Estimation of Coherence using the Median is Robust against EEG Artefacts

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***Abstract*— Coherence is a mathematical measure of correlation in the frequency domain, commonly used to quantify the oscillatory synchrony of bio-signals such as the electroencephalogram (EEG). In biomedical applications, such as assessment of functional connectivity, reliable estimation of coherence is of paramount importance for studying the function of complex brain networks, as well as their disruption in neurological disorders. A major challenge for robust estimation of coherence measures is the presence of artefacts. Here, we propose an alternative method for finding coherence by estimating auto- and cross-spectral densities based on the median or trimmed-mean values across trials, rather than the mean. The variance of the average cortico-cortical coherence measures, i.e., the inter-individual variability, was taken as a measure of robustness and tested on resting-state recordings from 34 healthy individuals, both without screening, as well as after screening by a statistical thresholding artefact rejection. The variability of average coherence in individual channels and frequency bands decreased by using the median-based estimation of coherence. Averaged across all channels and frequency bands, the variability of coherence estimates based on median was significantly lower than mean-based estimates for both unscreened data ($F = 9.28$, $p = 0.003$, $1\text{-}\beta_{0.05} = 0.98$) and screened data ($F = 6.58$, $p = 0.01$, $1\text{-}\beta_{0.05} = 0.91$). Moreover, the variability for median-based estimates was almost identical for unscreened and screened data ($F = 0.004$, $p = 0.95$), suggesting that coherence based on median without artefact rejection might be sufficient for robust estimation of coherence.**

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I. INTRODUCTION

In the frequency analysis of bio-signals, coherence is a principal measure that estimates the linear correlation (bounded between 0 and 1) between two signals recorded using the same or different modalities [1]. In other words, it is a measure of the consistency of the relative phase of oscillation (weighted by amplitude) between the signals in a given frequency band [2]. It is used to show if two regions have coherent or synchronous activity and hence, if those regions are functionally connected.

A major application of coherence-based measures is studying the information transfer between brain regions, which is of particular interest in neuroscience [3]. For instance, inspecting the differences of neural communication patterns between controls and patients leads us to better understand the underlying mechanisms responsible for disease development [4]. Over the last three decades, studies have shown that brain regions specialized for processing certain types of information are functionally connected forming distinctive brain networks. In addition to this, studying interrelations between muscle and brain activity is becoming pivotal for better understanding of the mechanisms of muscle control by the central nervous system [5]. In order to measure the aforementioned physiological activities, various imaging techniques are commonly employed, including: Electroencephalography (EEG), Magneto-encephalography (MEG) and Electromyography (EMG). The measurements from these techniques allow us to calculate the cortico-cortical, cortico-muscular and inter-muscular connectivity and synchrony between distant brain and muscle areas. This can be assessed in the frequency domain using coherence [3].

Due to its wide range of applications in neurophysiology it is imperative to have reliable estimation of coherence measures. However, one of the greatest challenges is the potentially low signal-to-noise ratio (SNR) in the recorded bio-signals such as EEG. The engineering term SNR here corresponds to the ratio of the desired neurophysiological signal and the unwanted signal that is inevitably recorded together with the desired signal. This unwanted signal can be another physiological signal (e.g. electro-ocular or electro-cardiac artefacts) or noise originating from other recording points (e.g. the recording unit itself or bad contact at the recording site). Low SNR can result in falsely high (or low) coherence between signals, since noise with similar (or dissimilar) frequency content can be induced in both signals. Therefore, it is crucial to minimize the presence of noise in data or minimize its effects when estimating coherence measures.

In this report, we present an alternative, more robust method for finding coherence measures, by estimating the expected values of the cross-spectral and auto-spectral densities using the median or trimmed-mean of Fourier Transform values across trials, as opposed to the mean. To test the hypothesis that the coherence based on these approaches is more robust, we compare the inter-subject variability of cortico-cortical coherence measures achieved by the new and traditional methods, using resting-state 128-channels EEG data collected from 34 heathy subjects. To gauge the level of robustness against artefacts in each case, the comparisons of the new and traditional methods were performed both without screening, as well as after screening by a statistical thresholding artefact rejection.

II. Methods

A. Data

We used resting-state EEG data from 34 heathy controls recorded for a study in the elderly (f/m: 19/15; age: 58.1 ± 13.9 in the range 30-78). Data were recorded using Biosemi ActiveTwo system (BioSemi B.V., Amsterdam, The Netherlands) from 128 scalp electrodes for 6 minutes per subject, sampled at 512 Hz. In the experiment, subjects were instructed to relax and to maintain their gaze on a fixation cross placed in front of them. The recordings and the use of data were approved by the Research Ethics Committee of Beaumont Hospital, Dublin, Ireland (REC reference: 13/102) and the subjects gave written informed consent. The experimental procedure conformed to the Declaration of Helsinki.

B. Signal Processing

For signal analysis, we used bipolar channels formed by subtracting adjacent electrodes along the tangential directions, to capture the superficial cortical brain activity and to minimize the effects of volume conduction and deeper brain sources. For each of 125 bipolar channels, data were filtered offline using an equiripple filter between 1 and 105 Hz, divided into 750 ms epochs, multiplied by a Hann window, and subsequently fed to the screening method to remove the contaminated epochs from the data (as required). For each subject, the 125×125 pairwise coherence matrix was calculated from the cross-spectral matrix in 8 frequency bands δ (2–4 Hz), θ (5 – 7 Hz), α (α_l: 8-10 Hz, α_h: 11-13 Hz), β (β_l: 14-20 Hz, β_h: 21-30 Hz), and γ (γ_l: 31-47 Hz, γ_h: 53-97 Hz), using either median, trimmed-mean or mean (3 methods) on the screened or unscreened data (2 conditions). Finally, the variance (Var) between subjects was calculated in all six cases for the median coherence of each channel (with other channels) in each frequency band, as well as for the average coherence across all channels and frequency bands, and the results were subsequently compared.

C. Screening Method

We elected to use the FASTER algorithm [6], which is fully-automated, accounts for major types of artefacts and provides high sensitivity and specificity [6]. More specifically, we used an adapted and simplified version of FASTER to exclude contaminated chunks of data from analysis: Contaminated epochs were detected using 4 statistical features of the data. Z-score was applied to each of these 4 statistical parameters and the threshold of ±3.5 was used to determine the bad epochs. The detection for each feature was done in four steps by calculating the Z-score of: each channel, each epoch (containing all the channels), mean channel and mean epoch. If the threshold was reached in the two first steps, single epochs were marked as bad; while in the third step the epoch was marked as bad through all channel; and in the fourth step, the whole channel was marked as bad. The 4 statistical features included: The amplitude range, mean shift, variance, and mean spectral power in 0-2 Hz and 20-40 Hz frequency bands. Finally, if 28 out of 128 channels in an epoch were marked as bad the whole epoch was discarded.

D. Coherence Estimation

The real-valued coherence $|C_{xy}(f)|^2$ is the magnitude squared of the complex-valued coherency $C_{xy}(f)$, defined as following [7], [8]:

$$C_{xy}(f) = S_{xy}(f) / \sqrt{S_{xx}(f)S_{yy}(f)} \tag{1}$$

Where S_{xy} is the expected value of the cross-spectral density between $x(t)$ and $y(t)$ signals, whereas S_{xx} and S_{yy} are the expected values of the auto-spectral densities calculated for those signals. Further, S_{xy} is typically estimated as:

$$S_{xy} = \frac{1}{N}\sum_{n=1}^{N} X_n Y_n^* =$$
$$= \frac{1}{N}\sum_{n=1}^{N} real(X_n Y_n^*) + j\frac{1}{N}\sum_{n=1}^{N} imag(X_n Y_n^*) \tag{2}$$

Where X and Y are Fourier transformations of the corresponding time-domain signals x and y; * shows the conjugate, and n refers to a trial (time segment) number. Similarly, the auto-spectra S_{xx} and S_{yy} are estimated as:

$$S_{xx} = \frac{1}{N}\sum_{n=1}^{N} |X_n|^2 \tag{3}$$

$$S_{yy} = \frac{1}{N}\sum_{n=1}^{N} |Y_n|^2 \tag{4}$$

Note that (2), (3) and (4) involve calculating mean values across trials when determining the values of auto- and cross-spectral densities. Mean value as a statistical measure, is sensitive to outliers (possibly originating from artefacts); therefore, we define new estimates of the auto- and cross-spectra as follows.

1) Trimmed mean (TM)

This statistical measure calculates the mean value after discarding a given percentile of the samples equally at the left and right tails. This procedure discards the outliers and can potentially give a better estimation of the sample's central tendency. In this case the estimates of the spectra would be:

$$S_{xy} = TM\{real(X_n Y_n^*)\} + j \cdot TM\{imag(X_n Y_n^*)\} \tag{5}$$

$$S_{xx} = TM\{|X_n|^2\} \tag{6}$$

$$S_{yy} = TM\{|Y_n|^2\} \tag{7}$$

2) Median (Md)

This statistical measure separates the higher half of the sample from the lower half. It takes either the middle value if the number of points in the data set is odd, or calculates the mean value between two middle points if the data has even number of points. In other words, the median is the 50 percentile value of the samples. In this case the estimates of the spectra would be:

$$S_{xy} = Md\{real(X_n Y_n^*)\} + j \cdot Md\{imag(X_n Y_n^*)\} \quad (8)$$

$$S_{xx} = Md\{|X_n|^2\} \quad (9)$$

$$S_{yy} = Md\{|Y_n|^2\} \quad (10)$$

Both of these measures give better estimates of central tendency than the ordinary mean measure. If there is no outlier present in the data and the distribution of the samples are normal, all three measure give approximately the same result.

E. Statistical Analysis

To make pair-wise comparisons between the variances in the 6 groups (3 coherence estimation methods × 2 screening conditions), Levene's test of equality of variances was used (which has tolerance against non-normal distribution of the data). If the null hypothesis of equal variances is rejected then it is concluded that there is a difference between the variances of the observed data sets. Additionally, the statistical power of the test ($1-\beta_{0.95}$) was calculated under the assumptions for an equivalent F-test.

F. Relationship between the Mean- and Median-based Coherence

In order to check for the validity of the results obtained using coherence based on median, we estimated the coherence using both the mean- and median-based method and compared the results. The coherence was estimated between two sets of simulated data: a 600 epochs × 512 time points with Gaussian standard normal random data and another similar-size set which was a linear sum of the first dataset (*r*) combined with a similar random dataset (1-*r*). *r* (as the true coherence) changed in 20 iterations from 0.05 to 1.

III. Results

A. Effects in Channels and Frequencies

First, the effect of the estimation method on coherence was qualitatively assessed in the unscreened data. Coh_M, Coh_{TM}, Coh_{Md} indicate the absolute coherence (i.e. connectivity) based on mean, trimmed mean and median, respectively. Fig. 1 shows the p-values plotted for positive difference between the variances of connectivity. The difference between variances was considerably higher for the results obtained using Coh_M compared to both Coh_{TM} and Coh_{Md}, while the difference between variances between Coh_{TM} and Coh_M was less pronounced. Overall, these results indicate that the coherence based on median gives the least variability in data and hence, is the most robust to artifacts in EEG data.

Next, the variances of each coherence estimation method were compared between the unscreened and screened data conditions. The p-values for positive difference between variances are plotted on Fig. 2. The results show that the variance of coherence calculated from the unscreened data is statistically higher compared to the coherence calculated from the screened data in the case of classical coherence, while in the cases of trimmed mean and median-based coherence the difference, although positive, is subtle.

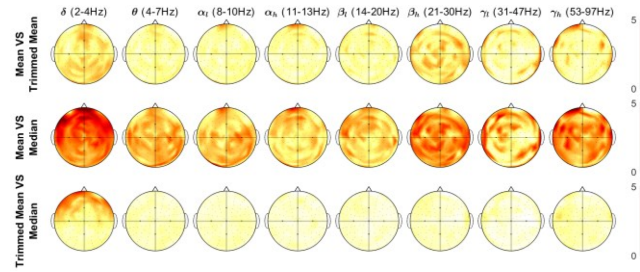

Figure 1. The differences of the inter-individual variability between coherence estimation methods, using unscreened data. The p-values for each channel and frequency band, plotted as –log(p), correspond to Levene's test of equality of variances. In all cases the plotted p-values correspond to positive differences.

C. Overall Effects

Fig. 3 displays the average variances and confidence intervals between subjects' connectivity for each estimation method and from both unscreened and screened data. The alpha value for Levine's test was set at $\alpha_{corrected} = 0.05/6 = 0.008$ using Bonferroni correction. Table I summarizes the comparisons of interest in the 6 groups studied.

TABLE I. Comparisons Between Combinations of Estimation Methods and Screening Conditions, Using the Levene's Test

Measure 1	Measure 2	F-value	p-value	$1-\beta_{0.95}$
Coh_M (NoScr)	Coh_M (Scr)	1.58	0.21	
Coh_M (NoScr)	Coh_{TM} (NoScr)	3.68	0.06	
Coh_M (NoScr)	Coh_{Md} (NoScr)	9.28	0.003*	0.98
Coh_{Md} (NoScr)	Coh_{Md} (Scr)	0.004	0.95	
Coh_{TM} (NoScr)	Coh_{Md} (NoScr)	3.11	0.08	
Coh_M (Scr)	Coh_{Md} (NoScr)	6.58	0.01^+	0.91

Finally, we compared the estimated coherences from the simulated data using both the mean- and median-based coherence and plotted their relationship (see Fig. 4). The figure shows that the median-based coherence gives slightly lower estimates compared to coherence based on mean in values around 0.5.

IV. Discussion

The aim of this study was to quantify the robustness of three different estimation methods for coherence in EEG data: (1) the classical coherence based on mean value, and two modified versions based on – (2) trimmed mean and (3) median value.

The results of the analysis support the hypothesis that the coherence based on the median value will give the least variability in results and therefore is the most robust against the artifacts in EEG data. It is interesting to note that the coherence based on median gave approximately the same results when applied to unscreened and screened data, suggesting that using coherence based on median on its own might be as efficient as a simple screening method. Furthermore, the reduction of variability by using median-

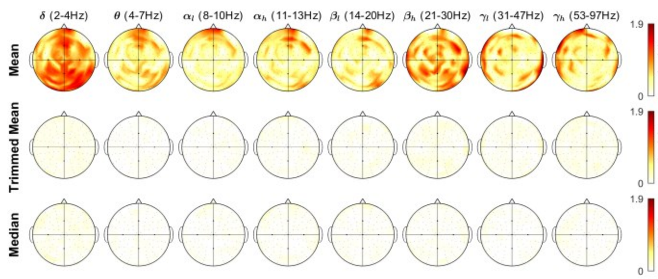

Figure 2. The differences of the inter-individual variability between the unscreened and screened data, separately plotted for the 3 estimation methods. The p-values for each channel and frequency band, plotted as –log(p), correspond to Levene's test of equality of variances. In all cases the plotted p-values correspond to positive differences.

based coherence was higher in unscreened data compared to screened data. Although, the latter difference was only marginally significant (after Bonferroni correction), it still suggests that the coherence based on median might be more efficient in reducing the inter-subject variability than a simple screening method. Another important finding was that our simple screening method indeed improved the results.

This result may be explained by the fact that the median and the mean values of non-normal distributions are not equal. Note that (2), (3) and (4) involve calculating the product of datasets that are assumed to be normally distributed. In the case of cross-spectral density, the resulting distribution can be approximated with the modified Bessel function of the second kind, while the distribution of the estimated auto-spectral density can be approximated with the χ^2 distribution with two degrees of freedom [9], [10]. The slightly different estimates by mean- vs. median-based estimates originates from the non-symmetric nature of these distributions; nevertheless, this small deviation (about 10%) cannot explain the notable reduction in the coherence values and inter-individual variance in real EEG data by the use of median-based coherence. This reduction is best attributed to the attenuation of the effects of artefacts.

These findings may be somewhat limited since we did not assess the exact sensitivity and specificity of our screening method by performing a complete performance testing. This could be performed on simulated data, where the nature of artifacts is limited to a few well-known types. Although the

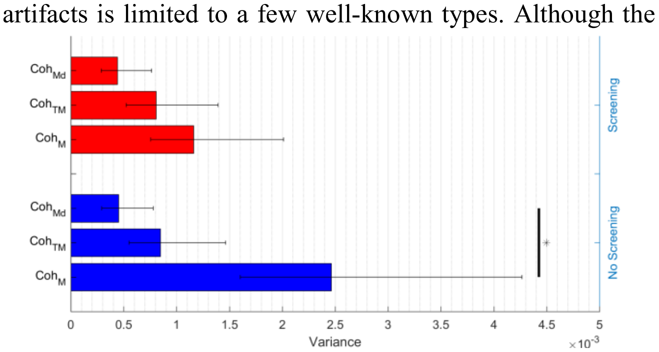

Figure 3. Comparison of variances using different screening conditions and coherence estimation methods. Coherence based on median has the lowest variability of results and it is statistically significant compared to the coherence based on mean in unscreened data ($F = 9.28$, $p = 0.003$, $1-\beta_{0.05} = 0.98$). Additionally, the variability for median-based estimates was almost identical for unscreened and screened data ($F = 0.004$, $p = 0.95$).

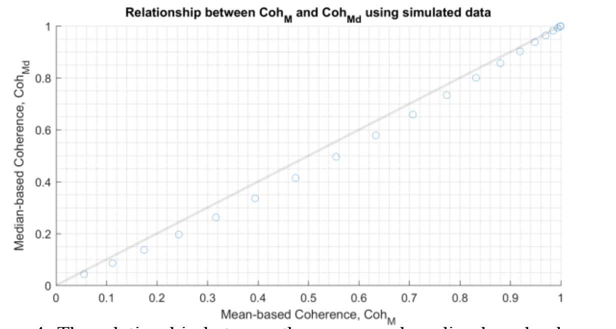

Figure 4. The relationship between the mean- and median-based coherence. Results from the simulated data show that the median-based coherence gives lower estimates in values around 0.5.

improvements by the coherence based on median on the screened data could be partially attributed to the limited performance of our screening method, it still indicates the utility of the proposed method as a simple useful approach that can be readily incorporated into analysis pipelines.

In this study we used only cortico-cortical coherence of EEG data to test our hypothesis. Although, we did not test it on any other modality, such as EMG or MEG, it is expected that due to shared mathematical and statistical principles, that coherence based on median would similarly result in decreased variability of results regardless of the physiological nature of the primary signal, thus extending its application to any other physiological signals and beyond.

V. Conclusion

The coherence measures estimated based on median are more robust against noise and artefact. Coherence based on median can be more efficient than simple subjective screening methods and simpler for use in analysis pipelines, especially in biomedical applications for studying the neurophysiological characteristics of the brain networks.

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