

Spectroscopy of B_s and D_s Mesons*

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We study B_s and D_s spectroscopy in quenched lattice QCD using the Fermilab approach to heavy quarks. We obtain results at four lattice spacings, a , using $O(a)$ -improved Wilson quarks. We compare and contrast the various methods for heavy quarks on the lattice, discussing which methods work best for different physical systems and the ease with which calculations may be performed.

1. METHODS FOR HEAVY QUARKS ON THE LATTICE

There are several ways of approximating heavy quarks in lattice QCD calculations, including Nonrelativistic QCD [1–3] (NRQCD), the static approximation [4,5], and the approach developed at Fermilab [6] which takes the uncorrected Wilson action as its leading approximation but adds correction operators which end up resembling those of NRQCD rather than those of the standard Symanzik improvement of the Wilson action [7].

NRQCD is based on an expansion in nonrelativistic operators (rotationally invariant but not Lorentz invariant) similar to that used in calculating relativistic corrections in the hydrogen atom. It can be thought of as arising from a discretization of the action arising from a Foldy-Wouthuysen-Tani transformation of the quark fields analogous to the one used in atomic physics.

$$\psi \rightarrow \exp(\theta D_i \gamma_i) \psi \quad (1)$$

leads to the action

$$\begin{aligned} \mathcal{D} + m &\rightarrow D_0 \gamma_0 + m \\ &+ \frac{\mathbf{D}^2}{2m} + \frac{(\mathbf{D}^2)^2}{8m^3} + \dots \end{aligned} \quad (2)$$

The pole mass term does not affect the dynamics of nonrelativistic systems and is conventionally dropped.

In B physics, the simplest of all the methods can be used, the static approximation. This approximates the heavy quark propagator with a simple Wilson line moving in the time direction, giving an action corresponding to the first term in Eqn. 2. It is clearly most useful for the heaviest quarks. It is not much used recently because it has a much worse signal to noise ratio than NRQCD, which is clear in retrospect but was not foreseen.

The third method can be thought of as arising from a partial FWT transformation:

$$\psi \rightarrow \exp(\theta' D_i \gamma_i) \psi, \quad (3)$$

and

$$\begin{aligned} \mathcal{D} + m &\rightarrow D_0 \gamma_0 + m + a_1 D_i \gamma_i \\ &+ a_2 \left(\frac{\mathbf{D}^2}{2m} + \frac{(\mathbf{D}^2)^2}{8m^3} + \dots \right), \end{aligned} \quad (4)$$

where $\theta' < \theta$, and a_1 and a_2 are between 0 and 1. This appears to be a crazy thing to do, producing an action which combines the defects of the transformed and the untransformed actions, which indeed it does. On the other hand, it turns out that this is the action we have been stuck with for a long time. The Wilson action has just this form at leading order. The $\bar{\psi} D^2 \psi$ term added to cure the doubling problem also contributes to the kinetic energy, as in Eqn. 4. Furthermore, perhaps surprisingly, the a_1 and a_2 implied by the Wilson action have the desired property that the nonrelativistic $\bar{\psi} D^2 \psi$ term takes over automatically from the Dirac style kinetic energy term

*Talks presented by P. Mackenzie and J. Simone.

$\bar{\psi}\mathcal{D}\psi$ as $\kappa \rightarrow 0$ in the Wilson action. The Wilson action automatically turns into a nonrelativistic action in the large mass limit.

The Wilson action does have the unwanted property that for large masses the pole mass does not equal the kinetic mass governing the energy momentum relation

$$\frac{1}{2M_{kin}} = \frac{dE}{dp^2}. \quad (5)$$

In nonrelativistic systems, the pole mass does not affect the dynamics and the kinetic mass governs the leading important term. Therefore, the pole mass term is normally simply omitted from the action in NRQCD, although there is no harm (and no benefit) in including it. Likewise, the Wilson action can be used without problems for nonrelativistic systems as long as M_{kin} and not the pole mass is used to set the quark mass. It is easy to find a Wilson style action which does satisfy $M_{pole} = M_{kin}$ by letting the hopping parameter for the time direction, κ_t , differ from the one in the spatial directions, κ_s .

The Wilson action corrected up to $\mathcal{O}(a)$ and the action of NRQCD both use one-hop time derivatives. When the quarks are heavy, this is a requirement, since two-hop time derivatives introduce new ghost states and imaginary energies when $ma > 1$. Therefore, the Wilson action applied to heavy paths cannot follow the conventional Symanzik program of using two-hop corrections at $\mathcal{O}(a^2)$, but must follow the NRQCD style of corrections which correct only the spatial interactions. The existence of the Hamiltonian and the transfer matrix ensures that this is possible.

The parameters of the action in this approach must have nontrivial mass dependence for large masses, just as do those of NRQCD. For large masses, the wave function normalization, the relation between the mass and the hopping parameter, etc., are completely different from their $m = 0$ values. When $m < 1$ (in lattice units), this mass dependence may be expanded in power series. For the Wilson and Sheikholeslami-Wohlert actions [8], this yields just the usual series of operators, with the same coefficients:

$$\mathcal{L} = m\bar{\psi}\psi + a_1 m^2 \bar{\psi}\psi + \dots$$

$$+ \bar{\psi}\mathcal{D}\psi + b_1 m \bar{\psi}\mathcal{D}\psi + b_2 m^2 \bar{\psi}\mathcal{D}\psi + \dots \\ + c_{sw} \bar{\psi}\sigma_{\mu\nu} F_{\mu\nu}\psi + c_2 m \bar{\psi}\sigma_{\mu\nu} F_{\mu\nu}\psi + \dots \quad (6)$$

Thus, while for $m > 1$ the action becomes very similar to NRQCD in its behavior, for $m < 1$ it may be regarded as an all orders in m resummation of the usual operators of the Wilson and SW actions. For hadrons containing charmed quarks, it is possible to do calculations with the Wilson and SW actions even with the old interpretation of the coefficients. However, since $am \approx 5a\Lambda_{QCD}$, the ability to sum up the required series in m exactly is likely to produce a much faster approach to the continuum limit.

2. HEAVY-LIGHT MESONS

Here we use this approach to calculate the spectrum of B_s and D_s mesons. The strong interactions of a meson containing a single heavy quark simplify in the infinite mass limit. The heavy-quark spin and the light-quark total angular momentum, j , are separately conserved. States with total spin $J = j \pm 1/2$ form degenerate-mass doublets. Hence, the hyperfine splitting between the vector, $J = 1$, and the pseudoscalar, $J = 0$, is zero. Heavy quark symmetry is only approximate at finite quark mass so the hyperfine splittings for D and B mesons are proportional to $1/m_Q$.

The sensitivity of the hyperfine splitting to the heavy-quark mass makes spectroscopy of the B and D mesons an important check on the procedure used to determine the bare charm and bottom masses. The bare masses are important inputs to our programs to determine the renormalized quark masses and weak matrix elements of the D and B mesons[9,10].

Heavy-quark masses in this study are determined in quarkonia. The bare mass is found by demanding the experimental spin-averaged meson mass match the kinetic mass determined from the lattice energy-momentum dispersion relation[11]. Quark masses may also be determined by studying heavy-light mesons. These masses will differ from the masses in -onia since lattice spacing errors differ for the two methods[12]. Mass errors with either method however lead to bounded uncertainties in quantities

Table 1
Quenched gauge configuration ensembles.

beta	6.1	5.9	5.7	5.5
# configurations	100	350	300	500
$c_{sw} = \langle \text{plaq} \rangle^{-3/4}$	1.46	1.50	1.57	1.69
$a^{-1}(1\text{P-1S})$ (GeV)	$2.56 \left(\begin{smallmatrix} +16 \\ -12 \end{smallmatrix} \right)$	1.81(7)	1.16(3)	$0.75 \left(\begin{smallmatrix} +8 \\ -2 \end{smallmatrix} \right)$
length (fm)	1.9	1.7	2.0	2.1

such as hadron mass splittings and matrix elements. Moreover, quark-mass uncertainties in these quantities are overcome by extrapolation to zero lattice spacing.

3. CALCULATION DETAILS

We study pseudoscalar and vector mesons on the four ensembles of quenched Wilson glue described in Table 1. Lattice spacings are determined using the charmonium 1P-1S splitting. Spacings range from $a = 0.26$ to $a = 0.080$ fm. The coarsest lattice only provide a check upon cutoff effects; results are not used for continuum extrapolations.

We use Sheikholeslami Wohlert (SW) quarks with tree-level tadpole improvement. The clover coefficient, c_{sw} , is determined from the average plaquette (see Table 1). Calculations are done directly at the charm and bottom quark masses using. Results are fully $O(a)$ improved.

Bare strange-quark masses are determined from the light pseudoscalar spectrum using leading-order chiral perturbation theory and the experimental π and K masses. Meson masses are extracted from minimum χ^2 multistate fits to a matrix of meson correlators having Coulomb-gauge 1S- and 2S-smearred sources and sinks. The full covariance matrix is retained in fits.

4. SPECTROSCOPY

Fig. 1 shows the B_s and D_s hyperfine splittings for the lattices in Table 1. Results from the three finest lattices are extrapolated linearly to $a = 0$. The D_s slope has an error bound consistent with zero. Hence errors from the charm mass determination are barely discernible with present statistics. The B_s slope is more significant indicating larger errors in the bottom quark mass determi-

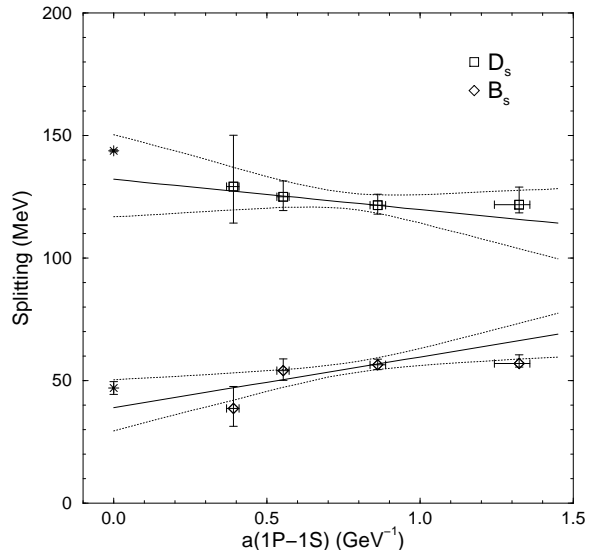


Figure 1. The D_s and B_s hyperfine splittings *vs.* the lattice spacing. Linear extrapolations (solid) with 68% confidence bounds (dashed) are shown. Experimental splittings (bursts) are indicated at $a = 0$.

nation. Reducing the lattice spacing decreases the B_s splitting. This is consistent with an underestimation of the bottom quark mass in $-onia$ [12].

The extrapolated B_s and D_s hyperfine splittings are consistent with experimental values. The central values however lie below experiment. This may be an indication that the quenched B_s and D_s hyperfine splittings are smaller than experiment. Note that the quenched hyperfine splittings in quarkonia lie significantly below experiment. Higher statistics are necessary to de-

termine if there is a similar quenching effect for heavy-light mesons.

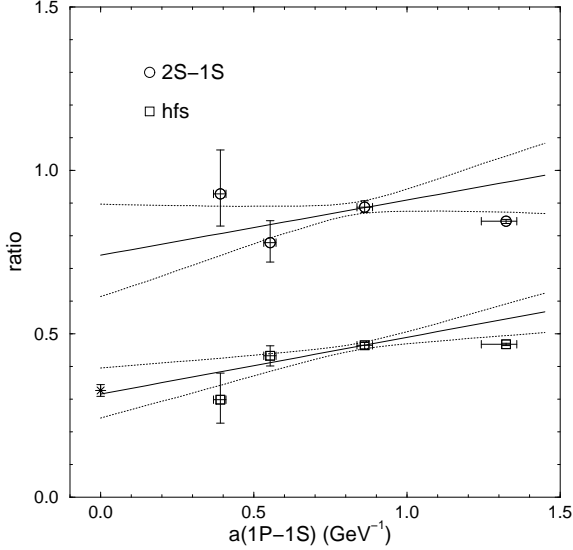


Figure 2. B_s splittings divided by D_s splittings. Hyperfine (hfs) and spin-averaged 2S-1S splitting ratios are shown. The experimental hfs ratio (burst) is shown at $a = 0$. Linear extrapolations with errors are shown.

The ratio of the B_s hyperfine splitting to the D_s splitting, R_{hfs} , is proportional to m_c/m_b according to heavy quark symmetry. The ratio is expected to be less sensitive to quenching errors than the individual splittings. The linear extrapolation in Fig. 2 yields $R_{hfs} = 0.32 \pm 0.08$. This compares well with the experimental value 0.327 ± 0.018 . This is evidence that our procedure for determining the bare quark masses leads to the correct continuum result.

We also study 2S states. Excited state statistical errors are an order of magnitude larger than ground state errors. Excited states are also subject to larger systematic errors. Combined errors are comparable to the spin splittings. Hence we report only the spin-averaged 2S splitting,

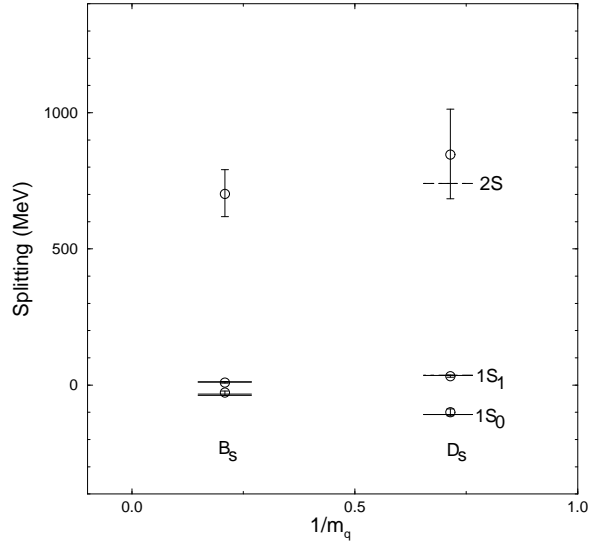


Figure 3. The D_s and B_s splittings in quenched QCD after extrapolation to zero lattice spacing (octagons). Experimental 1S levels (solid) are indicated. A potential model estimate of the D_s spin-averaged 2S-1S splitting (dashed) is also shown[13].

$m(2S-1S)$.

According to heavy quark symmetry splitting $m(2S-1S)$ is constant up to $O(1/m_Q)$ corrections. We check this in Fig. 2 by computing the ratio of 2S-1S splittings for B_s and D_s . The deviation from unity is consistent with $O(1/m_Q)$ effects.

We summarize our continuum B_s and D_s spectroscopy results in Fig. 3. Results are plotted in relation to $1/m_Q$ to emphasize the behavior of the splittings in the heavy quark limit. We compare to experimental values for the 1S states. A potential model inspired estimate for $m(2S-1S)$ is also shown[13]. Our lattice results are consistent with experiment.

Fig. 4 shows our most recent charmonium spectrum, which we will use in our discussion in the next section. It is mainly a statistical updating of our previous results, except for the χ_c states. These have been done using nonrelativistic opera-

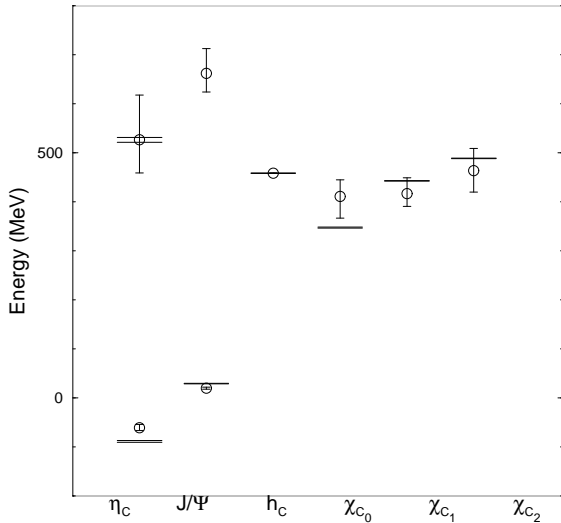


Figure 4. Charmonium spectrum from the $\beta = 6.1$ lattice.

tors for the first time rather than quark antiquark bilinears. This allows us to obtain a result for the χ_{c2} for the first time.

5. WHICH METHOD IS BEST WHERE?

The correction operators of NRQCD are simple and easy to organize. NRQCD quark propagators can be calculated almost instantaneously. Heavy quark actions which start from the Wilson action have the advantage that they continue to work as the $a \rightarrow 0$ limit is taken. However, correcting them to $\mathcal{O}(a^2)$ or to $\mathcal{O}(v^4)$ is messier than for NRQCD.

The action of NRQCD is an expansion around $m = \infty$ and is better and better behaved, the larger the quark mass. It breaks down in two distinct ways as $1/m$ becomes larger. When $1/(ma)$ gets larger than 1, perturbative calculability of coefficient functions breaks down due to factors of $1/(ma)$ in the diagrams. When Λ_{QCD}/m gets larger than 1, the convergence of the series of non-relativistic operators breaks down. This series

is very convergent for the b quark, and NRQCD works very well in B and Υ physics. The ψ system is reasonably nonrelativistic, too, as shown by the success of potential models. NRQCD works reasonably well for this system, too, although in some cases not as well as originally hoped.

For example, a recent investigation [14] of the effects of v^6 corrections in NRQCD studied their effect on the hyperfine splitting of charmonium. In results of a few years ago, the v^4 , a^2 improved NRQCD results lay much closer to the experimental answer than the $\mathcal{O}(a)$ improved relativistic Fermilab results. The interpretation seemed to be that the higher order corrections included in the NRQCD calculations made these results much more accurate than the less improved relativistic results. However, last year Trotter calculated some of the v^6 terms which contribute to the hyperfine splitting and found results which made a big jump back down toward zero. The actual situation seems to be that the v^2 expansion is very slow to converge for some quantities in the charmonium system. The ability of the harder to improve relativistic formalism to recover a Dirac-like form of the action in the $a \rightarrow 0$ limit is an advantage here.

For b quark physics, the situation is somewhat reversed. Here, the nonrelativistic expansion is well-converged after a few orders. (Potential models estimate that v^2 is about 1/4 in the J/ψ and about 1/10 in the Υ .) On the other hand, these corrections have to be included to achieve high accuracy. One does not have the option of replacing them and letting the relativistic $\bar{\psi}\not{D}\psi$ terms in the action of Ref. [6] take over when $ma < 1$. This would require an absurdly high lattice spacing.

A case where this plays a more significant role than had been expected is in the quantity $M_\Upsilon - 2M_B$. [15] In the quarkonium systems, the quark momentum $p \rightarrow m_b\alpha_s$ as $m_b \rightarrow \infty$. Therefore, $(pa)^2$ errors and $(p/m)^2$ errors also go to infinity in absolute terms, but approach fixed percentages of leading order physical quantities. In heavy-light systems on the other hand, the b quark's momentum $p \sim \Lambda_{QCD}$ as $m_b \rightarrow \infty$. $(pa)^2$ errors and $(p/m)^2$ errors due to the b quark approach constants in absolute terms.

Of the pole and kinetic masses in the equation

$$E = M_{pole} + \frac{1}{2M_{kin}}p^2 + \dots, \quad (7)$$

M_{kin} is more affected by v^2 errors than M_{pole} since it is higher order. For example, we can obtain the spin-averaged $2M_{HL} - M_{HH}$ splittings in the $D_s - \Upsilon$ and $B_s - \psi$ systems from the data from Figs. 1 – 4. We obtain 0.81 GeV in the charm system, compared with 1.08 GeV experimentally. We obtain -1.68 GeV in the bottom system compared with 1.35 GeV experimentally. Kronfeld used potential models to show that this was roughly what was to be expected. [12] The analogous effect in NRQCD improved only to $\mathcal{O}(a)$ is about 15%. [16]

It should be stressed that as a percentage error, no error in B or Υ physics is blowing up as $m \rightarrow \infty$. For example, the uncertainty induced in determinations of f_B from the 20% ambiguity of using M_B or M_Υ to determine the b quark mass is only of order a few per cent, as can be seen by examining a graph of $f_B\sqrt{m}$ vs. $1/m$. It is simply one of the p^2 errors known to be present in the $\mathcal{O}(a)$ improved theory. On the other hand, it is probably reduced from p_Υ^2 to p_B^2 by obtaining m_b from M_B rather than M_Υ in this case. It has been argued many times that quarkonia are nice systems to use for error analysis and parameter setting because of the tools that are available for understanding the errors. When those tools tell us that some errors in quarkonia are rather large, that strategy may sometimes have to be altered.

For pure B meson physics (not considering the Υ system), the situation is probably not as bad for “relativistic” heavy quark actions. (Relativistic is in quotes because for b systems, relativistic $\bar{\psi}\not{D}\psi$ terms in the action have almost no effect compared with the nonrelativistic $\bar{\psi}\mathbf{D}^2\psi$ terms.) In these systems, v^2 correction terms are probably negligible since $v^2 \sim (\Lambda_{QCD}/m_b)^2 < 1\%$. It is therefore possible that an $\mathcal{O}(a)$ improved heavy quark theory such as the heavy SW action may do an adequate job here. Perturbative corrections to decay constants, etc., are more annoying to calculate than for NRQCD or the $m = 0$ Wilson theory, but by no means intractable.

ACKNOWLEDGMENTS

This work is done in collaboration with A. El-Khadra, and A. Kronfeld. Calculations were performed on the ACPMAPS supercomputer built and operated by the Fermilab computing division. Fermilab is operated by University Research Association, Inc. under contract with the U.S. Department of Energy.

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