A perturbative determination of the parameters of an anisotropic quark action

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The parameters of a 3+1 anisotropic quark action with Symanzik-improved glue are determined at 1-loop in perturbation theory.

1. INTRODUCTION

The utility of the anisotropic lattice formulation in glueball spectroscopy [1] is well documented. In that case, using a lattice spacing which is finer in the temporal direction, a_t, than in the spatial, a_s, gives improved resolution of correlation-function decays with a minimal increase in computational workload. Our recent work has centred on the application of anisotropic lattices to the simulation of relativistic heavy quarks. We use a tree-level improved fermionic action, specifically formulated for large anisotropies on a 3+1 lattice, which is free of classical discretisation errors that scale with $a_s m_q$. If the cutoff effects remain well-defined beyond the tree level, it should be possible to accurately simulate heavy-quark systems on lattices for which $a_s \gg a_t$.

1.1. Anisotropy

Of critical importance is the dependence of the renormalised anisotropy $\xi_R = \frac{a_s}{a_t}$, determined from simulations, on both the bare anisotropy, ξ, and the bare quark mass. This report describes the tuning of the quark action parameters required to restore Lorentz symmetry (i.e. to restore the renormalised speed of light, $c_R \equiv \xi_R^{-1}\xi$, to its correct physical value of unity) at 1-loop in perturbation theory. An analytic study of this action in the Hamiltonian limit, $a_t \to 0$, [2] has yielded promising results. In that study, corrections to the speed of light parameter in the action were calculated at 1-loop order and found to depend almost linearly on the combination $a_s m_q$. The results of our numerical investigation have also been positive [3]. The variation of the renormalised anisotropy measured from the dispersion relations of pseudoscalar and vector mesons was shown to be mild over a range of quark masses from strange to charm. The success of that exploratory work has motivated this more comprehensive investigation of the quantum corrections to the fermionic action using perturbation theory.

1.2. Quark Action

The quark action keeps the usual Wilson term in the temporal direction while using a Hamber-Wu-like term to remove doublers in the spatial directions

$$\begin{aligned} S_q &= \bar{\psi}(\gamma_0\nabla_0 + \sum_i \mu_r \gamma_i \nabla_i (1 - \frac{1}{6}a_s^2 \triangle_i) \\ &\quad - \frac{r a_t}{2}(\triangle_0 - \frac{1}{2}\sigma_{i0}F_{i0}) + s a_s^2 \sum_i \triangle_i^2 \\ &\quad + m_0)\psi, \end{aligned} \qquad (1)$$

where $\mu_r = 1 + \frac{1}{2} r a_t m_0$, and r and s are constants which we set to 1 and 0.125 respectively. Tadpole improvement factors are contained in the definition of the lattice derivatives and the chromoelectric field. This action is classically improved to $\mathcal{O}(a_t, a_s^3)$ and further details of its construction can be found in Ref. [3].

1.3. Gauge Action

We use the Symanzik-improved gauge action introduced in Ref. [1]

$$S_w = \frac{\beta}{\xi}\left\{ \frac{5(1+\omega)}{3u_s^4}\Omega_s - \frac{5\omega}{3u_s^8}\Omega_s^{2t} - \frac{1}{12u_s^6}\Omega_s^R \right\}$$

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$$+\beta\xi\left\{\frac{4}{3u_s^2u_t^2}\Omega_t-\frac{1}{12u_s^4u_t^2}\Omega_t^R\right\}, \qquad (2)$$

where Ω_s and Ω_t are spatial and temporal plaquettes and Ω_s^R and Ω_t^R are 2×1 rectangles in the (i,j) and (i,t) planes respectively. Ω_s^{2t} is constructed from two spatial plaquettes separated by a single temporal link. In simulations, ω is chosen to avoid a critical point in the fundamental-adjoint action plane of $SU(3)$. In perturbation theory, ω appears only through vertices involving four or more gluons and its value is irrelevant to our calculation.

2. TUNING

To calculate the corrections to the fermionic action, we consider an expansion of the quark energy in powers of the spatial momentum

$$E^2(\mathbf{p})=M_1^2+\frac{M_1}{M_2}\mathbf{p}^2+O(p^4), \qquad (3)$$

where M_1 is the rest mass and M_2 is the kinetic mass. Setting $M_1=M_2$ restores Lorentz invariance. This is the mass-dependent improvement condition suggested in Ref. [4]. The deviation of the measured speed of light from unity can be attributed to two factors. Firstly, at non-zero quark mass, the speed of light is modified by lattice artifacts even in the isotropic formalism. Secondly, in this calculation, additional radiative corrections arise due to the different lattice spacing in the spatial and temporal directions and the inherent asymmetry of the actions used.

The relationship between M_1, M_2 and the action parameters may be determined to any order in perturbation theory by expanding the all-orders dispersion relation [5], corresponding to a pole in the momentum-space full quark propagator, in powers of the coupling. At the tree level, tuning the action parameters to satisfy the improvement condition amounts to a redefinition of μ_r,

$$\mu_r^{(0)} = \sqrt{\frac{a_tm_0(2+a_tm_0)}{2\ln(1+a_tm_0)}}, \qquad (4)$$

which leaves the action unchanged at $\mathcal{O}(a_t)$. Similarly, it is clear from the form of the action and the quark dispersion relation that higher-order radiative corrections can be also be absorbed into μ_r. The expression for the 1-loop coefficient is

$$\mu_r^{(1)}=\Big(\frac{e^{2a_tM_1^{(0)}}}{\mu_r^{(0)}}-\mu_r^{(0)}\Big)\frac{M_1^{(1)}}{2M_1^{(0)}}+\frac{\mu_r^{(0)}}{2}Z_{M_2}^{(1)}, \qquad (5)$$

where the kinetic mass renormalisation factor,

$$Z_{M_2}=\frac{\mu_r^2M_2}{\sinh(a_tM_1)}e^{-a_tM_1}, \qquad (6)$$

gives a measure of the difference between M_1 and M_2 due to radiative corrections. At 1-loop order, $M_1^{(1)}$ and $Z_{M_2}^{(1)}$ are independent of $\mu_r^{(1)}$ and Eq. 5 is a closed expression for the 1-loop correction to the action. The 1-loop coefficients of M_1 and Z_{M_2} have been calculated for a number of different quark actions in Refs. [2] and [5]. As these are physical quantities, each term in their perturbative expansion must be infrared finite and gauge-invariant. Eq. 5 makes explicit the gauge invariance of $\mu_r^{(1)}$ which serves as an important check in our calculation.

In the limit $m_0\to 0$, the 1-loop coefficient reduces to the expression used in Ref. [6].

3. DETAILS OF THE CALCULATION

The calculation of $\mu_r^{(1)}$ reduces to a determination of the 1-loop quark self-energy and its derivatives with respect to external momenta. Details can be found in Refs. [2,5]. Expressions for the propagators and vertices were obtained using Mathematica. The spin algebra required to obtain the loop integrands was initially performed with Mathematica but later checked using an automated approach. To find derivatives, automatic differentiation [7] was used where analytic expressions would have been too unwieldy. We used the mean link in Landau gauge to implement tadpole improvement. Loop integrals were evaluated with Vegas [8]. Although the 1-loop coefficient is infrared finite, this depends on the cancellation of divergences in its constituent pieces. To calculate the integrals numerically, we introduced a small gluon mass, λ, as an intermediate regulator. For a given quark mass and bare anisotropy, we calculated $\mu_r^{(1)}$ over a range of gluon mass values and extrapolated to $\lambda=0$ to obtain a final result.

4. PRELIMINARY RESULTS

We present results for $\xi = 6$ which was the bare anisotropy examined in Ref. [3]. The 1-loop values given below are coefficients of g^2. In this initial study we did not include a chromoelectric term in the fermionic action.

For Wilson-type quarks there is an additive renormalisation of the bare quark mass. We find that at $\xi = 6$, the 1-loop critical bare mass corresponding to massless quarks is given by $a_t m_c^{(1)} = -0.008688(1)$.

Fig. 1 shows the 1-loop rest mass as a function of the tree-level rest mass. $a_t M_1^{(1)}$ varies

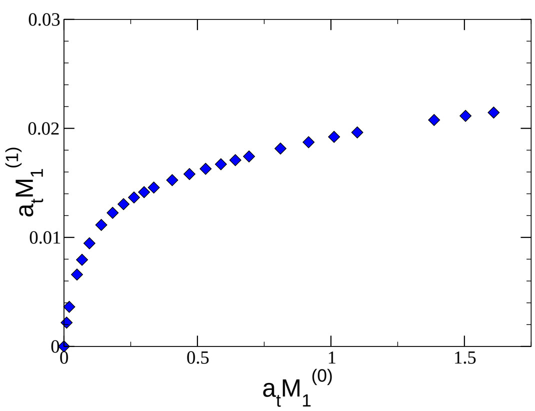

Figure 1: The 1-loop rest mass renormalisation at $\xi = 6$.

smoothly and remains small over a range of values of $a_t M_1^{(0)}$. The main result, however, is the mass dependence of $\mu_r^{(1)}$ which is plotted in Fig. 2. The 1-loop correction to μ_r is extremely small and appears to vary linearly with the mass. This dependence is very weak and the value of $\mu_r^{(1)}$ changes by less than 0.1 over the range considered.

OUTLOOK AND DEVELOPMENTS

Preliminary results for the speed of light renormalisation are in qualitative agreement with our previous work. We plan to carry out a detailed comparison of perturbative and non-perturbative

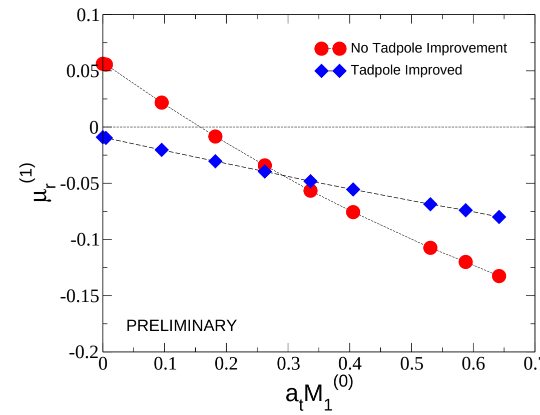

Figure 2: Speed of light renormalisation plotted as a function of the tree-level rest mass at $\xi = 6$.

tuning of the fermionic action in the near future. Ultimately, this work will extend to a Symanzik-type perturbative improvement program for the quark action including chromomagnetic interactions.

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