Risk-Based Optimization of Design Load on Membrane Structures with Predetermined Service Life

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ABSTRACT: “A membrane structure” can be a good candidate of a “Kigen-tsuki building,” because its dominant design load is wind or snow. The object of this research is to discuss the merit of designing a membrane structure as a “Kigen-tsuki building”.

1. INTRODUCTION
In recent years, the social needs of buildings with limited service-lives have been increasing for the purpose of efficient use of land leasehold, experimental verification of new construction methods, etc (Kimura U, 2013). However, in the current Building Standards Act of Japan, a building with a service life longer than a year is designed in principle as an ordinary building. In order to meet such social needs, a “Kigen-tsuki Building,” a building designed with a predetermined service life and conditions of use, and, in principle, disassembled after the expiration of the period of the predetermined service life, has been proposed and Recommendations for design of such buildings was published (AIJ, 2013). Because the possibility that a building with a service life shorter than that of an ordinary building encounters a rare event during its service life is lower, the design load could be reduced as long as the safety level equivalent to that of an ordinary building can be achieved. One of the ideas to achieve the safety level is to execute Preliminary Preventive Cares (PPC) such as installation of temporary supports against wind and/or snow load on the basis of well-developed weather forecast.

The dominant load of a “membrane structure” is wind or snow, and thus, a membrane structure can be a good candidate of a “Kigen-tsuki building.” The object of this research is to discuss the merit of designing a membrane structure as a “Kigen-tsuki building”. To achieve this objective, first, a method for estimating the limit state probability of the steel frame designed as a Kigen-tsuki Building is developed by taking into account the conditional distribution of the daily maximum wind speed and ground snow weight given that a PPC is executed as a function of the accuracy of the forecast. The method also takes into account the possibility of the failure of conducting PPC owing to uncertainty in the forecast. Then an approximation formula for estimating the residual strength of membrane materials considering its deterioration owing to UV.

2. RISKS ASSOCIATED WITH LOAD REDUCTION AND EXPECTED TOTAL COST
Risks associated with the reduction of design loads on a Kigen-tsuki Building are classified into the following three types (Mori et al., 2012).

$C_{r_1}$: Cost of PPC including loss owing to restricted use of the building (per time),

$C_{r_2}$: Loss of failure owing to improper conduct of PPC,

$C_{r_3}$: Loss owing to the occurrence of load exceeding the design load of ordinary buildings.
Since it is difficult to treat $C_1$ and $C_2$ with clear distinctions, they are treated together as $C_T = C_1 + C_2$. Then, the expected total cost, $C_T$, can be expressed as the sum of the initial cost, $C_I$ the expected cost associated with PPC, and the expected cost of failure in serviceability and safety.

$$C_T = C_I + C_{FS} \cdot t_L + C_{FU} \cdot P_{FU} + C_{tr} \cdot E[N_{tr}]$$  \hspace{1cm} (1)

where $t_L$ is the pre-determined service life (year), $C_{FS}$ and $C_{FU}$ are losses owing to the exceedance of the serviceability limit state and ultimate limit state, respectively, $P_{FS}$ and $P_{FU}$ are the serviceability limit state probability and ultimate limit state probability, respectively, $N_{tr}$ is the number of times that PPC is conducted during $t_L$, and $E[\cdot]$ is the expectation operator. The reference period of $P_{tr}$ is set to be 1 year, while that of $P_{tr}$ is set to $t_L$ years. According to the principle of minimizing the total expected cost, the design load can be determined so that $C_T$ in Eq. (1) is minimized.

3. DESIGN LOAD AND TRIGGER LEVEL

3.1. Evaluation method

3.1.1. PPC and Posterior probability distribution of wind speed and ground snow weight

Because snow on roof in ordinary areas and a strong wind last mostly one day, it is assumed that PPC is applied on a daily basis when the weather forecast value exceeds the trigger level, the predetermined forecast level at which PPC is executed. Because of errors in forecasting, PPC cannot always be executed even if the actual daily maximum wind speed or the ground snow weight, $X_e$, exceeds the trigger level. Here, it is assumed that the probability of executing PPC when the trigger level is set to $X_{tr}$ given $X_e = x$ can be expressed as

$$F_{tr}(x; x_{tr}) = P[\text{triggered}|X_e = x] = \Phi\left(\frac{x - x_{tr}}{V_{tr} \cdot x_{tr}}\right)$$  \hspace{1cm} (2)

where $\Phi(\cdot)$ is the standard normal probability distribution function (CDF), and $V_{tr}$ is the accuracy of the weather forecast expressed by its standard error divided by the actual value, corresponding to its coefficient of variation (COV).

According to the theorem of total probability, the probability that PPC is triggered on a certain day, $p_{tr}$, can be expressed as

$$p_{tr} = \int_0^\infty F_{tr}(x; x_{tr}) f_{X_e}(x) dx$$  \hspace{1cm} (3)

where $f_{X_e}(x)$ is the probability density function of the daily maximum value, $X_e$. Then $E[N_{tr}]$ in Eq. (1) can be expressed as

$$E[N_{tr}] = r_d \cdot t_L \cdot p_{tr}$$  \hspace{1cm} (4)

where $r_d$ is the number of days that a trigger may take during a year. The conditional CDF of $X_e$ given that PPC is triggered can be expressed as

$$F_{X_e|tr}(x) = P[X_e \leq x|\text{triggered}]$$

$$= \int_0^x P[\text{triggered}|X_e = x] \cdot f_{X_e}(x) \, dx$$

$$p_{tr} = \frac{\int_0^x F_{tr}(x; x_{tr}) \cdot f_{X_e}(x) \, dx}{p_{tr}}$$  \hspace{1cm} (5)

Likewise, the posterior CDF of $X_e$ given that PPC is not triggered can be expressed as

$$F_{X_e|\neg tr}(x) = P[X_e \leq x|\text{not triggered}]$$

$$= \int_0^x \left(1 - F_{tr}(x; x_{tr})\right) \cdot f_{X_e}(x) \, dx$$

$$= \frac{F_{X_e}(x) - \int_0^x F_{tr}(x; x_{tr}) \cdot f_{X_e}(x) \, dx}{1 - p_{tr}}$$  \hspace{1cm} (6)

Fig. 1 shows the unconditional (solid line) and conditional CDF of the daily maximum wind speed given that PPC is triggered (dashed line) or not triggered (dotted line). It is assumed that $V_X = 0.15$, $x_{tr} = 16$ m/s, $V_{tr} = 0.1$ and the daily maximum wind speed is described by a Gumbel distribution with mean equal to 7.1 m/s and COV equal to 0.48.
3.1.2. Wind load model

Wind load, \( W \) (N), can be expressed as
\[
W = 0.5 \rho (X_{E_0})^2 C_D G_D A \tag{7}
\]
where \( X \) is the 10-minute average wind speed (m/s) at 10 m above the ground, \( \rho \) is the air density, which is equal to 1.2 kg/m\(^3\), \( E_0 \) is the vertical profile coefficient, \( C_D \) is the wind force coefficient, \( G_D \) is the gust response factor, and \( A \) is the projected area (m\(^2\)). Eq. (7) can be rewritten as
\[
W = a \cdot B \cdot X^2 \tag{8}
\]
where \( a \) is a normalization coefficient making the median of \( B \) equal to unity. Then \( B \) can be expressed as
\[
B = \frac{1}{a} \cdot \rho \cdot (E_0)^2 \cdot C_D \cdot G_D \cdot A \tag{9}
\]
Assuming that \( A \) is a deterministic value, and that \( \rho, E_0, C_D, \) and \( G_D \) are lognormally distributed, \( B \) is also lognormally distributed with logarithmic standard deviation, \( \sigma_{\ln B} \), as
\[
\sigma_{\ln B} = \sqrt{\sigma_{\ln \rho}^2 + 2 \sigma_{\ln E_0}^2 + \sigma_{\ln C_D}^2 + \sigma_{\ln G_D}^2} \tag{10}
\]

3.1.3. Snow load model

The snow load \( S \) (N/m\(^2\)) can be expressed as
\[
S = X \cdot \mu_0 \cdot R_{env} \tag{11}
\]
where \( X \) is the ground snow weight (N/m\(^2\)), \( \mu_0 \) is the roof shape coefficient, and \( R_{env} \) is the environmental coefficient.

Like Eq. (8), Eq. (11) can be rewritten as
\[
S = a \cdot B \cdot X \tag{12}
\]

where
\[
B = \frac{1}{a} \cdot \mu_0 \cdot R_{env} \tag{13}
\]

The logarithmic standard deviation of \( B \) can be expressed as
\[
\sigma_{\ln B} = \sqrt{\sigma_{\ln \mu_0}^2 + \sigma_{\ln R_{env}}^2} \tag{14}
\]

The forecasted ground snow weight is evaluated by multiplying the forecasted ground snow depth with the unit snow weight.

3.1.4. Strength model

The safety of ordinary buildings against wind and snow in Japan is in general ensured not by ultimate strength design but by allowable stress design (ASD). Considering such situations, the probability characteristics of strength for the serviceability limit state and the ultimate limit state are modeled as follows:

The allowable stress for short term loading, \( R_n \), is expressed as
\[
R_n = D_n + L_n + P_n \tag{15}
\]
where \( D_n, L_n \) and \( P_n \) are the design dead load, design live load and design wind (or snow) load considered in ASD.

In a steel structure in Japan, \( R_n \) is the smaller between the nominal yield strength, \( R_S \), and 70% of the nominal tensile strength, \( R_T \). Here, it is assumed that \( R_n \) is the 5% quantile value of \( R_S \) that is considered as the strength for the serviceability limit state. Then the strength for the ultimate limit state is modeled as
\[
R_T = \frac{R_S}{0.7} \tag{16}
\]

Assuming that \( R_S \) is lognormal distributed with a COV equal to \( V_R \), then the mean of \( R_S, \mu_{R_S} \), can be expressed as
\[
\mu_{R_S} = \alpha_R \cdot R_n \tag{17}
\]
where
\[
\alpha_R = \frac{\sqrt{1 + V_R^2}}{\exp \left\{ \Phi^{-1}(0.05) \sqrt{\ln(1 + V_R^2)} \right\}} \tag{18}
\]

From Eqs. (15), (17) and (18)
where $\mu_D$ and $\mu_L$ are the means of $D$ and $L$, respectively, and $\mu_{pa}$ is the mean of the annual maximum of the wind load or snow load.

It is assumed that the design load formulae in ASD of an ordinary building is based on the 50 years recurrence value of the wind speed or ground snow weight. Because the wind load is proportional to the square of the design wind ground snow weight.

The limit state probability of a structural member, $P_f$, can be expressed as

$$P_f = 1 - \prod_{i=1}^{r_d} (R_{PPC} - (D + L + P_i) > 0)$$

where $r_d$ is the reference period (days), $R_{PPC}$ is the strength when either PPC is conducted, $R$, or not conducted, $R_0$. $P_i$ is the load owing to the $i$-th day maximum wind speed or ground snow weight.

By substituting Eqs. (8) and (12) into Eq. (22) and assuming that the daily maximum wind speed or ground snow weight are statistically independent of each other, $P_f$ can be expressed as

$$P_f = 1 - \prod_{i=1}^{r_d} (R_{PPC} - (D + L + W(\text{or } S)) > 0)$$

$$= 1 - \int ... \{F_{X_{r}}[g(r_{PPC}, d, l, b, a)]\}^{r_d} \cdot f_{R}(r) \, dr$$

where $f_{R}(r)$ is the joint probability density function of $R, R_0, D, L,$ and $B$.

$g(r, d, l, b, a)$ is expressed as follows:

**Wind load**

$$g(r, d, l, b, a) = \exp \left[ \frac{\ln(r - d - l) - \ln b - \ln a}{2} \right]$$

**Snow load**

$$g(r, d, l, b, a) = \exp[\ln(r - d - l) - \ln b - \ln a]$$

$f_{X_{r}}[g(r_{PPC}, d, l, b, a)]$ can be estimated using its conditional CDF expressed by Eqs. (5) and (6) and the theorem of total probability as

$$F_{X_{r}}[g(r_{PPC}, d, l, b, a)] = F_{X_{r}}[g(r_{0}, d, l, b, a)] \cdot p_{tr}$$

+ $F_{X_{r}}[g(r, d, l, b, a)] \cdot (1 - p_{tr})$
Table 1: Probability model of wind speed and ground snow weight

<table>
<thead>
<tr>
<th></th>
<th>CDF</th>
<th>COV ($\nu_x$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual max of wind speed</td>
<td>Gumbel or Fréchet</td>
<td>0.1, 0.2, or 0.3</td>
</tr>
<tr>
<td>Annual max of ground snow weight</td>
<td>Gumbel</td>
<td>0.8, 1.0, or 1.2</td>
</tr>
</tbody>
</table>

Table 2: Parameters related to wind and snow load.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air density, $\rho$</td>
<td>0.10</td>
</tr>
<tr>
<td>Vertical profile coefficient, $E_v$</td>
<td>0.10</td>
</tr>
<tr>
<td>Wind force coefficient, $C_D$</td>
<td>0.15</td>
</tr>
<tr>
<td>Gust response factor, $G_D$</td>
<td>0.15</td>
</tr>
<tr>
<td>Roof shape coefficient, $\mu_0$</td>
<td>0.15</td>
</tr>
<tr>
<td>Environment coefficient, $R_{env}$</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 3: Probability model of loads and resistance.

<table>
<thead>
<tr>
<th>CDF</th>
<th>Nominal mean</th>
<th>Mean</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Normal</td>
<td>1.0</td>
<td>$\mu_D$ 0.1</td>
</tr>
<tr>
<td>$L$</td>
<td>LN</td>
<td>0.4</td>
<td>$0.75\mu_0$ 0.4</td>
</tr>
<tr>
<td>$R$</td>
<td>LN</td>
<td>5% low limit</td>
<td>Eq. (19) 0.1</td>
</tr>
</tbody>
</table>

Table 4: Cost model.

<table>
<thead>
<tr>
<th>Cost component</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost, $C_I$</td>
<td>1.0</td>
</tr>
<tr>
<td>Parameters related to initial construction cost, $C_{ia}$</td>
<td>0.01, 0.03, 0.05, or 0.1</td>
</tr>
<tr>
<td>Losses owing to the exceedance of the serviceability limit state, $C_{FS}$</td>
<td>0.1 or 0.3</td>
</tr>
<tr>
<td>Losses owing to the exceedance of the ultimate limit state, $C_U$</td>
<td>2.0</td>
</tr>
<tr>
<td>Cost of PPC including loss owing to limited use of the building, $C_{br}$</td>
<td>0.001, 0.004, or 0.007</td>
</tr>
</tbody>
</table>

is described by either a Gumbel or Fréchet distribution, and the statistics listed in Table 1 are considered here.

According to statistical data from 1961 to 2015, it snows more than 50 mm above the ground for 1.5 days on average in ordinary area such as Nagoya, Tokyo, and Yokohama from December to February (JMA, 2014). Then, it is assumed here that the events of snow can be modeled by a Poisson process with mean occurrence rate, $\lambda$ equal to 1.5 days / 90 days. Then, the CDF of the $r_a$ day maximum ground snow weight can be expressed as

$$F_{X_0}(x) = \sum_{i=1}^{\infty} (F_{X_S}(x))^i \cdot \frac{(\lambda \cdot r_a)^i e^{-\lambda r_a}}{i!}$$

$$= \exp \left[ -\lambda \cdot r_a \cdot \left( 1 - F_{X_S}(x) \right) \right]$$

where $F_{X_S}(x)$ is the CDF of the ground snow weight when it snows more than 50 mm.

It is assumed that all snow melts after the end of each snowfall. The 90 day (1 year) maximum ground snow weight generally has a Gumbel distribution and is expressed as

$$F_X(x) = \exp \left[ -\lambda \cdot 90 \cdot \left( 1 - F_{X_S}(x) \right) \right]$$

From Eq. (28), $F_{X_S}(x)$ can be expressed as

$$F_{X_S}(x) = 1 - \frac{\exp\left[ -\alpha(x - u) \right]}{\lambda \cdot 90}$$

Parameters in Eqs. (7) and (11) other than wind speed and ground snow weight are listed in Table 2. It is assumed that these parameters are lognormally distributed with a median equal to unity.

Probability models of the strength and dead load and live load are shown in Table 3. Considering low-rise steel structures, which are often candidates for a Kigen-tsuki Building, it is further assumed that the ratio, $\mu_{\text{LD}}/\mu_D = 0.75$ and the ratio $\mu_{\text{LD}}/(\mu_D + \mu_{\text{LD}}) = 1.0$ (Itoda et al., 2004). It should be noted that the ratio, $\mu_{\text{LD}}/(\mu_D + \mu_{\text{LD}})$ was separately examined in the range of 0.8 to 2.0, and it was confirmed that the influence of the ratio on the following conclusions is very low.

The design wind load and snow load are determined on the basis of the return period of wind speed or ground snow weight, $r_p$, equal to either 2, 3, 5, 10, 15, 20, 25, 30, 35, 45 or 50 years. In the following calculation, the 50-year recurrence wind speed is set to 32 m/s, snow depth is set to 30 cm. It is also assumed that the accuracy
of the weather forecast, $V_{tr}$, for the wind speed and snow depth is 0.1 or 0.2 (Shimizu and Omasa, 2006).

### 3.2.2. Cost model

Because the initial cost, $C_i$, is approximately a linear function of the design load (Kanda, 2008), the following initial cost model is assumed.

$$C_i = C_{ia} \cdot P_d \cdot t_5 + C_{ib}$$  \hspace{2cm} (30)

where $P_d$ and $P_{50}$ are the design load on a membrane structure as Kigen-tsuki Building and an ordinary building, respectively. The dimensionless cost models shown in Table 4 are considered here.

### 3.2.3. Results

Fig.2 shows an example of the dependence of (a) $P_{fs}$ and (b) $P_{fu}$ on the ratio of the trigger level, $x_{tr}$, to the design wind speed, $x_d$, which is determined on the basis of a return period equal to 2, 5, 15, 25, or 35 years. The design wind speed, $x_d$, normalized by the 50-year recurrence wind speed, $x_{50}$, is also presented in the figure. Here, it is assumed that the annual maximum wind speed is described by a Gumbel distribution with a COV equal to 0.2, $V_{r} = 0.2$, $V_{tr} = 0.1$, and $t_L = 10$ years. When $x_{tr}$ is sufficiently smaller than the design wind speed, $P_{fs}$ and $P_{fu}$ are nearly constant; however, it starts to increase at a certain value of $x_{tr} = x_d$. The value of the ratio where $P_{fs}$ starts to increase varies depending on $x_d$, and it decreases as $x_d$ becomes smaller. By contrast, the value where $P_{fu}$ starts to increase is not significantly dependent on $x_d$. To ensure the same safety level with an ordinary building, PPC must be conducted before $P_{fu}$ rises.

Fig.3 shows the dependence of $C_{tr} \cdot E[N_{tr}]$ (dotted-dashed line), $C_{fs} \cdot P_{fs} \cdot t_L$ (dashed line), $C_{fu} \cdot P_{fu}$ (dotted line), and their sum (solid line) on the ratio ($x_{tr}/x_d$). Then the same probability model considered in Fig.2 and the design wind speed is determined on the basis of $r_p = 5$ years, $t_L = 1$ year, $C_{fs} = 0.3$, and $C_{tr} = 0.001$. When $x_{tr}/x_d$ is up to 0.8, $P_{fu}$ starts to increase when $x_{tr}/x_d$ is about 0.6.
Because $C_I$ is constant, $x_{tr}$ that minimizes the sum of these three costs yields the minimum expected total cost. The optimum $x_{tr}$ in this example is about 70% of $x_d$.

Fig. 4 shows the expected total cost, $C_T$, using the same probability model considered in Fig. 3 but with various design wind speeds, which are determined on the basis of $t_P = 2, 5, 15, 25, 35, \text{ or } 45$ years. $C_{fa}$ for the initial cost is assumed to be 0.1. It can be found in the figure that a design wind speed of $t_P = 15$ years and a trigger level of $x_{tr} = 0.85x_d$ yield the minimum total expected cost.

Figs. 5 and 6 show the optimum design wind speed and ground snow weight, respectively on a Kigen-tsuki Building with a service life, $t_L$. The associated trigger levels are also presented in the figures. Here, it is assumed that the COV of the annual wind speed is equal to 0.2 or 0.3, the COV of the annual ground snow weight is equal to 0.8 or 1.2, $V_R = 0.2, V_{tr} = 0.1, C_{fa} = 0.1, 0.03 \text{ or } 0.05, C_{tr} = 0.004 \text{ or } 0.007, \text{ and } C_{FS} = 0.1 \text{ or } 0.3$. $x_{fa}$ and $x_{tr}$ are indicated by ● and ■ as a ratio of 50-year recurrence value, respectively. Because PPC is not conducted when the optimum design load is based on the 50-year recurrence value, the optimum trigger level is not shown. Compared with the basic case in Figs. 5(a) and 6(a), the optimum design load decreases when $V_X$ is large [Figs. 5(b) and 6(b)] or $C_{FS}$ is small [Figs. 5(c) and 6(c)]. By contrast, when $C_{tr}$ is large [Figs. 5(d) and 6(d)], not only the optimum design load but also the trigger level increases. This is because the priority here is to reduce the cost of PPC than the risk of exceeding service ability limit state. When $C_{fa}$ is small [Figs. 5(e) and 6(e)], the optimum design load increases, because $C_I$ cannot be reduced significantly even if the design load is reduced. In the case of the snow load in Fig. 6, the design load reduction rate is larger than the wind load because the COV of the ground snow weight is larger than that of the wind speed and the period during snowfall is as short as 90 days.

4. DETERIORATION FUNCTION OF MEMBRANE MATERIAL

In the above chapter, we described design load reduction, focusing on the steel frame of Kigen-tsuki building. The other main material in membrane structure, that is membrane, deteriorates with time and such deterioration should be considered in the risk assessment. The rate of the deterioration depends on the type of membrane material; the strength could be
halved in 10 years. Fig.7 shows residual strength rate, $T_D$ as a function of the total ultraviolet dose factor, $UV_T$. The deterioration data of membrane materials collected from tent warehouses with age from 7.6 to 20 years at 15 locations throughout Japan are indicated by ●. Since $T_D$ falls exponentially, we propose an approximation formula as a function of $UV_T$ as

$$T_D = 100 \cdot e^{-UV_T}$$  \hspace{1cm} (31)$$

where $UV_T$ can be estimated

$$UV_T = UV_a \cdot t_L \cdot L_\theta$$  \hspace{1cm} (32)$$

where $UV_a$ is the annual UV index shown in Fig.8, normalized by the value in Tokyo; $t_L$ indicates the elapsed years of the membrane material; $L_\theta$ is the angle index. shown in Table 5, which is determined from the inclination angle of the membrane material and the angle to the south direction.

5. CONCLUSION
A method for calculating the optimum design load of membrane structures with predetermined service life is presented. Because currently the deterioration of the membrane it out of scope of this method, it will be extended to consider the deterioration for which estimation formula is presented in Chapter 4.

6. REFERENCES


