light propagating in this direction forms a cone of light within the crystal and a cylinder of light upon exiting the crystal [(4)]. Conversely the optic axis of the ray surface defines the cone of external conical refraction. The resulting conically refracted ray has a unique polarisation and phase distribution due to the infinite amount of points on the wave surface. The polarisation distribution is shown in fig (2). At the waist of the input beam a sharp double ringed structure appears with a unique polarisation distribution of π/2 rotation of phase around the rings. The fine structure of the focal image plane reveals that this double ringed structure is separated by a dark Poggendorff ring this is due to the finite size of the input beam [(5)]. This occurs due to the finite size of the input beam giving a corresponding finite region around the optic axis resulting in the observed beam structure. The Poggendorff ring corresponds to the point of contact of the two sheeted wave surface (i.e. the optic axis) which has zero area hence zero intensity

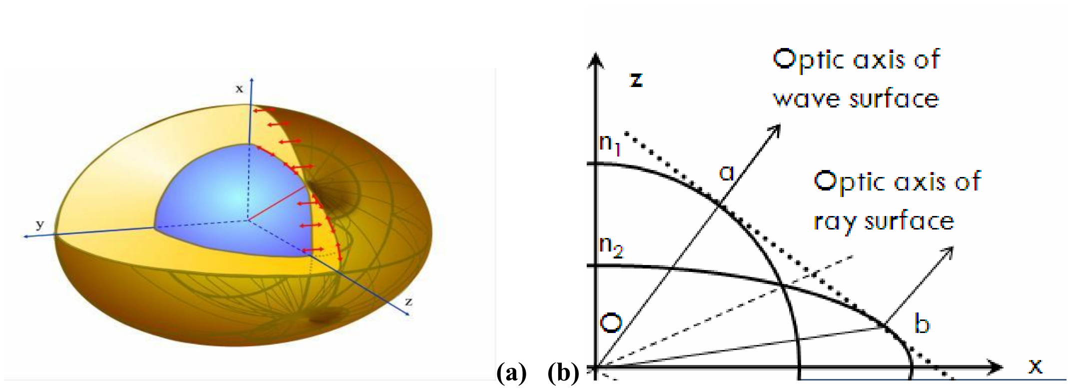


Figure 1: (a) Double sheeted wave surface with optic axis clearly defined (b) 2-D slice of the ray surface with the differences in refractive indices exaggerated. This is the cone of internal conical refraction. The optic axis of the wave surface defines this plane.

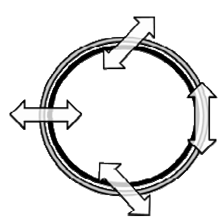


Figure 2 Distribution of polarization from cone of refraction.

Conical diffraction as first put forward by Beslki [(6)] and Berry [(7)] involves consideration of off axis waves i.e. the part of the dispersion surface around the conical point, Berry gives the form of the dispersion surface near the conical intersection as in fig(1-b) [7]. From geometric optics we know the phase and polarisation distributions fig (1). Knowing this the diffraction of the beam can thus be calculated with a transformation of a plane wave given by

$$\exp[ik(\vec{P}\cdot\vec{R} - z(1/2kP^2 \pm AP))]\vec{d}_{\pm}(\vec{P}) \tag{1}$$

With $d_{\pm}$ *(P)* is the polarisation of the plane wave, with two values due to the polarisation rotation around the optic axis. With the refractive index for a given wave direction is given by, with (A) being the semi angle of the crystal

$$n(k) = n_2(1 + A(k_x \pm k)) \tag{2}$$

With the z axis being the optic axis and k_x , k_y off axis wave vectors,

$$k = \sqrt{k_x^2 + k_y^2} \tag{3}$$

After using the paraxial treatment of the wave to describe the propagation of a beam through a biaxial medium the emergent field is given by:

$$\vec{E} = B_0\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} + B_1\begin{pmatrix} Cos\theta & Sin\theta \\ Sin\theta & -Cos\theta \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \tag{4}$$

Where: *e1 e2* are the input polarisation vectors. B_0 and B_1 are given by.

$$B_0 = \int dkk\exp\{-1/2k^2w^2\}\exp\{-1/2ikz\} \\ \cos(kR_0)J_0(kR) \tag{5}$$

$$B_1 = \int dkk\exp\{-1/2k^2w^2\}\exp\{-1/2ikz\} \\ \sin(kR_0)J_1(kR) \tag{6}$$

J_0 and J_1 are the Bessel functions of zero and first order. B_0 has the same polarization as the incident beam and B_1 has a phase change of polarization around the beam. For right circularly polarized incident light B_1 is left circular & vice versa. According to Berry [7] when *(R)* is close to (R_0) the integrals B_0 and B_1 are approximately equal resulting in the two beams being in close superposition. As a result the individual polarisations can be vector added to explain the polarisation distribution and presence of orbital angular momentum (OAM) of the beam as explained in Fig (3). We are interested in the case when the Bessel functions can be treated as slowly varying compared to the oscillatory terms. Applying the stationary phase method to the integrals (5, 6) then gives

$$B_0 \propto J_0(R, R_0),\ B_1 \propto J_1(R, R_0) \tag{7}$$

And for circular input light we have a superposition of Bessel beams with opposite circular polarizations.

$$\vec{E} = J_0(R, R_0)\begin{pmatrix}1\\ i\end{pmatrix} + J_1(R, R_0)\begin{pmatrix}Cos\Theta & Sin\Theta\\ Sin\Theta & -Cos\Theta\end{pmatrix}\begin{pmatrix}1\\ i\end{pmatrix} = J_0\hat{e}_R + e^{i\theta}J_1\hat{e}_L \tag{8}$$

Using equation (8) the polarisations of the individual beams can be mapped out for when $B_1 \approx B_0$ (beam waist of input Gaussian beam)

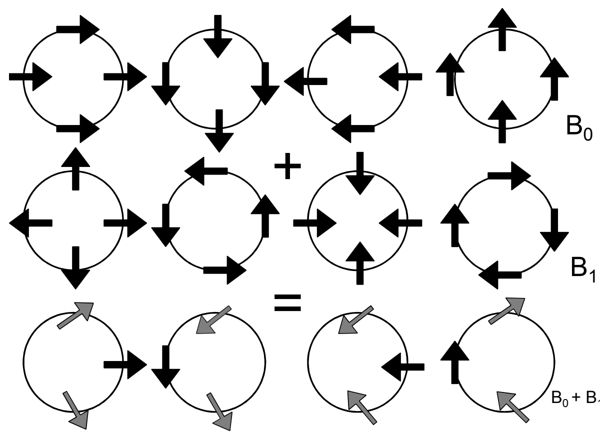


Figure 3: Polarisation distribution for one ring at beam waist, other ring is of opposite phase and polarisation direction. B_0 retains input polarisation and B_1 has opposite polarisation but with a phase change across beam. Over λ the resultant beam contains a ½ phase change around the beam resulting in a spiralling Poytings vector. This distribution only strictly holds true at $B_0 = B_1$.

The presence of orbital angular momentum in a conically refracted beam was first presented by Berry [(8)]. In this he predicts that the conical beam changes the solely spin input momentum into half integer purely orbital angular momentum hence exerting a torque on the biaxial medium. Orbital angular momentum can be defined by a rotation of phase around a beam axis [(3)], given by $e^{im\Theta}$, where (m) is the number of changes of phase around a beam axis resulting in orbital angular momentum of $m\hbar$ per photon. For the polarisation distribution of our conical beam it can be seen that B_0 contains no change of phase. Compared to B_1which contains one change of phase around beam axis. This results in a net orbital angular momentum of ½ $\hbar$ per photon. Due to the conical beam having a half integer OAM the electric field will have a spiralling wave front and will have an edge dislocation in its wave front. Diffraction of the beam away from the beam waist leads to $B_0 \neq B_1$. The diffraction of the conical beam form the beam waist results in a spreading of the beam and eventually a spike of intensity appears at what was the centre of the rings.

This is known as the axial spike. Separation of this axial spike reveals the characteristic Bessel beam distributions of zero J_0 and first J_1 order. These beams continue to spread and diverge as they propagate. By placing a lens at (f) away from the conical beam waist non diverging Bessel beams will be formed over the focal length of the lens. These non-diverging Bessel beams were first introduced by Durnin [(9)] as ways to form propagation-invariant optical beams given by

$$E(r,t) = E_0 J_0(kr)\exp(i(kz - \omega t)) \tag{9}$$

With J_0 the Bessel solutions of zero order and similarly for J_1. This is of the same kind as in equation (4) but with the addition of the first order beam with the polarisation modulation around the beam. However Bessel beams are not infinitely non-diverging they do have a maximum non-diverging length called z_{max}. The Bessel beam core diameter is given by (a)

$$a = \frac{J_0 \lambda}{\sin\theta . 2\pi} \tag{10}$$

Where J_0 is 2.405 and Θ is given by

$$\tan(\theta) = \frac{r}{f} \tag{11}$$

Where (r) is the radius of the dark Poggendorff ring in the focal image plane and (f) is the focal length of the collecting lens.

EXPERIMENTAL

The formation of the conical beam is achieved by focusing a beam of light down the optic axis of a biaxial crystal. The input Gaussian beam is circularly polarised of either left or right handedness with a beam waist size of 18μm. A single crystal of monoclinic tungstate of KGd $(WO_2)_4$ [(10)] was used, with its respective refractive indices at 633nm given by [(11)]

$$n_1 = 2.01169,\ n_2 = 2.042198,\ n_3 = 2.09510$$

The laser is a HeNe with a power output of .3mW. The camera used as a Thor Labs DC310 B&W camera with 5.65μm pixel to pixel.

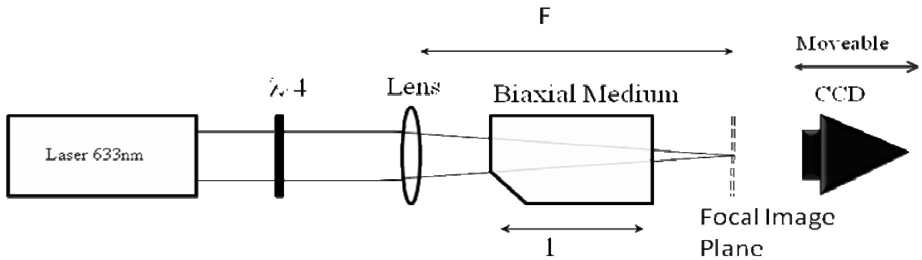


Figure 4: To form non diverging Bessel beams a lens is places at its focal length away from the focal image plane above.

The diameter of the rings at the focal image plane is given by the semi angle of the crystal and the length of the crystal. R is the radius of the Poggendorff rings upon exiting the crystal, resulting in a theoretical ring radius of 5.9015x10-4m.

$$R = Al \tag{12}$$

Using numerical calculation the experimental intensity distributions can be compared to the theoretical intensity distributions using equation (4)

	Theoretical	**Experimental**
Semi Angle	19.6mRad	20.04mRad
Ring Radius	5.9012E-4m	6.012E-4m

Table 1: Conical refraction properties due to crystal dimensions compared with theory

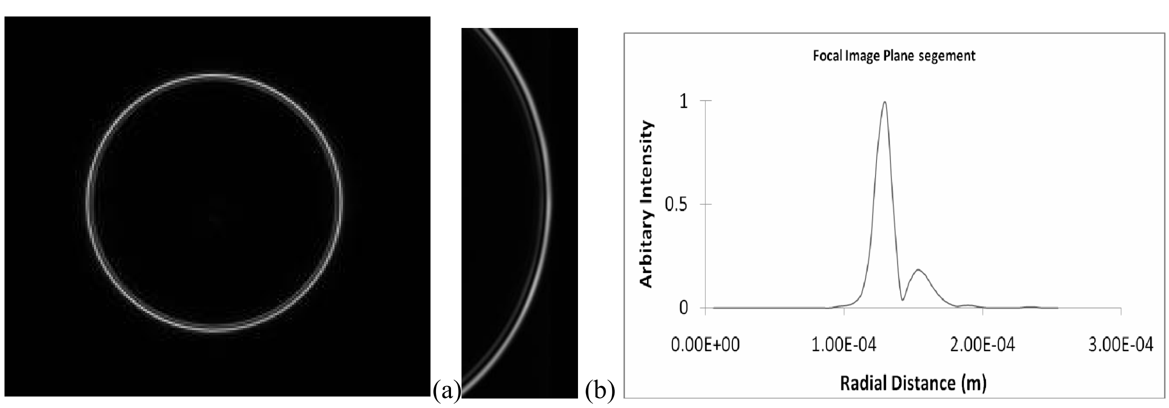


Figure 5: Focal image plane with the respective intensity distribution, (b) magnified by 2.5 with the intensity distribution across rings segment

RESULTS

The generation of propagation invariant Bessel beams results in a central core of width (a) whose dimensions remains constant over z_{max}. For a core size (a) of 6μm a lens with a focal length of 8mm was used. Profiling the beams gives z_{max} to be .36(±.05)mm per core over a distance of 1.1mm. Fig 6 shows the beam profiles and surface plots of intensity. Fig 7 shows the Bessel beams core intensity over the 1.1mm range along the propagation direction. It clearly shows the presence of two beam core due to presence of the double ring structure in the focal image plane. Fig 8 shows the beam profile of one of the cores over .36mm which has a constant core width of 5.7μm.

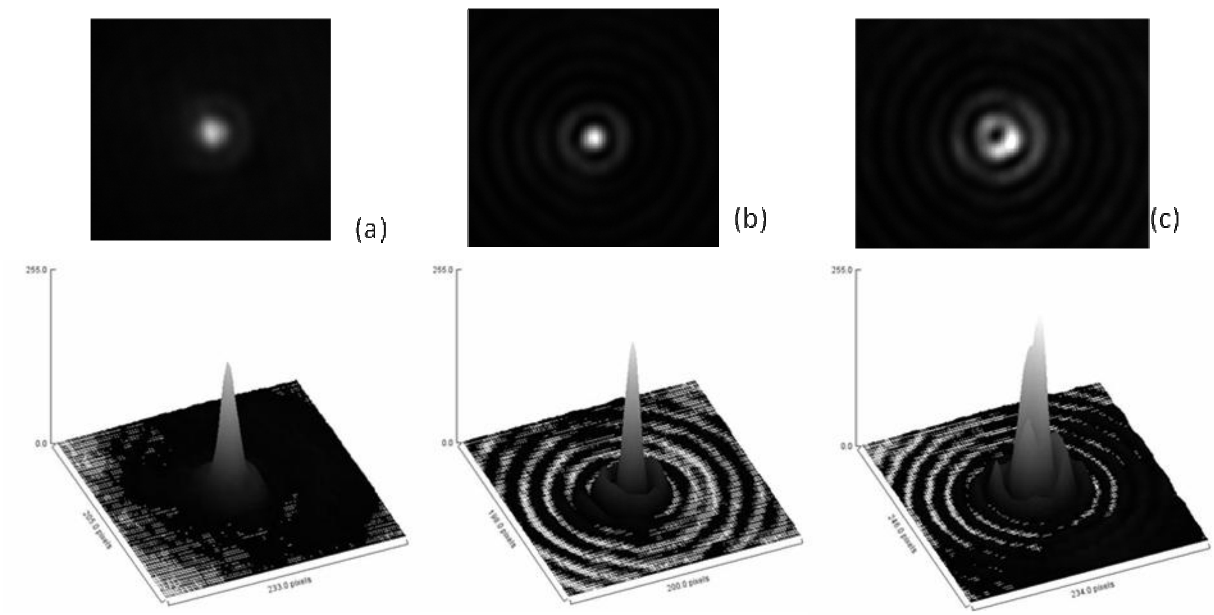
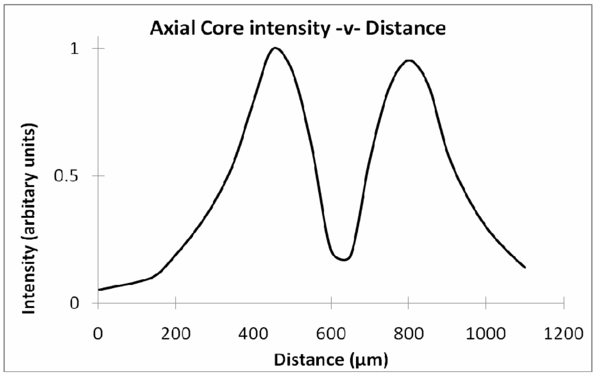
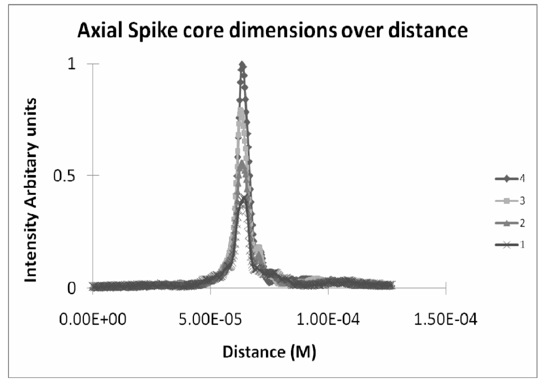


Figure 6: Beam profile plot including 3-D surface plot of intensity at $z_{max}/2$



(7)



(8)

Figure 8: Plot of Intensity of Bessel Beam cores along propagation direction.

Figure 8: Pixel line intensity across axial spike over 4 segments of 50µm apart.

DISCUSSION

The formation of zero and first order Bessel beams by conical diffraction is clearly demonstrated via diffraction of the sharp double ring structure from the beam waist. These beams which are in superposition can easily be separated due to their individual polarisation distribution revealing two beams, one with only spin angular momentum and a beam with a phase change across the beam giving it ħ optical orbital angular momentum. However when they are in superposition the conical beam contains only ½ ħ OAM with only ½ phase change around the beam. These beams can also be easily modified via a lens to form non diverging beam cores over long distances compared to Gaussian beam propagation. Along the beam axis two beam cores are created of opposite phase separated by a region of low core intensity. This Bessel beam core has a region of zero intensity along the propagation direction due to the presence of the Poggendorff dark ring in the focal image plane. Both cores are of equal length and intensity which is due to the fact that both inner and outer rings contains equal amount of energy [7]. A beam core width of 5.7um was formed with two beam cores of .36mm in length. Trapping experiments are planned to provide a viable optical trapping system that is easily switch able between beam of zero, one and ½ ħ OAM. These Bessel beams have also been gaining interest in various fields [12] [13] [14]. Recent work has shown interest in these half integer states with regards to quantum entanglement and quantum information processing.

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