On the Value of Information forecasting with multi-information systems

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ABSTRACT: The deterioration of structural components together with the climate and mobility increasing demands undermine civil structures' safety and functionality performance. Planning of maintenance activities is necessary, along with budget optimization. In this context, inspections, destructive and non-destructive tests, and Structural Health Monitoring (SHM) can increase the knowledge about the structures' safety and functionality, improving prioritization and the selection of the optimal interventions.

This paper aims to present a method to investigate different information strategies for a generic multi-component redundant structure. A structural model capable of redistributing the loads and accounting for correlation, component behavior characteristics, and sensor configuration, is adopted in the study. The modeling of structural integrity management accounts for a set of actions, considered to be representative for the maintenance action domain. Information strategies are modelled by probabilistic structural system characteristics accounting for diverse measurement and monitoring strategies. Value of Information (VoI) from Bayesian decision theory is used to quantify the effects collected observations have in structural integrity management. The approach is exemplified with analyses of brittle and ductile Daniels systems revealing effects of structural system characteristics on the VoI.

1. INTRODUCTION

The management of aging civil structures and infrastructures is a growing concern in modern societies. Common users must be ensured of adequate safety and functionality performance of the vast structural portfolio despite the budget constraints. In this context, making decisions in structural integrity management reveals a crucial issue as they can have relevant social and economic consequences. Inspections, destructive and non-destructive testing, and Structural Health Monitoring (SHM) systems can be useful in providing a significant aid in structural integrity management. However, obtaining information on the structural state entails a cost that may not be worth the benefit achieved by its use. In these terms, the collection of information can be optimized by investigating if, when, and which type of information provides the best support in integrity management. Value of Information (VoI) by the Bayesian decision theory can be used

in this process of optimization. Raiffa and Schlaifer (1961) formulated VoI as a method to quantify the expected utility gained by information. Recently, VoI analysis has been used in various contexts, such as environmental and emergency management, see e.g., Giordano et al. (2020) and Klerk et al. (2019), reliability-based inspection and maintenance planning, see e.g., Bismut et al. (2022) and Zhang et al. (2022), and sensor arrangement, e.g., Malings and Pozzi (2018) and Long et al. (2020). A comprehensive review is presented in Zhang et al. (2021).
The quantification of the value of Structural Health Information (SHI) has been increasingly reported in the scientific production of the last years, also in conjunction with the COST Action TU1402, see e.g., Miraglia et al. (2016).Furthermore, a potential for scientifically directing the development of information technologies, among SHM technologies, has been identified in SHI value quantification. Thöns, Irman, and Limongelli (2021) provide a description and an alignment of the Decision Value Analysis (DVA) and the technology readiness levels (TRLs), commonly used to assess the maturity of a technology, see Sadin et al. (1989). In DVA three main phases are identified, namely the decision value forecasting, the decision value analysis, and technology value quantification constituting a stepwise approach to technology development. Among these, information-supported decision value forecasting allows for the screening of technologies to observe structural characteristics, and the selection of the optimal ones for integrity management. In these terms, VoI forecasting allows for a generic modelling of information and the evaluation of an information strategy even before the development of the specific acquirement technology has started.

The purpose of this paper is to present a method, built upon Thöns et al. (2021), to investigate various information strategies for a generic multi-component redundant structure. The study adopts a generic structural model that can redistribute loads and accounts for correlation, component behavior characteristics, and sensor configurations. A wide set of actions is considered in structural integrity management to be representative of the broad maintenance actions domain. Information strategies are evaluated and ranked, for diverse structural assumptions and sensor setups. Findings and considerations from the presented method can be used to drive innovation in the creation and development of information technology. Section 2 reports the decision scenario and the decision problem analytical formulation. Section 3 presents the decision problem models, including the structural system, information, actions, and utility models. Section 4 illustrates an exemplary case study for which information-supported decision value forecasting is performed. Finally, considerations and conclusions on the method are presented in Section 5.

2. DECISION SCENARIO

The decision scenario encompasses two distinct decision steps, namely the selection of the monitoring strategy and the selection of the action to implement in the integrity management of a generic structure. The different information strategies i provide observations O_i on the structure condition state X_l. Actions a_k are selected based on the collected observation O_i and their implementation is characterized by uncertainty Y_k. Finally, structural states X_l are associated with utilities u. A decision tree carrying the possible decision and chance alternatives is depicted in Figure 1.

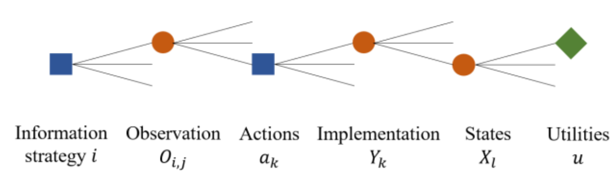

Figure 1: Decision Tree

Decision nodes are represented by blue boxes, chance nodes by orange circles, and utilities by green diamonds.

2.1. Decision analytical formulation

The decision analytical formulation allows for the estimation of the VoI deriving from a specific information strategy and takes into account how observation affects structural integrity management. Taking basis in Benjamin and Cornell (1970) and Raiffa and Schlaifer (1961), Table 2 contains the objective functions for a prior utility, which does not account for information and, for a pre-posterior utility in the normal form, which accounts for predicted observations. In the decision problem, the information strategy and action are selected such that the prior utility and the pre-posterior utility are maximized according to the expected utility theorem. This is equivalent to the minimization of the total expected costs if only costs and consequences are modeled.

Table 1: Decision problem analytical formulation

	Objective function
Prior	$U_{prior} = \max_{a_k} E_{X_l}[u(X_l, a_k, Y_k)] = \max_{a_k} \sum_{X_l} P(X_l \mid a_k, Y_k) \cdot u(X_l, a_k, Y_k) - c_{a_k}$
Pre-posterior	$U_{pre-post} = \max_{i,a_k} E_{X_l \mid a_k, Y_k}[E_{O_{i,j} \mid X_l}[u(X_l, a_k, Y_k)] = \max_{i,a_k} \sum_{X_l} P(X_l \mid a_k, Y_k) \cdot \sum_{O_{i,j}} P(O_{i,j} \mid X_l) \cdot u(X_l, a_k, Y_k) - c_{a_k} - n_j \cdot c_i$

X_l are the system states, $O_{i,j}$ are the observations, i.e., the observation collected on the *j-th* structural component by the *i-th* information strategy, a_k indicates an action k that can be implemented on the structure with an implementation uncertainty Y_k, c_{a_k} is the cost of the action k, c_i is cost of single obtained observation on any component by the *i-th* information strategy, and n_j is the number of monitored components.

In order compare the different information strategies, a normalized VoI, herein simply referred as VoI, is defined as follows:

$$VoI = \frac{U_{pre-post} - U_{prior}}{U_{prior}} \qquad (1)$$

3. MODELS

This section encompasses the system state model, the information model, the action and the utility models for the generic case of a redundant parallel system.

3.1. System State model

Any structural system is characterized by a capacity, i.e., resistance which decreases over time due to deterioration, and by a demand, i.e., the effect of the load which will be herein addressed as load. A general limit state equation can be written as in Eq. (2)

$$g_X = M_R \cdot R_0 \cdot (1 - M_D \cdot D) - M_S \cdot S \qquad (2)$$

where R_0 indicates the initial resistance of the system, D is the damage, S is the load, M_R, M_D and M_S are the respective model uncertainties. The term $R_0 \cdot (1 - M_D \cdot D)$ represents a model of the current resistance of a system characterized by an initial resistance reduced by damage, and overall affected by a model uncertainty M_R. Herein, we refer to $R = R_0 \cdot (1 - M_D \cdot D)$ as resistance, i.e., current resistance. Two structural states are considered: X_1 for $g_X < 0$ and X_2 for $g_X \geq 0$. The probability of the structure to be in one of the two structural states can be written as $P(X_1) = P(g_X < 0)$ and $P(X_2) = P(g_X \geq 0)$.
As described in Section 1, the analysis presented in the paper aims to investigate different information strategies for a multi-component redundant structure that is herein modeled as a Daniels system, see Daniels (1945). Daniels systems allow for modeling system correlation, redundancy, and system components behavior. The limit state equations relevant to the two different component behaviors are reported below: Eq. (3) describes a system with *n* ductile components, and Eq. (4) a system with *n* brittle components.

$$g_X = \sum_j^n M_{R_j} \cdot R_{0_j} \cdot \left(1 - M_{D_j} \cdot D_j\right) - \sum_j^n M_{S_j} \cdot S_j \qquad (3)$$

where the initial resistance R_{0_j}, the damage D_j and the load S_j, and the model uncertainties M_{R_j}, M_{D_j} and M_{S_j} refer to the *j-th* structural component.

$$g_X = \max_{i=1\dots n}\left((n-i+1)\cdot m_{R_j}\cdot r_{0_j}\cdot \left(1-m_{D_j}\cdot d_j\right)\right) - \sum_j^n M_{S_j}\cdot S_j \qquad (4)$$

where m_{R_j}, r_j, m_{D_j}, d_j are individual realizations of the resistance model M_{R_j}, of the initial resistance distribution R_{0_j}, of the damage model M_{D_j} and of the damage distribution D_j, allowing for the modeling of the brittle structural resistance, see Gollwitzer and Rackwitz (1990).

3.2. Information Model

Visual inspections, destructive and non-destructive tests, and SHM are commonly performed to enhance the knowledge of the system states allowing for efficient support in integrity management. Information on the to the random variables of the limit state function - resistance, damage, or load - can be obtained directly through measurements, or indirectly by measuring related quantities. For example, resistance may be measured directly by performing a destructive test on a sample, or indirectly from the stiffness, if the relation between resistance and stiffness is known, see e.g., Kamariotis et al. (2023).

Measurements make available parts of the full probabilistic model of a random variable, according to the measured temporal and spatial boundaries. One approach of distinguishing between measurable and non-measurable parts of a random variable is to model precise observations as realizations of the model uncertainty, normalized to the model predictions and subjected to the measurement uncertainty M_U. The term ‘precise’ indicates observations not affected by measurement uncertainty. This modelling approach takes basis in the definition and determination of the model uncertainties, see JCSS (2001) part 3.09.

In the decision analytical formulation, observations are considered by adapting the limit state function g_X thereby allowing for a new estimation of the structural states’ probabilities. In the current study, observations are assumed to be taken on the system resistance (initial resistance reduced by damage), damage, and load.

Precise observations of the resistance are modeled as realizations of the distribution M_R, damage precise observations are modeled as realizations of M_D, and load precise observations are modeled as realizations of M_S. For a structure composed by a single component in the case an observation on the current resistance is collected, Eq. (2) is written as:

$$g_{X|m_R} = M_U\cdot m_R\cdot R_0\cdot(1-M_D\cdot D) - M_S\cdot S \qquad (5)$$

where M_U is the measurement uncertainty and is equal to 1 in case of precise observation, and m_R is the model-predicted observation normalized to the current resistance $R_0\cdot(1-M_D\cdot D)$. In the case of a multi-component structure, if observations are performed on selected components – e.g. sensor are installed only on selected components – a consistent model must be adopted. An observation $O_{i,j}$ by the *i*-th information strategy on the *j*-th investigated component is defined as $O_{R,j} = M_U\cdot m_{R_j}$. In the case a resistance observation is obtained only on the first component $O_{R,1}$, the posterior resistance model uncertainty is written as $M_{R_1}|O_{R,1} = M_U\cdot m_{R_1}$. The system limit state equations, i.e., Eq. (3) in the case of ductile components and Eq.(4) in the case of brittle components, considering observations on resistance, can be written as follows.

$$g_{X|O_{R,j}} = \sum_j^n M_{R_j}|O_{R,j}\cdot R_{0_j}\cdot\left(1-M_{D_j}\cdot D_j\right) - \sum_j^n M_{S_j}\cdot S_j \qquad (6)$$

$$g_{X|o_{R,j}} = \max_{i=1\ldots n}\left((n-i+1)\cdot m_{R_j}|o_{R,j}\cdot r_{0_j}\cdot\left(1-m_{D_j}\cdot d_j\right)\right)-\sum_j^n M_{S_j}\cdot S_j \quad (7)$$

Eq.(6) and Eq.(7) can be rewritten for the cases observation is collected on the system damage or load by substituting the relative component model uncertainty, i.e., M_{D_j} or M_{S_j} by the collected normalized observation, i.e., m_{D_j} or m_{S_j}. In this study, resistance, damage, and load observations are analyzed independently from each other. Furthermore, observations are collected on a varying number of components. In the first case, the observation is obtained on one component, in the second case on two components, etc.

3.3. Action and Utility Model

In the field of structural integrity management, several actions can be implemented to preserve or enhance the structural performance. As the main scope of this paper includes the investigation of the value of different information strategies, decision alternatives are defined to be representative of the wide range of possible interventions on a structure. Therefore, five distinct action options are defined and their effect, cost, and implementation uncertainty are exemplarily quantified or taken from Thöns (2022). See Table 3.

Table 2: Action types, effects, costs, and implementation uncertainties

Action Effect	Cost (% c_F)	Implementation uncertainty
a_0 - Do Nothing		
-	-	-
$g_{X_1\|a_0} = M_R\cdot R_0\cdot(1-M_D\cdot D)-M_S\cdot S$		
a_1 - Strengthening		
$1.5\cdot R$	2 %	$Y_1 = Tr(0.95, 1.05, 1.10)$
$g_{X_1\|a_1,Y_1} = M_R\cdot 1.5\cdot R_0\cdot(1-M_D\cdot D)\cdot Y_1-M_S\cdot S$		
a_2 - Repair		
$M_D\cdot D=0$	10 %	$Y_2 = Tr(0.90, 0.95, 1.00)$
$g_{X_1\|a_2,Y_2} = M_R\cdot R_0\cdot Y_2-M_S\cdot S$		
a_3 – Load reduction		
$S/2$	1.0 %	$Y_3 = N(1, 0.1)$
$g_{X_1\|a_3,Y_3} = M_R\cdot R_0\cdot(1-M_D\cdot D)-M_S\cdot S/2\cdot Y_3$		
a_4 – Consequence reduction		
$c_{F,red} = 0{,}85\cdot c_F$	1.0 %	$Y_4 = Tr(0.85, 0.95, 1.05)$
$g_{X_1(t)\|a_4,Y_4} = M_R\cdot R_0\cdot(1-M_D\cdot D)-M_S\cdot S\le 0$ $c_{F,red}=0{,}85\cdot c_F\cdot Y_4$		

Action a_0 corresponds to the “do nothing” action. It does not result in any cost or effect on the system. Action a_1, i.e., “strengthening”, a_2, i.e., “repair” and a_3, i.e., “load reduction”, have a direct effect respectively on the components’ resistance, damage, and load and are therefore referred as system state actions. Action a_4 , i.e., consequence reduction, reduces the consequences of a failure event, in terms of costs, rather than the probability of the failure event to occur. This type of action is referred to as utility actions, see e.g., Thöns (2022).

The utility model exemplarily accounts for a cost of failure, the cost associated with the structure being in state X_1, equal to 1 ($c_F = 1$). For simplicity reasons, action costs, information costs and utilities are calculated as percentages of the failure cost. The same single observation cost is considered for the three different information strategies $c_i = 0.01\cdot c_F$. Multiple observations are considered by multiplying the cost of a single observation by the number of monitored components.

4. EXEMPLARY CASE STUDY

In this exemplary case study, the information strategies, i.e., single, or multiple observations of components’ resistance, load, or damage, are analyzed for Daniels systems with brittle and ductile component behavior. The Daniels system is composed of five components. VoI comparisons are performed (i) in between the same information strategy investigating how VoI varies according to the number of monitored components, and (ii) among the three information strategies. The analysis considers both precise observations as representative of the maximum benefit which can be obtained from information in integrity management, and imprecise observations to account for the effect of a generic measurement uncertainty on the value of information. The probabilistic model uncertainty models assumed in the analysis are taken from JCSS (2001) and are reported in Table 4.

Table 3: Probabilistic Model

Variable	Distribution	Mean	Variance
M_R	Lognormal	1.0	0.05
R_{mat}	Lognormal	1.0	0.05
M_D	Lognormal	1.0	0.1
D	Lognormal	0.2	0.02
M_S	Lognormal	1.0	0.1
S	Gumbel	1.0	0.2
M_U	Normal	1.0	0.03
γ	-	1.5	

The system material resistance R_{mat} is modeled as fully correlated as dependent on the system components fabrication and production. Structural components are designed such that their initial resistance $R_0 = \gamma \cdot A \cdot R_{mat}$, where γ is safety factor and A represents the components geometrical properties, are designed for a structural reliability equal to $P(X_2) = 10^{-5}$ according to standards, see e.g. Eurocode - Basis of structural design (2002). Damage can be modeled as fully correlated for system damage phenomena or non-correlated for localized. For the sake of simplicity, in this analysis, damage is assumed to be fully correlated. Load is fully correlated as the external load distributes equally to the system components. The model uncertainties, i.e., M_R, M_D and M_S are assumed to be exemplarily non-correlated as hardly any reference can be found in literature. The measurement uncertainty M_U is considered not to be correlated for simplicity reasons.

4.1. Value of the information strategy given one observation

The value of each information strategy is firstly analyzed for the case in which a single component is observed. The resistance, damage, and load information strategies are compared considering the two cases of precise and imprecise observations. Figure 2 reports the VoI of the three information strategies when a Daniels system with five ductile components is assumed.

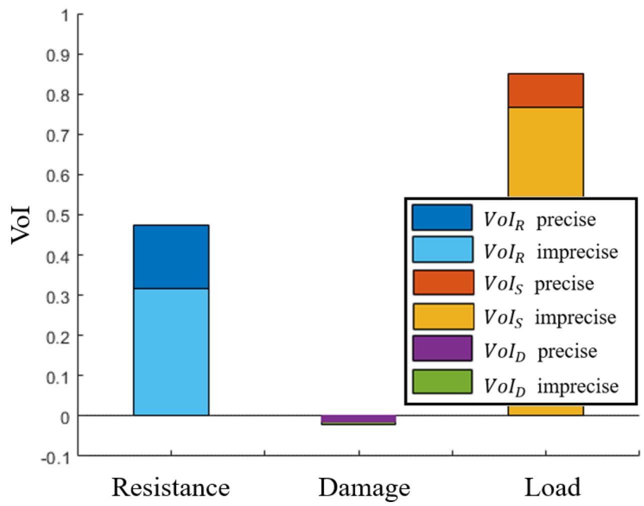

Figure 2: Value of a single information system, ductile components.

The VoI of the load information strategy (VoI_S) with both precise and imprecise observations has the highest value. This indicates that for a single observation and a Daniels system with ductile components, the load information strategy leads to the largest benefit in the structural integrity management. The resistance information strategy results in a lower VoI, i.e., $VoI_R < VoI_S$, and the related integrity management is potentially more sensitive to measurement uncertainty, i.e. the gap between VoI_R precise and VoI_R imprecise is larger than in the other cases. The damage information strategy has a very low VoI, i.e., VoI_D is negative due to the information cost.

Figure 3 reports the VoI of the three information strategies when a Daniels system with brittle components is assumed. The load information strategy results again in the highest VoI. It is observed that the VoI is less sensitive to the measurement uncertainty, i.e., the gap between precise and imprecise VoI is generally lower compared to the ductile Daniels system (see Figure 2).

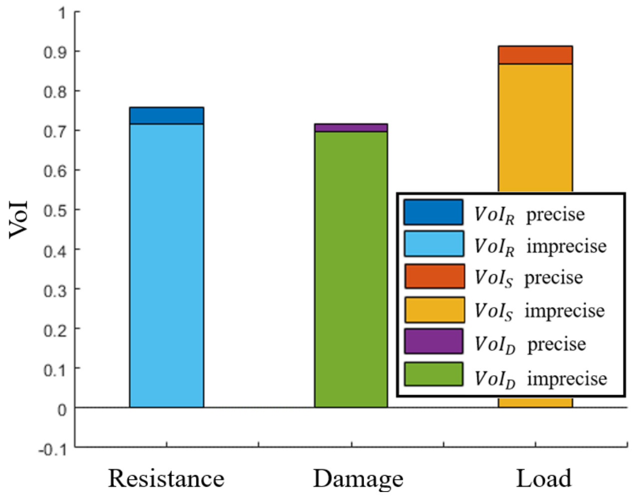

Figure 3: Value of a single information system, brittle components.

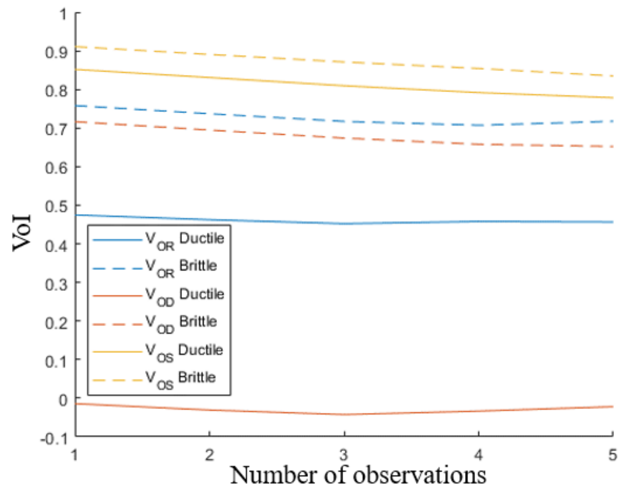

Figure 4: Values of multi-information systems deployed on a Daniels system.

In the analysis, the VoI of the three information strategies VoI_R , VoI_D and VoI_S is significantly higher than in the previous analysis with ductile components Figure 2. The reason may be the fact that the same observation may induce the estimation of different structural reliabilities and risks for brittle and ductile components, which consequently may lead to the selection of different optimal actions. It should be noted that the system reliability is generally higher for Daniels systems composed of ductile components rather than brittle components, see Gollwitzer and Rackwitz (1990), and the relationship between structural reliability and VoI may be furtherly investigated.

To summarize: the load information strategy is highly valuable for both components' behaviors. The resistance information strategy results in a higher VoI for the brittle component behavior, whereas the value of the damage information strategy significantly depends on the assumption on the structural component behavior.

4.2. Value of the information strategy given multiple observations

This section encompasses the analysis of the VoI for the three information strategies when multiple observations are collected on the five structural components. Figure 4 shows the results of the analysis performed accounting for both the ductile and brittle components behaviors.

In Figure 4, it is noted that the VoI generally decreases with the number of observations collected, except for the case of the damage information strategy and ductile components. The decrease occurs as the calculated VoI includes the information cost. Furthermore, it is noted that the VoI_R and VoI_D ductile curves have local minima for which the VoI reaches its lowest value among the 5 components. This indicates the number of observations that have the lowest effect on the structural integrity management, e.g., three damage observations for ductile components. The difference between brittle and ductile component behaviors is the highest for the damage information strategy as observed previously in Section 4.1.

5. CONCLUSIONS

This paper contains the modelling and value of information analysis of different information strategies for a generic multi-component redundant structure.

The study shows that structural system characteristics influence the VoI for different measurement and monitoring strategies. The VoI analyses include resistance, damage and load information acquirement strategies and a set of structural integrity management action such as repair, strengthening, load reduction and consequence reduction. The optimal number of monitored components is found to be depending both on the structural characteristics and the information acquirement strategy. Furthermore, in the exemplary case study, it is found that the number of VoI-optimal observations may be very low or may first decrease and then increase with an increasing number of sensors. Further analysis is required to identify the conditions for this varying behavior. Moreover, the influence of the structural reliability on VoI should be investigated accounting for both the component behaviors and correlation.

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