Risk management strategies and probabilistic failure pressure model development for pipelines with crack-like defect

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**ABSTRACT:** Optimal inspection and maintenance planning for deteriorating pipelines is necessary for cost-effective risk management decisions. The objective of this study is to develop a framework for determining the optimal inspection and maintenance planning for deteriorating pipelines considering failure probabilities. The framework is developed based on a decision tree model by using analytical methods to evaluate events and considers the inspection schedules and the threshold for repair as the design variables. To evaluate the probability of failure, an accurate probabilistic failure pressure model for thin-walled pipelines containing crack-like defects is proposed. The failure pressure prediction model is developed by adding a correction factor to the modified Ln-Secant method, a semi-empirical model developed previously. It is found that the proposed model provides unbiased and more accurate predictions. To illustrate the proposed framework, a simple problem of a steel pipeline with an initial crack is used, where a stochastic model of crack propagation is assumed. The case study indicates that finding an optimal inspection and maintenance planning is a compromise of the costs of inspection, repair, and failure.

1. **INTRODUCTION**

Risk assessment for pipelines with anomalies is critical, since anomalies weaken the pipeline and make it more vulnerable to failure that can lead to tremendous consequences (both economically and environmentally). Usually, inspection and repair actions are performed to ensure pipeline integrity. However, inspections and repairs can be costly throughout the service life of pipelines if not well planned.

In the literature, frameworks have been proposed to optimize the inspection and repairs strategies for deteriorating pipelines. For example, Arzaghi et al. (2017) developed a risk-based decision-making framework for the maintenance scheduling of subsea pipelines, with the aim of maximizing the expected utilities for maintenance alternatives every year of facility operation. Xie and Tian (2018) proposed a risk-based pipeline framework to determine the inspection time by finding the optimal probability of failure threshold by minimizing the total cost rate and the consecutive inspection time is updated using the latest inspection data. However, the framework is not based on the expected total life-cycle cost which typically is the interest of decision makers. Gomes and Beck (2014a) suggested a framework for optimal inspection planning and repair under crack propagation by minimizing the expected total cost of initial construction, inspection, maintenance, and failure costs, where the expected numbers of failures and repairs are calculated using a decision trees concept with Monte Carlo simulation. Li et al. (2019) proposed a framework where the expected number of failures was calculated based on the renewal theory; however, the impact of repair actions on the probability of failure is not considered. Zou et al. (2019) suggested a
framework that considers the impact of maintenance actions on the probability of failure, but the failure cost is calculated based on probability of only one failure occurrence during the service life. There is a need to develop a reliable framework that is not too complex to use in the risk management decisions practically.

In addition, an accurate failure pressure model plays a critical role in the assessment of probability of failure. For pipelines with single crack-like defect, the failure pressure has been assessed using models like the original Ln-Sec (Kiefner et al., 1973), modified Ln-Sec (Kiefner, 2008), CorLAS™ (Polasik et al., 2016), and failure assessment diagram (FAD) methods such as API 579 (API 579-1/ASME FFS-1, 2016) and BS 7910 (BS 7910, 2013). Several studies have compared the performance of the existing models. For instance, Yan et al. (2014) compared 4 models (i.e., original Ln-Sec, CorLAS, BS 7910-version 2013, and API 579-version 2016) using 112 full scale burst test data. They found that the CorLAS model had the best performance, and the original Ln-Sec, BS 7910, and API 579 are in general conservative. The authors also conducted prediction comparison of several existing models such as the original Ln-Sec, modified Ln-Sec, CorLAS™, API 579, and BS 7910 based on the mean and standard deviation of the ratio of the predicted to the actual burst failure pressures (Kere et al., 2022). It is found that most of existing models are conservative, which is not suitable to use in the risk management of pipelines.

There are two objectives in this study: to develop an accurate probabilistic failure pressure model for thin-walled pipelines containing single crack-like defects and to develop a decision framework that is easy to use for determining the optimal inspection and maintenance planning for deteriorating pipelines. Firstly, the probabilistic failure pressure model is developed by adding a correction factor to an existing prediction model using a multivariate linear regression based on a database established in this study. Next, the framework is developed based on a decision tree model by using analytical methods to evaluate events. The optimum design variables are determined by minimizing the expected total life-cycle cost, which consists of construction, inspection, repair, and failure costs. Lastly, a case study of a steel pipeline with an initial crack is used to illustrate the proposed framework.

2. MODEL DEVELOPMENT FOR PRESSURE PREDICTION

Based on a study by Kere et al. (2022), it is found that the modified Ln-Sec model provides the best prediction performance among the existing models they studied, although the model displays a big prediction variance. Here, the failure pressure, \( P_f \), is modeled by adding a correction factor to the modified Ln-Sec model, \( \hat{P}_{mod \ Ln-Sec} \), which is expressed as follows:

\[
\hat{P}_f = \alpha \cdot \hat{P}_{mod \ Ln-Sec}
\]  

(1)

\[
\hat{P}_{mod \ Ln-Sec} = \frac{2t\sigma_f}{D} \cdot \frac{1 - A/A_0}{1 - (A/A_0)M_T^{-1}} \cdot \frac{\cos^{-1}(e^{-x})}{\cos^{-1}(e^{-y})}
\]  

(2)

\[
x = \frac{\pi K^2_{mat}}{8c_e \sigma_f^2} \quad \text{&} \quad y = x(1 - a/d_w)^{-1}
\]  

(3)

where \( \alpha \) = correction factor; \( D \) = outside diameter of the pipe; \( a \) = depth of crack; \( d_w \) = wall thickness of the pipe; \( 2c \) = crack length; \( 2c_e = A/a \) equivalent length of crack; \( A \) = actual area of the surface crack along its length; \( A_0 = 2c \cdot d_w \); \( \sigma_f \) = yield strength of the pipe material; \( \sigma_y = 68.95 \) MPa = flow stress of the pipe material; \( M_T \) = folias or bulging factor of pipe, and \( K_{mat} \) = fracture toughness of the pipe material. If actual data of \( K_{mat} \) is not available, it can be approximated using the following empirical expression:

\[
K_{mat}^2 = C_v E/A_c
\]  

(4)

where \( C_v \) = upper shelf energy determined from tests of Charpy V-notch impact specimens, \( A_c \) = cross-sectional area of the Charpy specimen used, and \( E = \text{Young’s modulus of the pipe material} \).

The correction factor, \( \alpha \), is modeled using a multivariate linear regression formulation in this study as follows:
\[ \alpha = \beta_0 + \sum_{i=1}^{m} \beta_i z_i + \sigma \varepsilon \]  

where \( \beta_i \) = model parameters; \( Z = \{z_i\} \) = independent variables; and \( \sigma \varepsilon \) = residual model error in which \( \sigma \) is the standard deviation of the model error (assumed to be constant) and \( \varepsilon \) is the standard normal random variable (i.e., normality assumption). Four normalized variables and their 2\textsuperscript{nd} order interaction among these four variables are used here to construct \( Z \), as shown below:

\[
Z = \begin{pmatrix}
D & \sigma_u & a & a \\
\sigma_y & d_w & c & \cdot \\
\sigma_y & d_w & c & \cdot \\
\cdot & \cdot & \cdot & \cdot
\end{pmatrix}
\]

(6)

where \( \sigma_u \) = ultimate strength of the pipe material. Considering all the 2\textsuperscript{nd} order interaction among the four basic variables, a total of 14 variables are resulted in \( Z \).

When considering all the 14 variables in Eq. (5), the model is a full model. An all-possible-subset model selection is adopted to eliminate the ones that do not contribute statistically significantly to the prediction (Sheather, 2009). In addition, a maximum model size of five (i.e., five variables in a model) is considered to avoid complex model formulations, and the model performance for each model size is compared using the model error standard deviation, \( \sigma \). The model with the lowest \( \sigma \) is the most desirable model.

A comprehensive database of pipelines with single crack-like defect (including both experimental and numerical data), which consists of the data collected (a total of 122 data points) from the literature is used for the model development. After model selection, it was found that the model with size 5 is the best model overall compared with other sizes models. Table 1 shows the variables selected and the statistics of the corresponding model parameters in the final model. It is worth noting that the data ranges used for the model development are \( D/d_w \) in [22 100], \( a/d_w \) in [0.19 0.99], and \( a/c \) in [0.0032 0.5140].

The prediction performance of the proposed model is compared with the modified Ln-Sec model through the mean, standard deviation of the ratio of the predicted to the actual burst failure pressures, \( P_{\text{pred}}/P_{\text{act}} \), as shown in Figure 1, where the cross refers to mean and the horizontal lines refer to mean \( \pm 1 \) standard deviation. Figure 1 indicates that the modified Ln-Sec model overestimates the burst pressure, while the proposed model provides unbiased prediction. Also, the proposed model shows smaller variability in \( P_{\text{pred}}/P_{\text{act}} \). Therefore, one can conclude that the correction factor proposed improves the modified Ln-Sec model for the failure pressure prediction of a pipe with longitudinally oriented single crack-like defect.

![Figure 1: Comparison of \( P_{\text{pred}}/P_{\text{act}} \) using the modified Ln-Sec model (Mod Ln-sec) and the proposed model (PM)](image)

### Table 1: Variables and model parameter statistics for the correction factor

<table>
<thead>
<tr>
<th>Model Parameters</th>
<th>Mean</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 ) (Intercept)</td>
<td>1.960</td>
<td>0.167</td>
</tr>
<tr>
<td>( \beta_1 ) ( D/d_w )</td>
<td>-0.020</td>
<td>0.006</td>
</tr>
<tr>
<td>( \beta_2 ) ( \sigma_u/\sigma_y )</td>
<td>-0.701</td>
<td>0.115</td>
</tr>
<tr>
<td>( \beta_3 ) ( D/d_w \cdot \sigma_u/\sigma_y )</td>
<td>0.019</td>
<td>0.004</td>
</tr>
<tr>
<td>( \beta_4 ) ( D/d_w \cdot a/d_w )</td>
<td>-0.007</td>
<td>0.001</td>
</tr>
<tr>
<td>( \beta_5 ) ( D/d_w \cdot a/c )</td>
<td>-0.007</td>
<td>0.003</td>
</tr>
</tbody>
</table>

3. FRAMEWORK DEVELOPMENT

The proposed decision framework aims to determine the optimal inspection interval and repair planning for deteriorating pipelines by minimizing the expected total cost, which consists of construction, inspection, repair, and failure costs. Considering the number of failures that
could happen during the pipeline service life, the
expected total cost, $E[C_T]$, can be calculated using
the following expression:

$$E[C_T] = C_0 + \sum_{k=0}^{n} P_{f,k} \cdot E[C_T|k \text{ failures}]$$  (7)

where $C_0 =$ cost of initial construction; $P_{f,k} =$ probability of $k$ number of failures occurrence during the service life, and $E[C_T|k \text{ failures}] =$ expected total cost given $k$ number of failures occurrence. In this framework, the decision
variables considered are: (1) the inspection times
and (2) defect size threshold for repair. Also, the
following assumptions are made:

- The time interval between successive inspections is constant.
- Inspection detection error is negligible.
- At inspection, a repair action is performed if
  the defect size exceeds a threshold.
- If failure occurs, a replacement will be put in place.
- After repair/replacement action, the pipe is
  restored to its initial state.

### 3.1. Expected total cost given no failure

When $k = 0$, the expected total cost during a
service life $t_s$, is calculated using the following
expression:

$$C_{NF}(t_s) = P_{f,0} \cdot (E[C_{in}|0 \text{ failure}] + E[C_T|0 \text{ failure}])$$  (8)

where $C_{NF}(t_s) =$ expected total costs of inspection
and repair given no failure during $t_s; E[C_{in}] 0 \text{ failure} =$ expected total cost of inspection
and repair given no failure, respectively. Figure 2 shows a decision tree where there are various scenarios (branches) due to possible failure occurrences and repair actions. Each branch corresponds to a unique defect growth path due to the repair/replacement actions. If the number of inspections prior to inspection time $t_i$ is $n_{in,i}$, then the total number of branches, $n_b$ will be $2^{n_{in,i}}$. Thus, the expected costs of
inspection and repair can be calculated as follows, respectively:

$$P_{f,0} \cdot E[C_{in}|0 \text{ failure}] = \sum_{i=1}^{n_{in}} \frac{C_i}{(1 + \gamma)^{t_i}} \cdot P_{f,0}(t_i)$$  (9)

$$P_{f,0} \cdot E[C_{rep}|0 \text{ failure}] = \sum_{i=1}^{n_{in}} \sum_{j=1}^{n_b} \frac{C_R}{(1 + \gamma)^{t_i}} \cdot P_{f,0}(t_i) \cdot P(R_{i,j})$$  (10)

where $n_{in} =$ total number of inspections during $t_i;$
$C_i$ and $C_R =$ unit costs of inspections and repairs,
respectively; $t_i =$ time of inspection; $\gamma =$ discount
rate; $P_{f,0}(t_i) =$ probability of no failure
occurrence before time $t_i$ where $P(t_i) =$ probability
of failure at time $t_i$; $P(R_{i,j}) =$ probability of no
failure occurrence in each branch $j$ before time $t_i$;
and $P(R_{i,j}) =$ probability of performing a repair
action at $t_i$ based on the defect size resulting from
branch $j$.

The failure here refers to the burst failure of
a pipeline. The probability of failure, $P_f$, is defined
as the conditional probability of attaining or
exceeding prescribed limit states given a set of
boundary variables, and can be written as:

$$P_f(t) = \int_{C(t) - D \leq 0} f(X) dX$$  (11)

where $f(X)$ is the joint probability density function of
a vector of random variables, $X; C(t) - D \leq 0$
refers to the failure domain in which $C$ is the
pressure capacity of the pipe, and $D$ is the demand
(that is the operating pressure of the pipe). This
probability is assessed by conducting a reliability
analysis such as Monte Carlo simulations or First
Order Reliability Methods (FORM).

If $T_f$ is the time at which failure occurs, then
the event of $T_f > t$ is equivalent to the event of $C(t) > D$; thus, $P(T_f \leq t) = P(C(t) \leq D) = P(t)$. In other
words, $P_f(t)$ can be considered as the cumulative
distribution function (CDF) of $T_f$. Furthermore,
the evolution of $C(t)$, thus $P(t)$, depends on the
defect time evolution that is impacted by the
maintenance actions (i.e., repair or no repair). As
an illustration, the decision tree shown in Figure 2
is used to show how the CDF of $T_f$, $F_{T_f}(t)$, is derived:

For $0 < t \leq t_1$

$$F_{T_f}(t) = P(T_f \leq t) = P(C(t) \leq D)$$ (12)

For $t_1 < t \leq t_2$

$$F_{T_f}(t) = P(T_f \leq t|R_{1,1}) \cdot P(R_{1,1}) + P(T_f \leq t|\bar{R}_{1,1}) \cdot P(\bar{R}_{1,1})$$ (13)

where

$$P(T_f \leq t|R_{1,1}) = P(T_f \leq t_1|R_{1,1}) + P(t_1 < T_f \leq t|R_{1,1})$$ (13a)

$$P(T_f \leq t_1|R_{1,1}) = P(C(t_1) \leq D)$$ (13b)

$$P(t_1 < T_f \leq t|R_{1,1}) = \left(1 - P(T_f \leq t_1|R_{1,1})\right) \cdot P(T_f \leq (t - t_1)|R_{1,1})$$ (13c)

$$P(T_f \leq t|\bar{R}_{1,1}) = P(C(t) \leq D)$$ (13d)

$$P(R_{1,1}) = P(d_i \geq d_r)$$ (13e)

where $d_i$ = defect size at the $i^{th}$ inspection and $d_r$ = defect size threshold for repair.

Figure 2: Decision tree ($F$: failure, $\bar{F}$: survival)

3.2. Expected total cost given one failure occurrence

The expected total cost considering only one failure occurrence during $t_s$ can be interpreted as the sum of the expected cost of one failure occurrence at time $T_f$ and the expected total cost given no failure during $[0 \ T_f]$ and during the remaining of the service life, $(t_s - T_f)$, both of which can be calculated using Eq. (8) by setting $t_s$ to be the last inspection time before failure and $t_s - T_f$, respectively. Since $T_f$ is a random variable, the product of the probability of one failure occurrence and the expected total cost considering only one failure occurrence can be calculated using the probability density function (PDF), $f(t)$, of $T_f$ as follows:

$$P_{f,1} \cdot E[C_f|1 \ failure] = \sum_{i=1}^{n_{in}} P(A_i)$$

$$= \left[ \begin{array}{c}
C_{NF}((i-1)\Delta t) \\
\cdot + \int_{(i-1)\Delta t}^{i\Delta t} \frac{C_F}{(1 + \gamma)^{\frac{t}{\Delta t}}} \cdot f_{T_f1}(t_{f1})dt_{f1} \\
\cdot + \int_{(i-1)\Delta t}^{i\Delta t} C_{NF}(t_{s} - t_{f1}) \cdot f_{T_f1}(t_{f1})dt_{f1} + P(A_s) \\
\cdot + \int_{n_{in}\Delta t}^{t_{s}} \frac{C_F}{(1 + \gamma)^{\frac{t}{\Delta t}}} \cdot f_{T_f1}(t_{f1})dt_{f1} \\
\end{array} \right]$$ (14)

$$P(A_i) = P\left((i-1)\Delta t < T_{f1} \leq i\Delta t \cap T_{f2} > t_s\right)$$ (14a)

$$= \int_{t_s}^{\infty} \left( \int_{(i-1)\Delta t}^{i\Delta t} f_{T_f1,T_f2}(t_{f1}, t_{f2})dt_{f1} \right) dt_{f2}$$ (14b)

$$P(A_s) = P(\bar{A}_s) = P(n_{in}\Delta t < T_{f1} \leq t_s \cap T_{f2} > t_s)$$ (14b)

$$= \int_{t_s}^{\infty} \left( \int_{n_{in}\Delta t}^{t_{s}} f_{T_f1,T_f2}(t_{f1}, t_{f2})dt_{f1} \right) dt_{f2}$$ (14b)

$$f_{T_f1,T_f2}(t_{f1}, t_{f2}) = f(t_{f1}) \cdot f(t_{f2} - t_{f1})$$ (14c)

where $T_{f1}$ and $T_{f2}$ = time of first failure and second failure, respectively, and they are random variables; $f_{T_f1,T_f2}(t_{f1}, t_{f2})$ = joint distribution of
The added model uncertainty, \( M_U \), is to account for the uncertainty on the deterministic variables of \( C, N, m, \) and \( Y \). The parameters used in the crack growth model are also listed in Table 2.

### Table 2: Distribution parameters of random variables

<table>
<thead>
<tr>
<th>Random variable</th>
<th>Mean</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside diameter of pipe, ( D ) (mm)</td>
<td>508*</td>
<td>25.4</td>
</tr>
<tr>
<td>Nominal wall thickness (mm), ( a_w )</td>
<td>5.7*</td>
<td>0.285</td>
</tr>
<tr>
<td>Yield strength, ( \sigma_y ) (MPa)</td>
<td>433*</td>
<td>12.99</td>
</tr>
<tr>
<td>Ultimate strength, ( \sigma_u ) (MPa)</td>
<td>618*</td>
<td>18.54</td>
</tr>
<tr>
<td>Estimated fracture toughness, ( K_{mat} ) (MPa²·m)</td>
<td>335.49</td>
<td>16.77</td>
</tr>
<tr>
<td>Operating Pressure, ( OP ) (MPa)</td>
<td>5.6</td>
<td>0.28</td>
</tr>
<tr>
<td>Material parameter, ( C )</td>
<td>2.17 \times 10^{-13}**</td>
<td>-</td>
</tr>
<tr>
<td>Material parameter, ( m )</td>
<td>3.0**</td>
<td>-</td>
</tr>
<tr>
<td>Scale parameter Weibull, ( A ) (MPa)</td>
<td>6.8</td>
<td>1.36</td>
</tr>
<tr>
<td>Shape parameter Weibull, ( B )</td>
<td>0.53</td>
<td>-</td>
</tr>
<tr>
<td>Geometry function, ( Y )</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Number of load cycles, ( N )</td>
<td>10⁶</td>
<td>-</td>
</tr>
<tr>
<td>Model uncertainty, ( M_u )</td>
<td>1**</td>
<td>0.18**</td>
</tr>
<tr>
<td>Initial crack depth, ( a_0 ) (mm)</td>
<td>0.5</td>
<td>-</td>
</tr>
<tr>
<td>Crack length, ( c ) (mm)</td>
<td>5</td>
<td>-</td>
</tr>
</tbody>
</table>

*(Hosseini et al., 2010) ; **(Friis-Hansen, 2000)

To determine the optimal inspection and maintenance planning for the deteriorating pipeline, the following unit costs are assumed: \( C_l = a_{ln} - C_0 \), \( C_r = a_r - C_0 \), and \( C_f = a_f - C_0 \), where \( a_{ln} (= 0.0177) \), \( a_r (= 0.243) \), and \( a_f (= 25) \) are multiplicative factors for inspection, repair, and failure, respectively. Those factor values are chosen based on the ranges presented in Gomes and Beck (2014b). In addition, the discount rate is
assumed to be 2% (0.02), the inspection interval, \(\Delta t\), is an integer number within the range of \([2, 10]\) years, and the repair threshold, \(d_r\) is within the range \([0, 20\%]\) of the wall thickness, \(d_w\). For a service life of 20 years, Figure 3 displays the expected total costs for different combination of \(\Delta t\) and \(d_r/d_w\). The lowest expected life-cycle cost occurs at \(\Delta t = 8\) years and \(d_r/d_w = 19\%\).

![Figure 3: Expected total costs versus inspection time](image)

Figure 4 shows for a given repair threshold (i.e., \(d_r/d_w = 13\%\)) how three different expected costs change with the value of the inspection interval. As expected, the expected total cost with no failure (only consists of inspection and repair costs), \(P_{0,0} E[CT]\) 0 failure, decreases with the increase of \(\Delta t\), since the frequency of inspection and repair decrease with the increase of \(\Delta t\). In addition, the expected total cost with one failure, \(P_{1,1} E[CT]\) 1 failure, increases with \(\Delta t\), since \(P_{1,1}\) increases with the increase of \(\Delta t\) due to the lower frequencies of inspection and repair actions.

Using Eq. (7) with \(n = 1\) (assuming probability of having two or more failure is negligible), the expected total cost, \(E[CT]\), is shown in the solid line in Figure 4. As shown in Figure 4, when \(\Delta t < 8\) years, the cost due to inspection and repair dominates \(E[CT]\); while when \(\Delta t > 8\) years, the failure cost starts to dominate. For this case study (with setting repair criteria of \(d_r/d_w = 13\%\)), it seems that when the inspection interval is between 4–8 years, one can effectively avoid failure occurrence without spending too much on the inspection and repair.

While the probability of having two or more failures is assumed to be negligible, one could follow the proposed framework to study the impact of adding those scenarios to the total cost. Nevertheless, the proposed framework provides a decision-making tool that analytically calculate failure, inspection, and repair cost.

![Figure 4: Different expected total costs versus inspection interval for \(d_r = 0.13d_w\)](image)

5. CONCLUSIONS

In this paper, a decision framework for determining the optimal inspection and maintenance planning for deteriorating pipelines has been proposed. The framework is developed based on a decision tree model by using analytical methods to evaluate events, where the time interval between inspections and the threshold for repair are set as the design variables. This framework is easy to implement for risk management. In addition, a probabilistic failure pressure model for thin-walled pipelines containing crack-like defects is proposed in order to accurately assess the probability of failure. A comparison study shows that the proposed model provides unbiased and more accurate predictions. A case study of a steel pipeline with an initial crack is then used to illustrate the proposed framework. Using the proposed framework, different components of costs (inspection, repair, and failure costs) can be easily examined, which can be useful for determining an optimal inspection and maintenance planning.
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